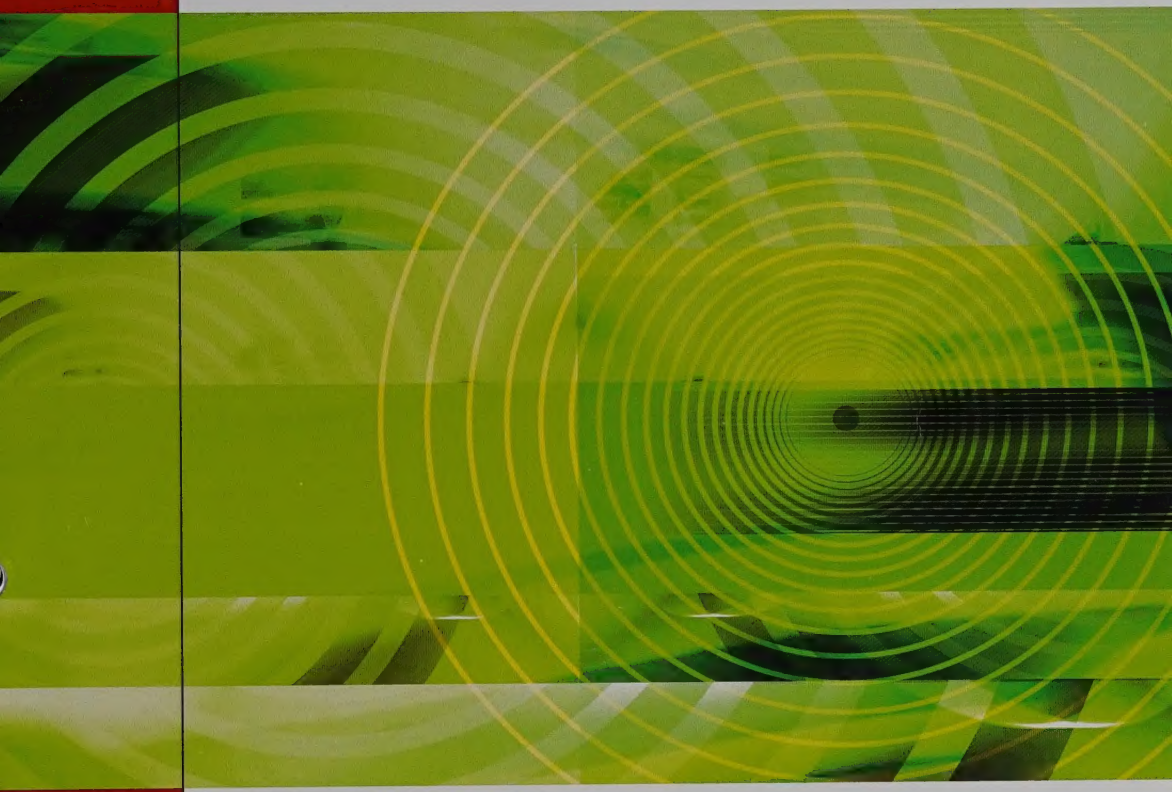
Fifth Edition

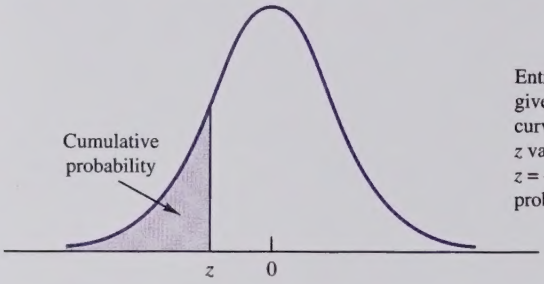
Anderson | Sweeney | Williams



Essentials of Modern Business Statistics

With Microsoft® Office Excel®

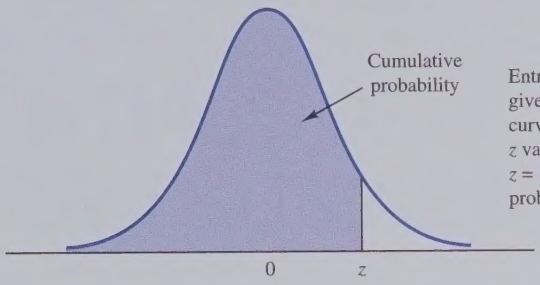
CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION



Entries in this table give the area under the curve to the left of the z value. For example, for $z = -.85$, the cumulative probability is .1977.

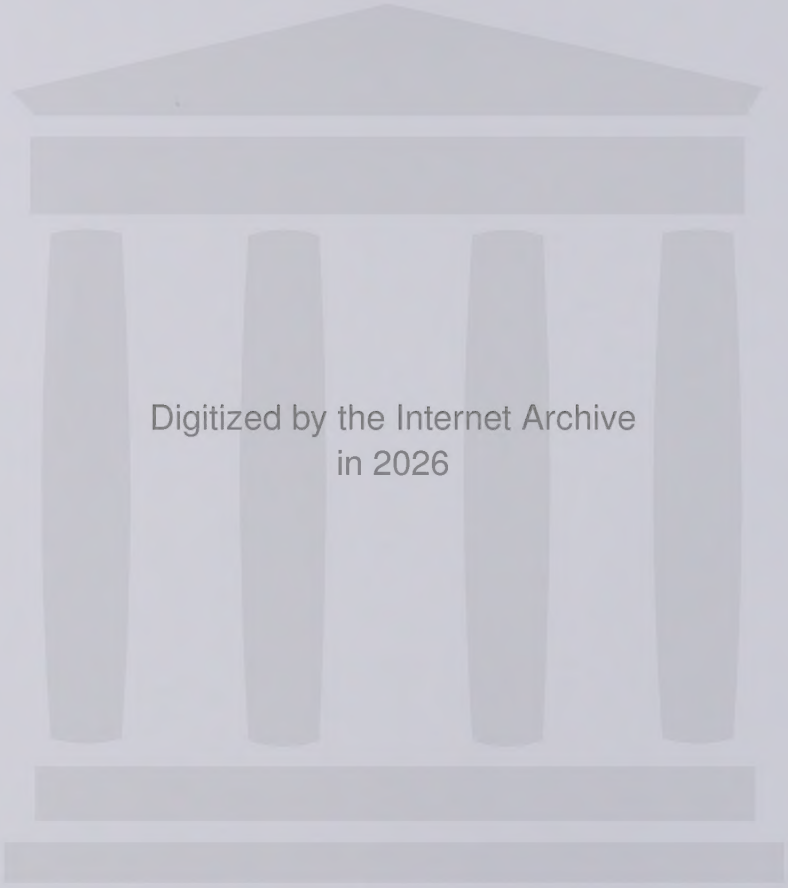
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION



Entries in the table give the area under the curve to the left of the z value. For example, for $z = 1.25$, the cumulative probability is .8944.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990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FIFTH EDITION

David R. Anderson | Dennis J. Sweeney
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Dedicated to
Krista, Justin, Mark, and Colleen
Mark, Linda, Brad, Tim, Scott, and Lisa
Cathy, David, and Kristin

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The purpose of *Essentials of Modern Business Statistics with Microsoft Office® Excel®* is to give students, primarily those in the fields of business administration and economics, a conceptual introduction to the field of statistics and its many applications. The text is applications oriented and written with the needs of the nonmathematician in mind; the mathematical prerequisite is knowledge of algebra.

Applications of data analysis and statistical methodology are an integral part of the organization and presentation of the text material. The discussion and development of each technique is presented in an applications setting, with the statistical results providing insights to decisions and solutions to problems.

Although the book is applications oriented, we have taken care to provide sound methodological development and to use notation that is generally accepted for the topic being covered. Hence, students will find that this text provides good preparation for the study of more advanced statistical material. A bibliography to guide further study is included as an appendix.

Use of Microsoft Excel for Statistical Analysis

Essentials of Modern Business Statistics with Microsoft Excel is first and foremost a statistics textbook that emphasizes statistical concepts and applications. But since most practical problems are too large to be solved using hand calculations, some type of statistical software package is required to solve these problems. There are several excellent statistical packages available today. However, because most students and potential employers value spreadsheet experience, many schools now use a spreadsheet package in their statistics courses. Microsoft Excel is the most widely used spreadsheet package in business as well as in colleges and universities. We have written *Essentials of Modern Business Statistics with Microsoft Excel* especially for statistics courses in which Microsoft Excel is used as the software package.

Excel has been integrated within each of the chapters and plays an integral part in providing an application orientation. Although we assume that readers using this text are familiar with Excel basics such as selecting cells, entering formulas, copying, and so on, we do not assume that readers are familiar with Excel 2010. As a result, we have included Appendix E, which provides an introduction to Excel 2010 and tools for statistical analysis.

Throughout the text the discussion of using Excel to perform a statistical procedure appears in a subsection immediately following the discussion of the statistical procedure. We believe that this style enables us to fully integrate the use of Excel throughout the text, but still maintain the primary emphasis on the statistical methodology being discussed. In each of these subsections, we use a standard format for using Excel for statistical analysis. There are three primary tasks: Enter Data, Enter Functions and Formulas, and Apply Tools. We believe a consistent framework for applying Excel helps users to focus on the statistical methodology without getting bogged down in the details of using Excel.

In presenting worksheet figures we often use a nested approach in which the worksheet shown in the background of the figure displays the formulas and the worksheet shown in the foreground shows the values computed using the formulas. Different colors and shades of colors are used to differentiate worksheet cells containing data, highlight cells containing Excel functions and formulas, and highlight material printed by Excel as a result of using one or more data analysis tools.

Use of StatTools

StatTools is a commercial Excel add-in which we and Palisade Corporation have made available to adopters of this text for free. StatTools extends the range of statistical and graphical options for Excel users. In an appendix to Chapter 1 we show how to download and install StatTools. Most chapters also include an appendix that shows the steps required to accomplish a statistical procedure using StatTools.

We have been very careful to make the use of StatTools completely optional. Users who want to teach using the standard tools available in Excel 2010 can continue to do so. But users who want additional statistical capabilities not available in Excel 2010 now have access to an industry standard statistics add-in that students will be able to continue to use in the workplace.

Changes in the Fifth Edition

We appreciate the acceptance and positive response to the previous editions of *Essentials of Modern Business Statistics with Microsoft Excel*. Accordingly, in making modifications for this new edition, we have maintained the presentation style and readability of those editions. The significant changes in the new edition are summarized here.

- **Microsoft Excel 2010.** Step-by-step instructions and screen captures show how to use the latest version of Excel to implement statistical procedures.
- **Introduction to Data Mining.** A new section in Chapter 1 introduces the field of data mining. We provide a brief overview of data mining and the concept of a data warehouse. We also describe how the fields of statistics and computer science join to make data mining operational and valuable.
- **Revised Chapter 2.** We have shortened the Excel coverage by moving the discussion of how to use Excel's PivotChart report to chapter appendixes. Appendix 2.1 shows how to use Excel's PivotChart report to summarize categorical data, and Appendix 2.2 shows how to use Excel's PivotChart report to summarize quantitative data.
- **Revised Sampling Material.** The introduction of Chapter 7 has been revised and now includes the concept of a sampled population and a frame. The distinction between sampling from a finite population and an infinite population has been clarified, with sampling from a process used to illustrate the selection of a random sample from an infinite population. A practical advice section stresses the importance of obtaining close correspondence between the sampled population and the target population.
- **Revised Introduction to Hypothesis Testing.** Section 9.1, Developing Null and Alternative Hypotheses, has been revised. A better set of guidelines has been developed for identifying the null and alternative hypotheses. The context of the situation and the purpose for taking the sample are key. In situations in which the focus is on finding evidence to support a research finding, the research hypothesis is the alternative hypothesis. In situations where the focus is on challenging an assumption, the assumption is the null hypothesis.
- **Section 13.8 – "Modeling Curvilinear Relationships."** This new section shows how curvilinear relationship can easily be handled using a multiple regression model. We illustrate the use of both Excel's Chart tools and Excel's Regression tool to fit a quadratic model.
- **Chapter 15 – "Time Series Analysis and Forecasting."** This new chapter, available on the text website, focuses on using the pattern in a time series plot to select and use an appropriate forecasting method. Section 15.1 discusses time series patterns, and

Section 15.2 introduces methods for measuring forecasting accuracy. Section 15.3 discusses moving averages and exponential smoothing. Section 15.4 introduces methods appropriate for a time series that exhibits a trend. Here we illustrate how regression analysis can be used for trend projection to model nonlinear relationships involving a quadratic trend. Section 15.5 then shows how dummy variables can be used to model seasonality in a forecasting equation. Section 15.6 discusses classical time series decomposition including the concept of deseasonalizing a time series.

- **New Examples and Exercises Based on Real Data.** We have added over 170 new examples and exercises based on real data and recently referenced sources of statistical information. Using data obtained from various data collection organizations and other sources, such as *The Wall Street Journal*, *USA Today*, *Fortune*, and *Barron's*, we have drawn upon actual studies to develop explanations and to create exercises that demonstrate many uses of statistics in business and economics. We believe the use of real data helps generate more student interest in the material and enables the student to learn about both the statistical methodology and its application.
- **New Case Problems.** We have added six new case problems to this edition. The new case problems appear in the chapters on descriptive statistics, hypothesis testing, regression analysis, and forecasting. The 24 case problems in the text provide students with the opportunity to analyze somewhat larger data sets and prepare managerial reports based on the results of their analysis.

Features and Pedagogy

Authors Anderson, Sweeney, and Williams have continued many of the features that appeared in previous editions. Important ones for students are noted here.

Statistics in Practice

Each chapter begins with a Statistics in Practice article that describes an application of the statistical methodology to be covered in the chapter.

Methods Exercises and Applications Exercises

The end-of-section exercises are split into two parts, Methods and Applications. The Methods exercises require students to use the formulas and make the necessary computations. The Applications exercises require students to use the chapter material in real-world situations. Thus, students first focus on the computational "nuts and bolts" and then move on to the subtleties of statistical application and interpretation.

Self-Test Exercises

Certain exercises are identified as self-test exercises. Completely worked-out solutions for those exercises are provided in Appendix D at the back of the book. Students can attempt the self-test exercises and immediately check the solution to evaluate their understanding of the concepts presented in the chapter.

Margin Annotations and Notes and Comments

Margin annotations that highlight key points and provide additional insights for the students are a key feature of this text. These annotations are designed to provide emphasis and enhance understanding of the terms and concepts being presented in the text.

At the end of many sections, we provide Notes and Comments designed to give the student additional insights about the statistical methodology and its application. Notes and Comments include warnings about or limitations of the methodology, recommendations for application, brief descriptions of additional technical considerations, and other matters.

Data Files Accompany the Text

Approximately 220 data files are available on the website that accompanies this text. The data sets are available in Excel 2010 format. WEBfile logos are used in the text to identify the data sets that are available on the website. Data sets for all case problems as well as data sets for larger exercises are included.

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- **More study options.** Students can choose how they read the textbook—via integrated digital eBook or by reading the print version.

Ancillary Learning Materials for Students

- Approximately 220 data sets are available on the website that accompanies this text. The webfiles are available in Excel 2010 format. WEBfile logos are used in the text to identify the data sets that are available on the website. Data sets for all case problems, as well as data sets for larger exercises, are included.
- **EasyStat Digital Tutor for Microsoft® Excel 2010.** These focused online tutorials will make it easier for students to learn how to use one of these well-known software products to perform statistical analysis. Each digital video demonstrates how the software can be used to perform a particular statistical procedure.

  The EasyStat videos for Excel 2010 will be available in early 2012. Students may purchase an online subscription for EasyStat Digital Tutor at www.cengagebrain.com.

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Essentials of Modern Business Statistics

With Microsoft® Office Excel® 5e

CHAPTER 1

Data and Statistics

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STATISTICS *in* PRACTICE

*BLOOMBERG BUSINESSWEEK**

NEW YORK, NEW YORK

With a global circulation of more than 1 million, *Bloomberg Businessweek* is one of the most widely read business magazines in the world. Bloomberg's 1700 reporters in 145 service bureaus around the world enable *Bloomberg Businessweek* to deliver a variety of articles of interest to the global business and economic community. Along with feature articles on current topics, the magazine contains articles on international business, economic analysis, information processing, and science and technology. Information in the feature articles and the regular sections helps readers stay abreast of current developments and assess the impact of those developments on business and economic conditions.

Most issues of *Bloomberg Businessweek,* formerly *BusinessWeek,* provide an in-depth report on a topic of current interest. Often, the in-depth reports contain statistical facts and summaries that help the reader understand the business and economic information. For example, the cover story for the March 3, 2011 issue discussed the impact of businesses moving their most important work to cloud computing; the May 30, 2011 issue included a report on the crisis facing the U.S. Postal Service; and the August 1, 2011 issue contained a report on why the debt crisis is even worse than you think. In addition, *Bloomberg Businessweek* provides a variety of statistics about the state of the economy, including production indexes, stock prices, mutual funds, and interest rates.

Bloomberg Businessweek also uses statistics and statistical information in managing its own business. For example, an annual survey of subscribers helps the company learn about subscriber demographics, reading habits, likely purchases, lifestyles, and so on. *Bloomberg Businessweek* managers use statistical summaries from the survey to provide better services to subscribers and advertisers. One recent North American subscriber survey indicated that 90% of *Bloomberg Businessweek* subscribers use a personal computer at home and that 64% of *Bloomberg Businessweek* subscribers are involved with computer purchases at work. Such statistics alert *Bloomberg Businessweek* managers to subscriber interest in articles about new developments in computers. The results of the subscriber survey are also made available to potential advertisers. The high percentage of subscribers using personal computers at home and the high percentage of subscribers involved with computer purchases at work would be an incentive for a computer manufacturer to consider advertising in *Bloomberg Businessweek.*

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Bloomberg Businessweek uses statistical facts and summaries in many of its articles.

In this chapter, we discuss the types of data available for statistical analysis and describe how the data are obtained. We introduce descriptive statistics and statistical inference as ways of converting data into meaningful and easily interpreted statistical information.

*The authors are indebted to Charlene Trentham, Research Manager, for providing this Statistics in Practice.

Frequently, we see the following types of statements in newspapers, magazines, and websites:

- The Dow Jones Industrial Average closed at 12,639 on May 6, 2011 (*Barron's,* May 9, 2011).
- A proposed interstate highway to Myrtle Beach would pay for itself in 4 years and bring about 22,000 jobs to South Carolina (*The Sun News,* May 13, 2011).

- The average amount spent on an airline ticket for business travel in the second quarter of 2011 was $447, up 9.3% over the same period last year (*The Wall Street Journal,* August 18, 2011).
- In January 2011, 586,271 people visited Hawaii by air, which was an increase of 12% over January, 2010 (Hawaii's Business Resource website, May 13, 2011).
- As of January, 2011, China held $1,155 billion of U.S. Treasuries. This was 26% of the U.S. Treasuries owned by foreigners (*Market Insights,* J.P. Morgan Asset Management, March 31, 2011).
- Last spring the Census Bureau found that 42% of American households had an income between $50,000 and $150,000 per year (*Ad Age* website, February 25, 2011).
- The percentage of adults who report using apps on their cell phones is 29% (*Delta Sky Magazine,* March, 2011).

The numerical facts in the preceding statements (12,639, 4, 22,000, $447, 9.3%, 586,271, 12%, $1,155 billion, 26%, 42%, $50,000, $150,000, 29%) are called statistics. In this usage, the term *statistics* refers to numerical facts such as index numbers, forecasts, percents, averages, and so on that help us understand a variety of business and economic situations. However, as you will see, the field, or subject, of statistics involves much more than numerical facts. In a broader sense, **statistics** is defined as the art and science of collecting, analyzing, presenting, and interpreting data. Particularly in business and economics, the information provided by collecting, analyzing, presenting, and interpreting data gives managers and decision makers a better understanding of the business and economic environment and thus enables them to make more informed and better decisions. In this text, we emphasize the use of statistics for business and economic decision making.

Chapter 1 begins with some illustrations of the applications of statistics in business and economics. In Section 1.2 we define the term *data* and introduce the concept of a data set. This section also introduces key terms such as *variables* and *observations,* discusses the difference between quantitative and categorical data, and illustrates the uses of cross-sectional and time series data. Section 1.3 discusses how data can be obtained from existing sources or through survey and experimental studies designed to obtain new data. The important role that the Internet now plays in obtaining data is also highlighted. The uses of data in developing descriptive statistics and in making statistical inferences are described in Sections 1.4 and 1.5. Section 1.6 discusses statistical analysis using Microsoft Excel, Section 1.7 provides an introduction to data mining, and Section 1.8 discusses ethical guidelines for statistical practice. A chapter-ending appendix includes an introduction to the add-in StatTools, which can be used to extend the statistical options for users of Microsoft Excel.

1.1 Applications in Business and Economics

In today's global business and economic environment, anyone can access vast amounts of statistical information. The most successful managers and decision makers understand the information and know how to use it effectively. In this section, we provide examples that illustrate some of the uses of statistics in business and economics.

Accounting

Public accounting firms use statistical sampling procedures when conducting audits for their clients. For instance, suppose an accounting firm wants to determine whether the amount of accounts receivable shown on a client's balance sheet fairly represents the actual amount of accounts receivable. Usually the large number of individual accounts receivable

makes reviewing and validating every account too time-consuming and expensive. As common practice in such situations, the audit staff selects a subset of the accounts called a sample. After reviewing the accuracy of the sampled accounts, the auditors draw a conclusion as to whether the accounts receivable amount shown on the client's balance sheet is acceptable.

Finance

Financial analysts use a variety of statistical information to guide their investment recommendations. In the case of stocks, the analysts review a variety of financial data, including price/earnings ratios and dividend yields. By comparing the information for an individual stock with information about the stock market averages, a financial analyst can begin to draw a conclusion as to whether an individual stock is over- or underpriced. For example, *The Wall Street Journal* (November 1, 2010) reported that the average dividend yield for the 30 stocks in the Dow Jones Industrial Average was 2.54%. Chevron showed a dividend yield of 3.5%. In this case, the statistical information on dividend yield indicates a higher dividend yield for Chevron than the average for the Dow Jones stocks. Therefore, a financial analyst might conclude that Chevron was underpriced. This and other information about Chevron would help the analyst make a buy, sell, or hold recommendation for the stock.

Marketing

Electronic scanners at retail checkout counters collect data for a variety of marketing research applications. For example, data suppliers such as ACNielsen and Information Resources, Inc., purchase point-of-sale scanner data from grocery stores, process the data, and then sell statistical summaries of the data to manufacturers. Manufacturers spend hundreds of thousands of dollars per product category to obtain this type of scanner data. Manufacturers also purchase data and statistical summaries on promotional activities such as special pricing and the use of in-store displays. Brand managers can review the scanner statistics and the promotional activity statistics to gain a better understanding of the relationship between promotional activities and sales. Such analyses often prove helpful in establishing future marketing strategies for the various products.

Production

Today's emphasis on quality makes quality control an important application of statistics in production. A variety of statistical quality control charts are used to monitor the output of a production process. In particular, an x-bar chart can be used to monitor the average output. Suppose, for example, that a machine fills cans with 12 ounces of a soft drink. Periodically, a production worker selects a sample of cans and computes the average number of ounces in the sample. This average, or x-bar value, is plotted on an x-bar chart. A plotted value above the chart's upper control limit indicates overfilling, and a plotted value below the chart's lower control limit indicates underfilling. The process is termed "in control" and allowed to continue as long as the plotted x-bar values fall between the chart's upper and lower control limits. Properly interpreted, an x-bar chart can help determine when adjustments are necessary to correct a production process.

Economics

Economists frequently provide forecasts about the future of the economy or some aspect of it. They use a variety of statistical information in making such forecasts. For instance, in forecasting inflation rates, economists use statistical information on such indicators as the Producer

Price Index, the unemployment rate, and manufacturing capacity utilization. Often these statistical indicators are entered into computerized forecasting models that predict inflation rates.

Applications of statistics such as those described in this section are an integral part of this text. Such examples provide an overview of the breadth of statistical applications. To supplement these examples, practitioners in the fields of business and economics provided chapter-opening Statistics in Practice articles that introduce the material covered in each chapter. The Statistics in Practice applications show the importance of statistics in a wide variety of business and economic situations.

1.2 Data

Data are the facts and figures collected, analyzed, and summarized for presentation and interpretation. All the data collected in a particular study are referred to as the **data set** for the study. Table 1.1 shows a data set containing information for 25 mutual funds that are part of the *Morningstar Funds 500* for 2008. Morningstar is a company that tracks over 7000 mutual funds and prepares in-depth analyses of 2000 of these. Its recommendations are followed closely by financial analysts and individual investors.

WEB file
Morningstar

Data sets such as Morningstar are available on the website for this text.

TABLE 1.1 DATA SET FOR 25 MUTUAL FUNDS

Fund Name	Fund Type	Net Asset Value ($)	5-Year Average Return (%)	Expense Ratio (%)	Morningstar Rank
American Century Intl. Disc	IE	14.37	30.53	1.41	3-Star
American Century Tax-Free Bond	FI	10.73	3.34	0.49	4-Star
American Century Ultra	DE	24.94	10.88	0.99	3-Star
Artisan Small Cap	DE	16.92	15.67	1.18	3-Star
Brown Cap Small	DE	35.73	15.85	1.20	4-Star
DFA U.S. Micro Cap	DE	13.47	17.23	0.53	3-Star
Fidelity Contrafund	DE	73.11	17.99	0.89	5-Star
Fidelity Overseas	IE	48.39	23.46	0.90	4-Star
Fidelity Sel Electronics	DE	45.60	13.50	0.89	3-Star
Fidelity Sh-Term Bond	FI	8.60	2.76	0.45	3-Star
Gabelli Asset AAA	DE	49.81	16.70	1.36	4-Star
Kalmar Gr Val Sm Cp	DE	15.30	15.31	1.32	3-Star
Marsico 21st Century	DE	17.44	15.16	1.31	5-Star
Mathews Pacific Tiger	IE	27.86	32.70	1.16	3-Star
Oakmark I	DE	40.37	9.51	1.05	2-Star
PIMCO Emerg Mkts Bd D	FI	10.68	13.57	1.25	3-Star
RS Value A	DE	26.27	23.68	1.36	4-Star
T. Rowe Price Latin Am.	IE	53.89	51.10	1.24	4-Star
T. Rowe Price Mid Val	DE	22.46	16.91	0.80	4-Star
Thornburg Value A	DE	37.53	15.46	1.27	4-Star
USAA Income	FI	12.10	4.31	0.62	3-Star
Vanguard Equity-Inc	DE	24.42	13.41	0.29	4-Star
Vanguard Sht-Tm TE	FI	15.68	2.37	0.16	3-Star
Vanguard Sm Cp Idx	DE	32.58	17.01	0.23	3-Star
Wasatch Sm Cp Growth	DE	35.41	13.98	1.19	4-Star

Source: Morningstar Funds 500 (2008).

Elements, Variables, and Observations

Elements are the entities on which data are collected. For the data set in Table 1.1 each individual mutual fund is an element: the element names appear in the first column. With 25 mutual funds, the data set contains 25 elements.

A **variable** is a characteristic of interest for the elements. The data set in Table 1.1 includes the following five variables:

- *Fund Type:* The type of mutual fund, labeled DE (Domestic Equity), IE (International Equity), and FI (Fixed Income)
- *Net Asset Value ($):* The closing price per share on December 31, 2007
- *5-Year Average Return (%):* The average annual return for the fund over the past 5 years
- *Expense Ratio:* The percentage of assets deducted each fiscal year for fund expenses
- *Morningstar Rank:* The overall risk-adjusted star rating for each fund; Morningstar ranks go from a low of 1-Star to a high of 5-Stars

Measurements collected on each variable for every element in a study provide the data. The set of measurements obtained for a particular element is called an **observation**. Referring to Table 1.1, we see that the set of measurements for the first observation (American Century Intl. Disc) is IE, 14.37, 30.53, 1.41, and 3-Star. The set of measurements for the second observation (American Century Tax-Free Bond) is FI, 10.73, 3.34, 0.49, and 4-Star, and so on. A data set with 25 elements contains 25 observations.

Scales of Measurement

Data collection requires one of the following scales of measurement: nominal, ordinal, interval, or ratio. The scale of measurement determines the amount of information contained in the data and indicates the most appropriate data summarization and statistical analyses.

When the data for a variable consist of labels or names used to identify an attribute of the element, the scale of measurement is considered a **nominal scale**. For example, referring to the data in Table 1.1, we see that the scale of measurement for the Fund Type variable is nominal because DE, IE, and FI are labels used to identify the category or type of fund. In cases where the scale of measurement is nominal, a numerical code as well as nonnumerical labels may be used. For example, to facilitate data collection and to prepare the data for entry into a computer database, we might use a numerical code by letting 1 denote Domestic Equity, 2 denote International Equity, and 3 denote Fixed Income. In this case the numerical values 1, 2, and 3 identify the category of fund. The scale of measurement is nominal even though the data appear as numerical values.

The scale of measurement for a variable is called an **ordinal scale** if the data exhibit the properties of nominal data and the order or rank of the data is meaningful. For example, Eastside Automotive sends customers a questionnaire designed to obtain data on the quality of its automotive repair service. Each customer provides a repair service rating of excellent, good, or poor. Because the data obtained are the labels—excellent, good, or poor—the data have the properties of nominal data. In addition, the data can be ranked, or ordered, with respect to the service quality. Data recorded as excellent indicate the best service, followed by good and then poor. Thus, the scale of measurement is ordinal. As another example, note that the Morningstar Rank for the data in Table 1.1 is ordinal data. It provides a rank from 1 to 5-Stars based on Morningstar's assessment of the fund's risk-adjusted return. Ordinal data can also be provided using a numerical code, such as your class rank in school.

The scale of measurement for a variable is an **interval scale** if the data have all the properties of ordinal data and the interval between values is expressed in terms of a fixed unit of measure. Interval data are always numerical. College admission SAT scores are an example of interval-scaled data. For example, three students with SAT math scores of 620, 550, and 470 can be ranked or ordered in terms of best performance to poorest performance in math. In addition, the differences between the scores are meaningful. For instance, student 1 scored $620 - 550 = 70$ points more than student 2, while student 2 scored $550 - 470 = 80$ points more than student 3.

The scale of measurement for a variable is a **ratio scale** if the data have all the properties of interval data and the ratio of two values is meaningful. Variables such as distance, height, weight, and time use the ratio scale of measurement. This scale requires that a zero value be included to indicate that nothing exists for the variable at the zero point. For example, consider the cost of an automobile. A zero value for the cost would indicate that the automobile has no cost and is free. In addition, if we compare the cost of $30,000 for one automobile to the cost of $15,000 for a second automobile, the ratio property shows that the first automobile is $30,000/$15,000 = 2 times, or twice, the cost of the second automobile.

Categorical and Quantitative Data

Data can be classified as either categorical or quantitative. Data that can be grouped by specific categories are referred to as **categorical data**. Categorical data use either the nominal or ordinal scale of measurement. Data that use numerical values to indicate how much or how many are referred to as **quantitative data**. Quantitative data are obtained using either the interval or ratio scale of measurement.

The statistical method appropriate for summarizing data depends on whether the data are categorical or quantitative.

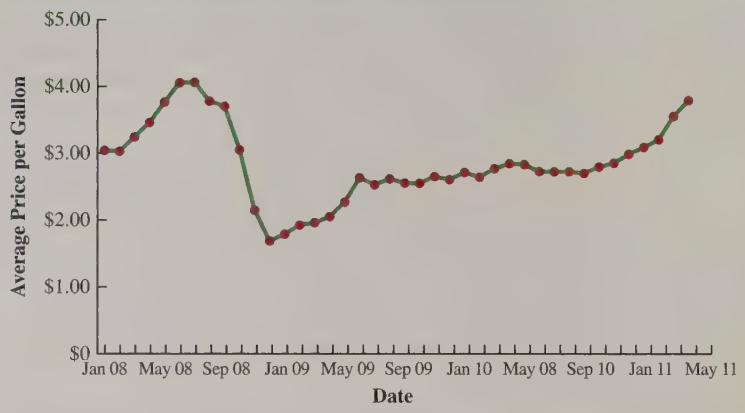
A **categorical variable** is a variable with categorical data, and a **quantitative variable** is a variable with quantitative data. The statistical analysis appropriate for a particular variable depends on whether the variable is categorical or quantitative. If the variable is categorical, the statistical analysis is limited. We can summarize categorical data by counting the number of observations in each category or by computing the proportion of the observations in each category. However, even when the categorical data are identified by a numerical code, arithmetic operations such as addition, subtraction, multiplication, and division do not provide meaningful results. Section 2.1 discusses ways for summarizing categorical data.

Arithmetic operations provide meaningful results for quantitative variables. For example, quantitative data may be added and then divided by the number of observations to compute the average value. This average is usually meaningful and easily interpreted. In general, more alternatives for statistical analysis are possible when data are quantitative. Section 2.2 and Chapter 3 provide ways of summarizing quantitative data.

Cross-Sectional and Time Series Data

For purposes of statistical analysis, distinguishing between cross-sectional data and time series data is important. **Cross-sectional data** are data collected at the same or approximately the same point in time. The data in Table 1.1 are cross-sectional because they describe the five variables for the 25 mutual funds at the same point in time. **Time series data** are data collected over several time periods. For example, the time series in Figure 1.1 shows the U.S. average price per gallon of regular gasoline between January 2008 and May 2011. Note that gasoline prices increased during the first half of 2008, reaching a high of $4.06 per gallon in July. However, prices decreased rapidly during the second half of 2008, reaching a low of $1.69 per gallon in December 2008. Throughout 2009 and

FIGURE 1.1 U.S. AVERAGE PRICE PER GALLON FOR REGULAR GASOLINE



Source: U.S. Energy Information Administration, May 2011.

the first five months of 2010, prices generally increased before stabilizing at a price of approximately $2.75 per gallon. Prices began to rise rapidly in the fourth quarter of 2010 and first quarter of 2011.

Graphs of time series data are frequently found in business and economic publications. Such graphs help analysts understand what happened in the past, identify any trends over time, and project future levels for the time series. The graphs of time series data can take on a variety of forms, as shown in Figure 1.2. With a little study, these graphs are usually easy to understand and interpret.

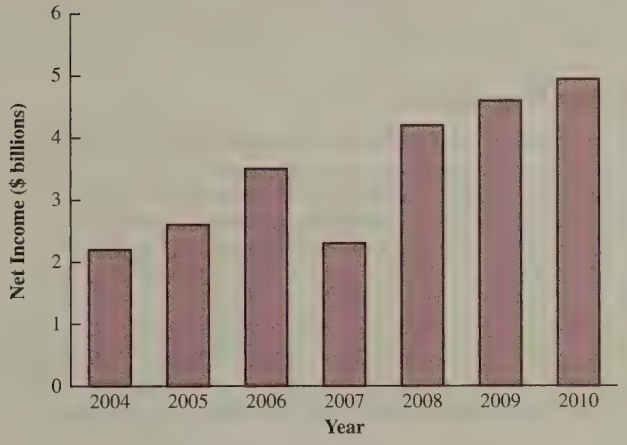
For example, Panel (A) in Figure 1.2 is a graph that shows the Dow Jones Industrial Average Index from 1998 to 2011. In April 1997, the popular stock market index was near 7000. Over the next 10 years the index rose to over 14,000 in July 2007. However, notice the sharp decline in the time series after the all-time high in 2007. By March 2009, poor economic conditions had caused the Dow Jones Industrial Average Index to return to the 7000 level of 1997. This was a scary and discouraging period for investors. By February 2011, the index was showing a recovery by reaching the 12,000 level.

The graph in Panel (B) shows the net income of McDonald's Inc. from 2004 to 2010. The declining economic conditions in 2008 and 2009 were actually beneficial to McDonald's as the company's net income rose to an all-time high. The growth in McDonald's net income showed that the company was thriving during the economic downturn as people were cutting back on the more expensive sit-down restaurants and seeking less-expensive alternatives offered by McDonald's.

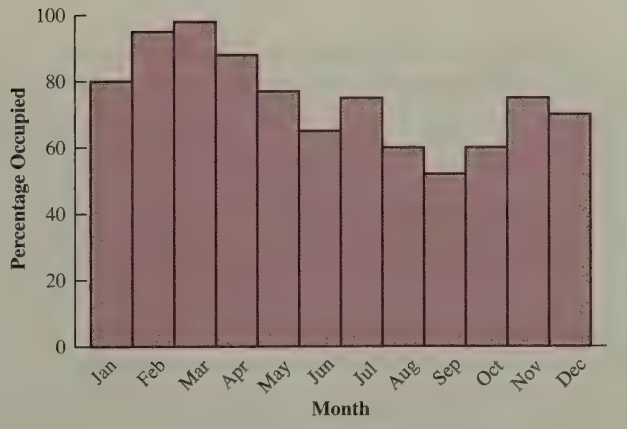
Panel (C) shows the time series for the occupancy rate of hotels in South Florida over a one-year period. The highest occupancy rates, 95% and 98%, occur during the months of February and March when the climate of South Florida is attractive to tourists. In fact, January to April of each year is typically the high-occupancy season for South Florida hotels. On the other hand, note the low occupancy rates during the months of August to October, with the lowest occupancy rate of 50% occurring in September. High temperatures and the hurricane season are the primary reasons for the drop in hotel occupancy during this period.

FIGURE 1.2 A VARIETY OF GRAPHS OF TIME SERIES DATA

(A) Dow Jones Industrial Average



(B) Net Income for McDonald's Inc.



(C) Occupancy Rate of South Florida Hotels

NOTES AND COMMENTS

1. An observation is the set of measurements obtained for each element in a data set. Hence, the number of observations is always the same as the number of elements. The number of measurements obtained for each element equals the number of variables. Hence, the total number of data items can be determined by multiplying the number of observations by the number of variables.
2. Quantitative data may be discrete or continuous. Quantitative data that measure how many (e.g., number of calls received in five minutes) are discrete. Quantitative data that measure how much (e.g., weight or time) are continuous because no separation occurs between the possible data values.

1.3 Data Sources

Data can be obtained from existing sources or from surveys and experimental studies designed to collect new data.

Existing Sources

In some cases, data needed for a particular application already exist. Companies maintain a variety of databases about their employees, customers, and business operations. Data on employee salaries, ages, and years of experience can usually be obtained from internal personnel records. Other internal records contain data on sales, advertising expenditures, distribution costs, inventory levels, and production quantities. Most companies also maintain detailed data about their customers. Table 1.2 shows some of the data commonly available from internal company records.

Organizations that specialize in collecting and maintaining data make available substantial amounts of business and economic data. Companies access these external data sources through leasing arrangements or by purchase. Dun & Bradstreet, Bloomberg, and Dow Jones & Company are three firms that provide extensive business database services to

TABLE 1.2 EXAMPLES OF DATA AVAILABLE FROM INTERNAL COMPANY RECORDS

Source	Some of the Data Typically Available
Employee records	Name, address, social security number, salary, number of vacation days, number of sick days, and bonus
Production records	Part or product number, quantity produced, direct labor cost, and materials cost
Inventory records	Part or product number, number of units on hand, reorder level, economic order quantity, and discount schedule
Sales records	Product number, sales volume, sales volume by region, and sales volume by customer type
Credit records	Customer name, address, phone number, credit limit, and accounts receivable balance
Customer profile	Age, gender, income level, household size, address, and preferences

clients. ACNielsen and Information Resources, Inc., built successful businesses collecting and processing data that they sell to advertisers and product manufacturers.

Data are also available from a variety of industry associations and special interest organizations. The Travel Industry Association of America maintains travel-related information such as the number of tourists and travel expenditures by states. Such data would be of interest to firms and individuals in the travel industry. The Graduate Management Admission Council maintains data on test scores, student characteristics, and graduate management education programs. Most of the data from these types of sources are available to qualified users at a modest cost.

The Internet continues to grow as an important source of data and statistical information. Almost all companies maintain websites that provide general information about the company as well as data on sales, number of employees, number of products, product prices, and product specifications. In addition, a number of companies now specialize in making information available over the Internet. As a result, one can obtain access to stock quotes, meal prices at restaurants, salary data, and an almost infinite variety of information.

Government agencies are another important source of existing data. For instance, the U.S. Department of Labor maintains considerable data on employment rates, wage rates, size of the labor force, and union membership. Table 1.3 lists selected governmental agencies and some of the data they provide. Most government agencies that collect and process data also make the results available through a website. Figure 1.3 shows the homepage for the U.S. Census Bureau website.

Statistical Studies

The largest experimental statistical study ever conducted is believed to be the 1954 Public Health Service experiment for the Salk polio vaccine. Nearly 2 million children in grades 1, 2, and 3 were selected from throughout the United States.

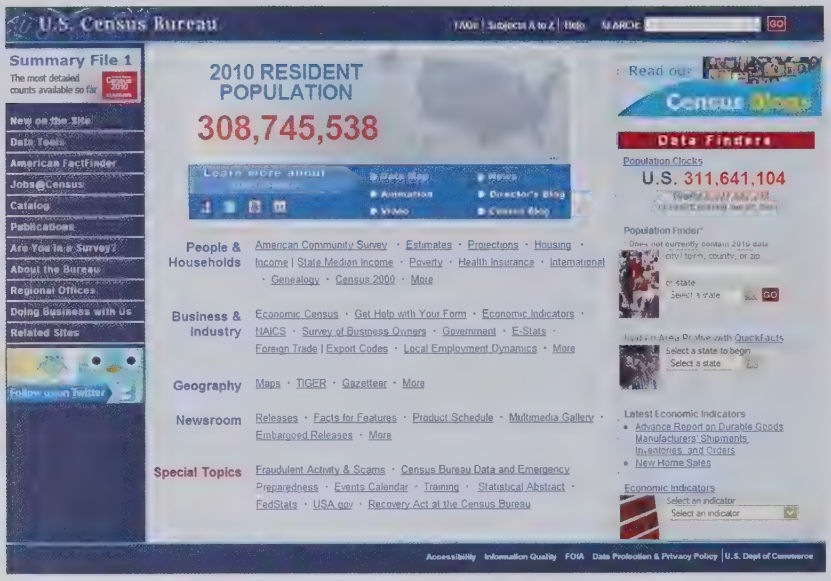
Sometimes the data needed for a particular application are not available through existing sources. In such cases, the data can often be obtained by conducting a statistical study. Statistical studies can be classified as either *experimental* or *observational.*

In an experimental study, a variable of interest is first identified. Then one or more other variables are identified and controlled so that data can be obtained about how they influence the variable of interest. For example, a pharmaceutical firm might be interested in conducting an experiment to learn about how a new drug affects blood pressure. Blood pressure is the variable of interest in the study. The dosage level of the new drug is another variable that is hoped to have a causal effect on blood pressure. To obtain data about the effect of the

TABLE 1.3 EXAMPLES OF DATA AVAILABLE FROM SELECTED GOVERNMENT AGENCIES

Government Agency	Some of the Data Available
Census Bureau	Population data, number of households, and household income
Federal Reserve Board	Data on the money supply, installment credit, exchange rates, and discount rates
Office of Management and Budget	Data on revenue, expenditures, and debt of the federal government
Department of Commerce	Data on business activity, value of shipments by industry, level of profits by industry, and growing and declining industries
Bureau of Labor Statistics	Consumer spending, hourly earnings, unemployment rate, safety records, and international statistics

FIGURE 1.3 U.S. CENSUS BUREAU HOMEPAGE



© Source: United States Census Bureau

new drug, researchers select a sample of individuals. The dosage level of the new drug is controlled, as different groups of individuals are given different dosage levels. "Before" and "after" data on blood pressure are collected for each group. Statistical analysis of the experimental data can help determine how the new drug affects blood pressure.

Nonexperimental, or observational, statistical studies make no attempt to control the variables that are thought to influence the variable of interest. A survey is perhaps the most common type of observational study. For instance, in a personal interview survey, research questions are first identified. Then a questionnaire is designed and administered to a sample of individuals. Some restaurants use observational studies to obtain data about customer opinions on the quality of food, quality of service, atmosphere, and so on. A customer opinion questionnaire used by Chops City Grill in Naples, Florida, is shown in Figure 1.4. Note that the customers who fill out the questionnaire are asked to provide ratings for 12 variables, including overall experience, greeting by hostess, manager (table visit), overall service, and so on. The response categories of excellent, good, average, fair, and poor provide categorical data that enable Chops City Grill management to maintain high standards for the restaurant's food and service.

Studies of smokers and nonsmokers are observational studies because researchers do not determine or control who will smoke and who will not smoke.

Anyone wanting to use data and statistical analysis as aids to decision making must be aware of the time and cost required to obtain the data. The use of existing data sources is desirable when data must be obtained in a relatively short period of time. If important data are not readily available from an existing source, the additional time and cost involved in obtaining the data must be taken into account. In all cases, the decision maker should consider the contribution of the statistical analysis to the decision-making process. The cost of data acquisition and the subsequent statistical analysis should not exceed the savings generated by using the information to make a better decision.

FIGURE 1.4 CUSTOMER OPINION QUESTIONNAIRE USED BY CHOPS CITY GRILL RESTAURANT IN NAPLES, FLORIDA

Chop's
CITY GRILL

Date: _____ Server Name: _____

Our customers are our top priority. Please take a moment to fill out our survey card, so we can better serve your needs. You may return this card to the front desk or return by mail. Thank you!

SERVICE SURVEY	Excellent	Good	Average	Fair	Poor
Overall Experience	❑	❑	❑	❑	❑
Greeting by Hostess	❑	❑	❑	❑	❑
Manager (Table Visit)	❑	❑	❑	❑	❑
Overall Service	❑	❑	❑	❑	❑
Professionalism	❑	❑	❑	❑	❑
Menu Knowledge	❑	❑	❑	❑	❑
Friendliness	❑	❑	❑	❑	❑
Wine Selection	❑	❑	❑	❑	❑
Menu Selection	❑	❑	❑	❑	❑
Food Quality	❑	❑	❑	❑	❑
Food Presentation	❑	❑	❑	❑	❑
Value for $ Spent	❑	❑	❑	❑	❑

What comments could you give us to improve our restaurant?

_____

_____

Thank you, we appreciate your comments. —The staff of Chops City Grill.

Data Acquisition Errors

Managers should always be aware of the possibility of data errors in statistical studies. Using erroneous data can be worse than not using any data at all. An error in data acquisition occurs whenever the data value obtained is not equal to the true or actual value that would be obtained with a correct procedure. Such errors can occur in a number of ways. For example, an interviewer might make a recording error, such as a transposition in writing the age of a 24-year-old person as 42, or the person answering an interview question might misinterpret the question and provide an incorrect response.

Experienced data analysts take great care in collecting and recording data to ensure that errors are not made. Special procedures can be used to check for internal consistency of the data. For instance, such procedures would indicate that the analyst should review the accuracy of data for a respondent shown to be 22 years of age but reporting 20 years of work experience. Data analysts also review data with unusually large and small values, called outliers, which are candidates for possible data errors. In Chapter 3 we present some of the methods statisticians use to identify outliers.

Errors often occur during data acquisition. Blindly using any data that happen to be available or using data that were acquired with little care can result in misleading information and bad decisions. Thus, taking steps to acquire accurate data can help ensure reliable and valuable decision-making information.

1.4 Descriptive Statistics

Most of the statistical information in newspapers, magazines, company reports, and other publications consists of data that are summarized and presented in a form that is easy for the reader to understand. Such summaries of data, which may be tabular, graphical, or numerical, are referred to as **descriptive statistics**.

Refer again to the data set in Table 1.1 showing data on 25 mutual funds. Methods of descriptive statistics can be used to provide summaries of the information in this data set. For example, a tabular summary of the data for the categorical variable Fund Type is shown in Table 1.4. A graphical summary of the same data, called a bar chart, is shown in Figure 1.5. These types of tabular and graphical summaries generally make the data easier to interpret. Referring to Table 1.4 and Figure 1.5, we can see easily that the majority of the mutual funds are of the Domestic Equity type. On a percentage basis, 64% are of the Domestic Equity type, 16% are of the International Equity type, and 20% are of the Fixed Income type.

A graphical summary of the data for the quantitative variable Net Asset Value, called a histogram, is provided in Figure 1.6. The histogram makes it easy to see that the net asset values range from $0 to $75, with the highest concentration between $15 and $30. Only one of the net asset values is greater than $60.

TABLE 1.4 FREQUENCIES AND PERCENT FREQUENCIES FOR MUTUAL FUND TYPE

Mutual Fund Type	Frequency	Percent Frequency
Domestic Equity	16	64
International Equity	4	16
Fixed Income	5	20
Totals	**25**	**100**

FIGURE 1.5 BAR CHART FOR MUTUAL FUND TYPE

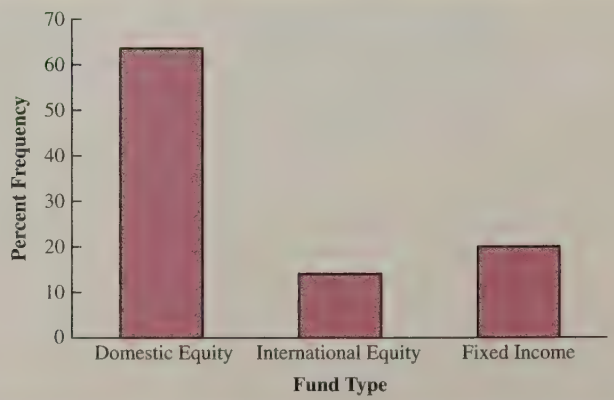
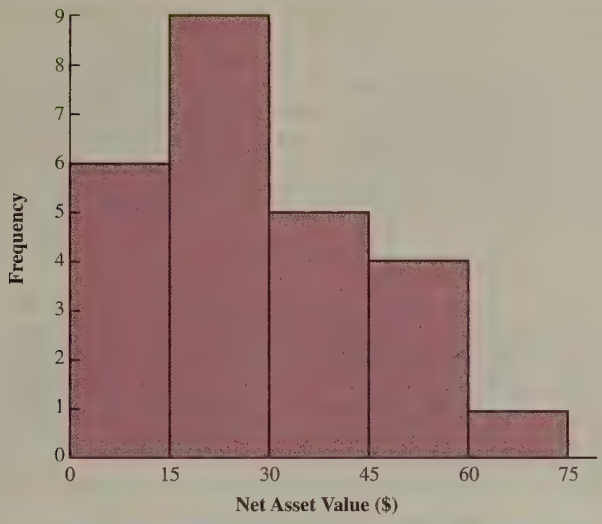


FIGURE 1.6 HISTOGRAM OF NET ASSET VALUE FOR 25 MUTUAL FUNDS



In addition to tabular and graphical displays, numerical descriptive statistics are used to summarize data. The most common numerical descriptive statistic is the average, or mean. Using the data on 5-Year Average Return for the mutual funds in Table 1.1, we can compute the average by adding the returns for all 25 mutual funds and dividing the sum by 25. Doing so provides a 5-year average return of 16.50%. This average demonstrates a measure of the central tendency, or central location, of the data for that variable.

There is a great deal of interest in effective methods for developing and presenting descriptive statistics. Chapters 2 and 3 devote attention to the tabular, graphical, and numerical methods of descriptive statistics.

1.5 Statistical Inference

Many situations require information about a large group of elements (individuals, companies, voters, households, products, customers, and so on). But, because of time, cost, and other considerations, data can be collected from only a small portion of the group. The larger group of elements in a particular study is called the **population**, and the smaller group is called the **sample**. Formally, we use the following definitions.

POPULATION

A population is the set of all elements of interest in a particular study.

SAMPLE

A sample is a subset of the population.

The U.S. government conducts a census every 10 years. Market research firms conduct sample surveys every day.

The process of conducting a survey to collect data for the entire population is called a **census**. The process of conducting a survey to collect data for a sample is called a **sample survey**. As one of its major contributions, statistics uses data from a sample to make estimates and test hypotheses about the characteristics of a population through a process referred to as **statistical inference**.

As an example of statistical inference, let us consider the study conducted by Norris Electronics. Norris manufactures a high-intensity lightbulb used in a variety of electrical products. In an attempt to increase the useful life of the lightbulb, the product design group developed a new lightbulb filament. In this case, the population is defined as all lightbulbs that could be produced with the new filament. To evaluate the advantages of the new filament, 200 bulbs with the new filament were manufactured and tested. Data collected from this sample showed the number of hours each lightbulb operated before filament burnout. See Table 1.5.

Suppose Norris wants to use the sample data to make an inference about the average hours of useful life for the population of all lightbulbs that could be produced with the new filament. Adding the 200 values in Table 1.5 and dividing the total by 200 provides the sample average lifetime for the lightbulbs: 76 hours. We can use this sample result to estimate that the average lifetime for the lightbulbs in the population is 76 hours. Figure 1.7 provides a graphical summary of the statistical inference process for Norris Electronics.

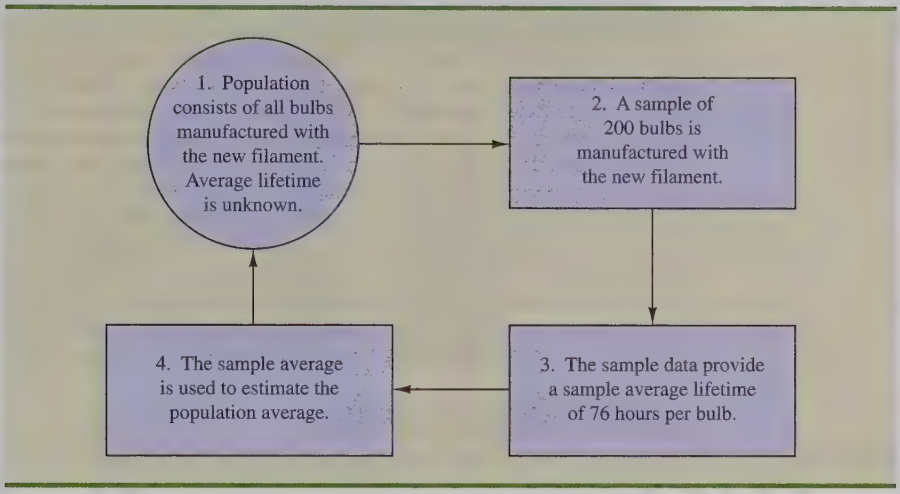
Whenever statisticians use a sample to estimate a population characteristic of interest, they usually provide a statement of the quality, or precision, associated with the estimate. For the Norris example, the statistician might state that the point estimate of the average lifetime for the population of new lightbulbs is 76 hours with a margin of error of ± 4 hours. Thus, an interval estimate of the average lifetime for all lightbulbs produced with the new

TABLE 1.5 HOURS UNTIL BURNOUT FOR A SAMPLE OF 200 LIGHTBULBS FOR THE NORRIS ELECTRONICS EXAMPLE

WEB file
Norris

107	73	68	97	76	79	94	59	98	57
54	65	71	70	84	88	62	61	79	98
66	62	79	86	68	74	61	82	65	98
62	116	65	88	64	79	78	79	77	86
74	85	73	80	68	78	89	72	58	69
92	78	88	77	103	88	63	68	88	81
75	90	62	89	71	71	74	70	74	70
65	81	75	62	94	71	85	84	83	63
81	62	79	83	93	61	65	62	92	65
83	70	70	81	77	72	84	67	59	58
78	66	66	94	77	63	66	75	68	76
90	78	71	101	78	43	59	67	61	71
96	75	64	76	72	77	74	65	82	86
66	86	96	89	81	71	85	99	59	92
68	72	77	60	87	84	75	77	51	45
85	67	87	80	84	93	69	76	89	75
83	68	72	67	92	89	82	96	77	102
74	91	76	83	66	68	61	73	72	76
73	77	79	94	63	59	62	71	81	65
73	63	63	89	82	64	85	92	64	73

FIGURE 1.7 THE PROCESS OF STATISTICAL INFERENCE FOR THE NORRIS ELECTRONICS EXAMPLE


1. Population consists of all bulbs manufactured with the new filament. Average lifetime is unknown.
2. A sample of 200 bulbs is manufactured with the new filament.
3. The sample data provide a sample average lifetime of 76 hours per bulb.
4. The sample average is used to estimate the population average.

filament is 72 hours to 80 hours. The statistician can also state how confident he or she is that the interval from 72 hours to 80 hours contains the population average.

1.6 Statistical Analysis Using Microsoft Excel

Because statistical analysis typically involves working with large amounts of data, computer software is frequently used to conduct the analysis. Often the data to be analyzed reside in a spreadsheet. Given the data management, data analysis, and presentation capabilities of modern spreadsheet packages, it is now possible to conduct statistical analyses using them. In this book we show how statistical analysis can be performed using Microsoft Excel. In selected cases where Excel does not contain statistical analysis functions or data analysis tools that can be used to perform a statistical procedure discussed in the text, we have included chapter appendixes that show how to use StatTools, an add-in for Excel that provides an extended range of statistical and graphical options. The appendix to Chapter 1 provides an introduction to StatTools.

We want to emphasize that this book is about statistics; it is not a book about spreadsheets. Thus, our focus is on showing the appropriate statistical procedures for collecting, analyzing, presenting, and interpreting data. Because Excel is widely available in business organizations, you can expect to put the knowledge gained here to use in the setting where you currently, or soon will, work. If, in the process of studying this material, you become more proficient in Excel, so much the better.

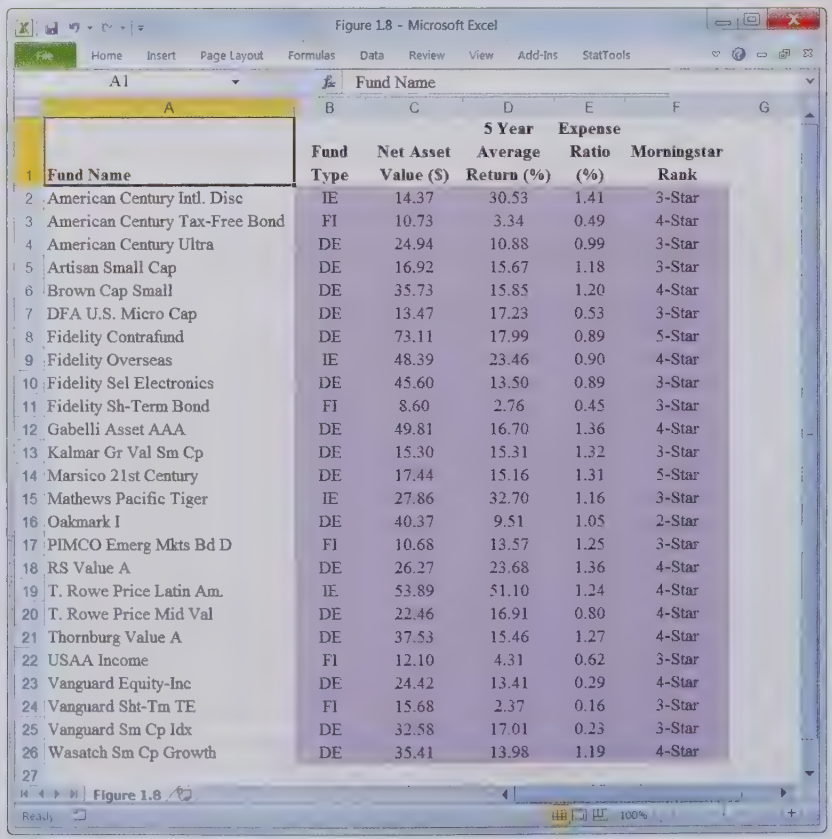
We begin most sections with an application scenario in which a statistical procedure is useful. After showing what the statistical procedure is and how it is used, we turn to showing how to implement the procedure using Excel. Thus, you should gain an understanding of what the procedure is, the situations in which it is useful, and how to implement it using the capabilities of Excel.

Data Sets and Excel Worksheets

Data sets are organized in Excel worksheets in much the same way as the data set for the 25 mutual fund companies that appears in Table 1.1. Figure 1.8 shows an Excel worksheet for that data set. Note that row 1 and column A contain labels. Cells B1:F1 contain the variable names and cells A2:A26 contain the observation names. Cells B2:F26 contain the data that were collected. A purple screen highlights the data. The data are the focus of the statistical analysis. Except for the headings in row 1, each row in the worksheet corresponds to an observation and each column corresponds to a variable. For instance, row 2 of the worksheet contains the data for the first observation, American Century Intl. Disc; row 3 contains the data for the second observation, American Century Tax-Free Bond; and so on. Thus, the names in column A provide a convenient way to refer to each of the 25 observations in the study. Note that column B of the worksheet contains the data for the variable Fund Type, column C contains the data for the variable Net Asset Value ($), and so on.

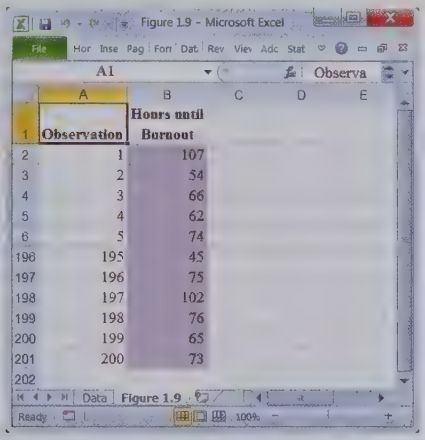
Suppose now that we want to use Excel to analyze the Norris Electronics data shown in Table 1.5. The data in Table 1.5 are organized into 10 columns with 20 data values in

FIGURE 1.8 EXCEL WORKSHEET FOR THE 25 MUTUAL FUNDS DATA SET



	A	B	C	D	E	F
1	Fund Name	Fund Type	Net Asset Value ($)	5 Year Average Return (%)	Expense Ratio (%)	Morningstar Rank
2	American Century Intl. Disc	IE	14.37	30.53	1.41	3-Star
3	American Century Tax-Free Bond	FI	10.73	3.34	0.49	4-Star
4	American Century Ultra	DE	24.94	10.88	0.99	3-Star
5	Artisan Small Cap	DE	16.92	15.67	1.18	3-Star
6	Brown Cap Small	DE	35.73	15.85	1.20	4-Star
7	DFA U.S. Micro Cap	DE	13.47	17.23	0.53	3-Star
8	Fidelity Contrafund	DE	73.11	17.99	0.89	5-Star
9	Fidelity Overseas	IE	48.39	23.46	0.90	4-Star
10	Fidelity Sel Electronics	DE	45.60	13.50	0.89	3-Star
11	Fidelity Sh-Term Bond	FI	8.60	2.76	0.45	3-Star
12	Gabelli Asset AAA	DE	49.81	16.70	1.36	4-Star
13	Kalmar Gr Val Sm Cp	DE	15.30	15.31	1.32	3-Star
14	Marsico 21st Century	DE	17.44	15.16	1.31	5-Star
15	Mathews Pacific Tiger	IE	27.86	32.70	1.16	3-Star
16	Oakmark I	DE	40.37	9.51	1.05	2-Star
17	PIMCO Emerg Mkts Bd D	FI	10.68	13.57	1.25	3-Star
18	RS Value A	DE	26.27	23.68	1.36	4-Star
19	T. Rowe Price Latin Am.	IE	53.89	51.10	1.24	4-Star
20	T. Rowe Price Mid Val	DE	22.46	16.91	0.80	4-Star
21	Thornburg Value A	DE	37.53	15.46	1.27	4-Star
22	USAA Income	FI	12.10	4.31	0.62	3-Star
23	Vanguard Equity-Inc	DE	24.42	13.41	0.29	4-Star
24	Vanguard Sht-Tm TE	FI	15.68	2.37	0.16	3-Star
25	Vanguard Sm Cp Idx	DE	32.58	17.01	0.23	3-Star
26	Wasatch Sm Cp Growth	DE	35.41	13.98	1.19	4-Star
27						

FIGURE 1.9 EXCEL WORKSHEET FOR THE NORRIS ELECTRONICS DATA SET



Note: Rows 7–195 are hidden.

	A	B
1	Observation	Hours until Burnout
2	1	107
3	2	54
4	3	66
5	4	62
6	5	74
196	195	45
197	196	75
198	197	102
199	198	76
200	199	65
201	200	73
202		

each column so that the data would fit nicely on a single page of the text. Even though the table has several columns, it shows data for only one variable (hours until burnout). In statistical worksheets, it is customary to put all the data for each variable in a single column. Refer to the Excel worksheet shown in Figure 1.9. To make it easier to identify each observation in the data set, we entered the heading Observation into cell A1 and the numbers 1–200 into cells A2:A201. The heading Hours until Burnout has been entered into cell B1, and the data for the 200 observations have been entered into cells B2:B201. Displaying a worksheet with this many rows on a single page of a textbook is not practical. In such cases we will hide selected rows to conserve space. In the Excel worksheet for the Norris Electronics problem, we have hidden rows 7 through 195 (observations 6 through 194) to conserve space.[1]

Using Excel for Statistical Analysis

In this text, we are careful to separate the discussion of a statistical procedure from the discussion of using Excel to implement the procedure. The material that discusses the use of Excel will usually be set apart in sections with headings such as Using Excel's COUNTIF Function to Construct a Frequency Distribution, Using Excel's Chart Tools to Construct Bar Charts and Pie Charts, and so on. In using Excel for statistical analysis, three tasks may be needed: Enter Data; Enter Functions and Formulas; and Apply Tools.

Enter Data: Select cell locations for the data and enter the data along with appropriate descriptive labels.

Enter Functions and Formulas: Select cell locations, enter Excel functions and formulas, and provide descriptive material to identify the results.

Apply Tools: Use Excel's tools for data management, data analysis, and presentation.

[1]To hide rows 7 through 195 in the Excel worksheet, first select rows 7 through 195. Then, right-click and choose the Hide option. To redisplay rows 7 through 195, just select rows 6 and 196, right-click, and select the Unhide option.

Our approach will be to describe how these tasks are performed each time we use Excel to implement a statistical procedure. It will always be necessary to enter data. But, depending on the complexity of the statistical analysis, only one of the last two tasks may be needed.

To illustrate how the discussion of using Excel will appear throughout the book, we will show how to use Excel's AVERAGE function to compute the average lifetime for the 200 burnout times in Table 1.5. Refer to Figure 1.10 as we describe the tasks involved. The worksheet shown in the foreground of Figure 1.10 displays the data for the problem and shows the results of the analysis. It is called a value worksheet. The worksheet shown in the background displays the Excel formula used to compute the average lifetime and is called the formula worksheet. A purple screen is used to highlight the data in both worksheets. In addition, a green screen is used to highlight functions and formulas in the formula worksheet and the corresponding results in the value worksheet.

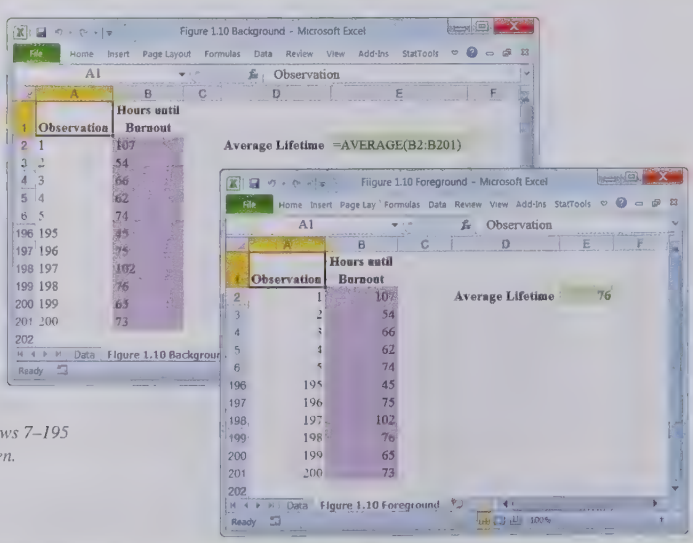
Enter Data: The labels Observation and Hours until Burnout are entered into cells A1:B1. The numbers 1–200 are entered into cells A2:A201 to identify each of the observations, and the data showing the hours until burnout for each observation are entered into cells B2:B201 of the worksheet.

Enter Functions and Formulas: Excel's AVERAGE function can be used to compute the average lifetime for the 200 lightbulbs. We can compute the average lifetime by entering the following formula into cell E2:

$$=\text{AVERAGE(B2:B201)}$$

To identify the result, the label Average Lifetime is entered into cell D2. Note that for this problem, the Apply Tools task was not required. The value worksheet shows that the value computed using the AVERAGE function is 76 hours.

FIGURE 1.10 COMPUTING THE AVERAGE LIFETIME OF LIGHTBULBS FOR NORRIS ELECTRONICS USING EXCEL'S AVERAGE FUNCTION



Note: Rows 7–195 are hidden.

1.7 Data Mining

With the aid of magnetic card readers, bar code scanners, and point-of-sale terminals, most organizations obtain large amounts of data on a daily basis. And, even for a small local restaurant that uses touch screen monitors to enter orders and handle billing, the amount of data collected can be significant. For large retail companies, the sheer volume of data collected is hard to conceptualize, and figuring out how to use these data effectively to improve profitability is a challenge. For example, mass retailers such as Walmart capture data on 20 to 30 million transactions every day, telecommunication companies such as France Telecom and AT&T generate over 300 million call records per day, and Visa processes 6800 payment transactions per second or approximately 600 million transactions per day. Storing and managing the transaction data is a significant undertaking.

The term *data warehousing* is used to refer to the process of capturing, storing, and maintaining the data. Computing power and data collection tools have reached the point where it is now feasible to store and retrieve extremely large quantities of data in seconds. Analysis of the data in the warehouse may result in decisions that will lead to new strategies and higher profits for the organization.

The subject of **data mining** deals with methods for developing useful decision-making information from large databases. Using a combination of procedures from statistics, mathematics, and computer science, analysts "mine the data" in the warehouse to convert it into useful information, hence the name *data mining*. Dr. Kurt Thearling, a leading practitioner in the field, defines data mining as "the automated extraction of predictive information from (large) databases." The two key words in Dr. Thearling's definition are *automated* and *predictive*. Data mining systems that are the most effective use automated procedures to extract information from the data using only the most general or even vague queries by the user. And data mining software automates the process of uncovering hidden predictive information that in the past required hands-on analysis.

The major applications of data mining have been made by companies with a strong consumer focus, such as retail businesses, financial organizations, and communication companies. Data mining has been successfully used to help retailers such as Amazon and Barnes & Noble determine one or more related products that customers who have already purchased a specific product are also likely to purchase. Then, when a customer logs on to the company's website and purchases a product, the website uses pop-ups to alert the customer about additional products that the customer is likely to purchase. In another application, data mining may be used to identify customers who are likely to spend more than $20 on a particular shopping trip. These customers may then be identified as the ones to receive special email or regular mail discount offers to encourage them to make their next shopping trip before the discount termination date.

Statistical methods play an important role in data mining, both in terms of discovering relationships in the data and predicting future outcomes. However, a thorough coverage of data mining and the use of statistics in data mining is outside the scope of this text.

Data mining is a technology that relies heavily on statistical methodology such as multiple regression, logistic regression, and correlation. But it takes a creative integration of all these methods and computer science technologies involving artificial intelligence and machine learning to make data mining effective. A significant investment in time and money is required to implement commercial data mining software packages developed by firms such as Oracle, Teradata, and SAS. The statistical concepts introduced in this text will be helpful in understanding the statistical methodology used by data mining software packages and enable you to better understand the statistical information that is developed.

Because statistical models play an important role in developing predictive models in data mining, many of the concerns that statisticians deal with in developing statistical models are also applicable. For instance, a concern in any statistical study involves the issue of model reliability. Finding a statistical model that works well for a particular sample of data does not necessarily mean that it can be reliably applied to other data. One of the common statistical

approaches to evaluating model reliability is to divide the sample data set into two parts: a training data set and a test data set. If the model developed using the training data is able to accurately predict values in the test data, we say that the model is reliable. One advantage that data mining has over classical statistics is that the enormous amount of data available allows the data mining software to partition the data set so that a model developed for the training data set may be tested for reliability on other data. In this sense, the partitioning of the data set allows data mining to develop models and relationships and then quickly observe if they are repeatable and valid with new and different data. On the other hand, a warning for data mining applications is that with so much data available, there is a danger of overfitting the model to the point that misleading associations and cause/effect conclusions appear to exist. Careful interpretation of data mining results and additional testing will help avoid this pitfall.

1.8 Ethical Guidelines for Statistical Practice

Ethical behavior is something we should strive for in all that we do. Ethical issues arise in statistics because of the important role statistics plays in the collection, analysis, presentation, and interpretation of data. In a statistical study, unethical behavior can take a variety of forms, including improper sampling, inappropriate analysis of the data, development of misleading graphs, use of inappropriate summary statistics, and/or a biased interpretation of the statistical results.

As you begin to do your own statistical work, we encourage you to be fair, thorough, objective, and neutral as you collect data, conduct analyses, make oral presentations, and present written reports containing information developed. As a consumer of statistics, you should also be aware of the possibility of unethical statistical behavior by others. When you see statistics in newspapers, on television, on the Internet, and so on, it is a good idea to view the information with some skepticism, always being aware of the source as well as the purpose and objectivity of the statistics provided.

The American Statistical Association, the nation's leading professional organization for statistics and statisticians, developed the report "Ethical Guidelines for Statistical Practice"[2] to help statistical practitioners make and communicate ethical decisions and assist students in learning how to perform statistical work responsibly. The report contains 67 guidelines organized into eight topic areas: Professionalism; Responsibilities to Funders, Clients, and Employers; Responsibilities in Publications and Testimony; Responsibilities to Research Subjects; Responsibilities to Research Team Colleagues; Responsibilities to Other Statisticians or Statistical Practitioners; Responsibilities Regarding Allegations of Misconduct; and Responsibilities of Employers Including Organizations, Individuals, Attorneys, or Other Clients Employing Statistical Practitioners.

One of the ethical guidelines in the professionalism area addresses the issue of running multiple tests until a desired result is obtained. Let us consider an example. In Section 1.5 we discussed a statistical study conducted by Norris Electronics involving a sample of 200 high-intensity lightbulbs manufactured with a new filament. The average lifetime for the sample, 76 hours, provided an estimate of the average lifetime for all lightbulbs produced with the new filament. However, consider this. Because Norris selected a sample of bulbs, it is reasonable to assume that another sample would have provided a different average lifetime.

Suppose Norris's management had hoped the sample results would enable them to claim that the average lifetime for the new lightbulbs was 80 hours or more. Suppose further that Norris's management decides to continue the study by manufacturing and testing

[2]American Statistical Association, "Ethical Guidelines for Statistical Practice," 1999.

repeated samples of 200 lightbulbs with the new filament until a sample mean of 80 hours or more is obtained. If the study is repeated enough times, a sample may eventually be obtained—by chance alone—that would provide the desired result and enable Norris to make such a claim. In this case, consumers would be misled into thinking the new product is better than it actually is. Clearly, this type of behavior is unethical and represents a gross misuse of statistics in practice.

Several ethical guidelines in the responsibilities and publications and testimony area deal with issues involving the handling of data. For instance, a statistician must account for all data considered in a study and explain the sample(s) actually used. In the Norris Electronics study the average lifetime for the 200 bulbs in the original sample is 76 hours; this is considerably less than the 80 hours or more that management hoped to obtain. Suppose now that after reviewing the results showing a 76-hour average lifetime, Norris discards all the observations with 70 or fewer hours until burnout, allegedly because these bulbs contain imperfections caused by startup problems in the manufacturing process. After discarding these lightbulbs, the average lifetime for the remaining lightbulbs in the sample turns out to be 82 hours. Would you be suspicious of Norris's claim that the lifetime for its lightbulbs is 82 hours?

If the Norris lightbulbs showing 70 or fewer hours until burnout were discarded to simply provide an average lifetime of 82 hours, there is no question that discarding the lightbulbs with 70 or fewer hours until burnout is unethical. But, even if the discarded lightbulbs contain imperfections due to startup problems in the manufacturing process—and, as a result, should not have been included in the analysis—the statistician who conducted the study must account for all the data that were considered and explain how the sample actually used was obtained. To do otherwise is potentially misleading and would constitute unethical behavior on the part of both the company and the statistician.

A guideline in the shared values section of the American Statistical Association report states that statistical practitioners should avoid any tendency to slant statistical work toward predetermined outcomes. This type of unethical practice is often observed when unrepresentative samples are used to make claims. For instance, in many areas of the country smoking is not permitted in restaurants. Suppose, however, a lobbyist for the tobacco industry interviews people in restaurants where smoking is permitted in order to estimate the percentage of people who are in favor of allowing smoking in restaurants. The sample results show that 90% of the people interviewed are in favor of allowing smoking in restaurants. Based on these sample results, the lobbyist claims that 90% of all people who eat in restaurants are in favor of permitting smoking in restaurants. In this case we would argue that only sampling persons eating in restaurants that allow smoking has biased the results. If only the final results of such a study are reported, readers unfamiliar with the details of the study (i.e., that the sample was collected only in restaurants allowing smoking) can be misled.

The scope of the American Statistical Association's report is broad and includes ethical guidelines that are appropriate not only for a statistician, but also for consumers of statistical information. We encourage you to read the report to obtain a better perspective of ethical issues as you continue your study of statistics and to gain the background for determining how to ensure that ethical standards are met when you start to use statistics in practice.

Summary

Statistics is the art and science of collecting, analyzing, presenting, and interpreting data. Nearly every college student majoring in business or economics is required to take a course in statistics. We began the chapter by describing typical statistical applications for business and economics.

Data consist of the facts and figures that are collected and analyzed. Four scales of measurement used to obtain data on a particular variable include nominal, ordinal, interval, and ratio. The scale of measurement for a variable is nominal when the data are labels or names used to identify an attribute of an element. The scale is ordinal if the data demonstrate the properties of nominal data and the order or rank of the data is meaningful. The scale is interval if the data demonstrate the properties of ordinal data and the interval between values is expressed in terms of a fixed unit of measure. Finally, the scale of measurement is ratio if the data show all the properties of interval data and the ratio of two values is meaningful.

For purposes of statistical analysis, data can be classified as categorical or quantitative. Categorical data use labels or names to identify an attribute of each element. Categorical data use either the nominal or ordinal scale of measurement and may be nonnumerical or numerical. Quantitative data are numerical values that indicate how much or how many. Quantitative data use either the interval or ratio scale of measurement. Ordinary arithmetic operations are meaningful only if the data are quantitative. Therefore, statistical computations used for quantitative data are not always appropriate for categorical data.

In Sections 1.4 and 1.5 we introduced the topics of descriptive statistics and statistical inference. Descriptive statistics are the tabular, graphical, and numerical methods used to summarize data. The process of statistical inference uses data obtained from a sample to make estimates or test hypotheses about the characteristics of a population. The last three sections of the chapter provide information on the role of computers in statistical analysis, an introduction to the relative new field of data mining, and a summary of ethical guidelines for statistical practice.

Glossary

Statistics The art and science of collecting, analyzing, presenting, and interpreting data.
Data The facts and figures collected, analyzed, and summarized for presentation and interpretation.
Data set All the data collected in a particular study.
Elements The entities on which data are collected.
Variable A characteristic of interest for the elements.
Observation The set of measurements obtained for a particular element.
Nominal scale The scale of measurement for a variable when the data are labels or names used to identify an attribute of an element. Nominal data may be nonnumerical or numerical.
Ordinal scale The scale of measurement for a variable if the data exhibit the properties of nominal data and the order or rank of the data is meaningful. Ordinal data may be nonnumerical or numerical.
Interval scale The scale of measurement for a variable if the data demonstrate the properties of ordinal data and the interval between values is expressed in terms of a fixed unit of measure. Interval data are always numerical.
Ratio scale The scale of measurement for a variable if the data demonstrate all the properties of interval data and the ratio of two values is meaningful. Ratio data are always numerical.
Categorical data Labels or names used to identify an attribute of each element. Categorical data use either the nominal or ordinal scale of measurement and may be nonnumerical or numerical.
Quantitative data Numerical values that indicate how much or how many of something. Quantitative data are obtained using either the interval or ratio scale of measurement.
Categorical variable A variable with categorical data.
Quantitative variable A variable with quantitative data.
Cross-sectional data Data collected at the same or approximately the same point in time.

Time series data Data collected over several time periods.
Descriptive statistics Tabular, graphical, and numerical summaries of data.
Population The set of all elements of interest in a particular study.
Sample A subset of the population.
Census A survey to collect data on the entire population.
Sample survey A survey to collect data on a sample.
Statistical inference The process of using data obtained from a sample to make estimates or test hypotheses about the characteristics of a population.
Data mining The process of using procedures from statistics and computer science to extract useful information from extremely large databases.

Supplementary Exercises

1. Discuss the differences between statistics as numerical facts and statistics as a discipline or field of study.

SELF test

2. The U.S. Department of Energy provides fuel economy information for a variety of motor vehicles. A sample of 10 automobiles is shown in Table 1.6 (Fuel Economy website, February 22, 2008). Data show the size of the automobile (compact, midsize, or large), the number of cylinders in the engine, the city driving miles per gallon, the highway driving miles per gallon, and the recommended fuel (diesel, premium, or regular).
   a. How many elements are in this data set?
   b. How many variables are in this data set?
   c. Which variables are categorical and which variables are quantitative?
   d. What type of measurement scale is used for each of the variables?

SELF test

3. Refer to Table 1.6.
   a. What is the average miles per gallon for city driving?
   b. On average, how much higher is the miles per gallon for highway driving as compared to city driving?
   c. What percentage of the cars have four-cylinder engines?
   d. What percentage of the cars use regular fuel?

4. Table 1.7 shows data for seven colleges and universities. The endowment (in billions of dollars) and the percentage of applicants admitted are shown (*USA Today*, February 3, 2008). The state each school is located in, the campus setting, and the NCAA Division for

TABLE 1.6 FUEL ECONOMY INFORMATION FOR 10 AUTOMOBILES

Car	Size	Cylinders	City MPG	Highway MPG	Fuel
Audi A8	Large	12	13	19	Premium
BMW 328Xi	Compact	6	17	25	Premium
Cadillac CTS	Midsize	6	16	25	Regular
Chrysler 300	Large	8	13	18	Premium
Ford Focus	Compact	4	24	33	Regular
Hyundai Elantra	Midsize	4	25	33	Regular
Jeep Grand Cherokee	Midsize	6	17	26	Diesel
Pontiac G6	Compact	6	15	22	Regular
Toyota Camry	Midsize	4	21	31	Regular
Volkswagen Jetta	Compact	5	21	29	Regular

TABLE 1.7 DATA FOR SEVEN COLLEGES AND UNIVERSITIES

School	State	Campus Setting	Endowment ($ billions)	% Applicants Admitted	NCAA Division
Amherst College	Massachusetts	Town: Fringe	1.7	18	III
Duke	North Carolina	City: Midsize	5.9	21	I-A
Harvard University	Massachusetts	City: Midsize	34.6	9	I-AA
Swarthmore College	Pennsylvania	Suburb: Large	1.4	18	III
University of Pennsylvania	Pennsylvania	City: Large	6.6	18	I-AA
Williams College	Massachusetts	Town: Fringe	1.9	18	III
Yale University	Connecticut	City: Midsize	22.5	9	I-AA

varsity teams were obtained from the National Center of Education Statistics website, February 22, 2008.

a. How many elements are in the data set?
b. How many variables are in the data set?
c. Which of the variables are categorical and which are quantitative?

5. Consider the data set in Table 1.7.
   a. Compute the average endowment for the sample.
   b. Compute the average percentage of applicants admitted.
   c. What percentage of the schools have NCAA Division III varsity teams?
   d. What percentage of the schools have a City: Midsize campus setting?

6. *Foreign Affairs* magazine conducted a survey to develop a profile of its subscribers (Foreign Affairs website, February 23, 2008). The following questions were asked.
   a. How many nights have you stayed in a hotel in the past 12 months?
   b. Where do you purchase books? Three options were listed: Bookstore, Internet, and Book Club.
   c. Do you own or lease a luxury vehicle? (Yes or No)
   d. What is your age?
   e. For foreign trips taken in the past three years, what was your destination? Seven international destinations were listed.

   Comment on whether each question provides categorical or quantitative data.

7. The Ritz-Carlton Hotel used a customer opinion questionnaire to obtain performance data about its dining and entertainment services (The Ritz-Carlton Hotel, Naples, Florida, February 2006). Customers were asked to rate six factors: Welcome, Service, Food, Menu Appeal, Atmosphere, and Overall Experience. Data were recorded for each factor, with 1 for Fair, 2 for Average, 3 for Good, and 4 for Excellent.
   a. The customer responses provided data for six variables. Are the variables categorical or quantitative?
   b. What measurement scale is used?

8. The *FinancialTimes*/Harris Poll is a monthly online poll of adults from six countries in Europe and the United States. A January poll included 1015 adults in the United States. One of the questions asked was, "How would you rate the Federal Bank in handling the credit problems in the financial markets?" Possible responses were Excellent, Good, Fair, Bad, and Terrible (Harris Interactive website, January 2008).
   a. What was the sample size for this survey?
   b. Are the data categorical or quantitative?
   c. Would it make more sense to use averages or percentages as a summary of the data for this question?

d. Of the respondents in the United States, 10% said the Federal Bank is doing a good job. How many individuals provided this response?

9. The Commerce Department reported receiving the following applications for the Malcolm Baldrige National Quality Award: 23 from large manufacturing firms, 18 from large service firms, and 30 from small businesses.
   a. Is type of business a categorical or quantitative variable?
   b. What percentage of the applications came from small businesses?

10. The Bureau of Transportation Statistics Omnibus Household Survey is conducted annually and serves as an information source for the U.S. Department of Transportation. In one part of the survey the person being interviewed was asked to respond to the following statement: "Drivers of motor vehicles should be allowed to talk on a hand-held cell phone while driving." Possible responses were strongly agree, somewhat agree, somewhat disagree, and strongly disagree. Forty-four respondents said that they strongly agree with this statement, 130 said that they somewhat agree, 165 said they somewhat disagree, and 741 said they strongly disagree with this statement (Bureau of Transportation website, August 2010).
   a. Do the responses for this statement provide categorical or quantitative data?
   b. Would it make more sense to use averages or percentages as a summary of the responses for this statement?
   c. What percentage of respondents strongly agree with allowing drivers of motor vehicles to talk on a hand-held cell phone while driving?
   d. Do the results indicate general support for or against allowing drivers of motor vehicles to talk on a hand-held cell phone while driving?

11. J.D. Power and Associates conducts vehicle quality surveys to provide automobile manufacturers with consumer satisfaction information about their products (Vehicle Quality Survey, January 2010). Using a sample of vehicle owners from recent vehicle purchase records, the survey asks the owners a variety of questions about their new vehicles, such as those shown below. For each question, state whether the data collected are categorical or quantitative and indicate the measurement scale being used.
   a. What price did you pay for the vehicle?
   b. How did you pay for the vehicle? (Cash, Lease, or Finance)
   c. How likely would you be to recommend this vehicle to a friend? (Definitely Not, Probably Not, Probably Will, and Definitely Will)
   d. What is the current mileage?
   e. What is your overall rating of your new vehicle? A 10-point scale, ranging from 1 for unacceptable to 10 for truly exceptional, was used.

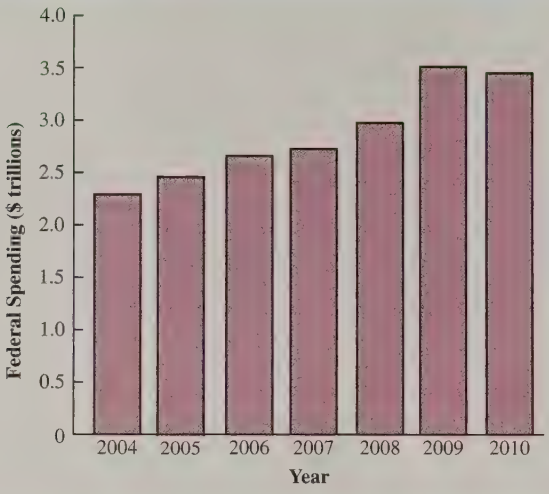
12. The Hawaii Visitors Bureau collects data on visitors to Hawaii. The following questions were among 16 asked in a questionnaire handed out to passengers during incoming airline flights.
   - This trip to Hawaii is my: 1st, 2nd, 3rd, 4th, etc.
   - The primary reason for this trip is: (10 categories, including vacation, convention, honeymoon)
   - Where I plan to stay: (11 categories, including hotel, apartment, relatives, camping)
   - Total days in Hawaii

   a. What is the population being studied?
   b. Is the use of a questionnaire a good way to reach the population of passengers on incoming airline flights?
   c. Comment on each of the four questions in terms of whether it will provide categorical or quantitative data.

SELF test

13. Figure 1.11 provides a bar chart showing the amount of federal spending for the years 2004 to 2010 (Congressional Budget Office website, May 15, 2011).
   a. What is the variable of interest?
   b. Are the data categorical or quantitative?
   c. Are the data time series or cross-sectional?
   d. Comment on the trend in federal spending over time.

FIGURE 1.11 FEDERAL SPENDING



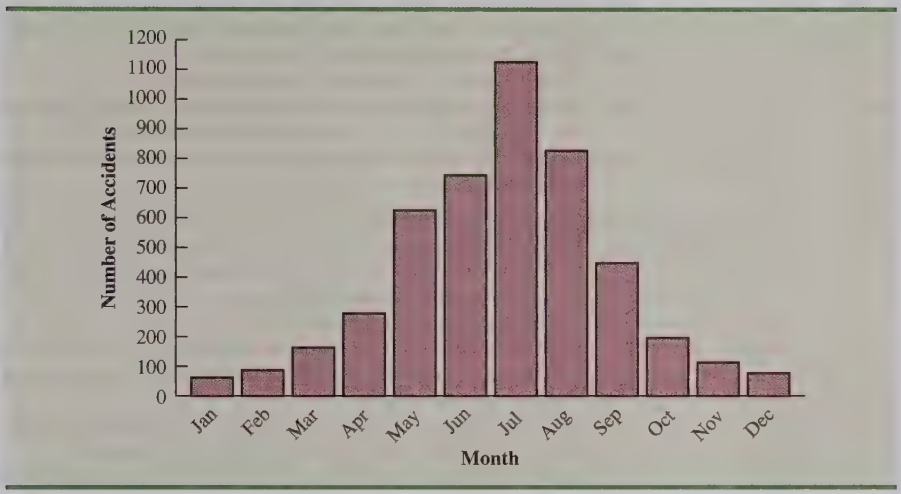
14. The following data show the number of rental cars in service for three rental car companies: Hertz, Avis, and Dollar. The data are for the years 2007–2010 and are in thousands of vehicles (Auto Rental News website, May 15, 2011).

	Cars in Service (1000s)			
Company	**2007**	**2008**	**2009**	**2010**
Hertz	327	311	286	290
Dollar	167	140	106	108
Avis	204	220	300	270

a. Construct a time series graph for the years 2007 to 2010 showing the number of rental cars in service for each company. Show the time series for all three companies on the same graph.
b. Comment on who appears to be the market share leader and how the market shares are changing over time.
c. Construct a bar chart showing rental cars in service for 2010. Is this chart based on cross-sectional or time series data?

15. Every year, the U.S. Coast Guard collects data and compiles statistics on reported recreational boating accidents. These statistics are derived from accident reports that are filed by the owners/operators of recreational vessels involved in accidents. Last year 4730 recreational boating accident reports were filed. Figure 1.12 provides a bar chart summarizing the number of accident reports that were filed each month (U.S. Coast Guard's Boating Safety Division website, August 2010).
a. Are the data categorical or quantitative?
b. Are the data time series or cross-sectional?
c. In what month were the most accidents reports filed? Approximately how many?

FIGURE 1.12 NUMBER OF RECREATIONAL BOATING ACCIDENTS



d. There were 61 accident reports filed in January and 76 accidents reports filed in December. What percentage of the total number of accident reports for the year was filed in these two months? Does this seem reasonable?
e. Comment on the overall shape of the bar graph.

16. The Energy Information Administration of the U.S. Department of Energy provided time series data for the U.S. average price per gallon of conventional regular gasoline between January 2008 and May 2011 (Energy Information Administration website, May 2010). Use the Internet to obtain the average price per gallon of conventional regular gasoline since May 2011.
    a. Extend the graph of the time series shown in Figure 1.1.
    b. What interpretations can you make about the average price per gallon of conventional regular gasoline since May 2011?
    c. Does the time series continue to show a summer increase in the average price per gallon? Explain.

17. A manager of a large corporation recommends a $10,000 raise be given to keep a valued subordinate from moving to another company. What internal and external sources of data might be used to decide whether such a salary increase is appropriate?

18. A random telephone survey of 1021 adults (aged 18 and older) was conducted by Opinion Research Corporation on behalf of CompleteTax, an online tax preparation and e-filing service. The survey results showed that 684 of those surveyed planned to file their taxes electronically (CompleteTax Tax Prep Survey 2010).
    a. Develop a descriptive statistic that can be used to estimate the percentage of all taxpayers who file electronically.
    b. The survey reported that the most frequently used method for preparing the tax return is to hire an accountant or professional tax preparer. If 60% of the people surveyed had their tax return prepared this way, how many people used an accountant or professional tax preparer?
    c. Other methods that the person filing the return often used include manual preparation, use of an online tax service, and use of a software tax program. Would the data for the method for preparing the tax return be considered categorical or quantitative?

19. A *Bloomberg Businessweek* North American subscriber study collected data from a sample of 2861 subscribers. Fifty-nine percent of the respondents indicated an annual income of $75,000 or more, and 50% reported having an American Express credit card.
    a. What is the population of interest in this study?
    b. Is annual income a categorical or quantitative variable?
    c. Is ownership of an American Express card a categorical or quantitative variable?
    d. Does this study involve cross-sectional or time series data?
    e. Describe any statistical inferences *Bloomberg Businessweek* might make on the basis of the survey.

20. A survey of 131 investment managers in *Barron's* Big Money poll revealed the following:
    - 43% of managers classified themselves as bullish or very bullish on the stock market.
    - The average expected return over the next 12 months for equities was 11.2%.
    - 21% selected health care as the sector most likely to lead the market in the next 12 months.
    - When asked to estimate how long it would take for technology and telecom stocks to resume sustainable growth, the managers' average response was 2.5 years.

    a. Cite two descriptive statistics.
    b. Make an inference about the population of all investment managers concerning the average return expected on equities over the next 12 months.
    c. Make an inference about the length of time it will take for technology and telecom stocks to resume sustainable growth.

21. A seven-year medical research study reported that women whose mothers took the drug DES during pregnancy were twice as likely to develop tissue abnormalities that might lead to cancer as were women whose mothers did not take the drug.
    a. This study involved the comparison of two populations. What were the populations?
    b. Do you suppose the data were obtained in a survey or an experiment?
    c. For the population of women whose mothers took the drug DES during pregnancy, a sample of 3980 women showed that 63 developed tissue abnormalities that might lead to cancer. Provide a descriptive statistic that could be used to estimate the number of women out of 1000 in this population who have tissue abnormalities.
    d. For the population of women whose mothers did not take the drug DES during pregnancy, what is the estimate of the number of women out of 1000 who would be expected to have tissue abnormalities?
    e. Medical studies often use a relatively large sample (in this case, 3980). Why?

22. The Nielsen Company surveyed consumers in 47 markets from Europe, Asia-Pacific, the Americas, and the Middle East to determine which factors are most important in determining where they buy groceries. Using a scale of 1 (low) to 5 (high), the highest rated factor was *good value for money*, with an average point score of 4.32. The second highest rated factor was *better selection of high-quality brands and products,* with an average point score of 3.78, and the lowest rated factor was *uses recyclable bags and packaging*, with an average point score of 2.71 (Nielsen website, February 24, 2008). Suppose that you have been hired by a grocery store chain to conduct a similar study to determine what factors customers at the chain's stores in Charlotte, North Carolina, think are most important in determining where they buy groceries.
    a. What is the population for the survey that you will be conducting?
    b. How would you collect the data for this study?

23. Pew Research Center is a nonpartisan polling organization that provides information about issues, attitudes, and trends shaping America. In a recent poll, Pew researchers found that 47% of American adult respondents reported getting at least some local news on their cell phone or tablet computer (Pew Research website, May 14, 2011). Further findings showed that 42% of respondents who own cell phones or tablet computers use those devices to check local weather reports and 37% use the devices to find local restaurants or other businesses.

a. One statistic reported concerned using cell phones or tablet computers for local news. What population is that finding applicable to?
b. Another statistic concerned using cell phones or tablet computers to check local weather reports and to find local restaurants. What population is this finding is applicable to?
c. Do you think the Pew researchers conducted a census or a sample survey to obtain their results? Why?
d. If you were a restaurant owner, would you find these results interesting? Why? How could you take advantage of this information?

24. A sample of midterm grades for five students showed the following results: 72, 65, 82, 90, 76. Which of the following statements are correct, and which should be challenged as being too generalized?
a. The average midterm grade for the sample of five students is 77.
b. The average midterm grade for all students who took the exam is 77.
c. An estimate of the average midterm grade for all students who took the exam is 77.
d. More than half of the students who take this exam will score between 70 and 85.
e. If five other students are included in the sample, their grades will be between 65 and 90.

25. Table 1.8 shows a data set containing information for 25 of the shadow stocks tracked by the American Association of Individual Investors. Shadow stocks are common stocks of

TABLE 1.8 DATA SET FOR 25 SHADOW STOCKS

Company	Exchange	Ticker Symbol	Market Cap ($ millions)	Price/ Earnings Ratio	Gross Profit Margin (%)
DeWolfe Companies	AMEX	DWL	36.4	8.4	36.7
North Coast Energy	OTC	NCEB	52.5	6.2	59.3
Hansen Natural Corp.	OTC	HANS	41.1	14.6	44.8
MarineMax, Inc.	NYSE	HZO	111.5	7.2	23.8
Nanometrics Incorporated	OTC	NANO	228.6	38.0	53.3
TeamStaff, Inc.	OTC	TSTF	92.1	33.5	4.1
Environmental Tectonics	AMEX	ETC	51.1	35.8	35.9
Measurement Specialties	AMEX	MSS	101.8	26.8	37.6
SEMCO Energy, Inc.	NYSE	SEN	193.4	18.7	23.6
Party City Corporation	OTC	PCTY	97.2	15.9	36.4
Embrex, Inc.	OTC	EMBX	136.5	18.9	59.5
Tech/Ops Sevcon, Inc.	AMEX	TO	23.2	20.7	35.7
ARCADIS NV	OTC	ARCAF	173.4	8.8	9.6
Qiao Xing Universal Tele.	OTC	XING	64.3	22.1	30.8
Energy West Incorporated	OTC	EWST	29.1	9.7	16.3
Barnwell Industries, Inc.	AMEX	BRN	27.3	7.4	73.4
Innodata Corporation	OTC	INOD	66.1	11.0	29.6
Medical Action Industries	OTC	MDCI	137.1	26.9	30.6
Instrumentarium Corp.	OTC	INMRY	240.9	3.6	52.1
Petroleum Development	OTC	PETD	95.9	6.1	19.4
Drexler Technology Corp.	OTC	DRXR	233.6	45.6	53.6
Gerber Childrenswear Inc.	NYSE	GCW	126.9	7.9	25.8
Gaiam, Inc.	OTC	GAIA	295.5	68.2	60.7
Artesian Resources Corp.	OTC	ARTNA	62.8	20.5	45.5
York Water Company	OTC	YORW	92.2	22.9	74.2

WEB file
Shadow02

smaller companies that are not closely followed by Wall Street analysts. The data set is also on the website that accompanies the text in the file named Shadow02.

a. How many variables are in the data set?
b. Which of the variables are categorical and which are quantitative?
c. For the Exchange variable, show the frequency and the percent frequency for AMEX, NYSE, and OTC. Construct a bar graph similar to Figure 1.5 for the Exchange variable.
d. Show the frequency distribution for the Gross Profit Margin using the five intervals: 0–14.9, 15–29.9, 30–44.9, 45–59.9, and 60–74.9. Construct a histogram similar to Figure 1.6.
e. What is the average price/earnings ratio?

Appendix An Introduction to StatTools

StatTools is a professional add-in that expands the statistical capabilities available with Microsoft Excel. StatTools software can be downloaded from the website that accompanies this text.

Excel does not contain statistical functions or data analysis tools to perform all the statistical procedures discussed in the text. StatTools is a Microsoft Excel statistics add-in that extends the range of statistical and graphical options for Excel users. Most chapters include a chapter appendix that shows the steps required to accomplish a statistical procedure using StatTools. For those students who want to make more extensive use of the software, StatTools offers an excellent Help facility. The StatTools Help system includes detailed explanations of the statistical and data analysis options available, as well as descriptions and definitions of the types of output provided.

Getting Started with StatTools

StatTools software may be downloaded and installed on your computer by accessing the website that accompanies this text. After downloading and installing the software, perform the following steps to use StatTools as an Excel add-in.

Step 1. Click the **Start** button on the taskbar and then point to **All Programs**
Step 2. Point to the folder entitled **Palisade DecisionTools**
Step 3. Click **StatTools for Excel**

These steps will open Excel and add the StatTools tab next to the Add-Ins tab on the Excel Ribbon. Alternately, if you are already working in Excel, these steps will make StatTools available.

Using StatTools

Before conducting any statistical analysis, we must create a StatTools data set using the StatTools Data Set Manager. Let us use the Excel worksheet for the mutual funds data set in Table 1.1 to show how this is done. The following steps show how to create a StatTools data set for the mutual funds data.

Step 1. Open the Excel file named Morningstar
Step 2. Select any cell in the data set (for example, cell A1)
Step 3. Click the **StatTools** tab on the Ribbon
Step 4. In the **Data** group, click **Data Set Manager**
Step 5. When StatTools asks if you want to add the range $A$1:$F$26 as a new StatTools data set, click **Yes**
Step 6. When the StatTools - Data Set Manager dialog box appears, click **OK**

FIGURE 1.13 THE STATTOOLS - DATA SET MANAGER DIALOG BOX

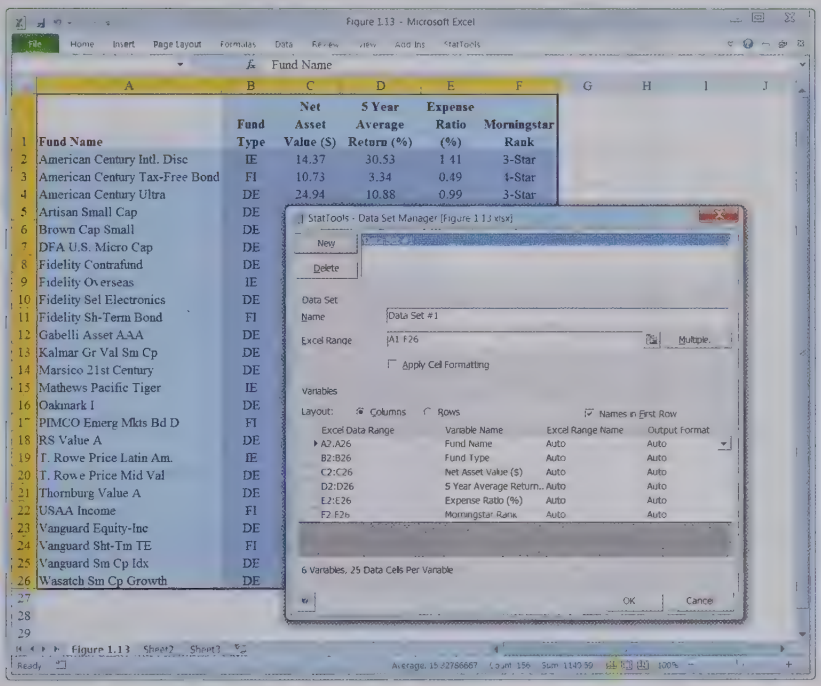


Figure 1.13 shows the StatTools - Data Set Manager dialog box that appears in step 6. By default, the name of the new StatTools data set is Data Set #1. You can replace the name Data Set #1 in step 6 with a more descriptive name. And, if you select the Apply Cell Format option, the column labels will be highlighted in blue and the entire data set will have outside and inside borders. You can always select the Data Set Manager at any time in your analysis to make these types of changes.

Recommended Application Settings

StatTools allows the user to specify some of the application settings that control such things as where statistical output is displayed and how calculations are performed. The following steps show how to access the StatTools - Application Settings dialog box.

Step 1. Click the **StatTools** tab on the Ribbon
Step 2. In the **Tools Group**, click **Utilities**
Step 3. Choose **Application Settings** from the list of options

Figure 1.14 shows that the StatTools - Application Settings dialog box has five sections: General Settings; Reports; Utilities; Data Set Defaults; and Analyses. Let us show how to make changes in the Reports section of the dialog box.

FIGURE 1.14 THE STATTOOLS - APPLICATION SETTINGS DIALOG BOX

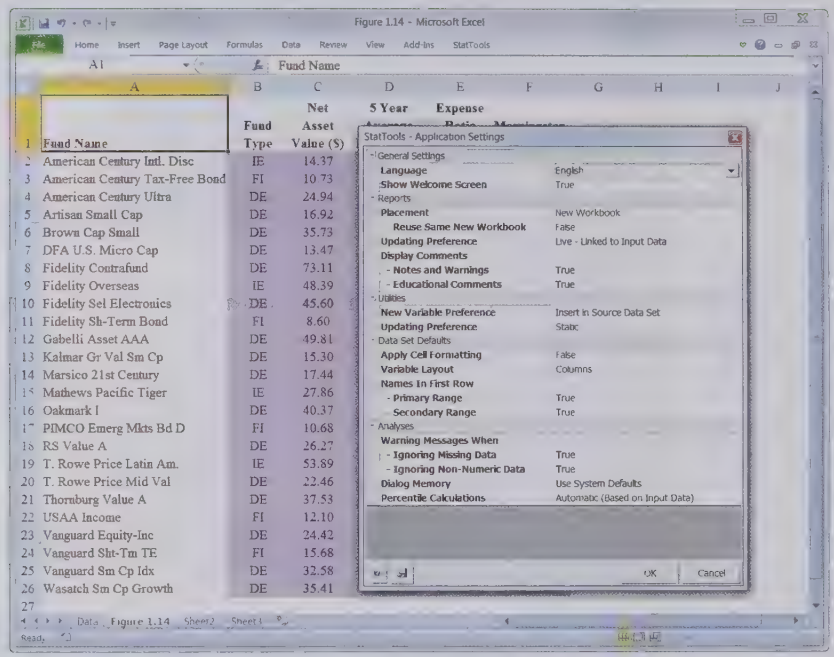


Figure 1.14 - Microsoft Excel
Home Insert Page Layout Formulas Data Review View Add-Ins StatTools
A1 Fund Name
Fund Name
Fund Type
Net Asset Value ($)
5 Year Expense
1 Fund Name Fund Type Net Asset Value ($)
2 American Century Intl. Disc IE 14.37
3 American Century Tax-Free Bond FI 10.73
4 American Century Ultra DE 24.94
5 Artisan Small Cap DE 16.92
6 Brown Cap Small DE 35.73
7 DFA U.S. Micro Cap DE 13.47
8 Fidelity Contrafund DE 73.11
9 Fidelity Overseas IE 48.39
10 Fidelity Sel Electronics DE 45.60
11 Fidelity Sh-Term Bond FI 8.60
12 Gabelli Asset AAA DE 49.81
13 Kalmar Gr Val Sm Cp DE 15.30
14 Marsico 21st Century DE 17.44
15 Mathews Pacific Tiger IE 27.86
16 Oakmark I DE 40.37
17 PIMCO Emerg Mkts Bd D FI 10.68
18 RS Value A DE 26.27
19 T. Rowe Price Latin Am. IE 53.89
20 T. Rowe Price Mid Val DE 22.46
21 Thornburg Value A DE 37.53
22 USAA Income FI 12.10
23 Vanguard Equity-Inc DE 24.42
24 Vanguard Sht-Tm TE FI 15.68
25 Vanguard Sm Cp Idx DE 32.58
26 Wasatch Sm Cp Growth DE 35.41
27
Data Figure 1.14 Sheet2 Sheet1
Ready
StatTools - Application Settings
General Settings
Language English
Show Welcome Screen True
Reports
Placement New Workbook
Reuse Same New Workbook False
Updating Preference Live - Linked to Input Data
Display Comments
- Notes and Warnings True
- Educational Comments True
Utilities
New Variable Preference Insert in Source Data Set
Updating Preference Static
Data Set Defaults
Apply Cell Formatting False
Variable Layout Columns
Names In First Row
- Primary Range True
- Secondary Range True
Analyses
Warning Messages When
- Ignoring Missing Data True
- Ignoring Non-Numeric Data True
Dialog Memory Use System Defaults
Percentile Calculations Automatic (Based on Input Data)
OK
Cancel

Figure 1.14 shows that the Placement option currently selected is **New Workbook**. Using this option, the StatTools output will be placed in a new workbook. But suppose you would like to place the StatTools output in the current (active) workbook. If you click the words **New Workbook**, a downward-pointing arrow will appear to the right. Clicking this arrow will display a list of all the placement options, including **Active Workbook**; we recommend using this option. Figure 1.14 also shows that the Updating Preferences option in the Reports section is currently **Live - Linked to Input Data**. With live updating, anytime one or more data values are changed, StatTools will automatically change the output previously produced; we also recommend using this option. Note that there are two options available under Display Comments: **Notes and Warnings** and **Educational Comments**. Because these options provide useful notes and information regarding the output, we recommend using both options. Thus, to include educational comments as part of the StatTools output, you will have to change the value of False for Educational Comments to True.

The StatTools - Application Settings dialog box contains numerous other features that enable you to customize the way that you want StatTools to operate. You can learn more about these features by clicking the Help icon located in the lower left-hand corner of the StatTools - Application Settings dialog box. When you have finished making changes in the application settings, click OK at the bottom of the dialog box and then click Yes when StatTools asks you if you want to save the new application settings.

CHAPTER 2

Descriptive Statistics: Tabular and Graphical Presentations

CONTENTS

STATISTICS IN PRACTICE: *COLGATE-PALMOLIVE COMPANY*

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- Using Excel's COUNTIF Function to Construct a Frequency Distribution
- Relative Frequency and Percent Frequency Distributions
- Using Excel to Construct Relative Frequency and Percent Frequency Distributions
- Bar Charts and Pie Charts
- Using Excel's Chart Tools to Construct a Bar Chart and a Pie Chart

2.2 SUMMARIZING QUANTITATIVE DATA
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- Using Excel's PivotTable Report to Construct a Frequency Distribution
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- Dot Plot
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- Cumulative Distributions
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2.3 EXPLORATORY DATA ANALYSIS: STEM-AND-LEAF DISPLAY

2.4 CROSSTABULATIONS AND SCATTER DIAGRAMS
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- Using Excel's PivotTable Report to Construct a Crosstabulation
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STATISTICS in PRACTICE

COLGATE-PALMOLIVE COMPANY*

NEW YORK, NEW YORK

The Colgate-Palmolive Company started as a small soap and candle shop in New York City in 1806. Today, Colgate-Palmolive employs more than 40,000 people working in more than 200 countries and territories around the world. Although best known for its brand names of Colgate, Palmolive, Ajax, and Fab, the company also markets Mennen, Hill's Science Diet, and Hill's Prescription Diet products.

The Colgate-Palmolive Company uses statistics in its quality assurance program for home laundry detergent products. One concern is customer satisfaction with the quantity of detergent in a carton. Every carton in each size category is filled with the same amount of detergent by weight, but the volume of detergent is affected by the density of the detergent powder. For instance, if the powder density is on the heavy side, a smaller volume of detergent is needed to reach the carton's specified weight. As a result, the carton may appear to be underfilled when opened by the consumer.

To control the problem of heavy detergent powder, limits are placed on the acceptable range of powder density. Statistical samples are taken periodically, and the density of each powder sample is measured. Data summaries are then provided for operating personnel so that corrective action can be taken if necessary to keep the density within the desired quality specifications.

A frequency distribution for the densities of 150 samples taken over a one-week period and a histogram are shown in the accompanying table and figure. Density levels above .40 are unacceptably high. The frequency distribution and histogram show that the operation is meeting its quality guidelines with all of the densities less than or equal to .40. Managers viewing these statistical summaries would be pleased with the quality of the detergent production process.

In this chapter, you will learn about tabular and graphical methods of descriptive statistics such as frequency distributions, bar charts, histograms, stem-and-leaf displays, crosstabulations, and others. The goal of these methods is to summarize data so that the data can be easily understood and interpreted.

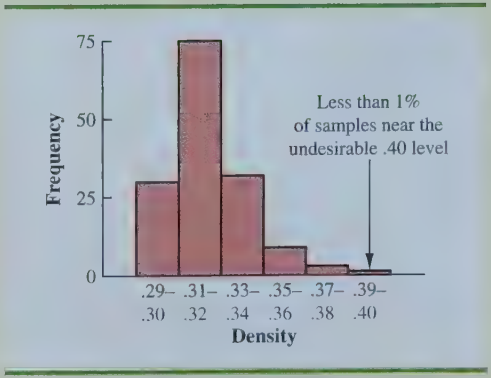
© Kurt Brady/Alamy

The Colgate-Palmolive Company uses statistical summaries to help maintain the quality of its products.

Frequency Distribution of Density Data

Density	Frequency
.29–.30	30
.31–.32	75
.33–.34	32
.35–.36	9
.37–.38	3
.39–.40	1
Total	150

Histogram of Density Data



*The authors are indebted to William R. Fowle, Manager of Quality Assurance, Colgate-Palmolive Company, for providing this Statistics in Practice.

As indicated in Chapter 1, data can be classified as either categorical or quantitative. **Categorical data** use labels or names to identify categories of like items. **Quantitative data** are numerical values that indicate how much or how many.

This chapter introduces tabular and graphical methods commonly used to summarize both categorical and quantitative data. Tabular and graphical summaries of data can be found in annual reports, newspaper articles, and research studies. Everyone is exposed to these types of presentations. Hence, it is important to understand how they are prepared and how they should be interpreted. We begin with tabular and graphical methods for summarizing data concerning a single variable. The last section introduces methods for summarizing data when the relationship between two variables is of interest.

Excel's wide variety of statistical functions and tools will be used extensively in this chapter. We will find the chart tools and PivotTable tools to be especially helpful. These tools can be used to provide tabular and graphical summaries for a single categorical or quantitative variable as well as provide crosstabulations and graphical presentation for data sets involving more than one variable.

2.1 Summarizing Categorical Data

Frequency Distribution

We begin the discussion of how tabular and graphical methods can be used to summarize categorical data with the definition of a **frequency distribution**.

FREQUENCY DISTRIBUTION

A frequency distribution is a tabular summary of data showing the number (frequency) of items in each of several nonoverlapping classes.

Let us use the following example to demonstrate the construction and interpretation of a frequency distribution for categorical data. Coca-Cola, Diet Coke, Dr. Pepper, Pepsi, and Sprite are five popular soft drinks. Assume that the data in Table 2.1 show the soft drink selected in a sample of 50 soft drink purchases.

WEB file
SoftDrink

TABLE 2.1 DATA FROM A SAMPLE OF 50 SOFT DRINK PURCHASES

Coca-Cola	Sprite	Pepsi
Diet Coke	Coca-Cola	Coca-Cola
Pepsi	Diet Coke	Coca-Cola
Diet Coke	Coca-Cola	Coca-Cola
Coca-Cola	Diet Coke	Pepsi
Coca-Cola	Coca-Cola	Dr. Pepper
Dr. Pepper	Sprite	Coca-Cola
Diet Coke	Pepsi	Diet Coke
Pepsi	Coca-Cola	Pepsi
Pepsi	Coca-Cola	Pepsi
Coca-Cola	Coca-Cola	Pepsi
Dr. Pepper	Pepsi	Pepsi
Sprite	Coca-Cola	Coca-Cola
Coca-Cola	Sprite	Dr. Pepper
Diet Coke	Dr. Pepper	Pepsi
Coca-Cola	Pepsi	Sprite
Coca-Cola	Diet Coke	

TABLE 2.2
FREQUENCY DISTRIBUTION OF SOFT DRINK PURCHASES

Soft Drink	Frequency
Coca-Cola	19
Diet Coke	8
Dr. Pepper	5
Pepsi	13
Sprite	5
Total	50

To develop a frequency distribution for these data, we count the number of times each soft drink appears in Table 2.1. Coca-Cola appears 19 times, Diet Coke appears 8 times, Dr. Pepper appears 5 times, Pepsi appears 13 times, and Sprite appears 5 times. These counts are summarized in the frequency distribution in Table 2.2.

This frequency distribution provides a summary of how the 50 soft drink purchases are distributed across the five soft drinks. This summary offers more insight than the original data shown in Table 2.1. Viewing the frequency distribution, we see that Coca-Cola is the leader, Pepsi is second, Diet Coke is third, and Sprite and Dr. Pepper are tied for fourth. The frequency distribution summarizes information about the popularity of the five soft drinks.

Let us examine how Excel can be used to count the number of times each soft drink occurs and construct a frequency distribution for the soft drink data in Table 2.1.

Using Excel's COUNTIF Function to Construct a Frequency Distribution

Two tasks are involved in using Excel's COUNTIF function to construct a frequency distribution: Enter Data and Enter Functions and Formulas. Refer to Figure 2.1 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet is in the foreground.

Enter Data: The label "Brand Purchased" and the data for the 50 soft drink purchases are entered into cells A1:A51.

Enter Functions and Formulas: Excel's COUNTIF function can be used to count the number of times each soft drink appears in cells A2:A51. We first entered a label and the soft drink names into cells C1:C6 and D1. Then, to count the number of times that Coca-Cola appears, we entered the following formula into cell D2:

```
=COUNTIF($A$2:$A$51,C2)
```

Appendix 2.1 shows how to use Excel's PivotChart Report to construct a frequency distribution for categorical data.

To count the number of times the other soft drinks appear, we copied the same formula into cells D3:D6.

The value worksheet, in the foreground of Figure 2.1, shows the values computed using these cell formulas; we see that the Excel worksheet shows the same frequency distribution that we developed in Table 2.2.

Relative Frequency and Percent Frequency Distributions

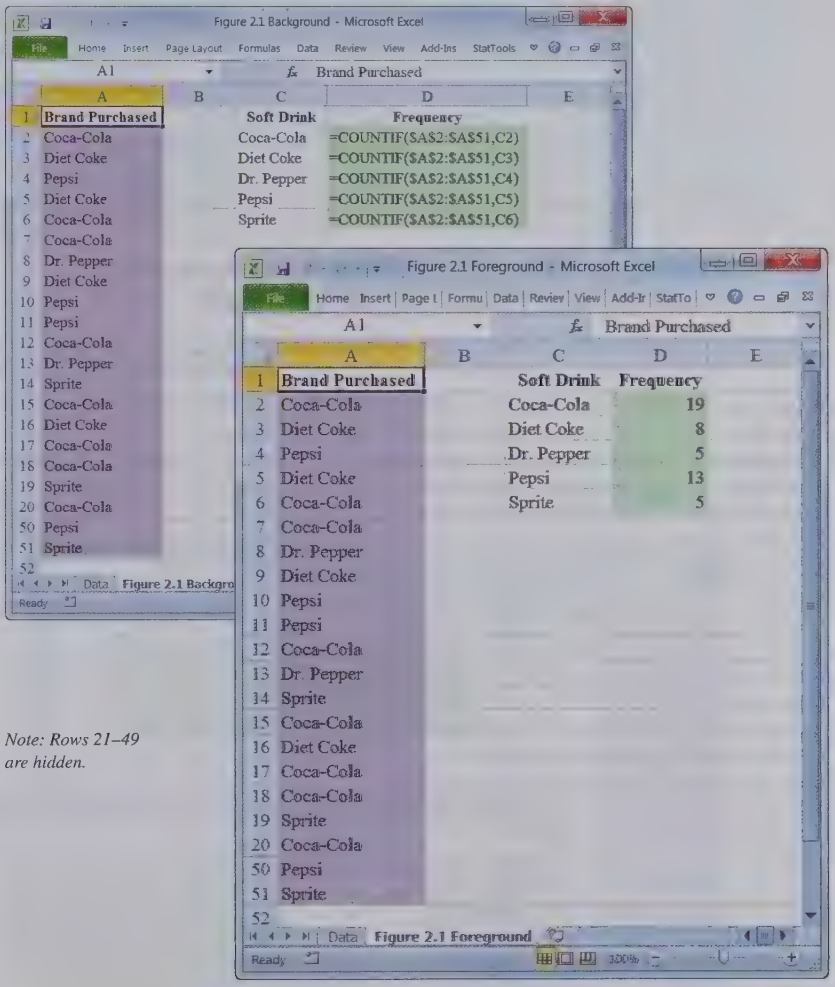
A frequency distribution shows the number (frequency) of items in each of several nonoverlapping classes. However, we are often interested in the proportion, or percentage, of items in each class. The *relative frequency* of a class equals the fraction or proportion of items belonging to a class. For a data set with n observations, the relative frequency of each class can be determined as follows:

RELATIVE FREQUENCY

$$\text{Relative frequency of a class} = \frac{\text{Frequency of the class}}{n} \qquad (2.1)$$

The *percent frequency* of a class is the relative frequency multiplied by 100.

FIGURE 2.1 FREQUENCY DISTRIBUTION FOR SOFT DRINK PURCHASES CONSTRUCTED USING EXCEL'S COUNTIF FUNCTION



Note: Rows 21–49 are hidden.

A **relative frequency distribution** gives a tabular summary of data showing the relative frequency for each class. A **percent frequency distribution** summarizes the percent frequency of the data for each class. Table 2.3 shows a relative frequency distribution and a percent frequency distribution for the soft drink data. In Table 2.3 we see that the relative frequency for Coca-Cola is 19/50 = .38, the relative frequency for Diet Coke is 8/50 = .16, and so on. From the percent frequency distribution, we see that 38% of the purchases were Coca-Cola, 16% of the purchases were Diet Coke, and so on. We can also note that 38% + 26% + 16% = 80% of the purchases were the top three soft drinks.

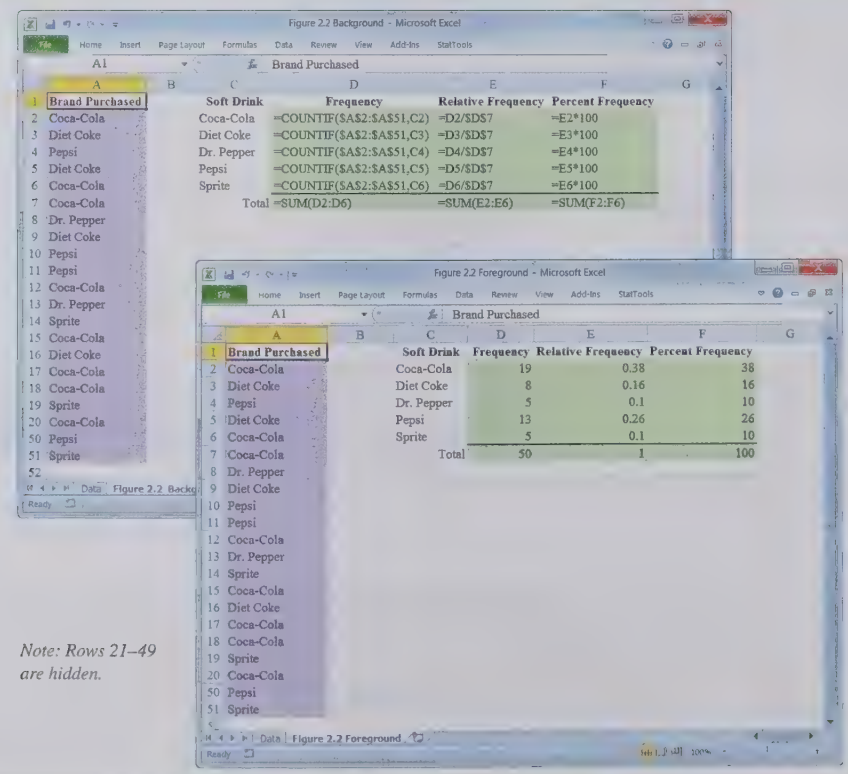
TABLE 2.3 RELATIVE FREQUENCY AND PERCENT FREQUENCY DISTRIBUTIONS OF SOFT DRINK PURCHASES

Soft Drink	Relative Frequency	Percent Frequency
Coca-Cola	.38	38
Diet Coke	.16	16
Dr. Pepper	.10	10
Pepsi	.26	26
Sprite	.10	10
Total	1.00	100

Using Excel to Construct Relative Frequency and Percent Frequency Distributions

Extending the worksheet shown in Figure 2.1, we can develop the relative frequency and percent frequency distributions shown in Table 2.3. Refer to Figure 2.2 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet is in the foreground.

FIGURE 2.2 RELATIVE FREQUENCY AND PERCENT FREQUENCY DISTRIBUTIONS OF SOFT DRINK PURCHASES CONSTRUCTED USING EXCEL



Note: Rows 21–49 are hidden.

Enter Data: The label "Brand Purchased" and the data for the 50 soft drink purchases are entered into cells A1:A51.

Enter Functions and Formulas: The information in cells C1:D6 is the same as in Figure 2.1. Excel's SUM function is used in cell D7 to compute the sum of the frequencies in cells D2:D6. The resulting value of 50 is the number of observations in the data set. To compute the relative frequency for Coca-Cola using equation (2.1), we entered the formula =D2/$D$7 into cell E2; the result, 0.38, is the relative frequency for Coca-Cola. Copying cell E2 to cells E3:E6 computes the relative frequencies for each of the other soft drinks.

To compute the percent frequency for Coca-Cola, we entered the formula =E2*100 into cell F2. The result, 38, indicates that 38% of the soft drink purchases were Coca-Cola. Copying cell F2 to cells F3:F6 computes the percent frequencies for each of the other soft drinks. Finally, copying cell D7 to cells E7:F7 computes the total of the relative frequencies (1.00) and the total of the percent frequencies (100).

Bar Charts and Pie Charts

A **bar chart** is a graphical device for depicting categorical data summarized in a frequency, relative frequency, or percent frequency distribution. On one axis of the graph (usually the horizontal axis), we specify the labels that are used for the classes (categories). A frequency, relative frequency, or percent frequency scale can be used for the other axis of the chart (usually the vertical axis). Then, using a bar of fixed width drawn above each class label, we extend the length of the bar until we reach the frequency, relative frequency, or percent frequency of the class. For categorical data, the bars should be separated to emphasize the fact that each class is separate. Figure 2.3 shows a bar chart of the frequency distribution for the 50 soft drink purchases. Note how the graphical presentation shows Coca-Cola, Pepsi, and Diet Coke to be the most preferred brands.

In quality control applications, bar charts are used to identify the most important causes of problems. When the bars are arranged in descending order of height from left to right with the most frequently occurring cause appearing first, the bar chart is called a Pareto diagram. *This diagram is named for its founder, Vilfredo Pareto, an Italian economist.*

The **pie chart** provides another graphical device for presenting relative frequency and percent frequency distributions for categorical data. To construct a pie chart, we first draw a circle to represent all the data. Then we use the relative frequencies to subdivide the circle into sectors, or parts, that correspond to the relative frequency for each class. For example, because a circle contains 360 degrees and Coca-Cola shows a relative

FIGURE 2.3 BAR CHART OF SOFT DRINK PURCHASES

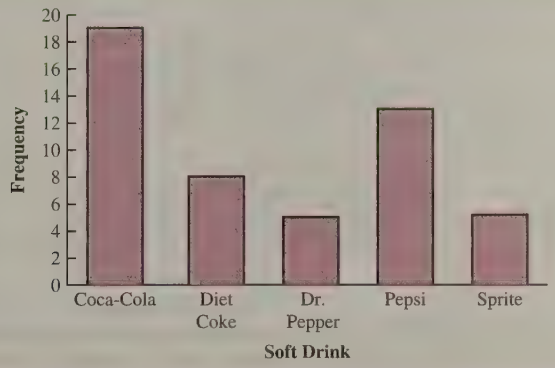
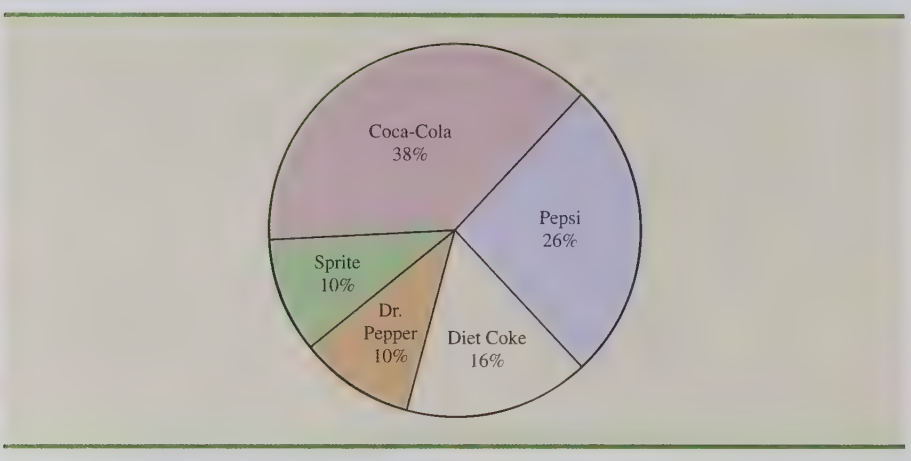


FIGURE 2.4 PIE CHART OF SOFT DRINK PURCHASES



frequency of .38, the sector of the pie chart labeled Coca-Cola consists of .38(360) = 136.8 degrees. The sector of the pie chart labeled Diet Coke consists of .16(360) = 57.6 degrees. Similar calculations for the other classes yield the pie chart in Figure 2.4. The numerical values shown for each sector can be frequencies, relative frequencies, or percent frequencies.

Using Excel's Chart Tools to Construct a Bar Chart and a Pie Chart

Appendix 2.1 shows how to use Excel's PivotChart Report to construct a bar chart for categorical data.

Excel's chart tools make it very easy to create a variety of graphical displays, including bar charts and pie charts. When such tools are used, a third task is needed for worksheet construction: **Apply Tools.** We illustrate by showing how to construct the bar chart for soft drink purchases. Refer to Figure 2.5 as we describe the tasks involved.

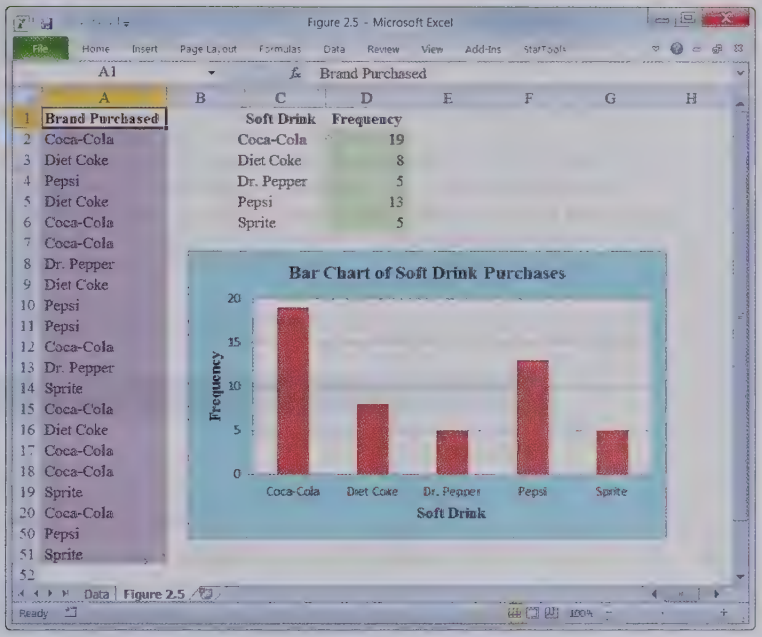
Enter Data: Same as in Figure 2.1

Enter Functions and Formulas: Same as in Figure 2.1

Apply Tools: The following steps describe how to use Excel's chart tools to construct a bar chart for the soft drink data using the frequency distribution appearing in cells C1:D6. The bar chart we will create uses the vertical axis to display the frequencies; in Excel this type of chart is referred to as a column chart.

Step 1. Select cells C2:D6
Step 2. Click the **Insert** tab on the Ribbon
Step 3. In the **Charts** group, click **Column**
Step 4. When the list of column chart subtypes appears,
Go to the **2-D Column** section
Click **Clustered Column** (the leftmost chart)
Step 5. In the **Chart Layouts** group, click the **More** button (the downward pointing arrow with a line over it) to display all the options
Step 6. Choose **Layout 9**

FIGURE 2.5 BAR CHART OF SOFT DRINK PURCHASES CONSTRUCTED USING EXCEL'S CHART TOOLS



Step 7. Click the **Chart Title** and replace it with **Bar Chart of Soft Drink Purchases**

Step 8. Click the **Horizontal (Category) Axis Title** and replace it with **Soft Drink**

Step 9. Click the **Vertical (Value) Axis Title** and replace it with **Frequency**

Step 10. Right-click the **Series 1 Legend Entry** and choose **Delete** from the list of options that appears

Step 11. Right-click the vertical axis and choose **Format Axis** from the options that appear

Step 12. When the Format Axis dialog box appears,
Go to the **Axis Options** section
Select **Fixed** for **Major Unit** and enter 5.0 in the corresponding box
Click **Close**

The resulting bar chart is shown in Figure 2.5.[1] If you prefer, you can display the bars on the horizontal axis by choosing **Bar** in step 3 instead of Column.

Excel's chart tools can also be used to develop a pie chart for the soft drink data in a similar fashion. The major difference is that in step 3 we would choose **Pie** in the Charts group.

[1]The bar chart in Figure 2.5 is slightly different from what was provided by Excel after clicking Close. Resizing an Excel chart is not difficult. First, select the chart. Small black squares, called sizing handles, will appear on the chart border. Click on the sizing handles and drag them to resize the figure to your preference.

NOTES AND COMMENTS

1. Often the number of classes in a frequency distribution is the same as the number of categories found in the data, as is the case for the soft drink purchase data in this section. The data involve only five soft drinks, and a separate frequency distribution class was defined for each one. Data that included all soft drinks would require many categories, most of which would have a small number of purchases. Most statisticians recommend that classes with smaller frequencies be grouped into an aggregate class called "other." Classes with frequencies of 5% or less would most often be treated in this fashion.
2. The sum of the frequencies in any frequency distribution always equals the number of observations. The sum of the relative frequencies in any relative frequency distribution always equals 1.00, and the sum of the percentages in a percent frequency distribution always equals 100.
3. Excel's PivotChart Report is a valuable tool for summarizing data and preparing charts. In Appendix 2.1, we show how it can be used to construct both a frequency distribution and a bar chart for categorical data.

Exercises

Methods

1. The response to a question has three alternatives: A, B, and C. A sample of 120 responses provides 60 A, 24 B, and 36 C. Show the frequency and relative frequency distributions.
2. A partial relative frequency distribution is given.

Class	Relative Frequency
A	.22
B	.18
C	.40
D	

   a. What is the relative frequency of class D?
   b. The total sample size is 200. What is the frequency of class D?
   c. Show the frequency distribution.
   d. Show the percent frequency distribution.

SELF test

3. A questionnaire provides 58 Yes, 42 No, and 20 no-opinion answers.
   a. In the construction of a pie chart, how many degrees would be in the section of the pie showing the Yes answers?
   b. How many degrees would be in the section of the pie showing the No answers?
   c. Construct a pie chart.
   d. Construct a bar chart.

Applications

WEB file

BestTV

4. The top four prime-time television shows were *Law & Order, CSI, Without a Trace,* and *Desperate Housewives* (Nielsen Media Research, January 1, 2007). Data indicating the preferred shows for a sample of 50 viewers follow.

DH	CSI	DH	CSI	L&O
Trace	CSI	L&O	Trace	CSI
CSI	DH	Trace	CSI	DH
L&O	L&O	L&O	CSI	DH
CSI	DH	DH	L&O	CSI

DH	Trace	CSI	Trace	DH
DH	CSI	CSI	L&O	CSI
L&O	CSI	Trace	Trace	DH
L&O	CSI	CSI	CSI	DH
CSI	DH	Trace	Trace	L&O

a. Are these data categorical or quantitative?
b. Provide frequency and percent frequency distributions.
c. Construct a bar chart and a pie chart.
d. On the basis of the sample, which television show has the largest viewing audience? Which one is second?

5. In alphabetical order, the six most common last names in the United States are Brown, Davis, Johnson, Jones, Smith, and Williams (U.S. Census Bureau website, November 1, 2010). Assume that a sample of 50 individuals with one of these last names provided the following data.

Brown	Williams	Williams	Williams	Brown
Smith	Jones	Smith	Johnson	Smith
Davis	Smith	Brown	Williams	Johnson
Johnson	Smith	Smith	Johnson	Brown
Williams	Davis	Johnson	Williams	Johnson
Williams	Johnson	Jones	Smith	Brown
Johnson	Smith	Smith	Brown	Jones
Jones	Jones	Smith	Smith	Davis
Davis	Jones	Williams	Davis	Smith
Jones	Johnson	Brown	Johnson	Davis

Summarize the data by constructing the following:
a. Relative and percent frequency distributions
b. A bar chart
c. A pie chart
d. Based on these data, what are the three most common last names?

6. The Nielsen Media Research television rating measures the percentage of television owners who are watching a particular television program. The highest-rated television program in television history was the *M*A*S*H Last Episode Special* shown on February 28, 1983. A 60.2 rating indicated that 60.2% of all television owners were watching this program. Nielsen Media Research provided the list of the 50 top-rated single shows in television history (*The New York Times Almanac,* 2006). The following data show the television network that produced each of these 50 top-rated shows.

ABC	ABC	ABC	NBC	CBS
ABC	CBS	ABC	ABC	NBC
NBC	NBC	CBS	ABC	NBC
CBS	ABC	CBS	NBC	ABC
CBS	NBC	NBC	CBS	NBC
CBS	CBS	CBS	NBC	NBC
FOX	CBS	CBS	ABC	NBC
ABC	ABC	CBS	NBC	NBC
NBC	CBS	NBC	CBS	CBS
ABC	CBS	ABC	NBC	ABC

a. Construct a frequency distribution, percent frequency distribution, and bar chart for the data.
b. Which network or networks have done the best in terms of presenting top-rated television shows? Compare the performance of ABC, CBS, and NBC.

SELF test

7. The Canmark Research Center Airport Customer Satisfaction Survey uses an online questionnaire to provide airlines and airports with customer satisfaction ratings for all aspects of the customers' flight experience (Airport Survey website, January 2010). After completing a flight, customers receive an email asking them to go to the website and rate a variety of factors, including the reservation process, the check-in process, luggage policy, cleanliness of gate area, service by flight attendants, food/beverage selection, on-time arrival, and so on. A five-point scale, with Excellent (E), Very Good (V), Good (G), Fair (F), and Poor (P), is used to record the customer ratings for each survey question. Assume that passengers on a Delta Airlines flight from Myrtle Beach, South Carolina, to Atlanta, Georgia, provided the following ratings for the question, "Please rate the airline based on your overall experience with this flight." The sample ratings are shown below.

WEB file
AirSurvey

E	E	G	V	V	E	V	V	V	E
E	G	V	E	E	V	E	E	E	V
V	V	V	F	V	E	V	E	G	E
G	E	V	E	V	E	V	V	V	V
E	E	V	V	E	P	E	V	P	V

a. Use a percent frequency distribution and a bar chart to summarize these data. What do these summaries indicate about the overall customer satisfaction with the Delta flight?
b. The online survey questionnaire enabled respondents to explain any aspect of the flight that failed to meet expectations. Would this be helpful information to a manager looking for ways to improve the overall customer satisfaction on Delta flights? Explain.

8. Data for a sample of 55 members of the Baseball Hall of Fame in Cooperstown, New York, are shown here. Each observation indicates the primary position played by the Hall of Famers: pitcher (P), catcher (H), 1st base (1), 2nd base (2), 3rd base (3), shortstop (S), left field (L), center field (C), and right field (R).

L	P	C	H	2	P	R	1	S	S	1	L	P	R	P
P	P	P	R	C	S	L	R	P	C	C	P	P	R	P
2	3	P	H	L	P	1	C	P	P	P	S	1	L	R
R	1	2	H	S	3	H	2	L	P					

a. Use frequency and relative frequency distributions to summarize the data.
b. What position provides the most Hall of Famers?
c. What position provides the fewest Hall of Famers?
d. What outfield position (L, C, or R) provides the most Hall of Famers?
e. Compare infielders (1, 2, 3, and S) to outfielders (L, C, and R).

9. The Pew Research Center's Social & Demographic Trends project found that 46% of U.S. adults would rather live in a different type of community than the one where they are living now (Pew Research Center, January 29, 2009). The national survey of 2260 adults asked: "Where do you live now?" and "What do you consider to be the ideal community?" Response options were City (C), Suburb (S), Small Town (T), or Rural (R). A representative portion of this survey for a sample of 100 respondents is as follows.

Where do you live now?

WEB file
LivingArea

S	T	R	C	R	R	T	C	S	T	C	S	C	S	T
S	S	C	S	S	T	T	C	C	S	T	C	S	T	C
T	R	S	S	T	C	S	C	T	C	T	C	T	C	R
C	C	R	T	C	S	S	T	S	C	C	C	R	S	C
S	S	C	C	S	C	R	T	T	T	C	R	T	C	R
C	T	R	R	C	T	C	C	R	T	T	R	S	R	T
T	S	S	S	S	S	C	C	R	T					

What do you consider to be the ideal community?

S	C	R	R	R	S	T	S	S	T	T	S	C	S	T
C	C	R	T	R	S	T	T	S	S	C	C	T	T	S
S	R	C	S	C	C	S	C	R	C	T	S	R	R	R
C	T	S	T	T	T	R	R	S	C	C	R	R	S	S
S	T	C	T	T	C	R	T	T	T	C	T	T	R	R
C	S	R	T	C	T	C	C	T	T	T	R	C	R	T
T	C	S	S	C	S	T	S	S	R					

a. Provide a percent frequency distribution for each question.
b. Construct a bar chart for each question.
c. Where are most adults living now?
d. Where do most adults consider the ideal community?
e. What changes in living areas would you expect to see if people moved from where they currently live to their ideal community?

WEB file
FedBank

10. The *Financial Times*/Harris Poll is a monthly online poll of adults from six countries in Europe and the United States. The poll conducted in January 2008 included 1015 adults. One of the questions asked was, "How would you rate the Federal Bank in handling the credit problems in the financial markets?" Possible responses were Excellent, Good, Fair, Bad, and Terrible (Harris Interactive website, January 2008). The 1015 responses for this question can be found in the data file named FedBank.
    a. Construct a frequency distribution.
    b. Construct a percent frequency distribution.
    c. Construct a bar chart for the percent frequency distribution.
    d. Comment on how adults in the United States think the Federal Bank is handling the credit problems in the financial markets.
    e. In Spain, 1114 adults were asked, "How would you rate the European Central Bank in handling the credit problems in the financial markets?" The percent frequency distribution obtained follows:

Rating	Percent Frequency
Excellent	0
Good	4
Fair	46
Bad	40
Terrible	10

Compare the results obtained in Spain with the results obtained in the United States.

2.2 Summarizing Quantitative Data

Frequency Distribution

TABLE 2.4
YEAR-END AUDIT TIMES (IN DAYS)

12	14	19	18
15	15	18	17
20	27	22	23
22	21	33	28
14	18	16	13

As defined in Section 2.1, a frequency distribution is a tabular summary of data showing the number (frequency) of items in each of several nonoverlapping classes. This definition holds for quantitative as well as categorical data. However, with quantitative data we must be more careful in defining the nonoverlapping classes to be used in the frequency distribution.

For example, consider the quantitative data in Table 2.4. These data show the time in days required to complete year-end audits for a sample of 20 clients of Sanderson and

Clifford, a small public accounting firm. The three steps necessary to define the classes for a frequency distribution with quantitative data are as follows:

1. Determine the number of nonoverlapping classes.
2. Determine the width of each class.
3. Determine the class limits.

Audit

Let us demonstrate these steps by developing a frequency distribution for the audit time data in Table 2.4.

Number of classes Classes are formed by specifying ranges that will be used to group the data. As a general guideline, we recommend using between 5 and 20 classes. For a small number of data items, as few as five or six classes may be used to summarize the data. For a larger number of data items, a larger number of classes is usually required. The goal is to use enough classes to show the variation in the data, but not so many classes that some contain only a few data items. Because the number of data items in Table 2.4 is relatively small ($n = 20$), we chose to develop a frequency distribution with five classes.

Making the classes the same width reduces the chance of inappropriate interpretations by the user.

Width of the classes The second step in constructing a frequency distribution for quantitative data is to choose a width for the classes. As a general guideline, we recommend that the width be the same for each class. Thus the choices of the number of classes and the width of classes are not independent decisions. A larger number of classes means a smaller class width, and vice versa. To determine an approximate class width, we begin by identifying the largest and smallest data values. Then, with the desired number of classes specified, we can use the following expression to determine the approximate class width.

$$\text{Approximate class width} = \frac{\text{Largest data value} - \text{Smallest data value}}{\text{Number of classes}} \qquad (2.2)$$

The approximate class width given by equation (2.2) can be rounded to a more convenient value based on the preference of the person developing the frequency distribution. For example, an approximate class width of 9.28 might be rounded to 10 simply because 10 is a more convenient class width to use in presenting a frequency distribution.

No single frequency distribution is best for a data set. Different people may construct different, but equally acceptable, frequency distributions. The goal is to reveal the natural grouping and variation in the data.

For the data involving the year-end audit times, the largest data value is 33 and the smallest data value is 12. Because we decided to summarize the data with five classes, using equation (2.2) provides an approximate class width of $(33 - 12)/5 = 4.2$. We therefore decided to round up and use a class width of five days in the frequency distribution.

In practice, the number of classes and the appropriate class width are determined by trial and error. Once a possible number of classes is chosen, equation (2.2) is used to find the approximate class width. The process can be repeated for a different number of classes. Ultimately, the analyst uses judgment to determine the combination of the number of classes and class width that provides the best frequency distribution for summarizing the data.

For the audit time data in Table 2.4, after deciding to use five classes, each with a width of five days, the next task is to specify the class limits for each of the classes.

TABLE 2.5
FREQUENCY DISTRIBUTION FOR THE AUDIT TIME DATA

Audit Time (days)	Frequency
10–14	4
15–19	8
20–24	5
25–29	2
30–34	1
Total	20

Class limits Class limits must be chosen so that each data item belongs to one and only one class. The *lower class limit* identifies the smallest possible data value assigned to the class. The *upper class limit* identifies the largest possible data value assigned to the class. In developing frequency distributions for categorical data, we did not need to specify class limits because each data item naturally fell into a separate class. But with quantitative data, such as the audit times in Table 2.4, class limits are necessary to determine where each data value belongs.

Using the audit time data in Table 2.4, we selected 10 days as the lower class limit and 14 days as the upper class limit for the first class. This class is denoted 10–14 in Table 2.5. The smallest data value, 12, is included in the 10–14 class. We then selected 15 days as the lower class limit and 19 days as the upper class limit of the next class. We continued

defining the lower and upper class limits to obtain a total of five classes: 10–14, 15–19, 20–24, 25–29, and 30–34. The largest data value, 33, is included in the 30–34 class. The difference between the lower class limits of adjacent classes is the class width. Using the first two lower class limits of 10 and 15, we see that the class width is $15 - 10 = 5$.

With the number of classes, class width, and class limits determined, a frequency distribution can be obtained by counting the number of data values belonging to each class. For example, the data in Table 2.4 show that four values—12, 14, 14, and 13—belong to the 10–14 class. Thus, the frequency for the 10–14 class is 4. Continuing this counting process for the 15–19, 20–24, 25–29, and 30–34 classes provides the frequency distribution in Table 2.5. Using this frequency distribution, we can observe the following:

1. The most frequently occurring audit times are in the class of 15–19 days. Eight of the 20 audit times belong to this class.
2. Only one audit required 30 or more days.

Other conclusions are possible, depending on the interests of the person viewing the frequency distribution. The value of a frequency distribution is that it provides insights about the data that are not easily obtained by viewing the data in their original unorganized form.

Class midpoint In some applications, we want to know the midpoints of the classes in a frequency distribution for quantitative data. The **class midpoint** is the value halfway between the lower and upper class limits. For the audit time data, the five class midpoints are 12, 17, 22, 27, and 32.

Using Excel's PivotTable Report to Construct a Frequency Distribution

Excel's FREQUENCY function can also be used to construct a frequency distribution for quantitative data.

Excel's PivotTable report is a powerful tool for summarizing data. In this section we show how to use Excel's PivotTable report to construct a frequency distribution for quantitative data by showing how to construct a frequency distribution for the audit time data.

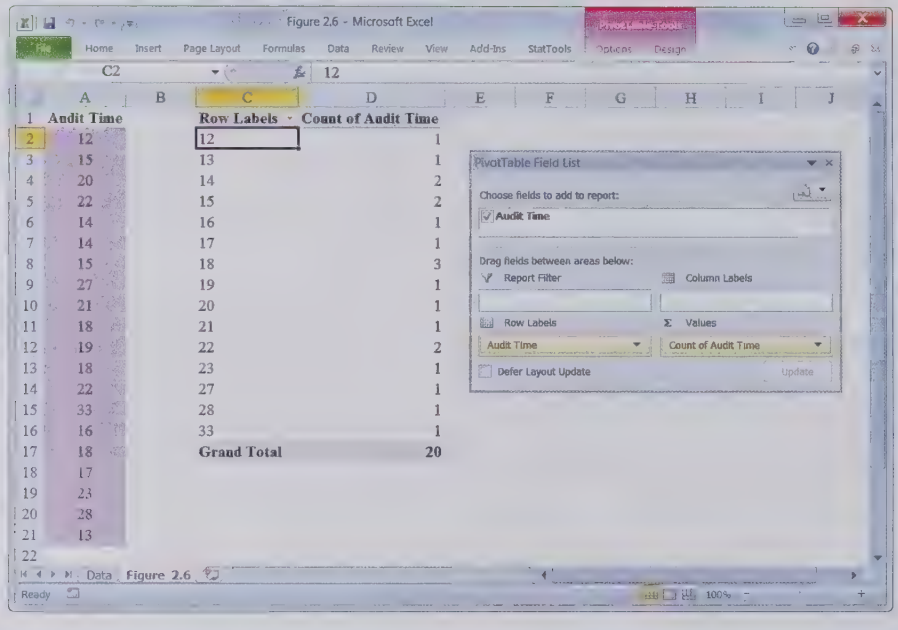
Enter Data: The label "Audit Time" and the 20 audit times are entered into cells A1:A21 of the Excel worksheet in Figure 2.6.

Enter Functions and Formulas: No functions and formulas are needed.

Apply Tools: The following steps describe how to use Excel's PivotTable report to construct a frequency distribution for the audit time data. When using Excel's PivotTable report, each column of data is referred to as a field. Thus, for the audit time example, the data appearing in cells A2:A21 and the corresponding label in cell A1 are referred to as the Audit Time field.

Step 1. Click the **Insert** tab on the Ribbon
Step 2. In the **Tables** group, click the icon above the word PivotTable
Step 3. When the **Create PivotTable** dialog box appears,
Choose **Select a table or range**
Enter A1:A21 in the **Table/Range** box
Choose **Existing Worksheet** as the location for the PivotTable
Enter C1 in the **Location** box
Click **OK**
Step 4. In the **PivotTable Field List,** go to **Choose Fields to add to report,**
Drag the **Audit Time** field to the **Row Labels** area
Drag the **Audit Time** field to the **Values** area
Step 5. Click on **Sum of Audit Time** in the **Values** area
Step 6. Click **Value Field Settings** from the list of options that appears

FIGURE 2.6 PIVOTTABLE FIELD LIST AND INITIAL PIVOTTABLE REPORT USED TO CONSTRUCT A FREQUENCY DISTRIBUTION FOR THE AUDIT TIME DATA



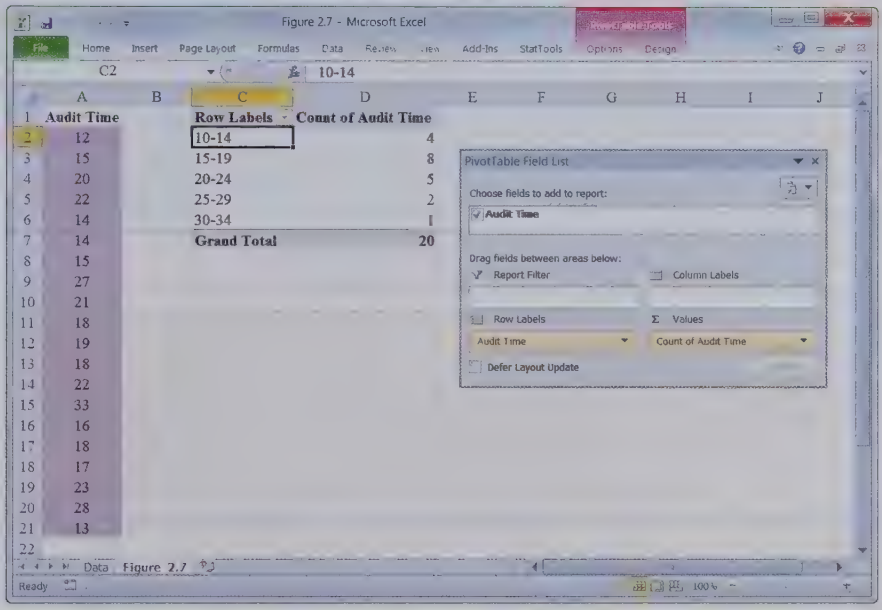
Step 7. When the Value Field Settings dialog appears,
Under **Summarize value field by,** choose **Count**
Click **OK**

Figure 2.6 shows the resulting PivotTable Field List and the corresponding PivotTable report. To construct the frequency distribution shown in Table 2.5, we must group the rows containing audit times. The following steps accomplish this.

Step 1. Right-click cell C2 in the PivotTable report or any other cell containing an audit time
Step 2. Choose **Group** from the list of options that appears
Step 3. When the **Grouping** dialog box appears,
Enter 10 in the **Starting at** box
Enter 34 in the **Ending at** box
Enter 5 in the **By** box
Click **OK**

Figure 2.7 shows the completed PivotTable Field List and PivotTable report. We see that with the exception of the column headings, the PivotTable report provides the same information as the frequency distribution shown in Table 2.5. And, if desired, we can change the labels to match the labels in Table 2.5 by selecting the cell and typing in the new label.

FIGURE 2.7 FREQUENCY DISTRIBUTION FOR THE AUDIT TIME DATA CONSTRUCTED USING EXCEL'S PIVOTTABLE REPORT



Relative Frequency and Percent Frequency Distributions

We define the relative frequency and percent frequency distributions for quantitative data in the same manner as for qualitative data. First, recall that the relative frequency is the proportion of the observations belonging to a class. With n observations,

$$\text{Relative frequency of class} = \frac{\text{Frequency of the class}}{n}$$

The percent frequency of a class is the relative frequency multiplied by 100.

Based on the class frequencies in Table 2.5 and with $n = 20$, Table 2.6 shows the relative frequency distribution and percent frequency distribution for the audit time data. Note that .40 of the audits, or 40%, required from 15 to 19 days. Only .05 of the audits, or 5%, required 30 or more days. Again, additional interpretations and insights can be obtained by using Table 2.6.

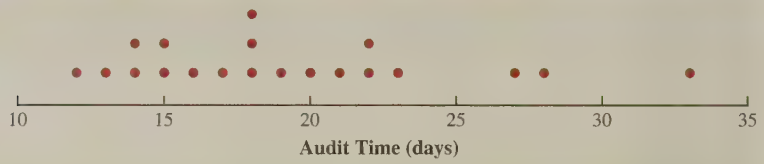
Dot Plot

One of the simplest graphical summaries of data is a **dot plot**. A horizontal axis shows the range for the data. Each data value is represented by a dot placed above the axis. Figure 2.8 is the dot plot for the audit time data in Table 2.4. The three dots located above 18 on the horizontal axis indicate that an audit time of 18 days occurred three times. Dot plots show the details of the data and are useful for comparing the distribution of the data for two or more variables.

TABLE 2.6 RELATIVE FREQUENCY AND PERCENT FREQUENCY DISTRIBUTIONS FOR THE AUDIT TIME DATA

Audit Time (days)	Relative Frequency	Percent Frequency
10–14	.20	20
15–19	.40	40
20–24	.25	25
25–29	.10	10
30–34	.05	5
Total	1.00	100

FIGURE 2.8 DOT PLOT FOR THE AUDIT TIME DATA

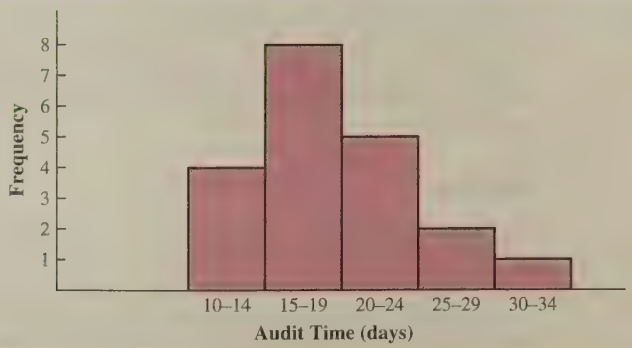


Histogram

A common graphical presentation of quantitative data is a **histogram**. This graphical summary can be prepared for data previously summarized in either a frequency, relative frequency, or percent frequency distribution. A histogram is constructed by placing the variable of interest on the horizontal axis and the frequency, relative frequency, or percent frequency on the vertical axis. The frequency, relative frequency, or percent frequency of each class is shown by drawing a rectangle whose base is determined by the class limits on the horizontal axis and whose height is the corresponding frequency, relative frequency, or percent frequency.

Figure 2.9 is a histogram for the audit time data. Note that the class with the greatest frequency is shown by the rectangle appearing above the class of 15–19 days. The height of

FIGURE 2.9 HISTOGRAM FOR THE AUDIT TIME DATA

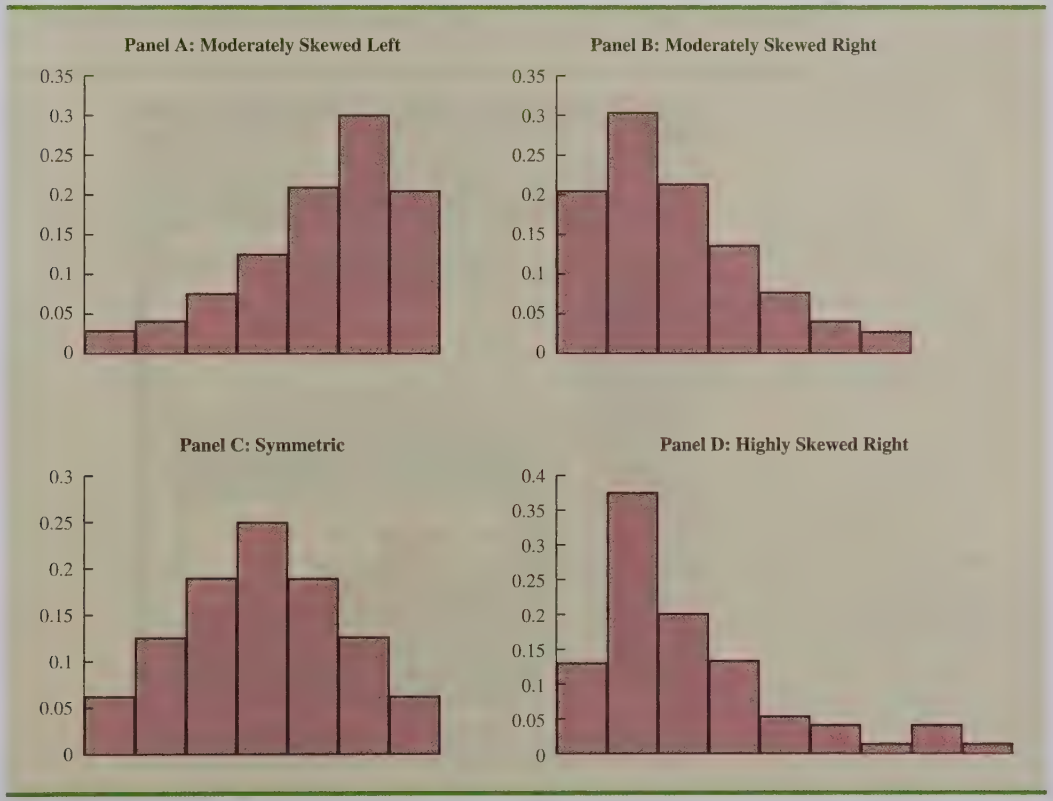


the rectangle shows that the frequency of this class is 8. A histogram for the relative or percent frequency distribution of these data would look the same as the histogram in Figure 2.9 with the exception that the vertical axis would be labeled with relative or percent frequency values.

As Figure 2.9 shows, the adjacent rectangles of a histogram touch one another. Unlike a bar graph, a histogram contains no natural separation between the rectangles of adjacent classes. This format is the usual convention for histograms. Because the classes for the audit time data are stated as 10–14, 15–19, 20–24, 25–29, and 30–34, one-unit spaces of 14 to 15, 19 to 20, 24 to 25, and 29 to 30 would seem to be needed between the classes. These spaces are eliminated when constructing a histogram. Eliminating the spaces between classes in a histogram for the audit time data helps show that all values between the lower limit of the first class and the upper limit of the last class are possible.

One of the most important uses of a histogram is to provide information about the shape, or form, of a distribution. Figure 2.10 contains four histograms constructed from relative frequency distributions. Panel A shows the histogram for a set of data moderately skewed to the left. A histogram is said to be skewed to the left if its tail extends farther to the left. This histogram is typical for exam scores, with no scores above 100%, most of the scores above 70%, and only a few really low scores. Panel B shows the histogram for a set of data moderately skewed to the right. A histogram is said to be skewed to the right if its tail extends farther to the right. An example of this type of histogram would be for data such as housing prices; a few expensive houses create the skewness in the right tail.

FIGURE 2.10 HISTOGRAMS SHOWING DIFFERING LEVELS OF SKEWNESS



Panel C shows a symmetric histogram. In a symmetric histogram, the left tail mirrors the shape of the right tail. Histograms for data found in applications are never perfectly symmetric, but the histogram for many applications may be roughly symmetric. Data for SAT scores, heights and weights of people, and so on lead to histograms that are roughly symmetric. Panel D shows a histogram highly skewed to the right. This histogram was constructed from data on the amount of customer purchases over one day at a women's apparel store. Data from applications in business and economics often lead to histograms that are skewed to the right. For instance, data on housing prices, salaries, purchase amounts, and so on often result in histograms skewed to the right.

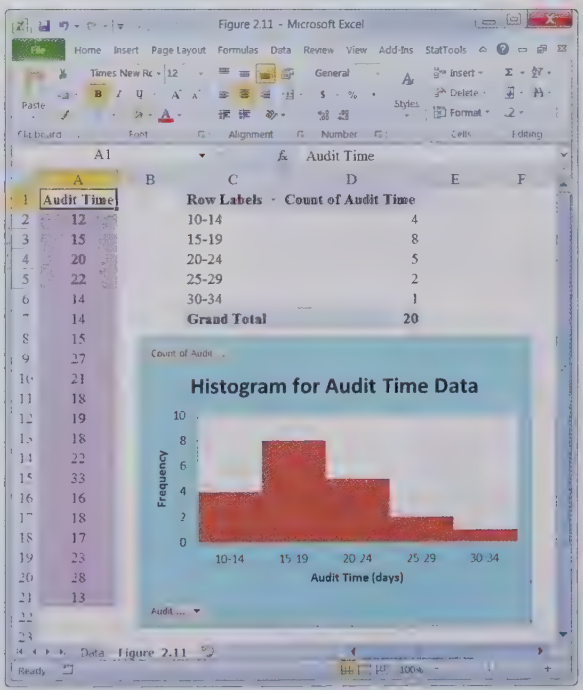
Using Excel's Chart Tools to Construct a Histogram

In Section 2.1 we showed how Excel's chart tools can be used to construct a bar chart for depicting categorical data summarized in a frequency distribution. Excel's chart tools can also be used to construct a histogram for depicting quantitative data summarized in a frequency distribution. We will illustrate the use of Excel's chart tools for constructing a histogram using the frequency distribution for the audit time data constructed using Excel's PivotTable Report. Refer to Figure 2.11 as we describe the tasks involved.

Enter Data: Same as in Figure 2.7

Enter Functions and Formulas: No functions and formulas are needed.

FIGURE 2.11 USING EXCEL'S CHART TOOLS TO CONSTRUCT A HISTOGRAM FOR THE AUDIT TIME DATA



Apply Tools: The following steps describe how to use Excel's chart tools to construct a histogram for the audit time data using the frequency distribution appearing in cells C1:D6.

Step 1. Select cells C2:D6
Step 2. Click the **Insert** tab on the Ribbon
Step 3. In the **Charts** group, click **Column**
Step 4. When the list of column chart types appears,
Go to the **2-D Column** section
Click **Clustered Column** (the leftmost chart)
Step 5. In the **Chart Layouts** group, click the **More** button (the downward pointing arrow with a line over it) to display all the options
Step 6. Choose **Layout 8**
Step 7. Click the **Chart Title** and replace it with **Histogram for Audit Time Data**
Step 8. Click the **Horizontal (Category) Axis Title** and replace it with **Audit Time (days)**
Step 9. Click the **Vertical (Value) Axis Title** and replace it with **Frequency**
Step 10. Right-click the vertical axis and choose **Format Axis** from the list of options that appear
Step 11. When the Format Axis dialog box appears,
Go to the **Axis Options** section
Select Fixed for the **Major Unit** and enter 1.0 in the corresponding box
Click **Close**

Figure 2.11 shows the completed histogram for the audit time data constructed using the PivotTable report output and Excel's chart tools. Because the histogram was constructed using the frequency distribution created using Excel's PivotTable report, the resulting chart contains a Value field button in the top left part of the chart and an Axis field button in the lower left part of the chart. If you want to remove either of both of these buttons, you right-click the button and select the option to remove that button or to remove all buttons shown on the chart.

Cumulative Distributions

A variation of the frequency distribution that provides another tabular summary of quantitative data is the **cumulative frequency distribution**. The cumulative frequency distribution uses the number of classes, class widths, and class limits developed for the frequency distribution. However, rather than showing the frequency of each class, the cumulative frequency distribution shows the number of data items with values *less than or equal to the upper class limit* of each class. The first two columns of Table 2.7 provide the cumulative frequency distribution for the audit time data.

TABLE 2.7 CUMULATIVE FREQUENCY, CUMULATIVE RELATIVE FREQUENCY, AND CUMULATIVE PERCENT FREQUENCY DISTRIBUTIONS FOR THE AUDIT TIME DATA

Audit Time (days)	Cumulative Frequency	Cumulative Relative Frequency	Cumulative Percent Frequency
Less than or equal to 14	4	.20	20
Less than or equal to 19	12	.60	60
Less than or equal to 24	17	.85	85
Less than or equal to 29	19	.95	95
Less than or equal to 34	20	1.00	100

To understand how the cumulative frequencies are determined, consider the class with the description "less than or equal to 24." The cumulative frequency for this class is simply the sum of the frequencies for all classes with data values less than or equal to 24. For the frequency distribution in Table 2.5, the sum of the frequencies for classes 10–14, 15–19, and 20–24 indicates that $4 + 8 + 5 = 17$ data values are less than or equal to 24. Hence, the cumulative frequency for this class is 17. In addition, the cumulative frequency distribution in Table 2.7 shows that four audits were completed in 14 days or less and 19 audits were completed in 29 days or less.

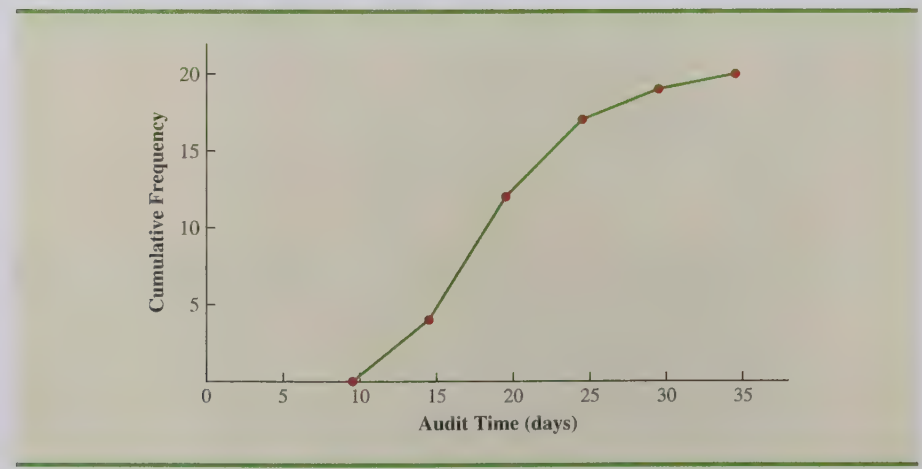
As a final point, we note that a **cumulative relative frequency distribution** shows the proportion of data items, and a **cumulative percent frequency distribution** shows the percentage of data items with values less than or equal to the upper limit of each class. The cumulative relative frequency distribution can be computed either by summing the relative frequencies in the relative frequency distribution or by dividing the cumulative frequencies by the total number of items. Using the latter approach, we found the cumulative relative frequencies in column 3 of Table 2.7 by dividing the cumulative frequencies in column 2 by the total number of items ($n = 20$). The cumulative percent frequencies were again computed by multiplying the relative frequencies by 100. The cumulative relative and percent frequency distributions show that .85 of the audits, or 85%, were completed in 24 days or less, .95 of the audits, or 95%, were completed in 29 days or less, and so on.

Ogive

A graph of a cumulative distribution, called an **ogive**, shows data values on the horizontal axis and either the cumulative frequencies, the cumulative relative frequencies, or the cumulative percent frequencies on the vertical axis. Figure 2.12 illustrates an ogive for the cumulative frequencies of the audit time data in Table 2.7.

The ogive is constructed by plotting a point corresponding to the cumulative frequency of each class. Because the classes for the audit time data are 10–14, 15–19, 20–24, and so on, one-unit gaps appear from 14 to 15, 19 to 20, and so on. These gaps are eliminated by plotting points halfway between the class limits. Thus, 14.5 is used for the 10–14 class, 19.5 is used for the 15–19 class, and so on. The "less than or equal to 14" class with a cumulative

FIGURE 2.12 OGIVE FOR THE AUDIT TIME DATA



frequency of 4 is shown on the ogive in Figure 2.12 by the point located at 14.5 on the horizontal axis and 4 on the vertical axis. The "less than or equal to 19" class with a cumulative frequency of 12 is shown by the point located at 19.5 on the horizontal axis and 12 on the vertical axis. Note that one additional point is plotted at the left end of the ogive. This point starts the ogive by showing that no data values fall below the 10–14 class. It is plotted at 9.5 on the horizontal axis and 0 on the vertical axis. The plotted points are connected by straight lines to complete the ogive.

NOTES AND COMMENTS

1. A bar chart and a histogram are essentially the same thing; both are graphical presentations of the data in a frequency distribution. A histogram is just a bar chart with no separation between bars. For some discrete quantitative data, a separation between bars is also appropriate. Consider, for example, the number of classes in which a college student is enrolled. The data may only assume integer values. Intermediate values such as 1.5, 2.73, and so on are not possible. With continuous quantitative data, however, such as the audit times in Table 2.4, a separation between bars is not appropriate.
2. The appropriate values for the class limits with quantitative data depend on the level of accuracy of the data. For instance, with the audit time data of Table 2.4 the limits used were integer values. If the data were rounded to the nearest tenth of a day (e.g., 12.3, 14.4, and so on), then the limits would be stated in tenths of days. For instance, the first class would be 10.0–14.9. If the data were recorded to the nearest hundredth of a day (e.g., 12.34, 14.45, and so on), the limits would be stated in hundredths of days. For instance, the first class would be 10.00–14.99.
3. An *open-end* class requires only a lower class limit or an upper class limit. For example, in the audit time data of Table 2.4, suppose two of the audits had taken 58 and 65 days. Rather than continue with the classes of width 5 with classes 35–39, 40–44, 45–49, and so on, we could simplify the frequency distribution to show an open-end class of "35 or more." This class would have a frequency of 2. Most often the open-end class appears at the upper end of the distribution. Sometimes an open-end class appears at the lower end of the distribution, and occasionally such classes appear at both ends.
4. The last entry in a cumulative frequency distribution always equals the total number of observations. The last entry in a cumulative relative frequency distribution always equals 1.00 and the last entry in a cumulative percent frequency distribution always equals 100.

Exercises

Methods

11. Consider the following data.

14	21	23	21	16
19	22	25	16	16
24	24	25	19	16
19	18	19	21	12
16	17	18	23	25
20	23	16	20	19
24	26	15	22	24
20	22	24	22	20

a. Develop a frequency distribution using classes of 12–14, 15–17, 18–20, 21–23, and 24–26.
b. Develop a relative frequency distribution and a percent frequency distribution using the classes in part (a).

SELF test

12. Consider the following frequency distribution.

Class	Frequency
10–19	10
20–29	14
30–39	17
40–49	7
50–59	2

Construct a cumulative frequency distribution and a cumulative relative frequency distribution.

13. Construct a histogram and an ogive for the data in exercise 12.

14. Consider the following data.

8.9	10.2	11.5	7.8	10.0	12.2	13.5	14.1	10.0	12.2
6.8	9.5	11.5	11.2	14.9	7.5	10.0	6.0	15.8	11.5

a. Construct a dot plot.
b. Construct a frequency distribution.
c. Construct a percent frequency distribution.

Applications

SELF test

15. A doctor's office staff studied the waiting times for patients who arrive at the office with a request for emergency service. The following data with waiting times in minutes were collected over a one-month period.

2 5 10 12 4 4 5 17 11 8 9 8 12 21 6 8 7 13 18 3

Use classes of 0–4, 5–9, and so on in the following:
a. Show the frequency distribution.
b. Show the relative frequency distribution.
c. Show the cumulative frequency distribution.
d. Show the cumulative relative frequency distribution.
e. What proportion of patients needing emergency service wait 9 minutes or less?

16. A shortage of candidates has required school districts to pay higher salaries and offer extras to attract and retain school district superintendents. The following data show the annual base salary ($1000s) for superintendents in 20 districts in the greater Rochester, New York, area (*The Rochester Democrat and Chronicle,* February 10, 2008).

187	184	174	185
175	172	202	197
165	208	215	164
162	172	182	156
172	175	170	183

Use classes of 150–159, 160–169, and so on in the following.
a. Show the frequency distribution.
b. Show the percent frequency distribution.
c. Show the cumulative percent frequency distribution.
d. Develop a histogram for the annual base salary.
e. Do the data appear to be skewed? Explain.
f. What percentage of the superintendents make more than $200,000?

17. The Dow Jones Industrial Average (DJIA) underwent one of its infrequent reshufflings of companies when General Motors and Citigroup were replaced by Cisco Systems and Travelers (*The Wall Street Journal,* June 8, 2009). At the time, the prices per share for the 30 companies in the DJIA were as follows:

WEB file
DJIAPrices Revised

Company	$/Share	Company	$/Share
3M	61	IBM	107
Alcoa	11	Intel	16
American Express	25	JPMorgan Chase	35
AT&T	24	Johnson & Johnson	56
Bank of America	12	Kraft Foods	27
Boeing	52	McDonald's	59
Caterpillar	38	Merck	26
Chevron	69	Microsoft	22
Cisco Systems	20	Pfizer	14
Coca-Cola	49	Procter & Gamble	53
DuPont	27	Travelers	43
ExxonMobil	72	United Technologies	56
General Electric	14	Verizon	29
Hewlett-Packard	37	Walmart Stores	51
Home Depot	24	Walt Disney	25

a. What is the highest price per share? What is the lowest price per share?
b. Using a class width of 10, develop a frequency distribution for the data.
c. Prepare a histogram. Interpret the histogram, including a discussion of the general shape of the histogram, the midprice range, and the most frequent price range.
d. Use the *The Wall Street Journal* or another newspaper to find the current price per share for these companies. Prepare a histogram of the data and discuss any changes since June 2009. What company has had the largest increase in the price per share? What company has had the largest decrease in the price per share?

WEB file
Holiday

18. NRF/BIG research provided results of a consumer holiday spending survey (*USA Today,* December 20, 2005). The following data provide the dollar amount of holiday spending for a sample of 25 consumers.

1200	850	740	590	340
450	890	260	610	350
1780	180	850	2050	770
800	1090	510	520	220
1450	280	1120	200	350

a. What is the lowest holiday spending? The highest?
b. Use a class width of $250 to prepare a frequency distribution and a percent frequency distribution for the data.
c. Prepare a histogram and comment on the shape of the distribution.
d. What observations can you make about holiday spending?

WEB file
LargeCorp

19. *Fortune* provides a list of America's largest corporations based on annual revenue. Shown below are the 50 largest corporations, with annual revenue expressed in billions of dollars (CNN Money website, January 15, 2010).

Corporation	Revenue	Corporation	Revenue
Amerisource Bergen	71	Berkshire Hathaway	108
Archer Daniels Midland	70	Boeing	61
AT&T	124	Cardinal Health	91
Bank of America	113	Caterpillar	51

(*Continued*)

Corporation	Revenue	Corporation	Revenue
Chevron	263	MetLife	55
Citigroup	112	Microsoft	60
ConocoPhillips	231	Morgan Stanley	62
Costco Wholesale	72	Pepsico	43
CVS Caremark	87	Pfizer	48
Dell	61	Procter & Gamble	84
Dow Chemical	58	Safeway	44
ExxonMobil	443	Sears Holdings	47
Ford Motors	146	State Farm Insurance	61
General Electric	149	Sunoco	52
Goldman Sachs	54	Target	65
Hewlett-Packard	118	Time Warner	47
Home Depot	71	United Parcel Service	51
IBM	104	United Technologies	59
JPMorgan Chase	101	UnitedHealth Group	81
Johnson & Johnson	64	Valero Energy	118
Kroger	76	Verizon	97
Lowe's	48	Walgreen	59
Marathon Oil	74	Walmart	406
McKesson	102	WellPoint	61
Medco Health	51	Wells Fargo	52

Summarize the data by constructing the following:

a. A frequency distribution (classes 0–49, 50–99, 100–149, and so on).
b. A relative frequency distribution.
c. A cumulative frequency distribution.
d. A cumulative relative frequency distribution.
e. What do these distributions tell you about the annual revenue of the largest corporations in America?
f. Show a histogram. Comment on the shape of the distribution.
g. What is the largest corporation in America and what is its annual revenue?

20. The *Golf Digest 50* lists the 50 professional golfers with the highest total annual income. Total income is the sum of both on-course and off-course earnings. Tiger Woods ranked first with a total annual income of $122 million. However, almost $100 million of this total was from off-course activities such as product endorsements and personal appearances. The 10 professional golfers with the highest *off-course* income are shown in the following table (Golf Digest website, February 2008).

Name	Off-Course Income ($1000s)
Tiger Woods	99,800
Phil Mickelson	40,200
Arnold Palmer	29,500
Vijay Singh	25,250
Ernie Els	24,500
Greg Norman	24,000
Jack Nicklaus	20,750
Sergio Garcia	14,500
Michelle Wie	12,500
Jim Furyk	11,000

The off-course income of all 50 professional golfers in the *Golf Digest 50* can be found on the website that accompanies the text. The income data are in $1000s. Use classes of

0–4999, 5000–9999, 10,000–14,999, and so on to answer the following questions. Include an open-ended class of 50,000 or more as the largest income class.

WEB file OffCourse

a. Construct a frequency distribution and percent frequency distribution of the annual off-course income of the 50 professional golfers.
b. Construct a histogram for these data.
c. Comment on the shape of the distribution of off-course income.
d. What is the most frequent off-course income class for the 50 professional golfers? Using your tabular and graphical summaries, what additional observations can you make about the off-course income of these 50 professional golfers?

21. The *Nielsen Home Technology Report* provided information about home technology and its usage. The following data are the hours of personal computer usage during one week for a sample of 50 persons.

WEB file Computer

4.1	1.5	10.4	5.9	3.4	5.7	1.6	6.1	3.0	3.7
3.1	4.8	2.0	14.8	5.4	4.2	3.9	4.1	11.1	3.5
4.1	4.1	8.8	5.6	4.3	3.3	7.1	10.3	6.2	7.6
10.8	2.8	9.5	12.9	12.1	0.7	4.0	9.2	4.4	5.7
7.2	6.1	5.7	5.9	4.7	3.9	3.7	3.1	6.1	3.1

Summarize the data by constructing the following:

a. A frequency distribution (use a class width of three hours)
b. A relative frequency distribution
c. A histogram
d. An ogive
e. Comment on what the data indicate about personal computer usage at home.

2.3 Exploratory Data Analysis: The Stem-and-Leaf Display

The techniques of **exploratory data analysis** consist of simple arithmetic and easy-to-draw graphs that can be used to summarize data quickly. One technique—referred to as a **stem-and-leaf display**—can be used to show both the rank order and shape of a data set simultaneously.

To illustrate the use of a stem-and-leaf display, consider the data in Table 2.8. These data result from a 150-question aptitude test given to 50 individuals recently interviewed for a position at Haskens Manufacturing. The data indicate the number of questions answered correctly.

To develop a stem-and-leaf display, we first arrange the leading digits of each data value to the left of a vertical line. To the right of the vertical line, we record the last digit for each

TABLE 2.8 NUMBER OF QUESTIONS ANSWERED CORRECTLY ON AN APTITUDE TEST

WEB file ApTest

112	72	69	97	107
73	92	76	86	73
126	128	118	127	124
82	104	132	134	83
92	108	96	100	92
115	76	91	102	81
95	141	81	80	106
84	119	113	98	75
68	98	115	106	95
100	85	94	106	119

data value. Based on the top row of data in Table 2.8 (112, 72, 69, 97, and 107), the first five entries in constructing a stem-and-leaf display would be as follows:

```
 6 | 9
 7 | 2
 8 |
 9 | 7
10 | 7
11 | 2
12 |
13 |
14 |
```

For example, the data value 112 shows the leading digits 11 to the left of the line and the last digit 2 to the right of the line. Similarly, the data value 72 shows the leading digit 7 to the left of the line and last digit 2 to the right of the line. Continuing to place the last digit of each data value on the line corresponding to its leading digit(s) provides the following:

```
 6 | 9 8
 7 | 2 3 6 3 6 5
 8 | 6 2 3 1 1 0 4 5
 9 | 7 2 2 6 2 1 5 8 8 5 4
10 | 7 4 8 0 2 6 6 0 6
11 | 2 8 5 9 3 5 9
12 | 6 8 7 4
13 | 2 4
14 | 1
```

With this organization of the data, sorting the digits on each line into rank order is simple. Doing so provides the stem-and-leaf display shown here.

```
 6 | 8 9
 7 | 2 3 3 5 6 6
 8 | 0 1 1 2 3 4 5 6
 9 | 1 2 2 2 4 5 5 6 7 8 8
10 | 0 0 2 4 6 6 6 7 8
11 | 2 3 5 5 8 9 9
12 | 4 6 7 8
13 | 2 4
14 | 1
```

The numbers to the left of the vertical line (6, 7, 8, 9, 10, 11, 12, 13, and 14) form the *stem*, and each digit to the right of the vertical line is a *leaf*. For example, consider the first row with a stem value of 6 and leaves of 8 and 9.

```
6 | 8 9
```

This row indicates that two data values have a first digit of six. The leaves show that the data values are 68 and 69. Similarly, the second row

$$7 \mid 2 \quad 3 \quad 3 \quad 5 \quad 6 \quad 6$$

indicates that six data values have a first digit of seven. The leaves show that the data values are 72, 73, 73, 75, 76, and 76.

To focus on the shape indicated by the stem-and-leaf display, let us use a rectangle to contain the leaves of each stem. Doing so, we obtain the following:

6	8 9
7	2 3 3 5 6 6
8	0 1 1 2 3 4 5 6
9	1 2 2 2 4 5 5 6 7 8 8
10	0 0 2 4 6 6 6 7 8
11	2 3 5 5 8 9 9
12	4 6 7 8
13	2 4
14	1

Rotating this page counterclockwise onto its side provides a picture of the data that is similar to a histogram with classes of 60–69, 70–79, 80–89, and so on.

Although the stem-and-leaf display may appear to offer the same information as a histogram, it has two primary advantages.

1. The stem-and-leaf display is easier to construct by hand.
2. Within a class interval, the stem-and-leaf display provides more information than the histogram because the stem-and-leaf shows the actual data.

Just as a frequency distribution or histogram has no absolute number of classes, neither does a stem-and-leaf display have an absolute number of rows or stems. If we believe that our original stem-and-leaf display condensed the data too much, we can easily stretch the display by using two or more stems for each leading digit. For example, to use two stems for each leading digit, we would place all data values ending in 0, 1, 2, 3, and 4 in one row and all values ending in 5, 6, 7, 8, and 9 in a second row. The following stretched stem-and-leaf display illustrates this approach.

In a stretched stem-and-leaf display, whenever a stem value is stated twice, the first value corresponds to leaf values of 0–4, and the second value corresponds to leaf values of 5–9.

6	8 9
7	2 3 3
7	5 6 6
8	0 1 1 2 3 4
8	5 6
9	1 2 2 2 4
9	5 5 6 7 8 8
10	0 0 2 4
10	6 6 6 7 8
11	2 3
11	5 5 8 9 9
12	4
12	6 7 8
13	2 4
13	
14	1

Note that values 72, 73, and 73 have leaves in the 0–4 range and are shown with the first stem value of 7. The values 75, 76, and 76 have leaves in the 5–9 range and are shown with the second stem value of 7. This stretched stem-and-leaf display is similar to a frequency distribution with intervals of 65–69, 70–74, 75–79, and so on.

The preceding example showed a stem-and-leaf display for data with as many as three digits. Stem-and-leaf displays for data with more than three digits are possible. For example, consider the following data on the number of hamburgers sold by a fast-food restaurant for each of 15 weeks.

1565	1852	1644	1766	1888	1912	2044	1812
1790	1679	2008	1852	1967	1954	1733	

A stem-and-leaf display of these data follows.

Leaf unit = 10

15	6
16	4 7
17	3 6 9
18	1 5 5 8
19	1 5 6
20	0 4

A single digit is used to define each leaf in a stem-and-leaf display. The leaf unit indicates how to multiply the stem-and-leaf numbers in order to approximate the original data. Leaf units may be 100, 10, 1, 0.1, and so on.

Note that a single digit is used to define each leaf and that only the first three digits of each data value have been used to construct the display. At the top of the display we have specified Leaf unit = 10. To illustrate how to interpret the values in the display, consider the first stem, 15, and its associated leaf, 6. Combining these numbers, we obtain 156. To reconstruct an approximation of the original data value, we must multiply this number by 10, the value of the *leaf unit*. Thus, $156 \times 10 = 1560$ is an approximation of the original data value used to construct the stem-and-leaf display. Although it is not possible to reconstruct the exact data value from this stem-and-leaf display, the convention of using a single digit for each leaf enables stem-and-leaf displays to be constructed for data having a large number of digits. For stem-and-leaf displays where the leaf unit is not shown, the leaf unit is assumed to equal 1.

Exercises

Methods

22. Construct a stem-and-leaf display for the following data.

70	72	75	64	58	83	80	82
76	75	68	65	57	78	85	72

SELF test

23. Construct a stem-and-leaf display for the following data.

11.3	9.6	10.4	7.5	8.3	10.5	10.0
9.3	8.1	7.7	7.5	8.4	6.3	8.8

24. Construct a stem-and-leaf display for the following data. Use a leaf unit of 10.

1161	1206	1478	1300	1604	1725	1361	1422
1221	1378	1623	1426	1557	1730	1706	1689

Applications

SELF test

25. A psychologist developed a new test of adult intelligence. The test was administered to 20 individuals, and the following data were obtained.

114	99	131	124	117	102	106	127	119	115
98	104	144	151	132	106	125	122	118	118

Construct a stem-and-leaf display for the data.

26. *Money magazine* listed top career opportunities for work that is enjoyable, pays well, and will still be around 10 years from now (*Money,* November 2009). Shown below are 20 top career opportunities, with the median pay and top pay for workers with two to seven years of experience in the field. Data are shown in thousands of dollars.

WEB file
Careers

Career	Median Pay	Top Pay
Account Executive	81	157
Certified Public Accountant	74	138
Computer Security Consultant	100	138
Director of Communications	78	135
Financial Analyst	80	109
Finance Director	121	214
Financial Research Analyst	66	155
Hotel General Manager	77	146
Human Resources Manager	72	111
Investment Banking	106	221
IT Business Analyst	83	119
IT Project Manager	99	140
Marketing Manager	77	126
Quality-Assurance Manager	80	122
Sales Representative	67	125
Senior Internal Auditor	76	106
Software Developer	79	116
Software Program Manager	110	152
Systems Engineer	87	130
Technical Writer	67	100

Develop a stem-and-leaf display for both the median pay and the top pay. Comment on what you learn about the pay for these careers.

27. Most major ski resorts offer family programs that provide ski and snowboarding instruction for children. The typical classes provide four to six hours on the snow with a certified instructor. The daily rate for a group lesson at 15 ski resorts follows (*The Wall Street Journal,* January 20, 2006).

Resort	Location	Daily Rate	Resort	Location	Daily Rate
Beaver Creek	Colorado	$137	Okemo	Vermont	$ 86
Deer Valley	Utah	115	Park City	Utah	145
Diamond Peak	California	95	Butternut	Massachusetts	75
Heavenly	California	145	Steamboat	Colorado	98
Hunter	New York	79	Stowe	Vermont	104
Mammoth	California	111	Sugar Bowl	California	100
Mount Sunapee	New Hampshire	96	Whistler-Blackcomb	British Columbia	104
Mount Bachelor	Oregon	83			

a. Develop a stem-and-leaf display for the data.
b. Interpret the stem-and-leaf display in terms of what it tells you about the daily rate for these ski and snowboarding instruction programs.

28. The 2004 Naples, Florida, half-marathon (13.1 miles) had 1228 registrants (*Naples Daily News*, January 17, 2004). Competition was held in six age groups. The following data show the ages for a sample of 40 individuals who participated in the marathon.

WEB file Marathon

49	33	40	37	56
44	46	57	55	32
50	52	43	64	40
46	24	30	37	43
31	43	50	36	61
27	44	35	31	43
52	43	66	31	50
72	26	59	21	47

a. Show a stretched stem-and-leaf display.
b. What age group had the largest number of runners?
c. What age occurred most frequently?
d. A *Naples Daily News* feature article emphasized the number of runners who were "20-something." What percentage of the runners were in the 20-something age group? What do you suppose was the focus of the article?

2.4 Crosstabulations and Scatter Diagrams

Crosstabulations and scatter diagrams are used to summarize data in a way that reveals the relationship between two variables.

Thus far in this chapter, we have focused on tabular and graphical methods used to summarize the data for *one variable at a time.* Often a manager or decision maker requires tabular and graphical methods that will assist in the understanding of the *relationship between two variables.* Crosstabulation and scatter diagrams are two such methods.

Crosstabulation

A **crosstabulation** is a tabular summary of data for two variables. Let us illustrate the use of a crosstabulation by considering the following application based on data from Zagat's Restaurant Review. The quality rating and the meal price data were collected for a sample of 300 restaurants located in the Los Angeles area. Table 2.9 shows the data for the first 10 restaurants. Data on a restaurant's quality rating and typical meal price are reported. Quality rating is a categorical variable with rating categories of good, very good, and excellent. Meal price is a quantitative variable that ranges from $10 to $49.

A crosstabulation of the data for this application is shown in Table 2.10. The left and top margin labels define the classes for the two variables. In the left margin, the row labels (good, very good, and excellent) correspond to the three classes of the quality rating variable. In the top margin, the column labels ($10–19, $20–29, $30–39, and $40–49) correspond to the four classes of the meal price variable. Each restaurant in the sample provides a quality rating and a meal price. Thus, each restaurant in the sample is associated with a cell appearing in one of the rows and one of the columns of the crosstabulation. For example, restaurant 5 is identified as having a very good quality rating and a meal price of $33. This restaurant belongs to the cell in row 2 and column 3 of Table 2.10. In constructing a crosstabulation, we simply count the number of restaurants that belong to each of the cells in the crosstabulation table.

TABLE 2.9 QUALITY RATING AND MEAL PRICE FOR 300 LOS ANGELES RESTAURANTS

WEB file
Restaurant

Restaurant	Quality Rating	Meal Price ($)
1	Good	18
2	Very Good	22
3	Good	28
4	Excellent	38
5	Very Good	33
6	Good	28
7	Very Good	19
8	Very Good	11
9	Very Good	23
10	Good	13
.	.	.
.	.	.
.	.	.

TABLE 2.10 CROSSTABULATION OF QUALITY RATING AND MEAL PRICE FOR 300 LOS ANGELES RESTAURANTS

	Meal Price				
Quality Rating	$10–19	$20–29	$30–39	$40–49	Total
Good	42	40	2	0	84
Very Good	34	64	46	6	150
Excellent	2	14	28	22	66
Total	78	118	76	28	300

In reviewing Table 2.10, we see that the greatest number of restaurants in the sample (64) have a very good rating and a meal price in the $20–29 range. Only two restaurants have an excellent rating and a meal price in the $10–19 range. Similar interpretations of the other frequencies can be made. In addition, note that the right and bottom margins of the crosstabulation provide the frequency distributions for quality rating and meal price separately. From the frequency distribution in the right margin, we see that data on quality ratings show 84 good restaurants, 150 very good restaurants, and 66 excellent restaurants. Similarly, the bottom margin shows the frequency distribution for the meal price variable.

Dividing the totals in the right margin of the crosstabulation by the total for that column provides a relative and percent frequency distribution for the quality rating variable.

Quality Rating	Relative Frequency	Percent Frequency
Good	.28	28
Very Good	.50	50
Excellent	.22	22
Total	1.00	100

From the percent frequency distribution we see that 28% of the restaurants were rated good, 50% were rated very good, and 22% were rated excellent.

Dividing the totals in the bottom row of the crosstabulation by the total for that row provides a relative and percent frequency distribution for the meal price variable.

Meal Price	Relative Frequency	Percent Frequency
$10–19	.26	26
$20–29	.39	39
$30–39	.25	25
$40–49	.09	9
Total	1.00	100

Note that the sum of the values in each column does not add exactly to the column total, because the values being summed are rounded. From the percent frequency distribution we see that 26% of the meal prices are in the lowest price class ($10–19), 39% are in the next higher class, and so on.

The frequency and relative frequency distributions constructed from the margins of a crosstabulation provide information about each of the variables individually, but they do not shed any light on the relationship between the variables. The primary value of a crosstabulation lies in the insight it offers about the relationship between the variables. A review of the crosstabulation in Table 2.10 reveals that higher meal prices are associated with the higher quality restaurants, and the lower meal prices are associated with the lower quality restaurants.

Converting the entries in a crosstabulation into row percentages or column percentages can provide more insight into the relationship between the two variables. For row percentages, the results of dividing each frequency in Table 2.10 by its corresponding row total are shown in Table 2.11. Each row of Table 2.11 is a percent frequency distribution of meal price for one of the quality rating categories. Of the restaurants with the lowest quality rating (good), we see that the greatest percentages are for the less expensive restaurants (50% have $10–19 meal prices and 47.6% have $20–29 meal prices). Of the restaurants with the highest quality rating (excellent), we see that the greatest percentages are for the more expensive restaurants (42.4% have $30–39 meal prices and 33.4% have $40–49 meal prices). Thus, we continue to see that the more expensive meals are associated with the higher quality restaurants.

A graph showing the information in a crosstabulation can also be developed to enhance the presentation. Figure 2.13 shows an Excel column chart for the results displayed in Table 2.11.

In practice, the final reports for many statistical studies include a large number of crosstabulations. In the Los Angeles restaurant survey, the crosstabulation is based on one categorical variable (quality rating) and one quantitative variable (meal price). Crosstabulations can also be developed when both variables are categorical and when both variables

TABLE 2.11 ROW PERCENTAGES FOR EACH QUALITY RATING CATEGORY

	Meal Price				
Quality Rating	$10–19	$20–29	$30–39	$40–49	Total
Good	50.0	47.6	2.4	0.0	100
Very Good	22.7	42.7	30.6	4.0	100
Excellent	3.0	21.2	42.4	33.4	100

FIGURE 2.13 EXCEL COLUMN CHART SHOWING ROW PERCENTAGES FOR EACH QUALITY RATING CATEGORY

are quantitative. When quantitative variables are used, however, we must first create classes for the values of the variable. For instance, in the restaurant example we grouped the meal prices into four classes ($10–19, $20–29, $30–39, and $40–49).

Using Excel's PivotTable Report to Construct a Crosstabulation

Excel's PivotTable Report provides an excellent tool for summarizing the data for two or more variables simultaneously. We will illustrate the use of Excel's PivotTable Report by showing how to develop a crosstabulation of quality ratings and meal prices for the sample of 300 restaurants located in the Los Angeles area.

Enter Data: The labels "Restaurant," "Quality Rating," and "Meal Price ($)" have been entered into cells A1:C1 of the worksheet shown in Figure 2.14. The data for each of the 300 restaurants in the sample have been entered into cells B2:C301.

Enter Functions and Formulas: No functions and formulas are needed.

Apply Tools: To use the PivotTable report to create a crosstabulation, we need to perform three tasks: Display the Initial PivotTable Field List and PivotTable Report; Set Up the PivotTable Field List; Finalize the PivotTable Report.

Display the Initial PivotTable Field List and PivotTable Report: Three steps are needed to display the initial PivotTable Field List and PivotTable report.

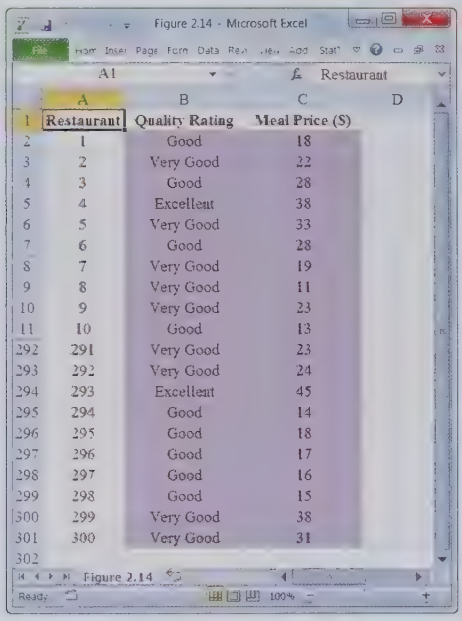
Step 1. Click the **Insert** tab on the Ribbon
Step 2. In the **Tables** group, click the icon above the word PivotTable
Step 3. When the **Create PivotTable** dialog box appears,
Choose **Select a Table or Range**
Enter A1:C301 in the **Table/Range** box
Choose **New Worksheet** as the location for the PivotTable Report
Click **OK**

The resulting initial PivotTable Field List and PivotTable Report are shown in Figure 2.15.

FIGURE 2.14 EXCEL WORKSHEET CONTAINING RESTAURANT DATA

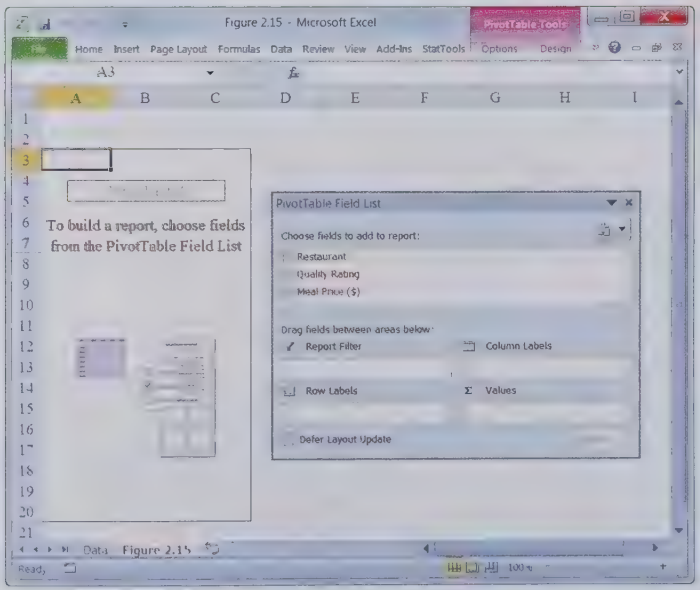
WEB file
Restaurant

Note: Rows 12–291 are hidden.



	A	B	C	D
1	Restaurant	Quality Rating	Meal Price ($)	
2	1	Good	18	
3	2	Very Good	22	
4	3	Good	28	
5	4	Excellent	38	
6	5	Very Good	33	
7	6	Good	28	
8	7	Very Good	19	
9	8	Very Good	11	
10	9	Very Good	23	
11	10	Good	13	
292	291	Very Good	23	
293	292	Very Good	24	
294	293	Excellent	45	
295	294	Good	14	
296	295	Good	18	
297	296	Good	17	
298	297	Good	16	
299	298	Good	15	
300	299	Very Good	38	
301	300	Very Good	31	
302				

FIGURE 2.15 INITIAL PIVOTTABLE FIELD LIST AND PIVOTTABLE FIELD REPORT FOR THE RESTAURANT DATA



Set Up the PivotTable Field List: Each of the three columns in Figure 2.14 (labeled Restaurant, Quality Rating, and Meal Price ($)) is considered a field by Excel. Fields may be chosen to represent rows, columns, or values in the body of the PivotTable Report. The following steps show how to use Excel's PivotTable Field List to assign the Quality Rating field to the rows, the Meal Price ($) field to the columns, and the Restaurant field to the body of the PivotTable report.

Step 1. In the **PivotTable Field List,** go to **Choose Fields to add to report**
Drag the **Quality Rating** field to the **Row Labels** area
Drag the **Meal Price ($)** field to the **Column Labels** area
Drag the **Restaurant** field to the **Values** area
Step 2. Click on **Sum of Restaurant** in the **Values** area
Step 3. Click **Value Field Settings** from the list of options that appear
Step 4. When the Value Field Settings dialog appears,
Under **Summarize value field by,** choose **Count**
Click **OK**

Figure 2.16 shows the completed PivotTable Field List and a portion of the PivotTable worksheet as it now appears.

Finalize the PivotTable Report: To complete the PivotTable Report, we need to group the columns representing meal prices and place the row labels for quality rating in the proper order. The following steps accomplish this.

Step 1. Right-click in cell B4 or any cell containing meal prices
Step 2. Choose **Group** from the list of options that appears

FIGURE 2.16 COMPLETED PIVOTTABLE FIELD LIST AND A PORTION OF THE PIVOTTABLE REPORT FOR THE RESTAURANT DATA (COLUMNS H:AK ARE HIDDEN)

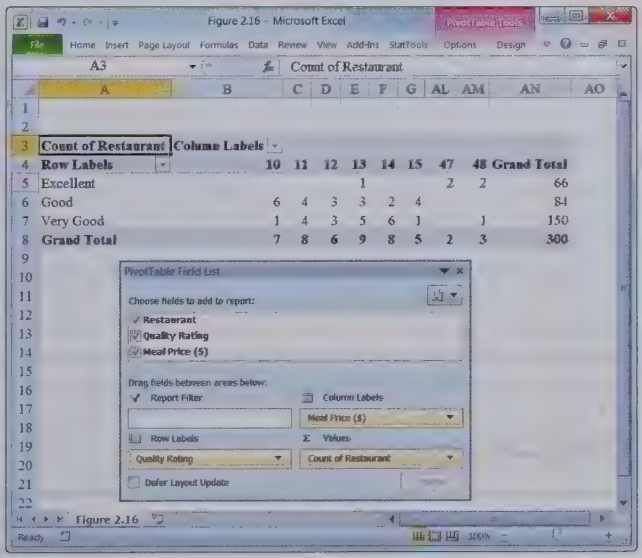
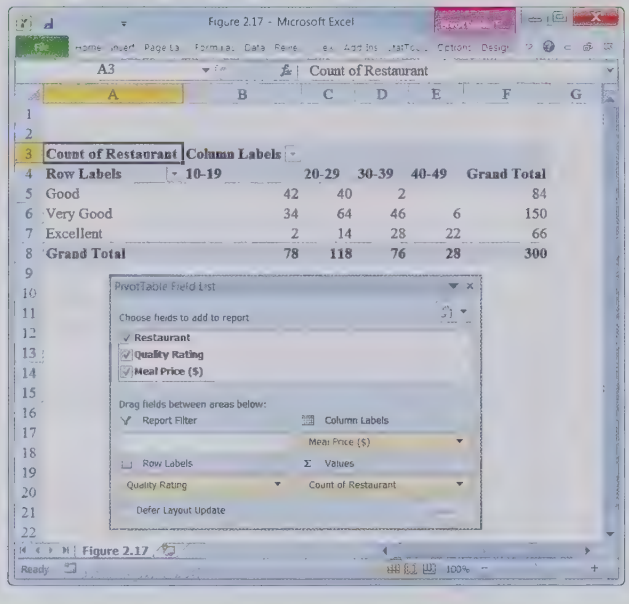


FIGURE 2.17 FINAL PIVOTTABLE REPORT FOR THE RESTAURANT DATA



Step 3. When the **Grouping** dialog box appears,
Enter 10 in the **Starting at** box
Enter 49 in the **Ending at** box
Enter 10 in the **By** box
Click **OK**

Step 4. Right-click on **Excellent** in cell A5

Step 5. Choose **Move** and click **Move "Excellent" to End**

The final PivotTable Report is shown in Figure 2.17. Note that it provides the same information as the crosstabulation shown in Table 2.10. We can now use Excel's chart tools to construct the column chart shown in Figure 2.13. Alternatively, Excel's PivotChart report can be used to create both the crosstabulation and corresponding column chart at the same time.

As we have seen, Excel's PivotTable report and PivotChart report are very useful in summarizing univariate and bivariate data. Once you have used these tools to create tabular and graphical summaries for one or two data sets, we think you will find that both tools are not only very easy to use, but they provide a very powerful option for quickly summarizing very complex data sets as well.

Simpson's Paradox

The data in two or more crosstabulations are often combined or aggregated to produce a summary crosstabulation showing how two variables are related. In such cases, we must be careful in drawing a conclusion because a conclusion based upon aggregate data can be reversed if we look at the unaggregated data. The reversal of conclusions based on aggregate

and unaggregated data is called **Simpson's paradox**. To provide an illustration of Simpson's paradox, we consider an example involving the analysis of verdicts for two judges in two different courts.

Judges Ron Luckett and Dennis Kendall presided over cases in Common Pleas Court and Municipal Court during the past three years. Some of the verdicts they rendered were appealed. In most of these cases the appeals court upheld the original verdicts, but in some cases those verdicts were reversed. For each judge a crosstabulation was developed based upon two variables: Verdict (upheld or reversed) and Type of Court (Common Pleas and Municipal). Suppose that the two crosstabulations were then combined by aggregating the type of court data. The resulting aggregated crosstabulation contains two variables: Verdict (upheld or reversed) and Judge (Luckett or Kendall). This crosstabulation shows the number of appeals in which the verdict was upheld and the number in which the verdict was reversed for both judges. The following crosstabulation shows these results along with the column percentages in parentheses next to each value.

	Judge		
Verdict	Luckett	Kendall	Total
Upheld	129 (86%)	110 (88%)	239
Reversed	21 (14%)	15 (12%)	36
Total (%)	150 (100%)	125 (100%)	275

A review of the column percentages shows that 86% of the verdicts were upheld for Judge Luckett, while 88% of the verdicts were upheld for Judge Kendall. From this aggregated crosstabulation, we conclude that Judge Kendall is doing the better job because a greater percentage of Judge Kendall's verdicts are being upheld.

The following unaggregated crosstabulations show the cases tried by Judge Luckett and Judge Kendall in each court; column percentages are shown in parentheses next to each value.

	Judge Luckett		
Verdict	Common Pleas	Municipal Court	Total
Upheld	29 (91%)	100 (85%)	129
Reversed	3 (9%)	18 (15%)	21
Total (%)	32 (100%)	118 (100%)	150

	Judge Kendall		
Verdict	Common Pleas	Municipal Court	Total
Upheld	90 (90%)	20 (80%)	110
Reversed	10 (10%)	5 (20%)	15
Total (%)	100 (100%)	25 (100%)	125

From the crosstabulation and column percentages for Judge Luckett, we see that the verdicts were upheld in 91% of the Common Pleas Court cases and in 85% of the Municipal Court cases. From the crosstabulation and column percentages for Judge Kendall, we see that the verdicts were upheld in 90% of the Common Pleas Court cases and in 80% of the Municipal Court cases. Thus, when we unaggregate the data, we see that Judge Luckett has a better record because a greater percentage of Judge Luckett's verdicts are being upheld in both courts. This result contradicts the conclusion we reached with the aggregated data crosstabulation that showed Judge Kendall had the better record. This reversal of conclusions based on aggregated and unaggregated data illustrates Simpson's paradox.

The original crosstabulation was obtained by aggregating the data in the separate crosstabulations for the two courts. Note that for both judges the percentage of appeals that resulted in reversals was much higher in Municipal Court than in Common Pleas Court. Because Judge Luckett tried a much higher percentage of his cases in Municipal Court, the aggregated data favored Judge Kendall. When we look at the crosstabulations for the two courts separately, however, Judge Luckett shows the better record. Thus, for the original crosstabulation, we see that the *type of court* is a hidden variable that cannot be ignored when evaluating the records of the two judges.

Because of the possibility of Simpson's paradox, realize that the conclusion or interpretation may be reversed depending on whether you are viewing unaggregated or aggregate crosstabulation data. Before drawing a conclusion, you may want to investigate whether the aggregate or unaggregate form of the crosstabulation provides the better insight and conclusion. Especially when the crosstabulation involves aggreagrated data, you should investigate whether a hidden variable could affect the results such that separate or unaggregated crosstabulations provide a different and possibly better insight and conclusion.

Scatter Diagram and Trendline

A **scatter diagram** is a graphical presentation of the relationship between two quantitative variables, and a **trendline** is a line that provides an approximation of the relationship. As an illustration, consider the advertising/sales relationship for a stereo and sound equipment store in San Francisco. On 10 occasions during the past three months, the store used weekend television commercials to promote sales at its stores. The managers want to investigate whether a relationship exists between the number of commercials shown and sales at the store during the following week. Sample data for the 10 weeks with sales in hundreds of dollars are shown in Table 2.12.

Figure 2.18 shows the scatter diagram and the trendline[2] for the data in Table 2.12. The number of commercials (x) is shown on the horizontal axis and the sales (y) are shown on the vertical axis. For week 1, $x = 2$ and $y = 50$. A point with those coordinates is plotted on the scatter diagram. Similar points are plotted for the other nine weeks. Note that during two of the weeks one commercial was shown, during two of the weeks two commercials were shown, and so on.

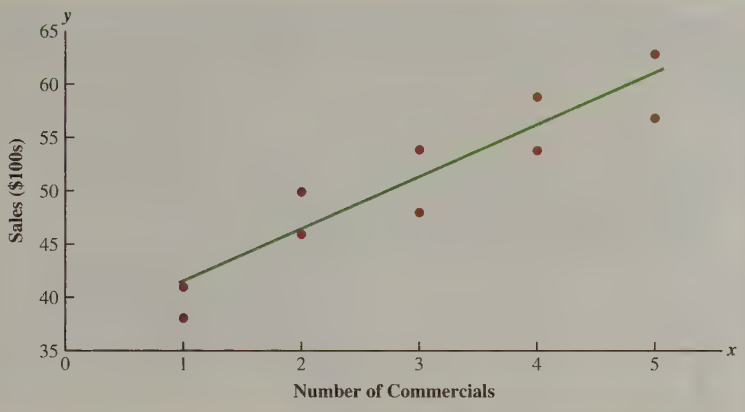
TABLE 2.12 SAMPLE DATA FOR THE STEREO AND SOUND EQUIPMENT STORE

WEB file
Stereo

Week	Number of Commercials x	Sales ($100s) y
1	2	50
2	5	57
3	1	41
4	3	54
5	4	54
6	1	38
7	5	63
8	3	48
9	4	59
10	2	46

[2]The equation of the trendline is $y = 36.15 + 4.95x$. The slope of the trendline is 4.95 and the y-intercept (the point where the line intersects the y-axis) is 36.15. We will discuss in detail the interpretation of the slope and y-intercept for a linear trendline in Chapter 14 when we study simple linear regression.

FIGURE 2.18 SCATTER DIAGRAM AND TRENDLINE FOR THE STEREO AND SOUND EQUIPMENT STORE



The completed scatter diagram in Figure 2.18 indicates a positive relationship between the number of commercials and sales. Higher sales are associated with a higher number of commercials. The relationship is not perfect in that all points are not on a straight line. However, the general pattern of the points and the trendline suggests that the overall relationship is positive.

Some general scatter diagram patterns and the types of relationships they suggest are shown in Figure 2.19. The top left panel depicts a positive relationship similar to the one for the number of commercials and sales example. In the top right panel, the scatter diagram shows no apparent relationship between the variables. The bottom panel depicts a negative relationship where y tends to decrease as x increases.

Using Excel's Chart Tools to Construct a Scatter Diagram and a Trendline

We can use Excel's chart tools to construct a scatter diagram and a trendline for the stereo and sound equipment store data. Refer to Figures 2.20 and 2.21 as we describe the tasks involved.

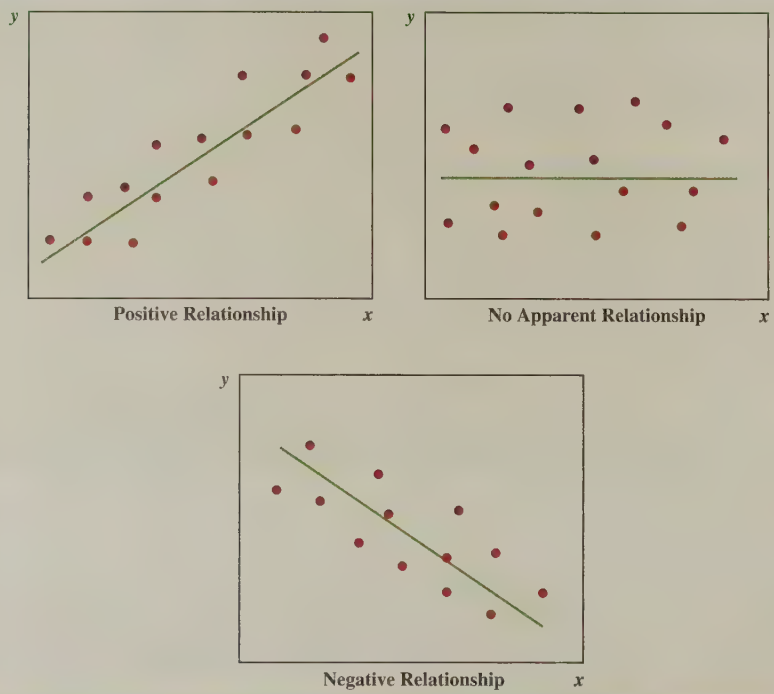
Enter Data: Appropriate labels and the sample data have been entered into cells A1:C11 of the worksheet shown in Figure 2.20.

Enter Functions and Formulas: No functions and formulas are needed.

Apply Tools: The following steps describe how to use Excel's chart tools to produce a scatter diagram from the data in the worksheet.

Step 1. Select cells B2:C11
Step 2. Click the **Insert** tab on the Ribbon
Step 3. In the **Charts** group, click **Scatter**
Step 4. When the list of scatter diagram subtypes appears, click **Scatter with only Markers** (the chart in the upper left corner)

FIGURE 2.19 TYPES OF RELATIONSHIPS DEPICTED BY SCATTER DIAGRAMS



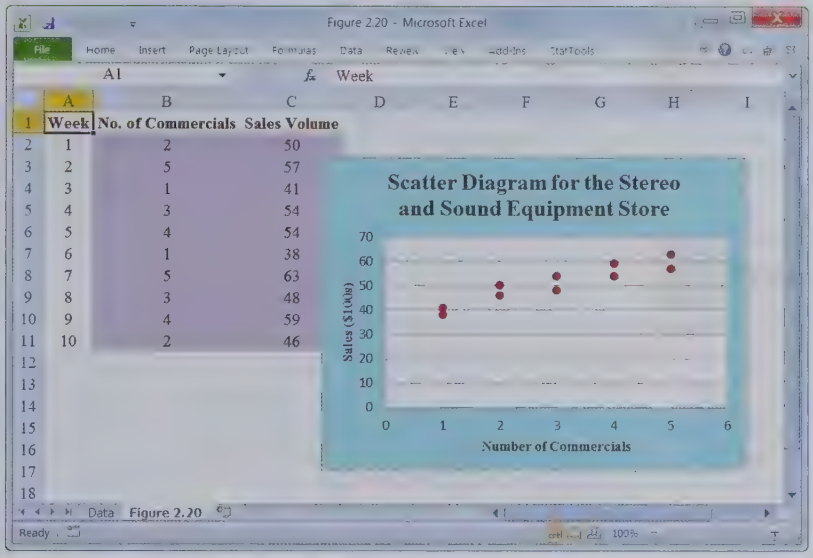
Step 5. In the **Chart Layouts** group, click **Layout 1**
Step 6. Select the **Chart Title** and replace it with **Scatter Diagram for the Stereo and Sound Equipment Store**
Step 7. Select the **Horizontal (Value) Axis Title** and replace it with **Number of Commercials**
Step 8. Select the **Vertical (Value) Axis Title** and replace it with **Sales ($100s)**
Step 9. Right-click the **Series 1 Legend Entry** and click **Delete**

The worksheet displayed in Figure 2.20 shows the scatter diagram produced by Excel. The following steps describe how to add a trendline.

Step 1. Position the mouse pointer over any data point in the scatter diagram and right-click to display a list of options
Step 2. Choose **Add Trendline**
Step 3. When the **Format Trendline** dialog box appears,
Select **Trendline Options**
Choose **Linear** from the **Trend/Regression Type** list
Click **Close**

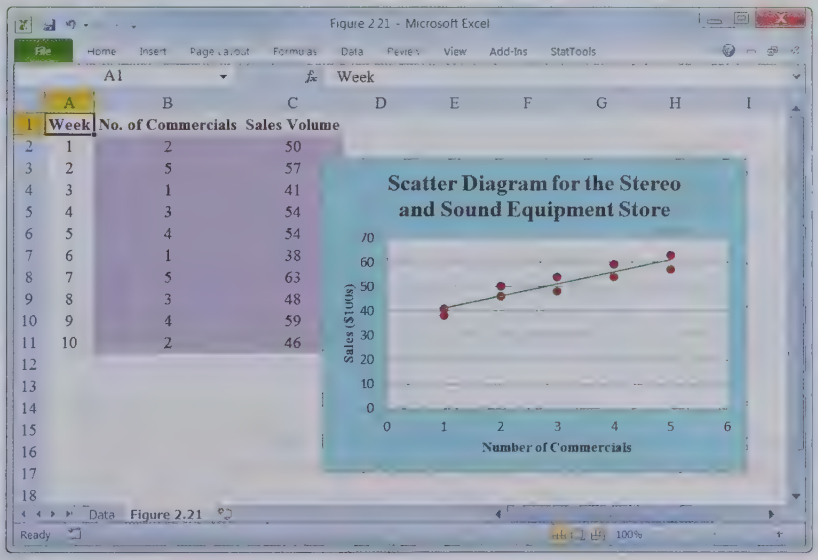
The worksheet displayed in Figure 2.21 shows the scatter diagram with the trendline added.

FIGURE 2.20 SCATTER DIAGRAM FOR THE STEREO AND SOUND EQUIPMENT STORE USING EXCEL'S CHART TOOLS



Week	No. of Commercials	Sales Volume
1	2	50
2	5	57
3	1	41
4	3	54
5	4	54
6	1	38
7	5	63
8	3	48
9	4	59
10	2	46

FIGURE 2.21 SCATTER DIAGRAM AND TRENDLINE FOR THE STEREO AND SOUND EQUIPMENT STORE USING EXCEL'S CHART TOOLS



Week	No. of Commercials	Sales Volume
1	2	50
2	5	57
3	1	41
4	3	54
5	4	54
6	1	38
7	5	63
8	3	48
9	4	59
10	2	46

Exercises

Methods

SELF test

29. The following data are for 30 observations involving two qualitative variables, x and y. The categories for x are A, B, and C; the categories for y are 1 and 2.

WEB file Crosstab

Observation	x	y	Observation	x	y
1	A	1	16	B	2
2	B	1	17	C	1
3	B	1	18	B	1
4	C	2	19	C	1
5	B	1	20	B	1
6	C	2	21	C	2
7	B	1	22	B	1
8	C	2	23	C	2
9	A	1	24	A	1
10	B	1	25	B	1
11	A	1	26	C	2
12	B	1	27	C	2
13	C	2	28	A	1
14	C	2	29	B	1
15	C	2	30	B	2

a. Develop a crosstabulation for the data, with x as the row variable and y as the column variable.
b. Compute the row percentages.
c. Compute the column percentages.
d. What is the relationship, if any, between x and y?

SELF test

30. The following 20 observations are for two quantitative variables, x and y.

WEB file Scatter

Observation	x	y	Observation	x	y
1	−22	22	11	−37	48
2	−33	49	12	34	−29
3	2	8	13	9	−18
4	29	−16	14	−33	31
5	−13	10	15	20	−16
6	21	−28	16	−3	14
7	−13	27	17	−15	18
8	−23	35	18	12	17
9	14	−5	19	−20	−11
10	3	−3	20	−7	−22

a. Develop a scatter diagram for the relationship between x and y.
b. What is the relationship, if any, between x and y?

Applications

31. The following crosstabulation shows household income by educational level of the head of household (*Statistical Abstract of the United States: 2008*).

	Household Income ($1000s)					
Educational Level	Under 25	25.0–49.9	50.0–74.9	75.0–99.9	100 or more	Total
Not H.S. graduate	4207	3459	1389	539	367	9961
H.S. graduate	4917	6850	5027	2637	2668	22099
Some college	2807	5258	4678	3250	4074	20067
Bachelor's degree	885	2094	2848	2581	5379	13787
Beyond bach. deg.	290	829	1274	1241	4188	7822
Total	13106	18490	15216	10248	16676	73736

a. Compute the row percentages and identify the percent frequency distributions of income for households in which the head is a high school graduate and in which the head holds a bachelor's degree.
b. What percentage of households headed by high school graduates earn $75,000 or more? What percentage of households headed by bachelor's degree recipients earn $75,000 or more?
c. Construct percent frequency histograms of income for households headed by persons with a high school degree and for those headed by persons with a bachelor's degree. Is any relationship evident between household income and educational level?

32. Refer again to the crosstabulation of household income by educational level shown in exercise 31.
a. Compute column percentages and identify the percent frequency distributions displayed. What percentage of the heads of households did not graduate from high school?
b. What percentage of the households earning $100,000 or more were headed by a person having schooling beyond a bachelor's degree? What percentage of the households headed by a person with schooling beyond a bachelor's degree earned over $100,000? Why are these two percentages different?
c. Compare the percent frequency distributions for those households earning "Under 25," "100 or more," and for "Total." Comment on the relationship between household income and educational level of the head of household.

33. Recently, management at Oak Tree Golf Course received a few complaints about the condition of the greens. Several players complained that the greens are too fast. Rather than react to the comments of just a few, the Golf Association conducted a survey of 100 male and 100 female golfers. The survey results are summarized here.

Male Golfers

	Greens Condition	
Handicap	Too Fast	Fine
Under 15	10	40
15 or more	25	25

Female Golfers

	Greens Condition	
Handicap	Too Fast	Fine
Under 15	1	9
15 or more	39	51

a. Combine these two crosstabulations into one, with Male and Female as the row labels and Too Fast and Fine as the column labels. Which group shows the highest percentage saying that the greens are too fast?
b. Refer to the initial crosstabulations. For those players with low handicaps (better players), which group (male or female) shows the highest percentage saying the greens are too fast?

c. Refer to the initial crosstabulations. For those players with higher handicaps, which group (male or female) shows the highest percentage saying the greens are too fast?
d. What conclusions can you draw about the preferences of men and women concerning the speed of the greens? Are the conclusions you draw from part (a) as compared with parts (b) and (c) consistent? Explain any apparent inconsistencies.

34. Table 2.13 shows a data set containing information for 45 mutual funds that are part of the *Morningstar Funds 500* for 2008. The data set includes the following five variables:

Fund Type: The type of fund, labeled DE (Domestic Equity), IE (International Equity), and FI (Fixed Income)

Net Asset Value ($): The closing price per share

5-Year Average Return (%): The average annual return for the fund over the past 5 years

Expense Ratio (%): The percentage of assets deducted each fiscal year for fund expenses

Morningstar Rank: The risk adjusted star rating for each fund; Morningstar ranks go from a low of 1-Star to a high of 5-Stars

a. Prepare a crosstabulation of the data on Fund Type (rows) and the average annual return over the past 5 years (columns). Use classes of 0–9.99, 10–19.99, 20–29.99, 30–39.99, 40–49.99, and 50–59.99 for the 5-Year Average Return (%).
b. Prepare a frequency distribution for the data on Fund Type.
c. Prepare a frequency distribution for the data on 5-Year Average Return (%).
d. How has the crosstabulation helped in preparing the frequency distributions in parts (b) and (c)?
e. What conclusions can you draw about the fund type and the average return over the past 5 years?

35. Refer to the data in Table 2.13.
a. Prepare a crosstabulation of the data on Fund Type (rows) and the expense ratio (columns). Use classes of .25–.49, .50–.74, .75–.99, 1.00–1.24, and 1.25–1.49 for Expense Ratio (%).
b. Prepare a percent frequency distribution for Expense Ratio (%).
c. What conclusions can you draw about fund type and the expense ratio?

36. Refer to the data in Table 2.13.
a. Prepare a scatter diagram with 5-Year Average Return (%) on the horizontal axis and Net Asset Value ($) on the vertical axis.
b. Comment on the relationship, if any, between the variables.

37. The U.S. Department of Energy's Fuel Economy Guide provides fuel efficiency data for cars and trucks (Fuel Economy website, September 8, 2012). A portion of the data for 149 compact, midsize, and large cars is shown in Table 2.14. The data set contains the following variables:

Size: Compact, Midsize, and Large

Displacement: Engine size in liters

Cylinders: Number of cylinders in the engine

Drive: All wheel (A), front wheel (F), and rear wheel (R)

Fuel Type: Premium (P) or regular (R) fuel

City MPG: Fuel efficiency rating for city driving in terms of miles per gallon

Hwy MPG: Fuel efficiency rating for highway driving in terms of miles per gallon

TABLE 2.13 FINANCIAL DATA FOR A SAMPLE OF 45 MUTUAL FUNDS

WEB file
MutualFunds

Fund Name	Fund Type	Net Asset Value ($)	5-Year Average Return (%)	Expense Ratio (%)	Morningstar Rank
Amer Cent Inc & Growth Inv	DE	28.88	12.39	0.67	2-Star
American Century Intl. Disc	IE	14.37	30.53	1.41	3-Star
American Century Tax-Free Bond	FI	10.73	3.34	0.49	4-Star
American Century Ultra	DE	24.94	10.88	0.99	3-Star
Ariel	DE	46.39	11.32	1.03	2-Star
Artisan Intl Val	IE	25.52	24.95	1.23	3-Star
Artisan Small Cap	DE	16.92	15.67	1.18	3-Star
Baron Asset	DE	50.67	16.77	1.31	5-Star
Brandywine	DE	36.58	18.14	1.08	4-Star
Brown Cap Small	DE	35.73	15.85	1.20	4-Star
Buffalo Mid Cap	DE	15.29	17.25	1.02	3-Star
Delafield	DE	24.32	17.77	1.32	4-Star
DFA U.S. Micro Cap	DE	13.47	17.23	0.53	3-Star
Dodge & Cox Income	FI	12.51	4.31	0.44	4-Star
Fairholme	DE	31.86	18.23	1.00	5-Star
Fidelity Contrafund	DE	73.11	17.99	0.89	5-Star
Fidelity Municipal Income	FI	12.58	4.41	0.45	5-Star
Fidelity Overseas	IE	48.39	23.46	0.90	4-Star
Fidelity Sel Electronics	DE	45.60	13.50	0.89	3-Star
Fidelity Sh-Term Bond	FI	8.60	2.76	0.45	3-Star
Fidelity	DE	39.85	14.40	0.56	4-Star
FPA New Income	FI	10.95	4.63	0.62	3-Star
Gabelli Asset AAA	DE	49.81	16.70	1.36	4-Star
Greenspring	DE	23.59	12.46	1.07	3-Star
Janus	DE	32.26	12.81	0.90	3-Star
Janus Worldwide	IE	54.83	12.31	0.86	2-Star
Kalmar Gr Val Sm Cp	DE	15.30	15.31	1.32	3-Star
Managers Freemont Bond	FI	10.56	5.14	0.60	5-Star
Marsico 21st Century	DE	17.44	15.16	1.31	5-Star
Mathews Pacific Tiger	IE	27.86	32.70	1.16	3-Star
Meridan Value	DE	31.92	15.33	1.08	4-Star
Oakmark I	DE	40.37	9.51	1.05	2-Star
PIMCO Emerg Mkts Bd D	FI	10.68	13.57	1.25	3-Star
RS Value A	DE	26.27	23.68	1.36	4-Star
T. Rowe Price Latin Am.	IE	53.89	51.10	1.24	4-Star
T. Rowe Price Mid Val	DE	22.46	16.91	0.80	4-Star
Templeton Growth A	IE	24.07	15.91	1.01	3-Star
Thornburg Value A	DE	37.53	15.46	1.27	4-Star
USAA Income	FI	12.10	4.31	0.62	3-Star
Vanguard Equity-Inc	DE	24.42	13.41	0.29	4-Star
Vanguard Global Equity	IE	23.71	21.77	0.64	5-Star
Vanguard GNMA	FI	10.37	4.25	0.21	5-Star
Vanguard Sht-Tm TE	FI	15.68	2.37	0.16	3-Star
Vanguard Sm Cp Idx	DE	32.58	17.01	0.23	3-Star
Wasatch Sm Cp Growth	DE	35.41	13.98	1.19	4-Star

TABLE 2.14 FUEL EFFICIENCY DATA FOR 311 CARS

Car	Size	Displacement	Cylinders	Drive	Fuel Type	City MPG	Hwy MPG
1	Compact	2.0	4	F	P	22	30
2	Compact	2.0	4	A	P	21	29
3	Compact	2.0	4	A	P	21	31
•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•
94	Midsize	3.5	6	A	R	17	25
95	Midsize	2.5	4	F	R	23	33
•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•
148	Large	6.7	12	R	P	11	18
149	Large	6.7	12	R	P	11	18

WEB file FuelData2012

The complete data set is contained in the file named FuelData2012.

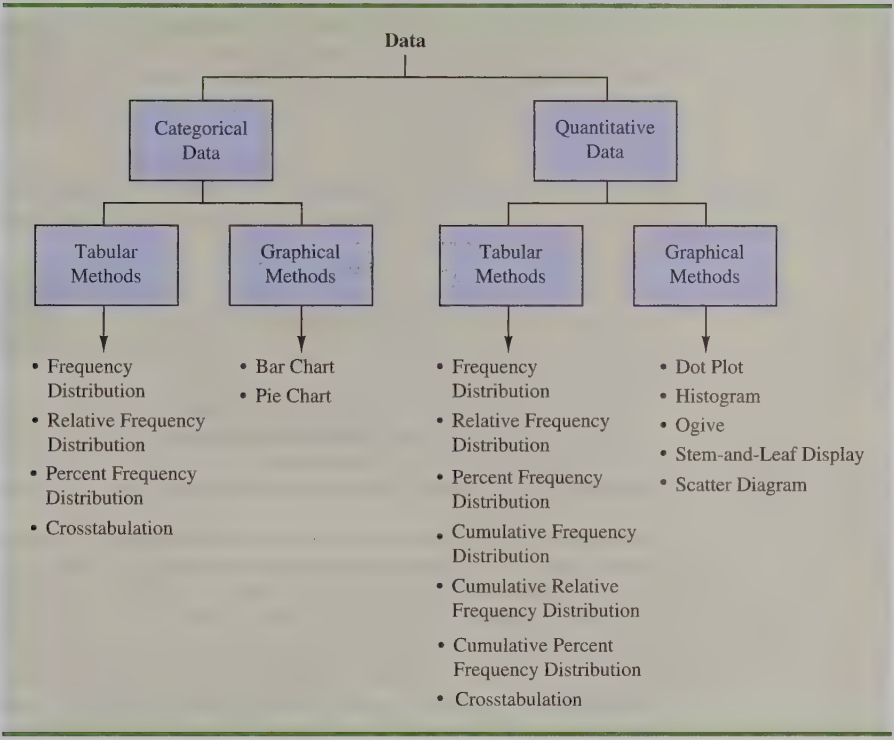
a. Prepare a crosstabulation of the data on Size (rows) and Hwy MPG (columns). Use classes of 15–19, 20–24, 25–29, 30–34, 35–39, and 40–44 for Hwy MPG.
b. Comment on the relationship beween Size and Hwy MPG.
c. Prepare a crosstabulation of the data on Drive (rows) and City MPG (columns). Use classes of 10–14, 15–19, 20–24, 25–29, 30–34, 35–39, and 40–44 for City MPG.
d. Comment on the relationship between Drive and City MPG.
e. Prepare a crosstabulation of the data on Fuel Type (rows) and City MPG (columns). Use classes of 10–14, 15–19, 20–24, 25–29, 30–34, 35–39, and 40–44 for City MPG.
f. Comment on the relationship between Fuel Type and City MPG.

38. Refer to exercise 37 and the data in the file named FuelData2012.
a. Prepare a crosstabulation of the data on Displacement (rows) and Hwy MPG (columns). Use classes of 1–2, 2–3, 3–4, 4–5, 5–6, and 6–7 for Displacement. Use classes of 15–19, 20–24, 25–29, 30–34, and 35–39 for Hwy MPG.
b. Comment on the relationship, if any, between Displacement and Hwy MPG.
c. Develop a scatter diagram of the data on Displacement and Hwy MPG. Use the vertical axis for Hwy MPG.
d. What does the scatter diagram developed in part (c) indicate about the relationship, if any, between Displacement and Hwy MPG?
e. In investigating the relationship between Displacement and Hwy MPG you developed a tabular summary of the data (crosstabulation) and a graphical summary (scatter diagram). In this case which approach do you prefer? Explain.

Summary

A set of data, even if modest in size, is often difficult to interpret directly in the form in which it is gathered. Tabular and graphical methods provide procedures for organizing and summarizing data so that patterns are revealed and the data are more easily interpreted. Frequency distributions, relative frequency distributions, percent frequency distributions, bar charts, and pie charts were presented as tabular and graphical procedures for summarizing categorical data. Frequency distributions, relative frequency distributions, percent frequency distributions, histograms, cumulative frequency distributions, cumulative relative

FIGURE 2.22 TABULAR AND GRAPHICAL METHODS FOR SUMMARIZING DATA



frequency distributions, cumulative percent frequency distributions, and ogives were presented as ways of summarizing quantitative data. A stem-and-leaf display provides an exploratory data analysis technique that can be used to summarize quantitative data. Crosstabulation was presented as a tabular method for summarizing data for two variables. The scatter diagram was introduced as a graphical method for showing the relationship between two quantitative variables. Figure 2.22 shows the tabular and graphical methods presented in this chapter.

With large data sets, computer software packages such as Excel and StatTools are essential in constructing tabular and graphical summaries of data.

Glossary

Categorical data Labels or names used to identify categories of like items.
Quantitative data Numerical values that indicate how much or how many.
Frequency distribution A tabular summary of data showing the number (frequency) of data values in each of several nonoverlapping classes.
Relative frequency distribution A tabular summary of data showing the fraction or proportion of data values in each of several nonoverlapping classes.
Percent frequency distribution A tabular summary of data showing the percentage of data values in each of several nonoverlapping classes.

Bar chart A graphical device for depicting categorical data that have been summarized in a frequency, relative frequency, or percent frequency distribution.
Pie chart A graphical device for presenting data summaries based on subdivision of a circle into sectors that correspond to the relative frequency for each class.
Class midpoint The value halfway between the lower and upper class limits.
Dot plot A graphical device that summarizes data by the number of dots above each data value on the horizontal axis.
Histogram A graphical presentation of a frequency distribution, relative frequency distribution, or percent frequency distribution of quantitative data constructed by placing the class intervals on the horizontal axis and the frequencies, relative frequencies, or percent frequencies on the vertical axis.
Cumulative frequency distribution A tabular summary of quantitative data showing the number of data values that are less than or equal to the upper class limit of each class.
Cumulative relative frequency distribution A tabular summary of quantitative data showing the fraction or proportion of data values that are less than or equal to the upper class limit of each class.
Cumulative percent frequency distribution A tabular summary of quantitative data showing the percentage of data values that are less than or equal to the upper class limit of each class.
Ogive A graph of a cumulative distribution.
Exploratory data analysis Methods that use simple arithmetic and easy-to-draw graphs to summarize data quickly.
Stem-and-leaf display An exploratory data analysis technique that simultaneously rank orders quantitative data and provides insight about the shape of the distribution.
Crosstabulation A tabular summary of data for two variables. The classes for one variable are represented by the rows; the classes for the other variable are represented by the columns.
Simpson's paradox Conclusions drawn from two or more separate crosstabulations that can be reversed when the data are aggregated into a single crosstabulation.
Scatter diagram A graphical presentation of the relationship between two quantitative variables. One variable is shown on the horizontal axis and the other variable is shown on the vertical axis.
Trendline A line that provides an approximation of the relationship between two quantitative variables.

Key Formulas

Relative Frequency

$$\frac{\text{Frequency of the class}}{n} \tag{2.1}$$

Approximate Class Width

$$\frac{\text{Largest data value} - \text{Smallest data value}}{\text{Number of classes}} \tag{2.2}$$

Supplementary Exercises

39. The Higher Education Research Institute at UCLA provides statistics on the most popular majors among incoming college freshmen. The five most popular majors are Arts and Humanities (A), Business Administration (B), Engineering (E), Professional (P), and Social Science (S) (*The New York Times Almanac,* 2006). A broad range of other (O) majors,

including biological science, physical science, computer science, and education, are grouped together. The majors selected for a sample of 64 college freshmen follow.

WEB file Major

S	P	P	O	B	E	O	E	P	O	O	B	O	O	O	A
O	E	E	B	S	O	B	O	A	O	E	O	E	O	B	P
B	A	S	O	E	A	B	O	S	S	O	O	E	B	O	B
A	E	B	E	A	A	P	O	O	E	O	B	B	O	P	B

a. Show a frequency distribution and percent frequency distribution.
b. Show a bar chart.
c. What percentage of freshmen select one of the five most popular majors?
d. What is the most popular major for incoming freshmen? What percentage of freshmen select this major?

WEB file GMSales

40. General Motors had a 23% share of the automobile industry with sales coming from eight divisions: Buick, Cadillac, Chevrolet, GMC, Hummer, Pontiac, Saab, and Saturn (*Forbes*, December 22, 2008). The data set GMSales shows the sales for a sample of 200 General Motors vehicles. The division for the vehicle is provided for each sale.
    a. Show the frequency distribution and the percent frequency distribution of sales by division for General Motors.
    b. Show a bar chart of the percent frequency distribution.
    c. Which General Motors division was the company leader in sales? What was the percentage of sales for this division? Was this General Motors' most important division? Explain.
    d. Due to the ongoing recession, high gasoline prices, and the decline in automobile sales, General Motors was facing bankruptcy in 2009. A government "bail-out" loan and a restructuring of the company were anticipated. Expectations were that General Motors could not continue to operate all eight divisions. Based on the percentage of sales, which of the eight divisions looked to be the best candidates for General Motors to discontinue? Which divisions looked to be the least likely candidates for General Motors to discontinue?

41. Dividend yield is the annual dividend paid by a company expressed as a percentage of the price of the stock (Dividend/Stock Price × 100). The dividend yield for the Dow Jones Industrial Average companies is shown in Table 2.15 (*The Wall Street Journal*, June 8, 2009).

TABLE 2.15 DIVIDEND YIELD FOR DOW JONES INDUSTRIAL AVERAGE COMPANIES

WEB file DYield

Company	Dividend Yield %	Company	Dividend Yield %
3M	3.6	IBM	2.1
Alcoa	1.3	Intel	3.4
American Express	2.9	JPMorgan Chase	0.5
AT&T	6.6	Johnson & Johnson	3.6
Bank of America	0.4	Kraft Foods	4.4
Boeing	3.8	McDonald's	3.4
Caterpillar	4.7	Merck	5.5
Chevron	3.9	Microsoft	2.5
Cisco Systems	0.0	Pfizer	4.2
Coca-Cola	3.3	Procter & Gamble	3.4
DuPont	5.8	Travelers	3.0
ExxonMobil	2.4	United Technologies	2.9
General Electric	9.2	Verizon	6.3
Hewlett-Packard	0.9	Walmart Stores	2.2
Home Depot	3.9	Walt Disney	1.5

a. Construct a frequency distribution and percent frequency distribution.
b. Construct a histogram.
c. Comment on the shape of the distribution.
d. What do the tabular and graphical summaries tell about the dividend yields among the Dow Jones Industrial Average companies?
e. What company has the highest dividend yield? If the stock for this company currently sells for $14 per share and you purchase 500 shares, how much dividend income will this investment generate in one year?

42. Approximately 1.5 million high school students take the Scholastic Aptitude Test (SAT) each year and nearly 80% of the college and universities without open admissions policies use SAT scores in making admission decisions (College Board, March 2009). The current version of the SAT includes three parts: reading comprehension, mathematics, and writing. A perfect combined score for all three parts is 2400. A sample of SAT scores for the combined three-part SAT is as follows:

WEB file
NewSAT

1665	1525	1355	1645	1780
1275	2135	1280	1060	1585
1650	1560	1150	1485	1990
1590	1880	1420	1755	1375
1475	1680	1440	1260	1730
1490	1560	940	1390	1175

a. Show a frequency distribution and histogram. Begin with the first class starting at 800 and use a class width of 200.
b. Comment on the shape of the distribution.
c. What other observations can be made about the SAT scores based on the tabular and graphical summaries?

43. The Pittsburgh Steelers defeated the Arizona Cardinals 27 to 23 in professional football's 43rd Super Bowl. With this win, its sixth championship, the Pittsburgh Steelers became the team with the most wins in the 43-year history of the event (*Tampa Tribune,* February 2, 2009). The Super Bowl has been played in eight different states: Arizona (AZ), California (CA), Florida (FL), Georgia (GA), Louisiana (LA), Michigan (MI), Minnesota (MN), and Texas (TX). Data in the following table show the state where the Super Bowls were played and the point margin of victory for the winning team.

WEB file
SuperBowl

Super Bowl	State	Won By Points	Super Bowl	State	Won By Points	Super Bowl	State	Won By Points
1	CA	25	16	MI	5	31	LA	14
2	FL	19	17	CA	10	32	CA	7
3	FL	9	18	FL	19	33	FL	15
4	LA	16	19	CA	22	34	GA	7
5	FL	3	20	LA	36	35	FL	27
6	FL	21	21	CA	19	36	LA	3
7	CA	7	22	CA	32	37	CA	27
8	TX	17	23	FL	4	38	TX	3
9	LA	10	24	LA	45	39	FL	3
10	FL	4	25	FL	1	40	MI	11
11	CA	18	26	MN	13	41	FL	12
12	LA	17	27	CA	35	42	AZ	3
13	FL	4	28	GA	17	43	FL	4
14	CA	12	29	FL	23			
15	LA	17	30	AZ	10			

a. Show a frequency distribution and bar chart for the state where the Super Bowl was played.
b. What conclusions can you draw from your summary in part (a)? What percentage of Super Bowls were played in the states of Florida or California? What percentage of Super Bowls were played in northern or cold-weather states?
c. Show a stretched stem-and-leaf display for the point margin of victory for the winning team. Show a histogram.
d. What conclusions can you draw from your summary in part (c)? What percentage of Super Bowls have been close games with the margin of victory less than 5 points? What percentage of Super Bowls have been won by 20 or more points?
e. The closest Super Bowl occurred when the New York Giants beat the Buffalo Bills. Where was this game played and what was the winning margin of victory? The biggest point margin in Super Bowl history occurred when the San Francisco 49ers beat the Denver Broncos. Where was this game played and what was the winning margin of victory?

44. Data from the U.S. Census Bureau provides the population by state in millions of people (*The World Almanac,* 2006).

WEB file
Population

State	Population	State	Population	State	Population
Alabama	4.5	Louisiana	4.5	Ohio	11.5
Alaska	0.7	Maine	1.3	Oklahoma	3.5
Arizona	5.7	Maryland	5.6	Oregon	3.6
Arkansas	2.8	Massachusetts	6.4	Pennsylvania	12.4
California	35.9	Michigan	10.1	Rhode Island	1.1
Colorado	4.6	Minnesota	5.1	South Carolina	4.2
Connecticut	3.5	Mississippi	2.9	South Dakota	0.8
Delaware	0.8	Missouri	5.8	Tennessee	5.9
Florida	17.4	Montana	0.9	Texas	22.5
Georgia	8.8	Nebraska	1.7	Utah	2.4
Hawaii	1.3	Nevada	2.3	Vermont	0.6
Idaho	1.4	New Hampshire	1.3	Virginia	7.5
Illinois	12.7	New Jersey	8.7	Washington	6.2
Indiana	6.2	New Mexico	1.9	West Virginia	1.8
Iowa	3.0	New York	19.2	Wisconsin	5.5
Kansas	2.7	North Carolina	8.5	Wyoming	0.5
Kentucky	4.1	North Dakota	0.6		

a. Develop a frequency distribution, a percent frequency distribution, and a histogram. Use a class width of 2.5 million.
b. Discuss the skewness in the distribution.
c. What observations can you make about the population of the 50 states?

45. *Drug Store News* (September 2002) provided data on annual pharmacy sales for the leading pharmacy retailers in the United States. The following data are annual sales in millions.

Retailer	Sales	Retailer	Sales
Ahold USA	$ 1,700	Medicine Shoppe	$ 1,757
CVS	12,700	Rite-Aid	8,637
Eckerd	7,739	Safeway	2,150
Kmart	1,863	Walgreens	11,660
Kroger	3,400	Walmart	7,250

a. Show a stem-and-leaf display.
b. Identify the annual sales levels for the smallest, medium, and largest drug retailers.
c. What are the two largest drug retailers?

46. The daily high and low temperatures for 20 cities follow (*USA Today,* March 3, 2006).

City	High	Low	City	High	Low
Albuquerque	66	39	Los Angeles	60	46
Atlanta	61	35	Miami	84	65
Baltimore	42	26	Minneapolis	30	11
Charlotte	60	29	New Orleans	68	50
Cincinnati	41	21	Oklahoma City	62	40
Dallas	62	47	Phoenix	77	50
Denver	60	31	Portland	54	38
Houston	70	54	St. Louis	45	27
Indianapolis	42	22	San Francisco	55	43
Las Vegas	65	43	Seattle	52	36

a. Prepare a stem-and-leaf display of the high temperatures.
b. Prepare a stem-and-leaf display of the low temperatures.
c. Compare the two stem-and-leaf displays and make comments about the difference between the high and low temperatures.
d. Provide a frequency distribution for both high and low temperatures.

47. Refer to the data set for high and low temperatures for 20 cities in exercise 46.
a. Develop a scatter diagram to show the relationship between the two variables, high temperature and low temperature.
b. Comment on the relationship between high and low temperatures.

48. One of the questions in a *Financial Times*/Harris Poll was, "How much do you favor or oppose a higher tax on higher carbon emission cars?" Possible responses were strongly favor, favor more than oppose, oppose more than favor, and strongly oppose. The following crosstabulation shows the responses obtained for 5372 adults surveyed in four countries in Europe and the United States (Harris Interactive website, February 27, 2008).

	Country					
Level of Support	Great Britain	Italy	Spain	Germany	United States	Total
Strongly favor	337	334	510	222	214	1617
Favor more than oppose	370	408	355	411	327	1871
Oppose more than favor	250	188	155	267	275	1135
Strongly oppose	130	115	89	211	204	749
Total	1087	1045	1109	1111	1020	5372

a. Construct a percent frequency distribution for the level of support variable. Do you think the results show support for a higher tax on higher carbon emission cars?
b. Construct a percent frequency distribution for the country variable.
c. Does the level of support among adults in the European countries appear to be different than the level of support among adults in the United States? Explain.

49. Western University has only one women's softball scholarship remaining for the coming year. The final two players that Western is considering are Allison Fealey and Emily Janson. The coaching staff has concluded that the speed and defensive skills are virtually identical for the two players, and that the final decision will be based on which player has the best batting average. Crosstabulations of each player's batting performance in their junior and senior years of high school are as follows:

	Allison Fealey	
Outcome	Junior	Senior
Hit	15	75
No Hit	25	175
Total At-Bats	40	250

	Emily Janson	
Outcome	Junior	Senior
Hit	70	35
No Hit	130	85
Total At Bats	200	120

A player's batting average is computed by dividing the number of hits a player has by the total number of at-bats. Batting averages are represented as a decimal number with three places after the decimal.

a. Calculate the batting average for each player in her junior year. Then calculate the batting average of each player in her senior year. Using this analysis, which player should be awarded the scholarship? Explain.

b. Combine or aggregate the data for the junior and senior years into one crosstabulation as follows:

	Player	
Outcome	Fealey	Janson
Hit		
No Hit		
Total At-Bats		

Calculate each player's batting average for the combined two years. Using this analysis, which player should be awarded the scholarship? Explain.

c. Are the recommendations you made in parts (a) and (b) consistent? Explain any apparent inconsistencies.

50. A survey of commercial buildings served by the Cincinnati Gas & Electric Company asked what main heating fuel was used and what year the building was constructed. A partial crosstabulation of the findings follows.

Year Constructed	Fuel Type				
	Electricity	Natural Gas	Oil	Propane	Other
1973 or before	40	183	12	5	7
1974–1979	24	26	2	2	0
1980–1986	37	38	1	0	6
1987–1991	48	70	2	0	1

a. Complete the crosstabulation by showing the row totals and column totals.

b. Show the frequency distributions for year constructed and for fuel type.

c. Prepare a crosstabulation showing column percentages.

d. Prepare a crosstabulation showing row percentages.

e. Comment on the relationship between year constructed and fuel type.

TABLE 2.16 DATA FOR A SAMPLE OF 50 *FORTUNE* 500 COMPANIES

Company	Stockholders' Equity ($1000s)	Market Value ($1000s)	Profit ($1000s)
AGCO	982.1	372.1	60.6
AMP	2698.0	12017.6	2.0
Apple Computer	1642.0	4605.0	309.0
Baxter International	2839.0	21743.0	315.0
Bergen Brunswick	629.1	2787.5	3.1
Best Buy	557.7	10376.5	94.5
Charles Schwab	1429.0	35340.6	348.5
.	.	.	.
.	.	.	.
.	.	.	.
Walgreen	2849.0	30324.7	511.0
Westvaco	2246.4	2225.6	132.0
Whirlpool	2001.0	3729.4	325.0
Xerox	5544.0	35603.7	395.0

WEB file Fortune

51. Table 2.16 contains a portion of the data in the file named Fortune. Data on stockholders' equity, market value, and profits for a sample of 50 *Fortune* 500 companies are shown.
    a. Prepare a crosstabulation for the variables Stockholders' Equity and Profit. Use classes of 0–200, 200–400, . . . , 1000–1200 for Profit, and classes of 0–1200, 1200–2400, . . . , 4800–6000 for Stockholders' Equity.
    b. Compute the row percentages for your crosstabulation in part (a).
    c. What relationship, if any, do you notice between Profit and Stockholders' Equity?
52. Refer to the data set in Table 2.16.
    a. Prepare a crosstabulation for the variables Market Value and Profit.
    b. Compute the row percentages for your crosstabulation in part (a).
    c. Comment on any relationship between the variables.
53. Refer to the data set in Table 2.16.
    a. Prepare a scatter diagram to show the relationship between the variables Profit and Stockholders' Equity.
    b. Comment on any relationship between the variables.
54. Refer to the data set in Table 2.16.
    a. Prepare a scatter diagram to show the relationship between the variables Market Value and Stockholders' Equity.
    b. Comment on any relationship between the variables.

Case Problem 1 Pelican Stores

Pelican Stores, a division of National Clothing, is a chain of women's apparel stores operating throughout the country. The chain recently ran a promotion in which discount coupons were sent to customers of other National Clothing stores. Data collected for a sample of 100 in-store credit card transactions at Pelican Stores during one day while the promotion was running are contained in the file named PelicanStores. Table 2.17 shows a portion of the data set. The Proprietary Card method of payment refers to charges made using a National Clothing charge card. Customers who made a purchase using a discount coupon are referred to as promotional customers and customers who made a purchase but did not use a discount coupon are referred to as regular customers. Because the

TABLE 2.17 DATA FOR A SAMPLE OF 100 CREDIT CARD PURCHASES AT PELICAN STORES

WEB file
PelicanStores

Customer	Type of Customer	Items	Net Sales	Method of Payment	Gender	Marital Status	Age
1	Regular	1	39.50	Discover	Male	Married	32
2	Promotional	1	102.40	Proprietary Card	Female	Married	36
3	Regular	1	22.50	Proprietary Card	Female	Married	32
4	Promotional	5	100.40	Proprietary Card	Female	Married	28
5	Regular	2	54.00	MasterCard	Female	Married	34
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
96	Regular	1	39.50	MasterCard	Female	Married	44
97	Promotional	9	253.00	Proprietary Card	Female	Married	30
98	Promotional	10	287.59	Proprietary Card	Female	Married	52
99	Promotional	2	47.60	Proprietary Card	Female	Married	30
100	Promotional	1	28.44	Proprietary Card	Female	Married	44

promotional coupons were not sent to regular Pelican Stores customers, management considers the sales made to people presenting the promotional coupons as sales it would not otherwise make. Of course, Pelican also hopes that the promotional customers will continue to shop at its stores.

Most of the variables shown in Table 2.17 are self-explanatory, but two of the variables require some clarification.

Items	The total number of items purchased
Net Sales	The total amount ($) charged to the credit card

Pelican's management would like to use this sample data to learn about its customer base and to evaluate the promotion involving discount coupons.

Managerial Report

Use the tabular and graphical methods of descriptive statistics to help management develop a customer profile and to evaluate the promotional campaign. At a minimum, your report should include the following:

1. Percent frequency distribution for key variables.
2. A bar chart or pie chart showing the number of customer purchases attributable to the method of payment.
3. A crosstabulation of type of customer (regular or promotional) versus net sales. Comment on any similarities or differences present.
4. A scatter diagram to explore the relationship between net sales and customer age.

Case Problem 2 Motion Picture Industry

The motion picture industry is a competitive business. More than 50 studios produce a total of 300 to 400 new motion pictures each year, and the financial success of each motion picture varies considerably. The opening weekend gross sales ($millions), the total gross sales ($millions), the number of theaters the movie was shown in, and the number of weeks the motion picture was in the top 60 for gross sales are common variables used to measure

TABLE 2.18 PERFORMANCE DATA FOR 10 MOTION PICTURES

Motion Picture	Opening Gross Sales ($millions)	Total Gross Sales ($millions)	Number of Theaters	Weeks in Top 60
Coach Carter	29.17	67.25	2574	16
Ladies in Lavender	0.15	6.65	119	22
Batman Begins	48.75	205.28	3858	18
Unleashed	10.90	24.47	1962	8
Pretty Persuasion	0.06	0.23	24	4
Fever Pitch	12.40	42.01	3275	14
Harry Potter and the Goblet of Fire	102.69	287.18	3858	13
Monster-in-Law	23.11	82.89	3424	16
White Noise	24.11	55.85	2279	7
Mr. and Mrs. Smith	50.34	186.22	3451	21

WEB file Movies

the success of a motion picture. Data collected for a sample of 100 motion pictures produced in 2005 are contained in the file named Movies. Table 2.18 shows the data for the first 10 motion pictures in this file.

Managerial Report

Use the tabular and graphical methods of descriptive statistics to learn how these variables contribute to the success of a motion picture. Include the following in your report.

1. Tabular and graphical summaries for each of the four variables along with a discussion of what each summary tells us about the motion picture industry.
2. A scatter diagram to explore the relationship between Total Gross Sales and Opening Weekend Gross Sales. Discuss.
3. A scatter diagram to explore the relationship between Total Gross Sales and Number of Theaters. Discuss.
4. A scatter diagram to explore the relationship between Total Gross Sales and Number of Weeks in the Top 60. Discuss.

Appendix 2.1 Using Excel's PivotChart Report to Summarize Categorical Data

WEB file SoftDrink

In Section 2.1 we showed how Excel's COUNTIF function can be used to develop a frequency distribution and how Excel's chart tools can be used to create bar and pie charts for categorical data. In this appendix we show how Excel's PivotChart report can also be used to develop a frequency distribution and graphical displays for categorical data. We will illustrate using the soft drink data in Figure 2.1. Refer to Figure 2.23 as we describe the tasks involved to develop a frequency distribution and a bar chart at the same time.

Enter Data: Same as in Figure 2.1

Enter Functions and Formulas: No functions and formulas are needed.

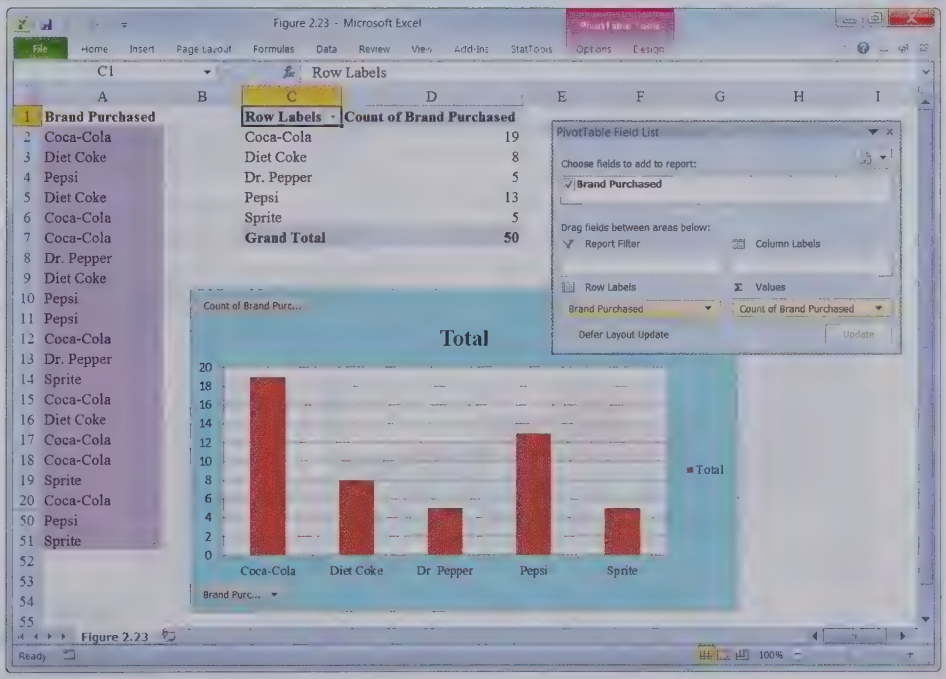
Apply Tools: When using Excel's PivotChart report, each column of data is referred to as a field. Thus, for the soft drink purchase example, the label in cell A1 and the data appearing in cells A2:A51 are referred to as the Brand Purchased field.

Step 1. Click the **Insert** tab on the Ribbon
Step 2. In the **Tables** group, click the word **PivotTable**
Step 3. Choose **PivotChart** from the options that appear
Step 4. When the **Create PivotTable with PivotChart** dialog box appears,
Choose **Select a table or range**
Enter A1:A51 in the **Table/Range** box
Choose **Existing Worksheet** as the location for the PivotTable and PivotChart
Enter C1 in the **Location** box
Click **OK**
Step 5. In the **PivotTable Field List,** go to **Choose Fields to add to report**
Drag the **Brand Purchased** field to the **Axis Fields (Categories)** area
Drag the **Brand Purchased** field to the **Values** area

The PivotChart contains a Value field button in the upper left corner of the chart and an Axis field button in the lower left corner. To remove either or both of these buttons, right-click either button and select the option to remove that button or all field buttons shown on the chart.

Figure 2.23 shows the resulting PivotTable and PivotChart. The PivotTable report (C1:D7) provides the frequency distribution for the soft drink data and the PivotChart provides the corresponding bar chart. If desired, we can change the labels in any cell in the frequency distribution by selecting the cell and typing in the new label. We can also use Excel's chart tools to reformat the bar chart.

FIGURE 2.23 USING EXCEL'S PIVOTCHART REPORT TO CONSTRUCT A FREQUENCY DISTRIBUTION AND A BAR CHART OF SOFT DRINK PREFERENCES



Appendix 2.2 Using Excel's PivotChart Report to Summarize Quantitative Data

WEB file Audit

In Section 2.2 we showed how Excel's PivotTable report can be used to construct a frequency distribution for quantitative data. Excel's PivotChart report can also be used to develop both a frequency distribution and a histogram at the same time. We will illustrate using the audit time data in Table 2.4. Refer to Figure 2.24 as we describe the tasks involved.

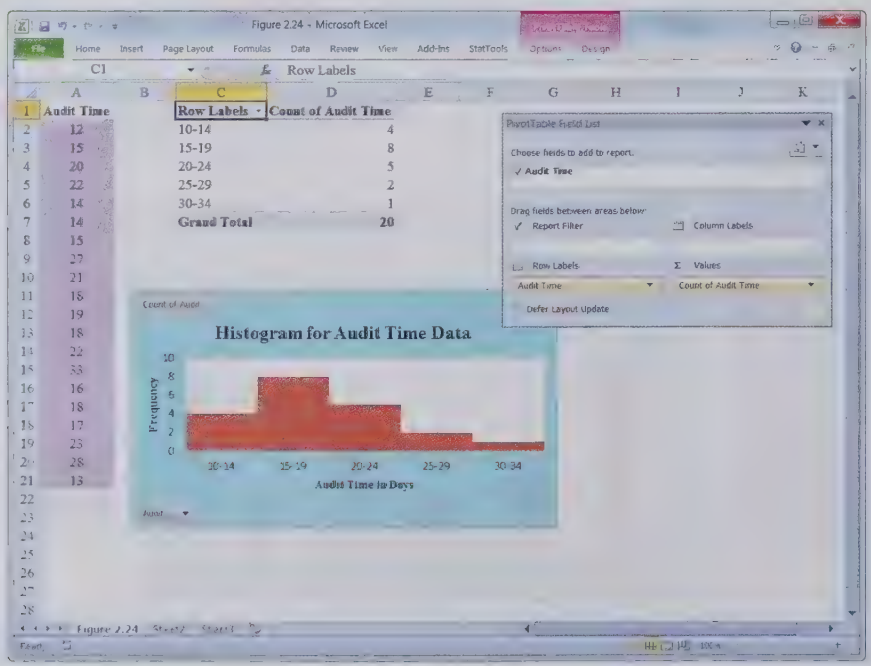
Enter Data: The label "Audit Time" and the 20 audit times are entered into cells A1:A21 of an Excel worksheet.

Enter Functions and Formulas: No functions and formulas are needed.

Apply Tools: The following steps describe how to use Excel's PivotChart report to construct a frequency distribution and a histogram for the audit time data.

Step 1. Click the **Insert** tab on the Ribbon
Step 2. In the **Tables** group, click the word **PivotTable**
Step 3. Choose **PivotChart** from the options that appear
Step 4. When the **Create PivotTable with PivotChart** dialog box appears,
Choose **Select a table or range**
Enter A1:A21 in the **Table/Range** box

FIGURE 2.24 USING EXCEL'S PIVOTCHART REPORT TO CONSTRUCT A FREQUENCY DISTRIBUTION AND A HISTOGRAM FOR THE AUDIT TIME DATA



Choose **Existing Worksheet** as the location for the PivotTable and PivotChart
Enter C1 in the **Location** box
Click **OK**

Step 5. In the **PivotTable Field List,** go to **Choose Fields to add to report**
Drag the **Audit Time** field to the **Axis Fields (Categories)** area
Drag the **Audit Time** field to the **Values** area

Step 6. Click **Sum of Audit Time** in the **Values** area

Step 7. Click **Value Field Settings** from the list of options that appears

Step 8. When the Value Field Settings dialog appears,
Under **Summarize value field by,** choose **Count**
Click **OK**

Step 9. Right-click cell C2 in the PivotTable report or any other cell containing an audit time

Step 10. Choose **Group** from the list of options that appears

Step 11. When the **Grouping** dialog box appears,
Enter 10 in the **Starting at** box
Enter 34 in the **Ending at** box
Enter 5 in the **By** box
Click **OK** (a PivotChart will appear)

Step 12. Click inside the resulting PivotChart

Step 13. Click the **Design** tab on the Ribbon

Step 14. In the **Chart Layouts** group, click the **More** button (the downward pointing arrow with a line over it) to display all the options

The PivotChart contains a Value field button in the upper left corner of the chart and an Axis field button in the lower left corner. To remove either or both of these buttons, right-click either button and select the option to remove that button or all field buttons shown on the chart.

Step 15. Choose **Layout 8**

Step 16. Select the **Chart Title** and replace it with **Histogram for Audit Time Data**

Step 17. Select the **Horizontal (Category) Axis Title** and replace it with **Audit Time in Days**

Step 18. Select the **Vertical (Value) Axis Title** and replace it with **Frequency**

Figure 2.24 shows the resulting PivotTable and PivotChart. We see that the PivotTable report provides the frequency distribution for the audit time data and the PivotChart provides the corresponding histogram. If desired, we can change the labels in any cell in the frequency distribution by selecting the cell and typing in the new label. We can also use Excel's chart tools as previously described to reformat the histogram.

Appendix 2.3 Using StatTools for Tabular and Graphical Presentations

In this appendix we show how StatTools can be used to construct a histogram and a scatter diagram.

Histogram

We use the audit time data in Table 2.4 to illustrate. Begin by using the Data Set Manager to create a StatTools data set for these data using the procedure described in the appendix in Chapter 1. The following steps will generate a histogram.

WEB file Audit

Step 1. Click the **StatTools** tab on the Ribbon

Step 2. In the **Analyses Group,** click **Summary Graphs**

Step 3. Choose the **Histogram** option

Step 4. When the StatTools - Histogram dialog box appears,
In the **Variables** section, select **Audit Time**
In the **Options** section,
Enter 5 in the **Number of Bins** box
Enter 9.5 in the **Histogram Minimum** box
Enter 34.5 in the **Histogram Maximum** box
Choose **Categorical** in the **X-Axis** box
Choose **Frequency** in the **Y-Axis** box
Click **OK**

A histogram for the audit time data similar to the histogram shown in Figure 2.9 will appear. The only difference is the histogram developed using StatTools shows the class midpoints on the horizontal axis.

Scatter Diagram

We use the stereo and sound equipment data in Table 2.12 to demonstrate the construction of a scatter diagram. Begin by using the Data Set Manager to create a StatTools data set for these data using the procedure described in the appendix in Chapter 1. The following steps will generate a scatter diagram.

Step 1. Click the **StatTools** tab on the Ribbon
Step 2. In the **Analyses Group**, click **Summary Graphs**
Step 3. Choose the **Scatterplot** option
Step 4. When the StatTools - Scatterplot dialog box appears,
In the **Variables** section,
In the column labeled **X**, select **No. of Commercials**
In the column labeled **Y**, select **Sales Volume**
Click **OK**

A scatter diagram similar to the one shown in Figure 2.20 will appear.

CHAPTER 3

Descriptive Statistics: Numerical Measures

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STATISTICS in PRACTICE

SMALL FRY DESIGN*

SANTA ANA, CALIFORNIA

Founded in 1997, Small Fry Design is a toy and accessory company that designs and imports products for infants. The company's product line includes teddy bears, mobiles, musical toys, rattles, and security blankets and features high-quality soft toy designs with an emphasis on color, texture, and sound. The products are designed in the United States and manufactured in China.

Small Fry Design uses independent representatives to sell the products to infant furnishing retailers, children's accessory and apparel stores, gift shops, upscale department stores, and major catalog companies. Currently, Small Fry Design products are distributed in more than 1000 retail outlets throughout the United States.

Cash flow management is one of the most critical activities in the day-to-day operation of this company. Ensuring sufficient incoming cash to meet both current and ongoing debt obligations can mean the difference between business success and failure. A critical factor in cash flow management is the analysis and control of accounts receivable. By measuring the average age and dollar value of outstanding invoices, management can predict cash availability and monitor changes in the status of accounts receivable. The company set the following goals: the average age for outstanding invoices should not exceed 45 days, and the dollar value of invoices more than 60 days old should not exceed 5% of the dollar value of all accounts receivable.

In a recent summary of accounts receivable status, the following descriptive statistics were provided for the age of outstanding invoices:

Mean	40 days
Median	35 days
Mode	31 days

*The authors are indebted to John A. McCarthy, President of Small Fry Design, for providing this Statistics in Practice.

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Small Fry Design uses descriptive statistics to monitor its accounts receivable and incoming cash flow.

Interpretation of these statistics shows that the mean or average age of an invoice is 40 days. The median shows that half of the invoices remain outstanding 35 days or more. The mode of 31 days, the most frequent invoice age, indicates that the most common length of time an invoice is outstanding is 31 days. The statistical summary also showed that only 3% of the dollar value of all accounts receivable was more than 60 days old. Based on the statistical information, management was satisfied that accounts receivable and incoming cash flow were under control.

In this chapter, you will learn how to compute and interpret some of the statistical measures used by Small Fry Design. In addition to the mean, median, and mode, you will learn about other descriptive statistics such as the range, variance, standard deviation, percentiles, and correlation. These numerical measures will assist in the understanding and interpretation of data.

In Chapter 2 we discussed tabular and graphical presentations used to summarize data. In this chapter, we present several numerical measures that provide additional alternatives for summarizing data.

We start by developing numerical summary measures for data sets consisting of a single variable. When a data set contains more than one variable, the same numerical measures

can be computed separately for each variable. However, in the two-variable case, we will also develop measures of the relationship between the variables.

Numerical measures of location, dispersion, shape, and association are introduced. If the measures are computed for data from a sample, they are called **sample statistics**. If the measures are computed for data from a population, they are called **population parameters**. In statistical inference, a sample statistic is referred to as the **point estimator** of the corresponding population parameter. In Chapter 7 we will discuss in more detail the process of point estimation.

3.1 Measures of Location

Mean

Perhaps the most important measure of location is the **mean**, or average value, for a variable. The mean provides a measure of central location for the data. If the data are for a sample, the mean is denoted by $\bar{x}$; if the data are for a population, the mean is denoted by the Greek letter μ.

In statistical formulas, it is customary to denote the value of variable x for the first observation by x_1, the value of variable x for the second observation by x_2, and so on. In general, the value of variable x for the ith observation is denoted by x_i. For a sample with n observations, the formula for the sample mean is as follows.

The sample mean $\bar{x}$ is a sample statistic.

SAMPLE MEAN

$$\bar{x} = \frac{\Sigma x_i}{n} \tag{3.1}$$

In the preceding formula, the numerator is the sum of the values of the n observations. That is,

$$\Sigma x_i = x_1 + x_2 + \cdots + x_n$$

The Greek letter Σ is the summation sign.

To illustrate the computation of a sample mean, let us consider the following class size data for a sample of five college classes.

46 54 42 46 32

We use the notation x_1, x_2, x_3, x_4, x_5 to represent the number of students in each of the five classes.

$$x_1 = 46 \quad x_2 = 54 \quad x_3 = 42 \quad x_4 = 46 \quad x_5 = 32$$

Hence, to compute the sample mean, we can write

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = \frac{46 + 54 + 42 + 46 + 32}{5} = 44$$

The sample mean class size is 44 students.

TABLE 3.1 MONTHLY STARTING SALARIES FOR A SAMPLE OF 12 BUSINESS SCHOOL GRADUATES

WEB file StartSalary

Graduate	Monthly Starting Salary ($)	Graduate	Monthly Starting Salary ($)
1	3450	7	3490
2	3550	8	3730
3	3650	9	3540
4	3480	10	3925
5	3355	11	3520
6	3310	12	3480

Another illustration of the computation of a sample mean is given in the following situation. Suppose that a college placement office sent a questionnaire to a sample of business school graduates requesting information on monthly starting salaries. Table 3.1 shows the collected data. The mean monthly starting salary for the sample of 12 business college graduates is computed as

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{x_1 + x_2 + \cdots + x_{12}}{12}$$
$$= \frac{3450 + 3550 + \cdots + 3480}{12}$$
$$= \frac{42{,}480}{12} = 3540$$

Equation (3.1) shows how the mean is computed for a sample with n observations. The formula for computing the mean of a population remains the same, but we use different notation to indicate that we are working with the entire population. The number of observations in a population is denoted by N and the symbol for a population mean is μ.

The sample mean $\bar{x}$ is a point estimator of the population mean μ.

POPULATION MEAN

$$\mu = \frac{\Sigma x_i}{N} \qquad (3.2)$$

Median

The **median** is another measure of central location. The median is the value in the middle when the data are arranged in ascending order (smallest value to largest value). With an odd number of observations, the median is the middle value. An even number of observations has no single middle value. In this case, we follow convention and define the median as the average of the values for the middle two observations. For convenience the definition of the median is restated as follows.

MEDIAN

Arrange the data in ascending order (smallest value to largest value).

(a) For an odd number of observations, the median is the middle value.
(b) For an even number of observations, the median is the average of the two middle values.

Let us apply this definition to compute the median class size for the sample of five college classes. Arranging the data in ascending order provides the following list.

32 42 46 46 54

Because $n = 5$ is odd, the median is the middle value. Thus the median class size is 46 students. Even though this data set contains two observations with values of 46, each observation is treated separately when we arrange the data in ascending order.

Suppose we also compute the median starting salary for the 12 business college graduates in Table 3.1. We first arrange the data in ascending order.

3310 3355 3450 3480 3480 3490 3520 3540 3550 3650 3730 3925

Middle Two Values

Because $n = 12$ is even, we identify the middle two values: 3490 and 3520. The median is the average of these values.

$$\text{Median} = \frac{3490 + 3520}{2} = 3505$$

The median is the measure of location most often reported for annual income and property value data because a few extremely large incomes or property values can inflate the mean. In such cases, the median is the preferred measure of central location.

Although the mean is the more commonly used measure of central location, in some situations the median is preferred. The mean is influenced by extremely small and large data values. For instance, suppose that one of the graduates (see Table 3.1) had a starting salary of $10,000 per month (maybe the individual's family owns the company). If we change the highest monthly starting salary in Table 3.1 from $3925 to $10,000 and recompute the mean, the sample mean changes from $3540 to $4046. The median of $3505, however, is unchanged, because $3490 and $3520 are still the middle two values. With the extremely high starting salary included, the median provides a better measure of central location than the mean. We can generalize to say that whenever a data set contains extreme values, the median is often the preferred measure of central location.

Mode

A third measure of location is the **mode**. The mode is defined as follows.

MODE

The mode is the value that occurs with greatest frequency.

To illustrate the identification of the mode, consider the sample of five class sizes. The only value that occurs more than once is 46. Because this value, occurring with a frequency

of 2, has the greatest frequency, it is the mode. As another illustration, consider the sample of starting salaries for the business school graduates. The only monthly starting salary that occurs more than once is $3480. Because this value has the greatest frequency, it is the mode.

Situations can arise for which the greatest frequency occurs at two or more different values. In these instances more than one mode exists. If the data contain exactly two modes, we say that the data are *bimodal*. If data contain more than two modes, we say that the data are *multimodal*. In multimodal cases the mode is almost never reported because listing three or more modes would not be particularly helpful in describing a location for the data.

Using Excel to Compute the Mean, Median, and Mode

Excel provides functions for computing the mean, median, and mode. We illustrate the use of these Excel functions by computing the mean, median, and mode for the starting salary data in Table 3.1. Refer to Figure 3.1 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet is in the foreground.

Enter Data: Labels and the starting salary data are entered into cells A1:B13 of the worksheet.

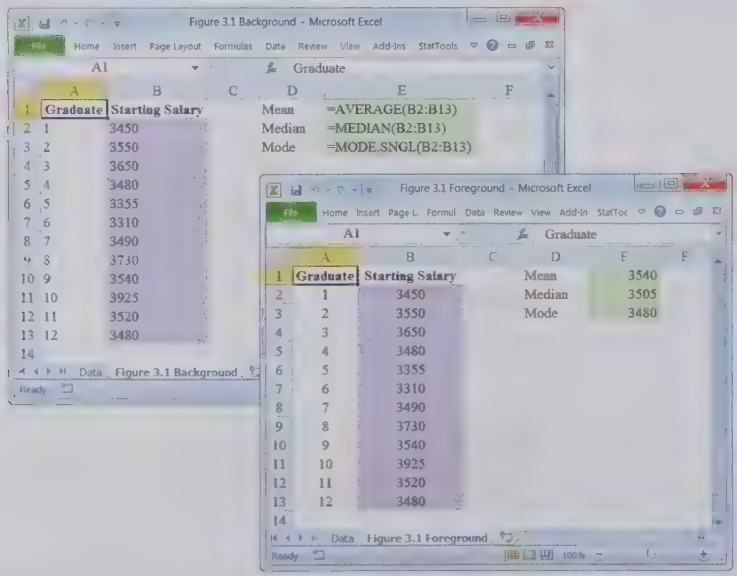
Enter Functions and Formulas: Excel's AVERAGE function can be used to compute the mean by entering the following formula into cell E1:

If the data are bimodal or multimodal, Excel's MODE.SNGL function will incorrectly identify a single mode.

$$=\text{AVERAGE(B2:B13)}$$

Similarly, the formulas =MEDIAN(B2:B13) and =MODE.SNGL(B2:B13) are entered into cells E2 and E3, respectively, to compute the median and the mode. The labels Mean, Median, and Mode are entered into cells D1:D3 to identify the output.

FIGURE 3.1 EXCEL WORKSHEET USED TO COMPUTE THE MEAN, MEDIAN, AND MODE FOR STARTING SALARIES



The formulas in cells E1:E3 are displayed in the formula worksheet in the background of Figure 3.1. The worksheet in the foreground shows the values computed using the Excel functions. Note that the mean (3540), median (3505), and mode (3480) are the same as we computed earlier.

Percentiles

A **percentile** provides information about how the data are spread over the interval from the smallest value to the largest value. For data that do not contain numerous repeated values, the pth percentile divides the data into two parts. Approximately p percent of the observations have values less than the pth percentile; approximately $(100 - p)$ percent of the observations have values greater than the pth percentile. The pth percentile is formally defined as follows.

PERCENTILE

The pth percentile is a value such that *at least* p percent of the observations are less than or equal to this value and *at least* $(100 - p)$ percent of the observations are greater than or equal to this value.

Colleges and universities frequently report admission test scores in terms of percentiles. For instance, suppose an applicant obtains a raw score of 54 on the verbal portion of an admission test. How this student performed in relation to other students taking the same test may not be readily apparent. However, if the raw score of 54 corresponds to the 70th percentile, we know that approximately 70% of the students scored lower than this individual and approximately 30% of the students scored higher than this individual.

The following procedure can be used to compute the pth percentile.

CALCULATING THE pTH PERCENTILE

Following these steps makes it easy to calculate percentiles.

Step 1. Arrange the data in ascending order (smallest value to largest value).

Step 2. Compute an index i

$$i = \left(\frac{p}{100}\right)n$$

where p is the percentile of interest and n is the number of observations.

Step 3. (a) If i *is not an integer, round up.* The next integer *greater* than i denotes the position of the pth percentile.

(b) If i *is an integer,* the pth percentile is the average of the values in positions i and $i + 1$.

As an illustration of this procedure, let us determine the 85th percentile for the starting salary data in Table 3.1.

Step 1. Arrange the data in ascending order.

3310 3355 3450 3480 3480 3490 3520 3540 3550 3650 3730 3925

Step 2.

$$i = \left(\frac{p}{100}\right)n = \left(\frac{85}{100}\right)12 = 10.2$$

Step 3. Because i is not an integer, *round up*. The position of the 85th percentile is the next integer greater than 10.2, the 11th position.

Returning to the data, we see that the 85th percentile is the data value in the 11th position, or 3730.

As another illustration of this procedure, let us consider the calculation of the 50th percentile for the starting salary data. Applying step 2, we obtain

$$i = \left(\frac{50}{100}\right)12 = 6$$

Because i is an integer, step 3(b) states that the 50th percentile is the average of the sixth and seventh data values; thus the 50th percentile is (3490 + 3520)/2 = 3505. Note that the *50th percentile is also the median.*

Quartiles

Quartiles are just specific percentiles; thus, the steps for computing percentiles can be applied directly in the computation of quartiles.

It is often desirable to divide data into four parts, with each part containing approximately one-fourth, or 25% of the observations. Figure 3.2 shows a data distribution divided into four parts. The division points are referred to as the **quartiles** and are defined as

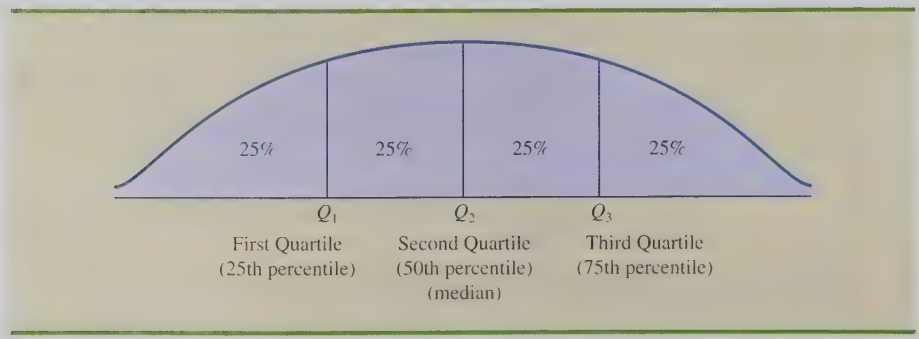
Q_1 = first quartile, or 25th percentile
Q_2 = second quartile, or 50th percentile (also the median)
Q_3 = third quartile, or 75th percentile

The starting salary data are again arranged in ascending order. We already identified Q_2, the second quartile (median), as 3505.

3310 3355 3450 3480 3480 3490 3520 3540 3550 3650 3730 3925

The computations of quartiles Q_1 and Q_3 require the use of the rule for finding the 25th and 75th percentiles. These calculations follow.

FIGURE 3.2 LOCATION OF THE QUARTILES



For Q_1,

$$i = \left(\frac{p}{100}\right)n = \left(\frac{25}{100}\right)12 = 3$$

Because i is an integer, step 3(b) indicates that the first quartile, or 25th percentile, is the average of the third and fourth data values; thus, $Q_1 = (3450 + 3480)/2 = 3465$.

For Q_3,

$$i = \left(\frac{p}{100}\right)n = \left(\frac{75}{100}\right)12 = 9$$

Again, because i is an integer, step 3(b) indicates that the third quartile, or 75th percentile, is the average of the ninth and tenth data values; thus, $Q_3 = (3550 + 3650)/2 = 3600$.

The quartiles divide the starting salary data into four parts, with each part containing 25% of the observations.

3310	3355	3450	3480	3480	3490	3520	3540	3550	3650	3730	3925
		$Q_1 = 3465$			$Q_2 = 3505$ (Median)			$Q_3 = 3600$			

We defined the quartiles as the 25th, 50th, and 75th percentiles. Thus, we computed the quartiles in the same way as percentiles. However, other conventions are sometimes used to compute quartiles, and the actual values reported for quartiles may vary slightly depending on the convention used. Nevertheless, the objective of all procedures for computing quartiles is to divide the data into four equal parts.

Using Excel to Compute Percentiles and Quartiles

Computer software packages do not all use the same method to compute percentiles and quartiles. The formula Excel uses to compute the location (L_p) of the pth percentile is

$$L_p = \left(\frac{p}{100}\right)n + \left(1 - \frac{p}{100}\right)$$

For instance, Excel would compute the location of the 85th percentile for the starting salary data as follows:

$$L_{85} = (.85)12 + (1 - .85) = 10.20 + .15 = 10.35$$

The value of $L_{85} = 10.35$ indicates that the 85th percentile is between the 10th and the 11th observations in rank order from the bottom up. It is the value of observation 10 (3650) plus .35 of the difference between observation 11 (3730) and observation 10. Therefore, the 85th percentile is $3650 + .35(3730 - 3650) = 3650 + .35(80) = 3678$.[1]

Computing percentiles and quartiles using Excel is easy. For instance, Excel's PERCENTILE.INC function can be used to compute the 85th percentile for the starting

[1]The value Excel computed for the 85th percentile does not strictly satisfy the definition of the 85th percentile because only 83% of the values are less than or equal to 3678. Our procedure would round up in this case to obtain 3730 as the value satisfying the definition of the 85th percentile. For larger data sets, the difference between the approximate value provided by Excel and the value computed using our three-step procedure is not of practical significance.

salary data by entering the following formula into any empty cell of the worksheet shown in Figure 3.1:

=PERCENTILE.INC(B2:B13,.85)

The value Excel provides is 3678. To compute a different percentile we simply change the value of .85. For instance, we can use Excel's PERCENTILE.INC function to compute the quartiles for the starting salary data by replacing .85 with values of .25 (25th percentile or first quartile), .50 (50th percentile or second quartile), and .75 (75th percentile or third quartile).

Alternatively, we can use Excel's QUARTILE.INC function to compute the quartiles. For example, to compute the first quartile for the starting salary data we would enter the following formula into an empty cell of the worksheet in Figure 3.1.

=QUARTILE.INC(B2:B13,1)

If the value of 1 in the QUARTILE.INC function is changed to 0, Excel computes the minimum value in the data set. If the value of 1 is changed to 4, Excel computes the maximum value.

The value Excel provides is 3472.5. We can compute the second quartile or median by replacing the value of 1 with 2, and the third quartile by replacing the value of 1 with 3.

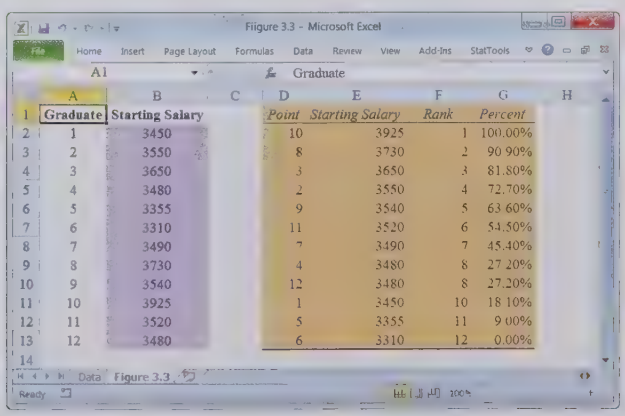
Computing percentiles and quartiles is useful for a data analyst who is interested in getting a feel for how the values in a data set are distributed. However, if you are a student who wants to know how the starting salary you received ranks compared to all the other starting salaries, then what you would like to do is to take your salary and compute its rank and percentile. Or perhaps you know your raw score on an exam and would like to know its rank and percentile. Excel's Rank and Percentile tool can be used to provide this information for an entire data set.

We illustrate the use of Excel's Rank and Percentile tool by computing the ranks and percentiles for the starting salaries in Table 3.1. Refer to Figure 3.3 as we present the tasks involved.

Enter Data: Labels and the starting salary data are entered into cells A1:B13.

Enter Functions and Formulas: No functions and formulas are needed.

FIGURE 3.3 USING EXCEL'S RANK AND PERCENTILE TOOL FOR STARTING SALARIES



	A	B	C	D	E	F	G	H
1	Graduate	Starting Salary		Point	Starting Salary	Rank	Percent	
2	1	3450		10	3925	1	100.00%	
3	2	3550		8	3730	2	90.90%	
4	3	3650		3	3650	3	81.80%	
5	4	3480		2	3550	4	72.70%	
6	5	3355		9	3540	5	63.60%	
7	6	3310		11	3520	6	54.50%	
8	7	3490		7	3490	7	45.40%	
9	8	3730		4	3480	8	27.20%	
10	9	3540		12	3480	8	27.20%	
11	10	3925		1	3450	10	18.10%	
12	11	3520		5	3355	11	9.00%	
13	12	3480		6	3310	12	0.00%	
14								

Apply Tools: The following steps will compute the rank and percentile for each observation.

Step 1. Click the **Data** tab on the Ribbon
Step 2. In the **Analysis** group, click **Data Analysis**
Step 3. Choose **Rank and Percentile** from the list of Analysis Tools
Step 4. When the Rank and Percentile dialog box appears,
Enter B1:B13 in the **Input Range** box
Select **Grouped by Columns**
Select **Labels in First Row**
Select **Output Range**
Enter D1 in the **Output Range** box
(Any cell where the upper left corner of the output is desired may be entered here.)
Click **OK**

The output from using the Rank and Percentile tool appears in cells D1:G13. Cells F2:F13 contain the rank of each observation. The highest salary is given a rank of 1 and the lowest salary is given a rank of 12. Cells G2:G13 show the percentile each salary represents. For instance, the percentile for 3450 is 18.1% because 2 of the other 11 salaries are smaller than 3450. The lowest salary is the 0th percentile and the highest salary is the 100th percentile. The percentiles increase by (1/11)100% as we move up from the lowest salary to the highest, except for ties. In the case of a tie, Excel gives the tied values the same rank and percentile. Note that the two observations with the same value (3480) both received a rank of 8 and a percentile of 27.20%.

NOTES AND COMMENTS

It is better to use the median than the mean as a measure of central location when a data set contains extreme values. Another measure, sometimes used when extreme values are present, is the *trimmed mean*. It is obtained by deleting a percentage of the smallest and largest values from a data set and then computing the mean of the remaining values. For example, the 5% trimmed mean is obtained by removing the smallest 5% and the largest 5% of the data values and then computing the mean of the remaining values. Using the sample with $n = 12$ starting salaries, $0.05(12) = 0.6$. Rounding this value to 1 indicates that the 5% trimmed mean would remove the 1 smallest data value and the 1 largest data value. The 5% trimmed mean using the 10 remaining observations is 3524.50.

Exercises

Methods

SELF test

1. Consider a sample with data values of 10, 20, 12, 17, and 16. Compute the mean and median.
2. Consider a sample with data values of 10, 20, 21, 17, 16, and 12. Compute the mean and median.
3. Consider a sample with data values of 27, 25, 20, 15, 30, 34, 28, and 25. Compute the 20th, 25th, 65th, and 75th percentiles.
4. Consider a sample with data values of 53, 55, 70, 58, 64, 57, 53, 69, 57, 68, and 53. Compute the mean, median, and mode.

Applications

5. The Dow Jones Travel Index reported what business travelers pay for hotel rooms per night in major U.S. cities (*The Wall Street Journal,* January 16, 2004). The average hotel room rates for 20 cities are as follows:

WEB file Hotels

Atlanta	$163	Minneapolis	$125
Boston	177	New Orleans	167
Chicago	166	New York	245
Cleveland	126	Orlando	146
Dallas	123	Phoenix	139
Denver	120	Pittsburgh	134
Detroit	144	San Francisco	167
Houston	173	Seattle	162
Los Angeles	160	St. Louis	145
Miami	192	Washington, D.C.	207

   a. What is the mean hotel room rate?
   b. What is the median hotel room rate?
   c. What is the mode?
   d. What is the first quartile?
   e. What is the third quartile?

6. During the 2007–2008 NCAA college basketball season, men's basketball teams attempted an all-time-high number of 3-point shots, averaging 19.07 shots per game (Associated Press Sports, January 24, 2009). In an attempt to discourage so many 3-point shots and encourage more inside play, the NCAA rules committee moved the 3-point line back from 19 feet, 9 inches to 20 feet, 9 inches at the beginning of the 2008–2009 basketball season. Shown in the following table are the 3-point shots taken and the 3-point shots made for a sample of 19 NCAA basketball games during the 2008–2009 season.

WEB file 3Points

3-Point Shots	Shots Made	3-Point Shots	Shots Made
23	4	17	7
20	6	19	10
17	5	22	7
18	8	25	11
13	4	15	6
16	4	10	5
8	5	11	3
19	8	25	8
28	5	23	7
21	7		

   a. What is the mean number of 3-point shots taken per game?
   b. What is the mean number of 3-point shots made per game?
   c. Using the closer 3-point line, players were making 35.2% of their shots. What percentage of shots were players making from the new 3-point line?
   d. What was the impact of the NCAA rules change that moved the 3-point line back to 20 feet, 9 inches for the 2008–2009 season? Would you agree with the Associated Press Sports article that stated, "Moving back the 3-point line hasn't changed the game dramatically"? Explain.

7. Endowment income is a critical part of the annual budgets at colleges and universities. A study by the National Association of College and University Business Officers

reported that the 435 colleges and universities surveyed held a total of $413 billion in endowments. The 10 wealthiest universities are shown below (*The Wall Street Journal*, January 27, 2009). Amounts are in billions of dollars.

University	Endowment ($billion)	University	Endowment ($billion)
Columbia	7.2	Princeton	16.4
Harvard	36.6	Stanford	17.2
M.I.T.	10.1	Texas	16.1
Michigan	7.6	Texas A&M	6.7
Northwestern	7.2	Yale	22.9

a. What is the mean endowment for these universities?
b. What is the median endowment?
c. What is the mode endowment?
d. Compute the first and third quartiles.
e. What is the total endowment at these 10 universities? These universities represent 2.3% of the 435 colleges and universities surveyed. What percentage of the total $413 billion in endowments is held by these 10 universities?
f. *The Wall Street Journal* reported that over a recent five-month period, a downturn in the economy has caused endowments to decline 23%. What is the estimate of the dollar amount of the decline in the total endowments held by these 10 universities? Given this situation, what are some of the steps you would expect university administrators to be considering?

SELF test

8. The cost of consumer purchases such as single-family housing, gasoline, Internet services, tax preparation, and hospitalization were provided in *The Wall-Street Journal* (January 2, 2007). Sample data typical of the cost of tax-return preparation by services such as H&R Block are shown below.

WEB file
TaxCost

120	230	110	115	160
130	150	105	195	155
105	360	120	120	140
100	115	180	235	255

a. Compute the mean, median, and mode.
b. Compute the first and third quartiles.
c. Compute and interpret the 90th percentile.

9. The National Association of Realtors provided data showing that home sales were the slowest in 10 years (Associated Press, December 24, 2008). Sample data with representative sales prices for existing homes and new homes follow. Data are in thousands of dollars:

Existing Homes	315.5	202.5	140.2	181.3	470.2	169.9	112.8	230.0	177.5
New Homes	275.9	350.2	195.8	525.0	225.3	215.5	175.0	149.5	

a. What is the median sales price for existing homes?
b. What is the median sales price for new homes?
c. Do existing homes or new homes have the higher median sales price? What is the difference between the median sales prices?
d. A year earlier the median sales price for existing homes was $208.4 thousand and the median sales price for new homes was $249 thousand. Compute the percentage

change in the median sales price of existing and new homes over the one-year period. Did existing homes or new homes have the larger percentage change in median sales price?

10. A panel of economists provided forecasts of the U.S. economy for the first six months of 2007 (*The Wall Street Journal,* January 2, 2007). The percent changes in the gross domestic product (GDP) forecasted by 30 economists are as follows.

2.6	3.1	2.3	2.7	3.4	0.9	2.6	2.8	2.0	2.4
2.7	2.7	2.7	2.9	3.1	2.8	1.7	2.3	2.8	3.5
0.4	2.5	2.2	1.9	1.8	1.1	2.0	2.1	2.5	0.5

a. What is the minimum forecast for the percent change in the GDP? What is the maximum?
b. Compute the mean, median, and mode.
c. Compute the first and third quartiles.
d. Did the economists provide an optimistic or pessimistic outlook for the U.S. economy? Discuss.

11. In automobile mileage and gasoline-consumption testing, 13 automobiles were road tested for 300 miles in both city and highway driving conditions. The following data were recorded for miles-per-gallon performance.

City:	16.2	16.7	15.9	14.4	13.2	15.3	16.8	16.0	16.1	15.3	15.2	15.3	16.2
Highway:	19.4	20.6	18.3	18.6	19.2	17.4	17.2	18.6	19.0	21.1	19.4	18.5	18.7

Use the mean, median, and mode to make a statement about the difference in performance for city and highway driving.

12. Walt Disney Company bought Pixar Animation Studios, Inc., in a deal worth $7.4 billion (*CNN Money* website, January 24, 2006). The animated movies produced by Disney and Pixar during the previous 10 years are listed in the following table. The box office revenues are in millions of dollars. Compute the total revenue, the mean, the median, and the quartiles to compare the box office success of the movies produced by both companies. Do the statistics suggest at least one of the reasons Disney was interested in buying Pixar? Discuss.

Disney Movies	Revenue ($millions)	Pixar Movies	Revenue ($millions)
Pocahontas	346	*Toy Story*	362
Hunchback of Notre Dame	325	*A Bug's Life*	363
Hercules	253	*Toy Story 2*	485
Mulan	304	*Monsters, Inc.*	525
Tarzan	448	*Finding Nemo*	865
Dinosaur	354	*The Incredibles*	631
The Emperor's New Groove	169		
Lilo & Stitch	273		
Treasure Planet	110		
The Jungle Book 2	136		
Brother Bear	250		
Home on the Range	104		
Chicken Little	249		

3.2 Measures of Variability

The variability in the delivery time creates uncertainty for production scheduling. Methods presented in this section help measure and understand variability.

In addition to measures of location, it is often desirable to consider measures of variability, or dispersion. For example, suppose that you are a purchasing agent for a large manufacturing firm and that you regularly place orders with two different suppliers. After several months of operation, you find that the mean number of days required to fill orders is 10 days for both of the suppliers. The histograms summarizing the number of working days required to fill orders from the suppliers are shown in Figure 3.4. Although the mean number of days is 10 for both suppliers, do the two suppliers demonstrate the same degree of reliability in terms of making deliveries on schedule? Note the dispersion, or variability, in delivery times indicated by the histograms. Which supplier would you prefer?

For most firms, receiving materials and supplies on schedule is important. The 7- or 8-day deliveries shown for J.C. Clark Distributors might be viewed favorably; however, a few of the slow 13- to 15-day deliveries could be disastrous in terms of keeping a workforce busy and production on schedule. This example illustrates a situation in which the variability in the delivery times may be an overriding consideration in selecting a supplier. For most purchasing agents, the lower variability shown for Dawson Supply, Inc., would make Dawson the preferred supplier.

We turn now to a discussion of some commonly used measures of variability.

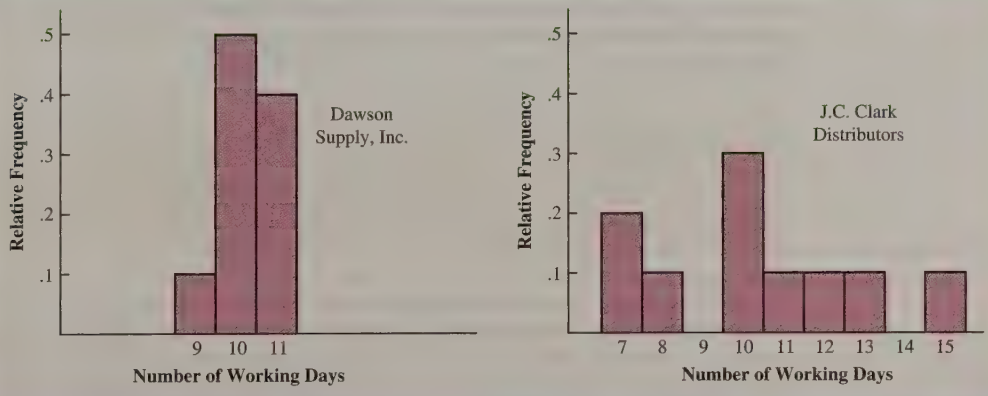
Range

The simplest measure of variability is the **range**.

RANGE

$$\text{Range} = \text{Largest value} - \text{Smallest value}$$

FIGURE 3.4 HISTORICAL DATA SHOWING THE NUMBER OF DAYS REQUIRED TO FILL ORDERS



Let us refer to the data on starting salaries for business school graduates in Table 3.1. The largest starting salary is 3925 and the smallest is 3310. The range is $3925 - 3310 = 615$.

Although the range is the easiest of the measures of variability to compute, it is seldom used as the only measure. The reason is that the range is based on only two of the observations and thus is highly influenced by extreme values. Suppose one of the graduates received a starting salary of $10,000 per month. In this case, the range would be $10{,}000 - 3310 = 6690$ rather than 615. This large value for the range would not be especially descriptive of the variability in the data because 11 of the 12 starting salaries are closely grouped between 3310 and 3730.

Interquartile Range

A measure of variability that overcomes the dependency on extreme values is the **interquartile range (IQR)**. This measure of variability is the difference between the third quartile, Q_3, and the first quartile, Q_1. In other words, the interquartile range is the range for the middle 50% of the data.

INTERQUARTILE RANGE

$$\text{IQR} = Q_3 - Q_1 \tag{3.3}$$

For the data on monthly starting salaries, the quartiles are $Q_3 = 3600$ and $Q_1 = 3465$. Thus, the interquartile range is $3600 - 3465 = 135$.

Variance

The **variance** is a measure of variability that utilizes all the data. The variance is based on the difference between the value of each observation (x_i) and the mean. The difference between each x_i and the mean ($\bar{x}$ for a sample, μ for a population) is called a *deviation about the mean*. For a sample, a deviation about the mean is written $(x_i - \bar{x})$; for a population, it is written $(x_i - \mu)$. In the computation of the variance, the deviations about the mean are *squared*.

If the data are for a population, the average of the squared deviations is called the *population variance*. The population variance is denoted by the Greek symbol σ^2. For a population of N observations and with μ denoting the population mean, the definition of the population variance is as follows.

POPULATION VARIANCE

$$\sigma^2 = \frac{\Sigma(x_i - \mu)^2}{N} \tag{3.4}$$

In most statistical applications, the data being analyzed are for a sample. When we compute a sample variance, we are often interested in using it to estimate the population variance σ^2. Although a detailed explanation is beyond the scope of this text, it can be shown that if the sum of the squared deviations about the sample mean is divided by $n - 1$, and not n, the resulting sample variance provides an unbiased estimate of the population variance. For this reason, the *sample variance*, denoted by s^2, is defined as follows.

TABLE 3.2 COMPUTATION OF DEVIATIONS AND SQUARED DEVIATIONS ABOUT THE MEAN FOR THE CLASS SIZE DATA

Number of Students in Class (x_i)	Mean Class Size ($\bar{x}$)	Deviation About the Mean ($x_i - \bar{x}$)	Squared Deviation About the Mean $(x_i - \bar{x})^2$
46	44	2	4
54	44	10	100
42	44	−2	4
46	44	2	4
32	44	−12	144
		0	256
		$\Sigma(x_i - \bar{x})$	$\Sigma(x_i - \bar{x})^2$

The sample variance s^2 is a point estimator of the population variance σ^2.

SAMPLE VARIANCE

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n - 1} \tag{3.5}$$

To illustrate the computation of the sample variance, we will use the data on class size for the sample of five college classes as presented in Section 3.1. A summary of the data, including the computation of the deviations about the mean and the squared deviations about the mean, is shown in Table 3.2. The sum of squared deviations about the mean is $\Sigma(x_i - \bar{x})^2 = 256$. Hence, with $n - 1 = 4$, the sample variance is

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n - 1} = \frac{256}{4} = 64$$

The variance is useful in comparing the variability of two or more variables.

Before moving on, let us note that the units associated with the sample variance often cause confusion. Because the values being summed in the variance calculation, $(x_i - \bar{x})^2$, are squared, the units associated with the sample variance are also *squared.* For instance, the sample variance for the class size data is $s^2 = 64$ (students)^2. The squared units associated with variance make it difficult to obtain an intuitive understanding and interpretation of the numerical value of the variance. We recommend that you think of the variance as a measure useful in comparing the amount of variability for two or more variables. In a comparison of the variables, the one with the largest variance shows the most variability. Further interpretation of the value of the variance may not be necessary.

As another illustration of computing a sample variance, consider the starting salaries listed in Table 3.1 for the 12 business school graduates. In Section 3.1, we showed that the sample mean starting salary was 3540. The computation of the sample variance ($s^2 = 27{,}440.91$) is shown in Table 3.3.

In Tables 3.2 and 3.3 we show both the sum of the deviations about the mean and the sum of the squared deviations about the mean. For any data set, the sum of the deviations about the mean will *always equal zero.* Note that in Tables 3.2 and 3.3, $\Sigma(x_i - \bar{x}) = 0$. The positive deviations and negative deviations cancel each other, causing the sum of the deviations about the mean to equal zero.

TABLE 3.3 COMPUTATION OF THE SAMPLE VARIANCE FOR THE STARTING SALARY DATA

Monthly Salary (x_i)	Sample Mean ($\bar{x}$)	Deviation About the Mean ($x_i - \bar{x}$)	Squared Deviation About the Mean $(x_i - \bar{x})^2$
3450	3540	−90	8,100
3550	3540	10	100
3650	3540	110	12,100
3480	3540	−60	3,600
3355	3540	−185	34,225
3310	3540	−230	52,900
3490	3540	−50	2,500
3730	3540	190	36,100
3540	3540	0	0
3925	3540	385	148,225
3520	3540	−20	400
3480	3540	−60	3,600
		0	301,850
		$\Sigma(x_i - \bar{x})$	$\Sigma(x_i - \bar{x})^2$

Using equation (3.5),

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n - 1} = \frac{301{,}850}{11} = 27{,}440.91$$

Standard Deviation

The **standard deviation** is defined to be the positive square root of the variance. Following the notation we adopted for a sample variance and a population variance, we use s to denote the sample standard deviation and σ to denote the population standard deviation. The standard deviation is derived from the variance in the following way.

The sample standard deviation s is a point estimator of the population standard deviation σ.

STANDARD DEVIATION

$$\text{Sample standard deviation} = s = \sqrt{s^2} \quad (3.6)$$

$$\text{Population standard deviation} = \sigma = \sqrt{\sigma^2} \quad (3.7)$$

Recall that the sample variance for the sample of class sizes in five college classes is $s^2 = 64$. Thus, the sample standard deviation is $s = \sqrt{64} = 8$. For the data on starting salaries, the sample standard deviation is $s = \sqrt{27{,}440.91} = 165.65$.

The standard deviation is easier to interpret than the variance because the standard deviation is measured in the same units as the data.

What is gained by converting the variance to its corresponding standard deviation? Recall that the units associated with the variance are squared. For example, the sample variance for the starting salary data of business school graduates is $s^2 = 27{,}440.91$ (dollars)^2. Because the standard deviation is the square root of the variance, the units of the variance, dollars squared, are converted to dollars in the standard deviation. Thus, the standard deviation of the starting salary data is \$165.65. In other words, the standard deviation is measured in the same units as the original data. For this reason the standard deviation is more

easily compared to the mean and other statistics that are measured in the same units as the original data.

Using Excel to Compute the Sample Variance and Sample Standard Deviation

Excel provides functions for computing the sample variance and sample standard deviation, which we will illustrate using the starting salary data. Refer to Figure 3.5 as we describe the steps involved. Figure 3.5 is an extension of Figure 3.1, where we showed how to use Excel functions to compute the mean, median, and mode. The formula worksheet is in the background; the value worksheet is in the foreground.

Enter Data: Labels and the starting salary data are entered into cells A1:B13 of the worksheet.

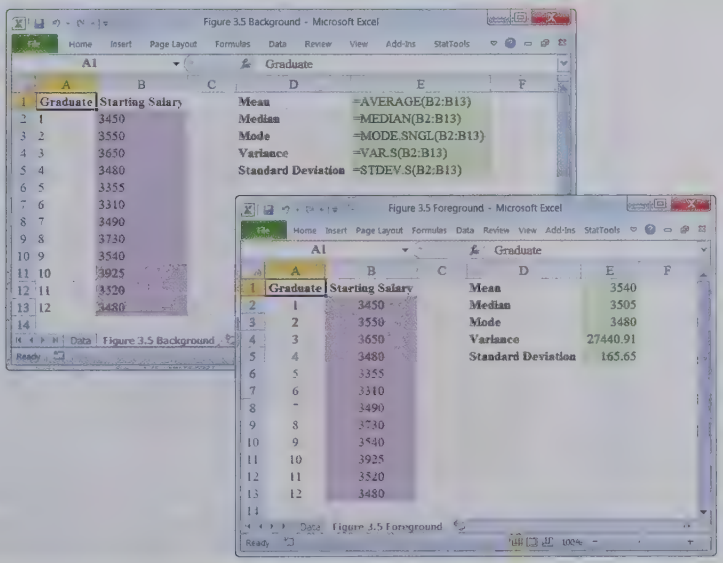
Enter Functions and Formulas: The Excel AVERAGE, MEDIAN, and MODE.SNGL functions are entered into cells E1:E3 as described earlier. Excel's VAR.S function can be used to compute the sample variance by entering the following formula into cell E4:

$$=\text{VAR.S(B2:B13)}$$

Similarly, the formula =STDEV.S(B2:B13) is entered into cell E5 to compute the sample standard deviation. Appropriate labels are entered into cells D1:D5 to identify the output.

The value worksheet, in the foreground, shows the values computed using the Excel functions. Note that the sample variance and sample standard deviation are the same as we computed earlier using the definitions.

FIGURE 3.5 EXCEL WORKSHEET USED TO COMPUTE THE SAMPLE VARIANCE AND THE SAMPLE STANDARD DEVIATION FOR STARTING SALARIES



Coefficient of Variation

The coefficient of variation is a relative measure of variability; it measures the standard deviation relative to the mean.

In some situations we may be interested in a descriptive statistic that indicates how large the standard deviation is relative to the mean. This measure is called the **coefficient of variation** and is usually expressed as a percentage.

COEFFICIENT OF VARIATION

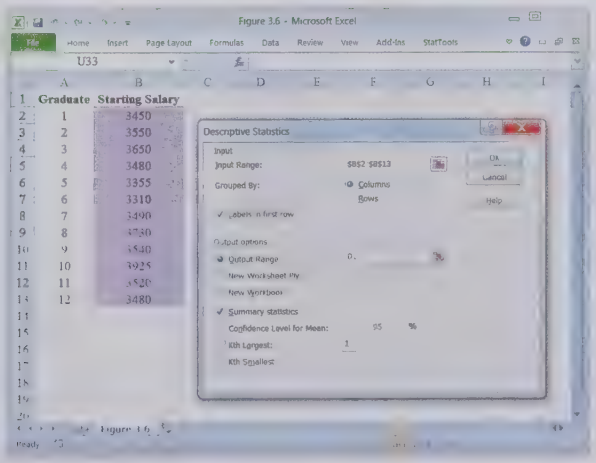
$$\left(\frac{\text{Standard deviation}}{\text{Mean}} \times 100\right)\% \tag{3.8}$$

For the class size data, we found a sample mean of 44 and a sample standard deviation of 8. The coefficient of variation is $[(8/44) \times 100]\% = 18.2\%$. In words, the coefficient of variation tells us that the sample standard deviation is 18.2% of the value of the sample mean. For the starting salary data with a sample mean of 3540 and a sample standard deviation of 165.65, the coefficient of variation, $[(165.65/3540) \times 100]\% = 4.7\%$, tells us the sample standard deviation is only 4.7% of the value of the sample mean. In general, the coefficient of variation is a useful statistic for comparing the variability of variables that have different standard deviations and different means.

Using Excel's Descriptive Statistics Tool

As we have seen, Excel provides statistical functions to compute descriptive statistics for a data set. These functions can be used to compute one statistic at a time (e.g., mean, variance, etc.). Excel also provides a variety of data analysis tools. One of these, called Descriptive Statistics, allows the user to compute a variety of descriptive statistics at once. We show here how it can be used to compute descriptive statistics for the starting salary data in Table 3.1. Refer to Figure 3.6 as we describe the tasks involved.

FIGURE 3.6 DIALOG BOX FOR EXCEL'S DESCRIPTIVE STATISTICS TOOL



Enter Data: Labels and the starting salary data are entered into cells A1:B13 of the worksheet.

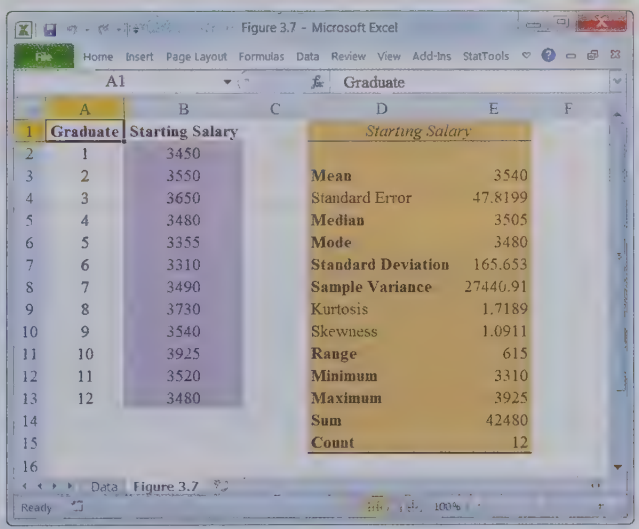
Enter Functions and Formulas: No functions and formulas are needed.

Apply Analysis Tools: The following steps describe how to use Excel's Descriptive Statistics tool for these data:

Step 1. Click the **Data** tab on the Ribbon
Step 2. In the **Analysis** group, click **Data Analysis**
Step 3. Choose **Descriptive Statistics** from the list of **Analysis Tools**
Step 4. When the Descriptive Statistics dialog box appears (see Figure 3.6),
Enter B1:B13 in the **Input Range** box
Select **Grouped By Columns**
Select **Labels in First Row**
Select **Output Range**
Enter D1 in the **Output Range** box (to identify the upper left corner of the section of the worksheet where the descriptive statistics will appear)
Select **Summary Statistics**
Click **OK**

Cells D1:E15 of Figure 3.7 show the descriptive statistics provided by Excel. A yellow screen is used to highlight the results. The boldfaced entries are the descriptive statistics we have covered. The descriptive statistics that are not boldfaced are either covered subsequently in the text or discussed in more advanced texts.

FIGURE 3.7 DESCRIPTIVE STATISTICS PROVIDED BY EXCEL FOR THE STARTING SALARY DATA



NOTES AND COMMENTS

1. The standard deviation is a commonly used measure of the risk associated with investing in stock and stock funds. It provides a measure of how monthly returns fluctuate around the long-run average return.
2. Rounding the value of the sample mean $\bar{x}$ and the values of the squared deviations $(x_i - \bar{x})^2$ may introduce errors when a calculator is used in the computation of the variance and standard deviation. To reduce rounding errors, we recommend carrying at least six significant digits during intermediate calculations. The resulting variance or standard deviation can then be rounded to fewer digits.
3. An alternative formula for the computation of the sample variance is

$$s^2 = \frac{\sum x_i^2 - n\bar{x}^2}{n - 1}$$

where $\sum x_i^2 = x_1^2 + x_2^2 + \cdots + x_n^2$.

Exercises

Methods

13. Consider a sample with data values of 10, 20, 12, 17, and 16. Compute the range and interquartile range.
14. Consider a sample with data values of 10, 20, 12, 17, and 16. Compute the variance and standard deviation.
15. Consider a sample with data values of 27, 25, 20, 15, 30, 34, 28, and 25. Compute the range, interquartile range, variance, and standard deviation.

Applications

16. A bowler's scores for six games were 182, 168, 184, 190, 170, and 174. Using these data as a sample, compute the following descriptive statistics:
    a. Range
    b. Variance
    c. Standard deviation
    d. Coefficient of variation
17. A home theater in a box is the easiest and cheapest way to provide surround sound for a home entertainment center. A sample of prices is shown here (*Consumer Reports Buying Guide,* 2004). The prices are for models with a DVD player and for models without a DVD player.

Models with DVD Player	Price	Models without DVD Player	Price
Sony HT-1800DP	$450	Pioneer HTP-230	$300
Pioneer HTD-330DV	300	Sony HT-DDW750	300
Sony HT-C800DP	400	Kenwood HTB-306	360
Panasonic SC-HT900	500	RCA RT-2600	290
Panasonic SC-MTI	400	Kenwood HTB-206	300

    a. Compute the mean price for models with a DVD player and the mean price for models without a DVD player. What is the additional price paid to have a DVD player included in a home theater unit?

b. Compute the range, variance, and standard deviation for the two samples. What does this information tell you about the prices for models with and without a DVD player?

18. Car rental rates per day for a sample of seven Eastern U.S. cities are as follows (*The Wall Street Journal,* January 16, 2004).

City	Daily Rate
Boston	$43
Atlanta	35
Miami	34
New York	58
Orlando	30
Pittsburgh	30
Washington, D.C.	36

a. Compute the mean, variance, and standard deviation for the car rental rates.
b. A similar sample of seven Western U.S. cities showed a sample mean car rental rate of $38 per day. The variance and standard deviation were 12.3 and 3.5, respectively. Discuss any difference between the car rental rates in Eastern and Western U.S. cities.

19. The *Los Angeles Times* regularly reports the air quality index for various areas of Southern California. A sample of air quality index values for Pomona provided the following data: 28, 42, 58, 48, 45, 55, 60, 49, and 50.
a. Compute the range and interquartile range.
b. Compute the sample variance and sample standard deviation.
c. A sample of air quality index readings for Anaheim provided a sample mean of 48.5, a sample variance of 136, and a sample standard deviation of 11.66. What comparisons can you make between the air quality in Pomona and that in Anaheim on the basis of these descriptive statistics?

20. The following data were used to construct the histograms of the number of days required to fill orders for Dawson Supply, Inc., and J.C. Clark Distributors (see Figure 3.4).

Dawson Supply Days for Delivery:	11	10	9	10	11	11	10	11	10	10
Clark Distributors Days for Delivery:	8	10	13	7	10	11	10	7	15	12

Use the range and standard deviation to support the previous observation that Dawson Supply provides the more consistent and reliable delivery times.

21. How do grocery costs compare across the country? Using a market basket of 10 items including meat, milk, bread, eggs, coffee, potatoes, cereal, and orange juice, *Where to Retire* magazine calculated the cost of the market basket in six cities and in six retirement areas across the country (*Where to Retire,* November/December 2003). The data with market basket cost to the nearest dollar are as follows:

City	Cost	Retirement Area	Cost
Buffalo, NY	$33	Biloxi-Gulfport, MS	$29
Des Moines, IA	27	Asheville, NC	32
Hartford, CT	32	Flagstaff, AZ	32
Los Angeles, CA	38	Hilton Head, SC	34
Miami, FL	36	Fort Myers, FL	34
Pittsburgh, PA	32	Santa Fe, NM	31

a. Compute the mean, variance, and standard deviation for the sample of cities and the sample of retirement areas.
b. What observations can be made based on the two samples?

WEB file
BackToSchool

22. The National Retail Federation reported that college freshmen spend more on back-to-school items than any other college group (*USA Today*, August 4, 2006). Sample data comparing the back-to-school expenditures for 25 freshmen and 20 seniors are shown in the data file BackToSchool.
    a. What is the mean back-to-school expenditure for each group? Are the data consistent with the National Retail Federation's report?
    b. What is the range for the expenditures in each group?
    c. What is the interquartile range for the expenditures in each group?
    d. What is the standard deviation for expenditures in each group?
    e. Do freshmen or seniors have more variation in back-to-school expenditures?

23. Scores turned in by an amateur golfer at the Bonita Fairways Golf Course in Bonita Springs, Florida, during 2005 and 2006 are as follows:

2005 Season:	74	78	79	77	75	73	75	77
2006 Season:	71	70	75	77	85	80	71	79

    a. Use the mean and standard deviation to evaluate the golfer's performance over the two-year period.
    b. What is the primary difference in performance between 2005 and 2006? What improvement, if any, can be seen in the 2006 scores?

24. The following times were recorded by the quarter-mile and mile runners of a university track team (times are in minutes).

Quarter-Mile Times:	.92	.98	1.04	.90	.99
Mile Times:	4.52	4.35	4.60	4.70	4.50

After viewing this sample of running times, one of the coaches commented that the quarter-milers turned in the more consistent times. Use the standard deviation and the coefficient of variation to summarize the variability in the data. Does the use of the coefficient of variation indicate that the coach's statement should be qualified?

3.3 Measures of Distribution Shape, Relative Location, and Detecting Outliers

We have described several measures of location and variability for data. In addition, it is often important to have a measure of the shape of a distribution. In Chapter 2 we noted that a histogram provides a graphical display showing the shape of a distribution. An important numerical measure of the shape of a distribution is called **skewness**.

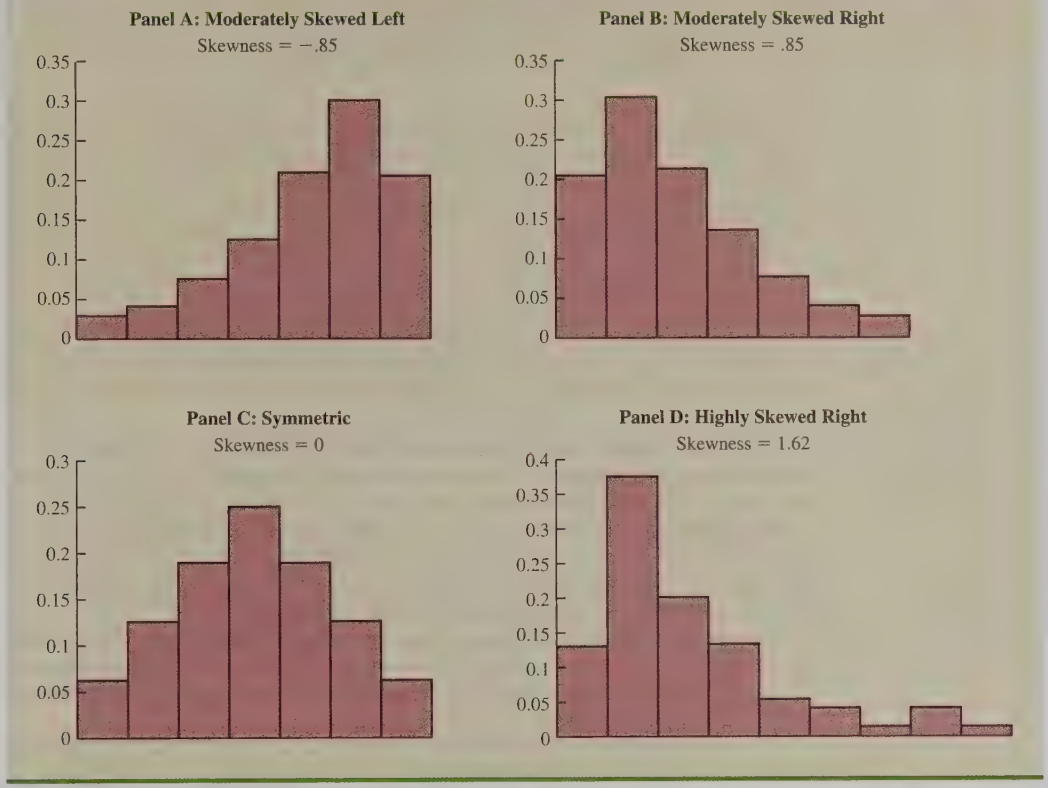
Distribution Shape

Shown in Figure 3.8 are four histograms constructed from relative frequency distributions. The histograms in Panels A and B are moderately skewed. The one in Panel A is skewed to the left; its skewness is −.85. The histogram in Panel B is skewed to the right; its skewness is +.85. The histogram in Panel C is symmetric; its skewness is zero. The histogram in Panel D is highly skewed to the right; its skewness is 1.62. The formula used to compute skewness is somewhat complex.[2] However, the skewness can be easily

[2]The formula for the skewness of sample data:

$$\text{Skewness} = \frac{n}{(n-1)(n-2)} \sum \left(\frac{x_i - \bar{x}}{s} \right)^3$$

FIGURE 3.8 HISTOGRAMS SHOWING THE SKEWNESS FOR FOUR DISTRIBUTIONS



computed using statistical software. For data skewed to the left, the skewness is negative; for data skewed to the right, the skewness is positive. If the data are symmetric, the skewness is zero.

For a symmetric distribution, the mean and the median are equal. When the data are positively skewed, the mean will usually be greater than the median; when the data are negatively skewed, the mean will usually be less than the median. The data used to construct the histogram in Panel D are customer purchases at a women's apparel store. The mean purchase amount is $77.60 and the median purchase amount is $59.70. The relatively few large purchase amounts tend to increase the mean, while the median remains unaffected by the large purchase amounts. The median provides the preferred measure of location when the data are highly skewed.

z-Scores

In addition to measures of location, variability, and shape, we are also interested in the relative location of values within a data set. Measures of relative location help us determine how far a particular value is from the mean.

By using both the mean and standard deviation, we can determine the relative location of any observation. Suppose we have a sample of n observations, with the values denoted by $x_1, x_2, \ldots, x_n$. In addition, assume that the sample mean, $\bar{x}$, and the sample standard deviation, s, are already computed. Associated with each value, x_i, is another value called its **z-score**. Equation (3.9) shows how the z-score is computed for each x_i.

z-SCORE

$$z_i = \frac{x_i - \bar{x}}{s} \tag{3.9}$$

where

z_i = the z-score for x_i
$\bar{x}$ = the sample mean
s = the sample standard deviation

The z-score is often called the *standardized value.* The z-score, z_i, can be interpreted as the *number of standard deviations x_i is from the mean $\bar{x}$.* For example, $z_1 = 1.2$ would indicate that x_1 is 1.2 standard deviations greater than the sample mean. Similarly, $z_2 = -.5$ would indicate that x_2 is .5, or 1/2, standard deviation less than the sample mean. A z-score greater than zero occurs for observations with a value greater than the mean, and a z-score less than zero occurs for observations with a value less than the mean. A z-score of zero indicates that the value of the observation is equal to the mean.

The z-score for any observation can be interpreted as a measure of the relative location of the observation in a data set. Thus, observations in two different data sets with the same z-score can be said to have the same relative location in terms of being the same number of standard deviations from the mean.

The z-scores for the class size data are computed in Table 3.4. Recall the previously computed sample mean, $\bar{x} = 44$, and sample standard deviation, $s = 8$. The z-score of -1.50 for the fifth observation shows it is farthest from the mean; it is 1.50 standard deviations below the mean.

Chebyshev's Theorem

Chebyshev's theorem enables us to make statements about the proportion of data values that must be within a specified number of standard deviations of the mean.

TABLE 3.4 z-SCORES FOR THE CLASS SIZE DATA

Number of Students in Class (x_i)	Deviation About the Mean ($x_i - \bar{x}$)	z-Score $\left(\frac{x_i - \bar{x}}{s}\right)$
46	2	2/8 = .25
54	10	10/8 = 1.25
42	−2	−2/8 = −.25
46	2	2/8 = .25
32	−12	−12/8 = −1.50

CHEBYSHEV'S THEOREM

At least $(1 - 1/z^2)$ of the data values must be within z standard deviations of the mean, where z is any value greater than 1.

Some of the implications of this theorem, with $z = 2, 3$, and 4 standard deviations, follow.

- At least .75, or 75%, of the data values must be within $z = 2$ standard deviations of the mean.
- At least .89, or 89%, of the data values must be within $z = 3$ standard deviations of the mean.
- At least .94, or 94%, of the data values must be within $z = 4$ standard deviations of the mean.

For an example using Chebyshev's theorem, suppose that the midterm test scores for 100 students in a college business statistics course had a mean of 70 and a standard deviation of 5. How many students had test scores between 60 and 80? How many students had test scores between 58 and 82?

For the test scores between 60 and 80, we note that 60 is two standard deviations below the mean and 80 is two standard deviations above the mean. Using Chebyshev's theorem, we see that at least .75, or at least 75%, of the observations must have values within two standard deviations of the mean. Thus, at least 75% of the students must have scored between 60 and 80.

Chebyshev's theorem requires $z > 1$; but z need not be an integer.

For the test scores between 58 and 82, we see that $(58 - 70)/5 = -2.4$ indicates 58 is 2.4 standard deviations below the mean and that $(82 - 70)/5 = +2.4$ indicates 82 is 2.4 standard deviations above the mean. Applying Chebyshev's theorem with $z = 2.4$, we have

$$\left(1 - \frac{1}{z^2}\right) = \left(1 - \frac{1}{(2.4)^2}\right) = .826$$

At least 82.6% of the students must have test scores between 58 and 82.

Empirical Rule

The empirical rule is based on the normal probability distribution, which will be discussed in Chapter 6. The normal distribution is used extensively throughout the text.

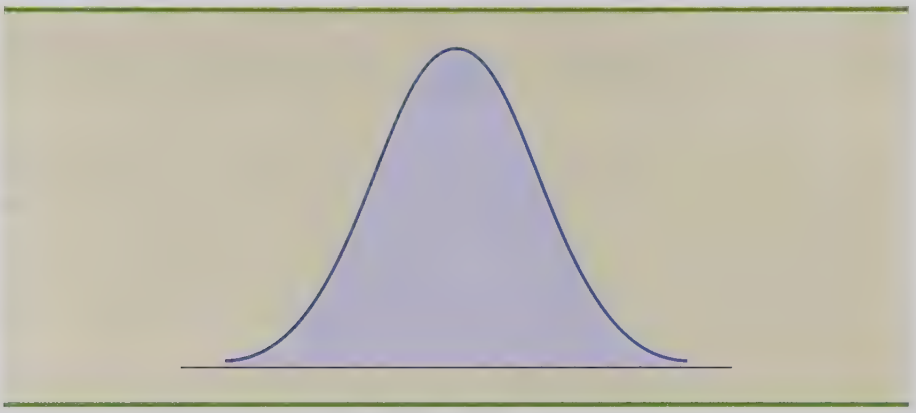
One of the advantages of Chebyshev's theorem is that it applies to any data set regardless of the shape of the distribution of the data. Indeed, it could be used with any of the distributions in Figure 3.8. In many practical applications, however, data sets exhibit a symmetric mound-shaped or bell-shaped distribution like the one shown in Figure 3.9. When the data are believed to approximate this distribution, the **empirical rule** can be used to determine the percentage of data values that must be within a specified number of standard deviations of the mean.

EMPIRICAL RULE

For data having a bell-shaped distribution:

- Approximately 68% of the data values will be within one standard deviation of the mean.
- Approximately 95% of the data values will be within two standard deviations of the mean.
- Almost all of the data values will be within three standard deviations of the mean.

FIGURE 3.9 A SYMMETRIC MOUND-SHAPED OR BELL-SHAPED DISTRIBUTION



For example, liquid detergent bottles are filled automatically on a production line. Filling weights frequently have a bell-shaped distribution. If the mean filling weight is 16 ounces and the standard deviation is .25 ounces, we can use the empirical rule to draw the following conclusions.

- Approximately 68% of the filled bottles will have weights between 15.75 and 16.25 ounces (within one standard deviation of the mean).
- Approximately 95% of the filled bottles will have weights between 15.50 and 16.50 ounces (within two standard deviations of the mean).
- Almost all filled bottles will have weights between 15.25 and 16.75 ounces (within three standard deviations of the mean).

Detecting Outliers

Sometimes a data set will have one or more observations with unusually large or unusually small values. These extreme values are called **outliers**. Experienced statisticians take steps to identify outliers and then review each one carefully. An outlier may be a data value that has been incorrectly recorded. If so, it can be corrected before further analysis. An outlier may also be from an observation that was incorrectly included in the data set; if so, it can be removed. Finally, an outlier may be an unusual data value that has been recorded correctly and belongs in the data set. In such cases it should remain.

It is a good idea to check for outliers before making decisions based on data analysis. Errors are often made in recording data and entering data into the computer. Outliers should not necessarily be deleted, but their accuracy and appropriateness should be verified.

Standardized values (z-scores) can be used to identify outliers. Recall that the empirical rule allows us to conclude that for data with a bell-shaped distribution, almost all the data values will be within three standard deviations of the mean. Hence, in using z-scores to identify outliers, we recommend treating any data value with a z-score less than -3 or greater than $+3$ as an outlier. Such data values can then be reviewed for accuracy and to determine whether they belong in the data set.

Refer to the z-scores for the class size data in Table 3.4. The z-score of -1.50 shows that the fifth class size is farthest from the mean. However, this standardized value is well within the -3 to $+3$ guideline for outliers. Thus, the z-scores do not indicate that outliers are present in the class size data.

NOTES AND COMMENTS

1. Chebyshev's theorem is applicable for any data set and can be used to state the minimum number of data values that will be within a certain number of standard deviations of the mean. If the data are known to be approximately bell-shaped, more can be said. For instance, the empirical rule allows us to say that *approximately* 95% of the data values will be within two standard deviations of the mean; Chebyshev's theorem allows us to conclude only that at least 75% of the data values will be in that interval.
2. Before analyzing a data set, statisticians usually make a variety of checks to ensure the validity of data. In a large study it is not uncommon for errors to be made in recording data values or in entering the values into a computer. Identifying outliers is one tool used to check the validity of the data.

Exercises

Methods

25. Consider a sample with data values of 10, 20, 12, 17, and 16. Compute the z-score for each of the five observations.

26. Consider a sample with a mean of 500 and a standard deviation of 100. What are the z-scores for the following data values: 520, 650, 500, 450, and 280?

SELF test

27. Consider a sample with a mean of 30 and a standard deviation of 5. Use Chebyshev's theorem to determine the percentage of the data within each of the following ranges:
    a. 20 to 40
    b. 15 to 45
    c. 22 to 38
    d. 18 to 42
    e. 12 to 48

28. Suppose the data have a bell-shaped distribution with a mean of 30 and a standard deviation of 5. Use the empirical rule to determine the percentage of data within each of the following ranges:
    a. 20 to 40
    b. 15 to 45
    c. 25 to 35

Applications

SELF test

29. The results of a national survey showed that on average, adults sleep 6.9 hours per night. Suppose that the standard deviation is 1.2 hours.
    a. Use Chebyshev's theorem to calculate the percentage of individuals who sleep between 4.5 and 9.3 hours.
    b. Use Chebyshev's theorem to calculate the percentage of individuals who sleep between 3.9 and 9.9 hours.
    c. Assume that the number of hours of sleep follows a bell-shaped distribution. Use the empirical rule to calculate the percentage of individuals who sleep between 4.5 and 9.3 hours per day. How does this result compare to the value that you obtained using Chebyshev's theorem in part (a)?

30. The Energy Information Administration reported that the mean retail price per gallon of regular grade gasoline was $2.81 (Energy Information Administration, November 2010). Suppose that the standard deviation was $.10 and that the retail price per gallon has a bell-shaped distribution.
    a. What percentage of regular grade gasoline sold between $2.71 and $2.91 per gallon?
    b. What percentage of regular grade gasoline sold between $2.71 and $3.01 per gallon?
    c. What percentage of regular grade gasoline sold for more than $3.01 per gallon?

31. The national average for the math portion of the College Board's SAT is 515 (*The World Almanac,* 2009). The College Board periodically rescales the test scores such that the standard deviation is approximately 100. Answer the following questions using a bell-shaped distribution and the empirical rule for the verbal test scores.
    a. What percentage of students have an SAT math score greater than 615?
    b. What percentage of students have an SAT math score greater than 715?
    c. What percentage of students have an SAT math score between 415 and 515?
    d. What percentage of students have an SAT math score between 315 and 615?

32. Many families in California are using backyard structures for home offices, art studios, and hobby areas as well as for additional storage. Suppose that the mean price for a customized wooden, shingled backyard structure is $3100. Assume that the standard deviation is $1200.
    a. What is the z-score for a backyard structure costing $2300?
    b. What is the z-score for a backyard structure costing $4900?
    c. Interpret the z-scores in parts (a) and (b). Comment on whether either should be considered an outlier.
    d. If the cost for a backyard shed-office combination built in Albany, California, is $13,000, should this structure be considered an outlier? Explain.

33. Florida Power & Light (FP&L) Company has enjoyed a reputation for quickly fixing its electric system after storms. However, during the hurricane seasons of 2004 and 2005, a new reality was that the company's historical approach to emergency electric system repairs was no longer good enough (*The Wall Street Journal,* January 16, 2006). Data showing the days required to restore electric service after seven hurricanes during 2004 and 2005 follow.

Hurricane	Days to Restore Service
Charley	13
Frances	12
Jeanne	8
Dennis	3
Katrina	8
Rita	2
Wilma	18

Based on this sample of seven, compute the following descriptive statistics:
    a. Mean, median, and mode
    b. Range and standard deviation
    c. Should Wilma be considered an outlier in terms of the days required to restore electric service?
    d. The seven hurricanes resulted in 10 million service interruptions to customers. Do the statistics show that FP&L should consider updating its approach to emergency electric system repairs? Discuss.

34. A sample of 10 NCAA college basketball game scores provided the following data (*USA Today*, January 26, 2004).

WEB file NCAA

Winning Team	Points	Losing Team	Points	Winning Margin
Arizona	90	Oregon	66	24
Duke	85	Georgetown	66	19
Florida State	75	Wake Forest	70	5
Kansas	78	Colorado	57	21
Kentucky	71	Notre Dame	63	8
Louisville	65	Tennessee	62	3
Oklahoma State	72	Texas	66	6
Purdue	76	Michigan State	70	6
Stanford	77	Southern Cal	67	10
Wisconsin	76	Illinois	56	20

a. Compute the mean and standard deviation for the points scored by the winning team.
b. Assume that the points scored by the winning teams for all NCAA games follow a bell-shaped distribution. Using the mean and standard deviation found in part (a), estimate the percentage of all NCAA games in which the winning team scores 84 or more points. Estimate the percentage of NCAA games in which the winning team scores more than 90 points.
c. Compute the mean and standard deviation for the winning margin. Do the data contain outliers? Explain.

35. The Associated Press Team Marketing Report listed the Dallas Cowboys as the team with the highest ticket prices in the National Football League (*USA Today*, October 20, 2009). Data showing the average ticket price for a sample of 14 teams in the National Football League are as follows.

WEB file NFLTickets

Team	Ticket Price	Team	Ticket Price
Atlanta Falcons	72	Green Bay Packers	63
Buffalo Bills	51	Indianapolis Colts	83
Carolina Panthers	63	New Orleans Saints	62
Chicago Bears	88	New York Jets	87
Cleveland Browns	55	Pittsburgh Steelers	67
Dallas Cowboys	160	Seattle Seahawks	61
Denver Broncos	77	Tennessee Titans	61

a. What is the mean ticket price?
b. The previous year, the mean ticket price was $72.20. What was the percentage increase in the mean ticket price for the one-year period?
c. Compute the median ticket price.
d. Compute the first and third quartiles.
e. Compute the standard deviation.
f. What is the z-score for the Dallas Cowboys ticket price? Should this price be considered an outlier? Explain.

3.4 Exploratory Data Analysis

In Chapter 2 we introduced the stem-and-leaf display as a technique of exploratory data analysis. Recall that exploratory data analysis enables us to use simple arithmetic and easy-to-draw pictures to summarize data. In this section we continue exploratory data analysis by considering five-number summaries and box plots.

Five-Number Summary

In a **five-number summary**, the following five numbers are used to summarize the data:

1. Smallest value
2. First quartile (Q_1)
3. Median (Q_2)
4. Third quartile (Q_3)
5. Largest value

The easiest way to develop a five-number summary is to first place the data in ascending order. Then it is easy to identify the smallest value, the three quartiles, and the largest value. The monthly starting salaries shown in Table 3.1 for a sample of 12 business school graduates are repeated here in ascending order.

3310	3355	3450	3480	3480	3490	3520	3540	3550	3650	3730	3925
		$Q_1 = 3465$			$Q_2 = 3505$ (Median)			$Q_3 = 3600$			

The median of 3505 and the quartiles $Q_1 = 3465$ and $Q_3 = 3600$ were computed in Section 3.1. Reviewing the data shows a smallest value of 3310 and a largest value of 3925. Thus the five-number summary for the salary data is 3310, 3465, 3505, 3600, 3925. Approximately one-fourth, or 25%, of the observations are between adjacent numbers in a five-number summary.

Box Plot

A **box plot** is a graphical summary of data that is based on a five-number summary. A key to the development of a box plot is the computation of the median and the quartiles, Q_1 and Q_3. The interquartile range, IQR $= Q_3 - Q_1$, is also used. Figure 3.10 is the box plot for the monthly starting salary data. The steps used to construct the box plot follow.

1. A box is drawn with the ends of the box located at the first and third quartiles. For the salary data, $Q_1 = 3465$ and $Q_3 = 3600$. This box contains the middle 50% of the data.
2. A vertical line is drawn in the box at the location of the median (3505 for the salary data).
3. By using the interquartile range, IQR $= Q_3 - Q_1$, *limits* are located. The limits for the box plot are 1.5(IQR) below Q_1 and 1.5(IQR) above Q_3. For the salary data, IQR $= Q_3 - Q_1 = 3600 - 3465 = 135$. Thus, the limits are $3465 - 1.5(135) = 3262.5$ and $3600 + 1.5(135) = 3802.5$. Data outside these limits are considered *outliers*.
4. The dashed lines in Figure 3.10 are called *whiskers*. The whiskers are drawn from the ends of the box to the smallest and largest values *inside the limits* computed in step 3. Thus, the whiskers end at salary values of 3310 and 3730.
5. Finally, the location of each outlier is shown with the symbol *. In Figure 3.10 we see one outlier, 3925.

Box plots provide another way to identify outliers. But they do not necessarily identify the same values as those with a z-score less than −3 or greater than +3. Either or both procedures may be used.

FIGURE 3.10 BOX PLOT OF THE STARTING SALARY DATA WITH LINES SHOWING THE LOWER AND UPPER LIMITS

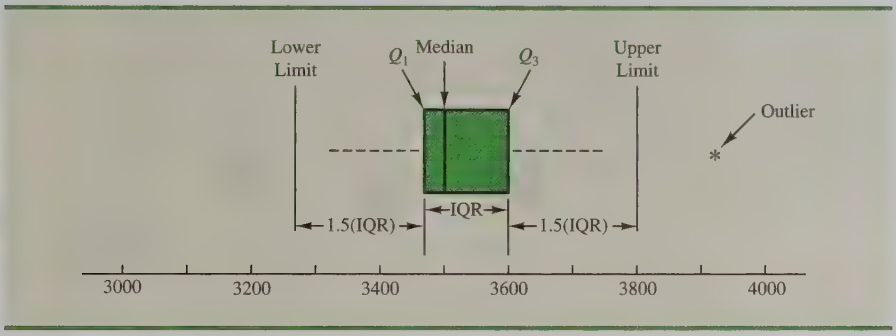
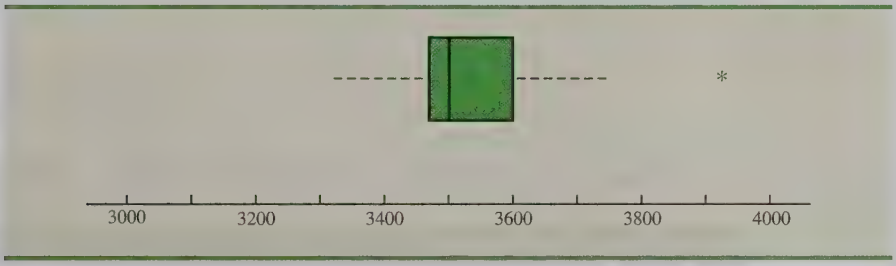


FIGURE 3.11 BOX PLOT OF MONTHLY STARTING SALARY DATA



In Figure 3.10 we included lines showing the location of the upper and lower limits. These lines were drawn to show how the limits are computed and where they are located. Although the limits are always computed, generally they are not drawn on the box plots. Figure 3.11 shows the usual appearance of a box plot for the salary data.

Comparative Analysis Using Box Plots

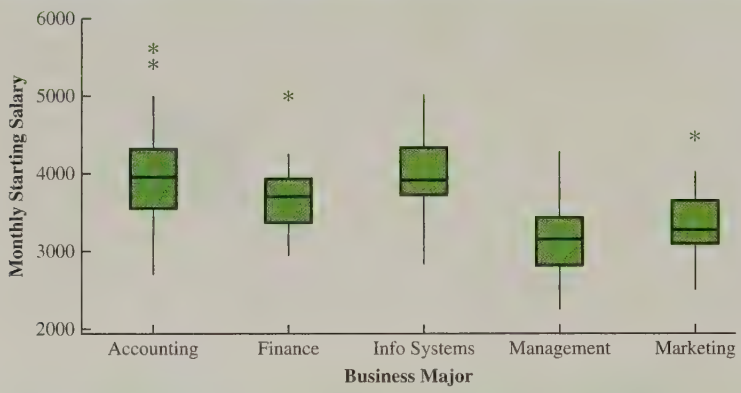
WEB file
MajorSalary

Box plots can also be used to provide a graphical summary of two or more groups and facilitate visual comparisons among the groups. For example, suppose the placement office decided to conduct a follow-up study to compare monthly starting salaries by the graduate's major: accounting, finance, information systems, management, and marketing. The major and starting salary data for a new sample of 111 recent business school graduates are shown in the data set named MajorSalary, and Figure 3.12 shows the box plots corresponding to each major. Note that major is shown on the horizontal axis and each box plot is shown vertically above the corresponding major. Displaying box plots in this manner is an excellent graphical technique for making comparisons among two or more groups.

What observations can we make about monthly starting salaries by major using the box plots in Figure 3.12? Specifically, we note the following:

- The higher salaries are in accounting; the lower salaries are in management and marketing.
- Based on the medians, accounting and information systems have similar and higher median salaries. Finance is next, with management and marketing showing lower median salaries.

FIGURE 3.12 BOX PLOTS OF MONTHLY STARTING SALARY BY MAJOR



- High salary outliers exist for accounting, finance, and marketing majors.
- Finance salaries appear to have the least variation, while accounting salaries appear to have the most variation.

Perhaps you can see additional interpretations based on these box plots.

NOTES AND COMMENTS

1. An advantage of the exploratory data analysis procedures is that they are easy to use; few numerical calculations are necessary. We simply sort the data values into ascending order and identify the five-number summary. The box plot can then be constructed. It is not necessary to compute the mean and the standard deviation for the data.
2. In the chapter appendix, we show how to construct a box plot for the starting salary data using StatTools.

Exercises

Methods

36. Consider a sample with data values of 27, 25, 20, 15, 30, 34, 28, and 25. Provide the five-number summary for the data.
37. Show the box plot for the data in exercise 36.
38. Show the five-number summary and the box plot for the following data: 5, 15, 18, 10, 8, 12, 16, 10, 6.
39. A data set has a first quartile of 42 and a third quartile of 50. Compute the lower and upper limits for the corresponding box plot. Should a data value of 65 be considered an outlier?

SELF test

Applications

40. Some of the most well-known food franchises and the number of retail locations for each are shown below (*The New York Times Almanac*, 2010).

WEB file
Franchise

Franchise	Locations	Franchise	Locations
Arby's	2,558	McDonald's	24,799
Baskin-Robbins	5,889	Papa John's	2,615
Dairy Queen	5,619	Pizza Hut	10,238
Domino's	8,053	Quiznos	5,110
Dunkin Donuts	8,082	Subway	29,612
Hardee's	1,397	Taco Bell	4,516
KFC Corp	11,553		

a. What is the largest franchise? How many retail locations does it have?
b. What is the median number of locations for the franchises?
c. Provide a five-number summary.
d. Are there any outliers?
e. Show the box plot.

41. Naples, Florida, hosts a half-marathon (13.1-mile race) in January each year. The event attracts top runners from throughout the United States as well as from around the world. In January 2009, 22 men and 31 women entered the 19–24 age class. Finish times in minutes are as follows (*Naples Daily News,* January 19, 2009). Times are shown in order of finish.

WEB file
Runners

Finish	Men	Women	Finish	Men	Women	Finish	Men	Women
1	65.30	109.03	11	109.05	123.88	21	143.83	136.75
2	66.27	111.22	12	110.23	125.78	22	148.70	138.20
3	66.52	111.65	13	112.90	129.52	23		139.00
4	66.85	111.93	14	113.52	129.87	24		147.18
5	70.87	114.38	15	120.95	130.72	25		147.35
6	87.18	118.33	16	127.98	131.67	26		147.50
7	96.45	121.25	17	128.40	132.03	27		147.75
8	98.52	122.08	18	130.90	133.20	28		153.88
9	100.52	122.48	19	131.80	133.50	29		154.83
10	108.18	122.62	20	138.63	136.57	30		189.27
						31		189.28

a. George Towett of Marietta, Georgia, finished in first place for the men and Lauren Wald of Gainesville, Florida, finished in first place for the women. Compare the first-place finish times for men and women. If the 53 men and women runners had competed as one group, in what place would Lauren have finished?
b. What is the median time for men and women runners? Compare men and women runners based on their median times.
c. Provide a five-number summary for both the men and the women.
d. Are there outliers in either group?
e. Show the box plots for the two groups. Did men or women have the most variation in finish times? Explain.

42. *Consumer Reports* provided overall customer satisfaction scores for AT&T, Sprint, T-Mobile, and Verizon cell-phone services in major metropolitan areas throughout the United States. The rating for each service reflects the overall customer satisfaction considering a variety of factors such as cost, connectivity problems, dropped calls, static interference, and customer support. A satisfaction scale from 0 to 100 was used, with 0

indicating completely dissatisfied and 100 indicating completely satisfied. The ratings for the four cell-phone services in 20 metropolitan areas are as shown (*Consumer Reports*, January 2009).

WEB file
CellService

Metropolitan Area	AT&T	Sprint	T-Mobile	Verizon
Atlanta	70	66	71	79
Boston	69	64	74	76
Chicago	71	65	70	77
Dallas	75	65	74	78
Denver	71	67	73	77
Detroit	73	65	77	79
Jacksonville	73	64	75	81
Las Vegas	72	68	74	81
Los Angeles	66	65	68	78
Miami	68	69	73	80
Minneapolis	68	66	75	77
Philadelphia	72	66	71	78
Phoenix	68	66	76	81
San Antonio	75	65	75	80
San Diego	69	68	72	79
San Francisco	66	69	73	75
Seattle	68	67	74	77
St. Louis	74	66	74	79
Tampa	73	63	73	79
Washington	72	68	71	76

a. Consider T-Mobile first. What is the median rating?
b. Develop a five-number summary for the T-Mobile service.
c. Are there outliers for T-Mobile? Explain.
d. Repeat parts (b) and (c) for the other three cell-phone services.
e. Show the box plots for the four cell-phone services on one graph. Discuss what a comparison of the box plots tells about the four services. Which service did *Consumer Reports* recommend as being best in terms of overall customer satisfaction?

43. The Philadelphia Phillies defeated the Tampa Bay Rays in the fifth game by a score of 4 to 3 to win the 2008 major league baseball World Series (*The Philadelphia Inquirer*, October 29, 2008). Earlier in the major league baseball playoffs, the Philadelphia Phillies defeated the Los Angeles Dodgers to win the National League Championship, while the Tampa Bay Rays defeated the Boston Red Sox to win the American League Championship. The file MLBSalaries contains the salaries for the 28 players on each of these four teams (USA Today Salary Database, October 2008). The data, shown in thousands of dollars, have been ordered from the highest salary to the lowest salary for each team.

WEB file
MLBSalaries

a. Analyze the salaries for the World Champion Philadelphia Phillies. What is the total payroll for the team? What is the median salary? What is the five-number summary?
b. Were there salary outliers for the Philadelphia Phillies? If so, how many and what were the salary amounts?
c. What is the total payroll for each of the other three teams? Develop the five-number summary for each team and identify any outliers.
d. Show the box plots of the salaries for all four teams. What are your interpretations? Of these four teams, does it appear that the team with the higher salaries won the league championships and the World Series?

TABLE 3.5 TWELVE-MONTH RETURN FOR MUTUAL FUNDS

Mutual Fund	Return (%)	Mutual Fund	Return (%)
Alger Capital Appreciation	23.5	Nations Small Company	21.4
Alger LargeCap Growth	22.8	Nations SmallCap Index	24.5
Alger MidCap Growth	38.3	Nations Strategic Growth	10.4
Alger SmallCap	41.3	Nations Value Inv	10.8
AllianceBernstein Technology	40.6	One Group Diversified Equity	10.0
Federated American Leaders	15.6	One Group Diversified Int'l	10.9
Federated Capital Appreciation	12.4	One Group Diversified Mid Cap	15.1
Federated Equity-Income	11.5	One Group Equity Income	6.6
Federated Kaufmann	33.3	One Group Int'l Equity Index	13.2
Federated Max-Cap Index	16.0	One Group Large Cap Growth	13.6
Federated Stock	16.9	One Group Large Cap Value	12.8
Janus Adviser Int'l Growth	10.3	One Group Mid Cap Growth	18.7
Janus Adviser Worldwide	3.4	One Group Mid Cap Value	11.4
Janus Enterprise	24.2	One Group Small Cap Growth	23.6
Janus High-Yield	12.1	PBHG Growth	27.3
Janus Mercury	20.6	Putnam Europe Equity	20.4
Janus Overseas	11.9	Putnam Int'l Capital Opportunity	36.6
Janus Worldwide	4.1	Putnam International Equity	21.5
Nations Convertible Securities	13.6	Putnam Int'l New Opportunity	26.3
Nations Int'l Equity	10.7	Strong Advisor Mid Cap Growth	23.7
Nations LargeCap Enhd. Core	13.2	Strong Growth 20	11.7
Nations LargeCap Index	13.5	Strong Growth Inv	23.2
Nation MidCap Index	19.5	Strong Large Cap Growth	14.5

44. A listing of 46 mutual funds and their 12-month total return percentage is shown in Table 3.5 (*Smart Money*, February 2004).
    a. What are the mean and median return percentages for these mutual funds?
    b. What are the first and third quartiles?
    c. Provide a five-number summary.
    d. Do the data contain any outliers? Show a box plot.

Measures of Association Between Two Variables

Thus far we have examined numerical methods used to summarize the data for *one variable at a time*. Often a manager or decision maker is interested in the *relationship between two variables*. In this section we present covariance and correlation as descriptive measures of the relationship between two variables.

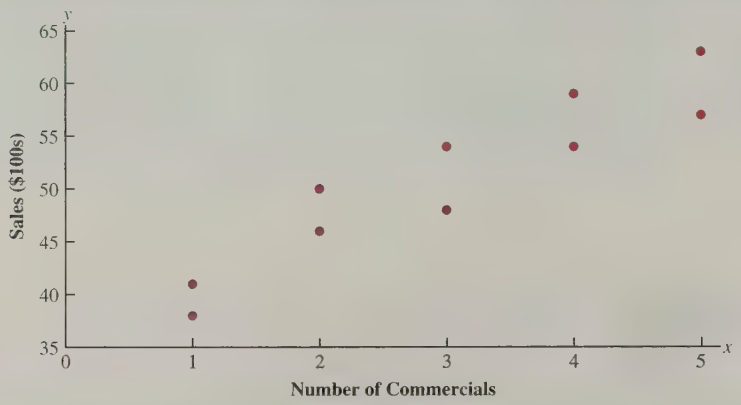
We begin by reconsidering the application concerning a stereo and sound equipment store in San Francisco as presented in Section 2.4. The store's manager wants to determine the relationship between the number of weekend television commercials shown and the sales at the store during the following week. Sample data with sales expressed in hundreds of dollars are provided in Table 3.6. It shows 10 observations ($n = 10$), one for each week. The scatter diagram in Figure 3.13 shows a positive relationship, with higher sales (y) associated with a greater number of commercials (x). In fact, the scatter diagram suggests

TABLE 3.6 SAMPLE DATA FOR THE STEREO AND SOUND EQUIPMENT STORE

WEB file
Stereo

Week	Number of Commercials x	Sales Volume ($100s) y
1	2	50
2	5	57
3	1	41
4	3	54
5	4	54
6	1	38
7	5	63
8	3	48
9	4	59
10	2	46

FIGURE 3.13 SCATTER DIAGRAM FOR THE STEREO AND SOUND EQUIPMENT STORE



that a straight line could be used as an approximation of the relationship. In the following discussion, we introduce **covariance** as a descriptive measure of the linear association between two variables.

Covariance

For a sample of size n with the observations (x_1, y_1), (x_2, y_2), and so on, the sample covariance is defined as follows:

SAMPLE COVARIANCE

$$s_{xy} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{n - 1} \qquad (3.10)$$

TABLE 3.7 CALCULATIONS FOR THE SAMPLE COVARIANCE

	x_i	y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})(y_i - \bar{y})$
	2	50	−1	−1	1
	5	57	2	6	12
	1	41	−2	−10	20
	3	54	0	3	0
	4	54	1	3	3
	1	38	−2	−13	26
	5	63	2	12	24
	3	48	0	−3	0
	4	59	1	8	8
	2	46	−1	−5	5
Totals	30	510	0	0	99

$$s_{xy} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{n - 1} = \frac{99}{10 - 1} = 11$$

This formula pairs each x_i with a y_i. We then sum the products obtained by multiplying the deviation of each x_i from its sample mean $\bar{x}$ by the deviation of the corresponding y_i from its sample mean $\bar{y}$; this sum is then divided by $n - 1$.

To measure the strength of the linear relationship between the number of commercials x and the sales volume y in the stereo and sound equipment store problem, we use equation (3.10) to compute the sample covariance. The calculations in Table 3.7 show the computation of $\Sigma(x_i - \bar{x})(y_i - \bar{y})$. Note that $\bar{x} = 30/10 = 3$ and $\bar{y} = 510/10 = 51$. Using equation (3.10), we obtain a sample covariance of

$$s_{xy} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{n - 1} = \frac{99}{9} = 11$$

The formula for computing the covariance of a population of size N is similar to equation (3.10), but we use different notation to indicate that we are working with the entire population.

POPULATION COVARIANCE

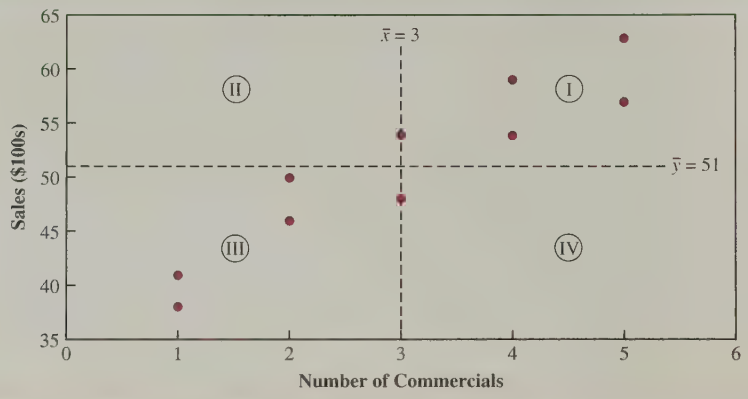
$$\sigma_{xy} = \frac{\Sigma(x_i - \mu_x)(y_i - \mu_y)}{N} \tag{3.11}$$

In equation (3.11) we use the notation μ_x for the population mean of the variable x and μ_y for the population mean of the variable y. The population covariance σ_{xy} is defined for a population of size N.

Interpretation of the Covariance

To aid in the interpretation of the sample covariance, consider Figure 3.14. It is the same as the scatter diagram of Figure 3.13 with a vertical dashed line at $\bar{x} = 3$ and a horizontal dashed line at $\bar{y} = 51$. The lines divide the graph into four quadrants. Points in quadrant I correspond to x_i greater than $\bar{x}$ and y_i greater than $\bar{y}$, points in quadrant II correspond to x_i

FIGURE 3.14 PARTITIONED SCATTER DIAGRAM FOR THE STEREO AND SOUND EQUIPMENT STORE



less than $\bar{x}$ and y_i greater than $\bar{y}$, and so on. Thus, the value of $(x_i - \bar{x})(y_i - \bar{y})$ must be positive for points in quadrant I, negative for points in quadrant II, positive for points in quadrant III, and negative for points in quadrant IV.

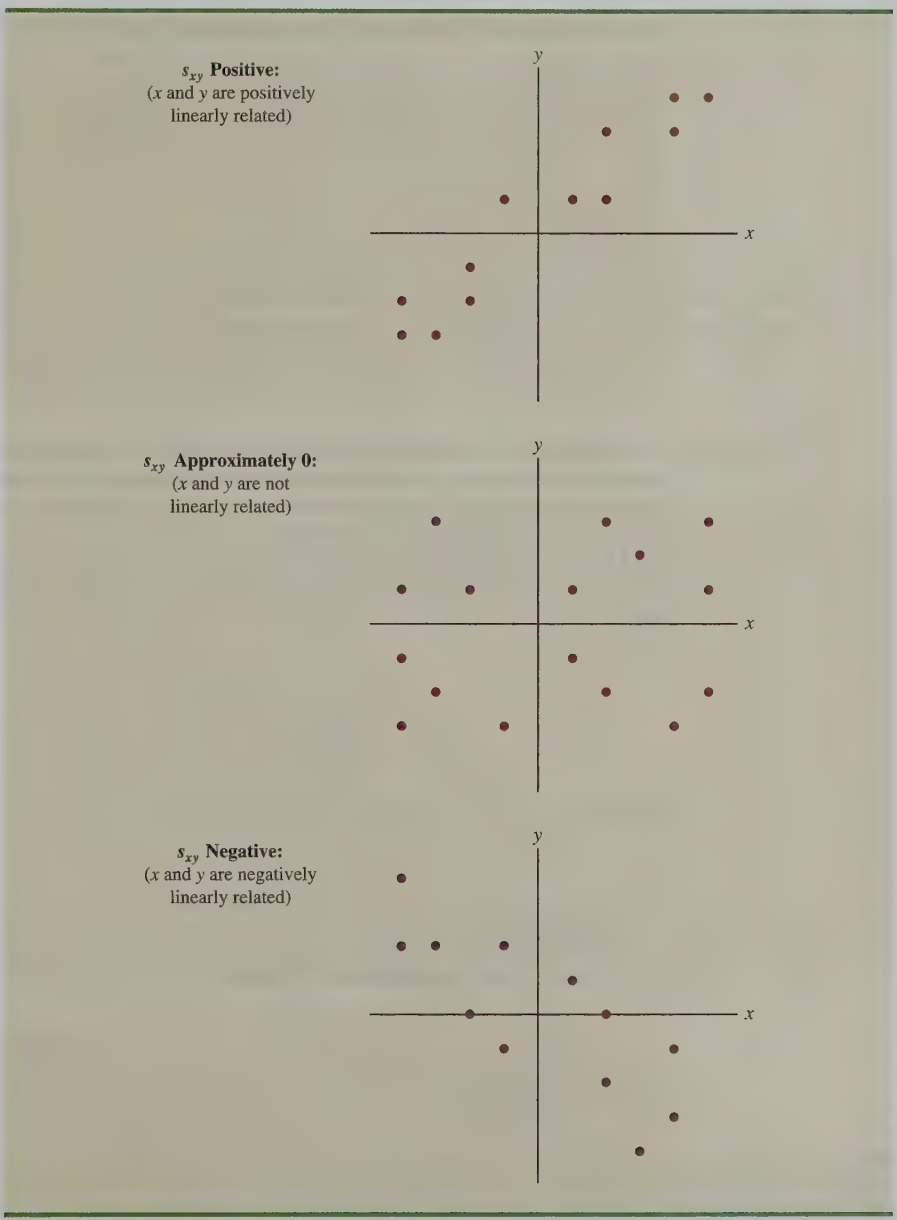
The covariance is a measure of the linear association between two variables.

If the value of s_{xy} is positive, the points with the greatest influence on s_{xy} must be in quadrants I and III. Hence, a positive value for s_{xy} indicates a positive linear association between x and y; that is, as the value of x increases, the value of y increases. If the value of s_{xy} is negative, however, the points with the greatest influence on s_{xy} are in quadrants II and IV. Hence, a negative value for s_{xy} indicates a negative linear association between x and y; that is, as the value of x increases, the value of y decreases. Finally, if the points are evenly distributed across all four quadrants, the value of s_{xy} will be close to zero, indicating no linear association between x and y. Figure 3.15 shows the values of s_{xy} that can be expected with three different types of scatter diagrams.

Referring again to Figure 3.14, we see that the scatter diagram for the stereo and sound equipment store follows the pattern in the top panel of Figure 3.15. As we should expect, the value of the sample covariance indicates a positive linear relationship with $s_{xy} = 11$.

From the preceding discussion, it might appear that a large positive value for the covariance indicates a strong positive linear relationship and that a large negative value indicates a strong negative linear relationship. However, one problem with using covariance as a measure of the strength of the linear relationship is that the value of the covariance depends on the units of measurement for x and y. For example, suppose we are interested in the relationship between height x and weight y for individuals. Clearly the strength of the relationship should be the same whether we measure height in feet or inches. Measuring the height in inches, however, gives us much larger numerical values for $(x_i - \bar{x})$ than when we measure height in feet. Thus, with height measured in inches, we would obtain a larger value for the numerator $\Sigma(x_i - \bar{x})(y_i - \bar{y})$ in equation (3.10)—and hence a larger covariance—when in fact the relationship does not change. A measure of the relationship between two variables that is not affected by the units of measurement for x and y is the **correlation coefficient**.

FIGURE 3.15 INTERPRETATION OF SAMPLE COVARIANCE



Correlation Coefficient

For sample data, the Pearson product moment correlation coefficient is defined as follows.

PEARSON PRODUCT MOMENT CORRELATION COEFFICIENT: SAMPLE DATA

$$r_{xy} = \frac{s_{xy}}{s_x s_y} \tag{3.12}$$

where

r_{xy} = sample correlation coefficient
s_{xy} = sample covariance
s_x = sample standard deviation of x
s_y = sample standard deviation of y

Equation (3.12) shows that the Pearson product moment correlation coefficient for sample data (commonly referred to more simply as the *sample correlation coefficient*) is computed by dividing the sample covariance by the product of the sample standard deviation of x and the sample standard deviation of y.

Let us now compute the sample correlation coefficient for the stereo and sound equipment store. Using the data in Table 3.7, we can compute the sample standard deviations for the two variables:

$$s_x = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{20}{9}} = 1.49$$

$$s_y = \sqrt{\frac{\Sigma(y_i - \bar{y})^2}{n-1}} = \sqrt{\frac{566}{9}} = 7.93$$

Now, because $s_{xy} = 11$, the sample correlation coefficient equals

$$r_{xy} = \frac{s_{xy}}{s_x s_y} = \frac{11}{(1.49)(7.93)} = .93$$

The formula for computing the correlation coefficient for a population, denoted by the Greek letter ρ_{xy} (rho, pronounced "row"), follows.

PEARSON PRODUCT MOMENT CORRELATION COEFFICIENT: POPULATION DATA

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \tag{3.13}$$

The sample correlation coefficient r_{xy} is a point estimator of the population correlation coefficient ρ_{xy}.

where

ρ_{xy} = population correlation coefficient
σ_{xy} = population covariance
σ_x = population standard deviation for x
σ_y = population standard deviation for y

Interpretation of the Correlation Coefficient

First let us consider a simple example that illustrates the concept of a perfect positive linear relationship. The scatter diagram in Figure 3.16 depicts the relationship between x and y based on the following sample data.

x_i	y_i
5	10
10	30
15	50

The straight line drawn through each of the three points shows a perfect linear relationship between x and y. In order to apply equation (3.12) to compute the sample correlation we must first compute s_{xy}, s_x, and s_y. Some of the computations are shown in Table 3.8.

FIGURE 3.16 SCATTER DIAGRAM DEPICTING A PERFECT POSITIVE LINEAR RELATIONSHIP

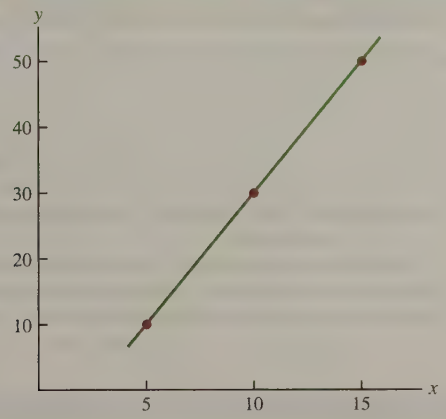


TABLE 3.8 COMPUTATIONS USED IN CALCULATING THE SAMPLE CORRELATION COEFFICIENT

	x_i	y_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$y_i - \bar{y}$	$(y_i - \bar{y})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
	5	10	−5	25	−20	400	100
	10	30	0	0	0	0	0
	15	50	5	25	20	400	100
Totals	30	90	0	50	0	800	200

$\bar{x} = 10 \quad \bar{y} = 30$

Using the results in this table, we find

$$s_{xy} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{n - 1} = \frac{200}{2} = 100$$

$$s_x = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n - 1}} = \sqrt{\frac{50}{2}} = 5$$

$$s_y = \sqrt{\frac{\Sigma(y_i - \bar{y})^2}{n - 1}} = \sqrt{\frac{800}{2}} = 20$$

$$r_{xy} = \frac{s_{xy}}{s_x s_y} = \frac{100}{5(20)} = 1$$

The correlation coefficient ranges from −1 to +1. Values close to −1 or +1 indicate a strong linear relationship. The closer the correlation is to zero, the weaker the relationship.

Thus, we see that the value of the sample correlation coefficient is 1.

In general, it can be shown that if all the points in a data set fall on a positively sloped straight line, the value of the sample correlation coefficient is +1; that is, a sample correlation coefficient of +1 corresponds to a perfect positive linear relationship between x and y. Moreover, if the points in the data set fall on a straight line having negative slope, the value of the sample correlation coefficient is −1; that is, a sample correlation coefficient of −1 corresponds to a perfect negative linear relationship between x and y.

Let us now suppose that a certain data set indicates a positive linear relationship between x and y but that the relationship is not perfect. The value of r_{xy} will be less than 1, indicating that the points in the scatter diagram are not all on a straight line. As the points deviate more and more from a perfect positive linear relationship, the value of r_{xy} becomes smaller and smaller. A value of r_{xy} equal to zero indicates no linear relationship between x and y, and values of r_{xy} near zero indicate a weak linear relationship.

For the data involving the stereo and sound equipment store, $r_{xy} = .93$. Therefore, we conclude that a strong positive linear relationship occurs between the number of commercials and sales. More specifically, an increase in the number of commercials is associated with an increase in sales.

In closing, we note that correlation provides a measure of linear association and not necessarily causation. A high correlation between two variables does not mean that changes in one variable will cause changes in the other variable. For example, we may find that the quality rating and the typical meal price of restaurants are positively correlated. However, simply increasing the meal price at a restaurant will not cause the quality rating to increase.

Using Excel to Compute the Covariance and Correlation Coefficient

Excel provides functions that can be used to compute the covariance and correlation coefficient. We show here how these functions can be used to compute the sample covariance and the sample correlation coefficient for the stereo and sound equipment store data. Refer to Figure 3.17 as we present the steps involved. The formula worksheet is in the background; the value worksheet is in the foreground.

Enter Data: Labels and data on commercials and sales are entered into cells A1:C11 of the worksheet.

Enter Functions and Formulas: Excel's covariance function, COVARIANCE.S, can be used to compute the sample covariance by entering the following formula into cell F1:

```
=COVARIANCE.S(B2:B11,C2:C11)
```

FIGURE 3.17 EXCEL WORKSHEET USED TO COMPUTE THE COVARIANCE AND CORRELATION COEFFICIENT

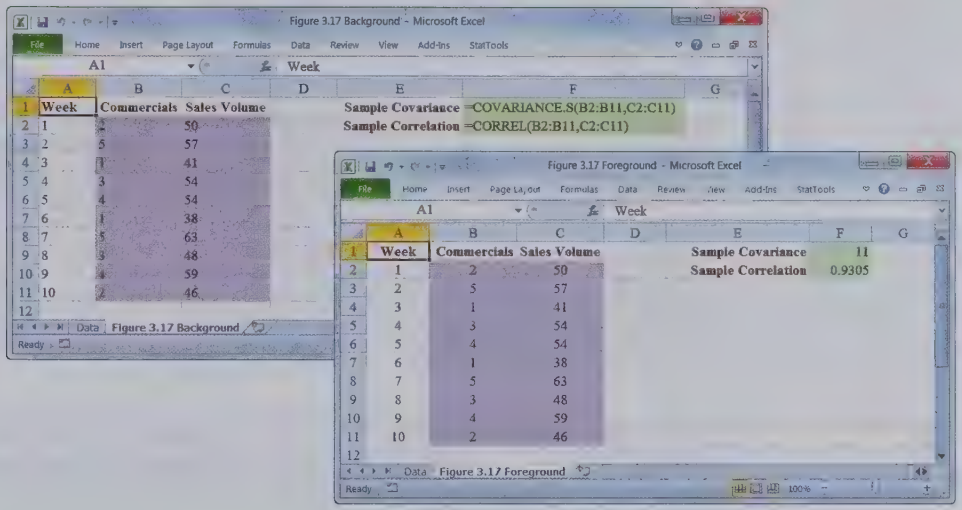


Figure 3.17 Background - Microsoft Excel
Sample Covariance =COVARIANCE.S(B2:B11,C2:C11)
Sample Correlation =CORREL(B2:B11,C2:C11)
Figure 3.17 Foreground - Microsoft Excel
Sample Covariance 11
Sample Correlation 0.9305

Similarly, the formula =CORREL(B2:B11,C2:C11) is entered into cell F2 to compute the sample correlation coefficient. The labels Sample Covariance and Sample Correlation are entered into cells E1 and E2 to identify the output.

The formulas in cells F1:F2 are displayed in the worksheet in the background of Figure 3.17. The worksheet in the foreground shows the values computed using the Excel functions. Note that, except for rounding, these values are the same as we computed earlier using equations (3.10) and (3.12).

Exercises

Methods

SELF test

45. Five observations taken for two variables follow.

x_i	4	6	11	3	16
y_i	50	50	40	60	30

a. Develop a scatter diagram with x on the horizontal axis.
b. What does the scatter diagram developed in part (a) indicate about the relationship between the two variables?
c. Compute and interpret the sample covariance.
d. Compute and interpret the sample correlation coefficient.

46. Five observations taken for two variables follow.

x_i	6	11	15	21	27
y_i	6	9	6	17	12

a. Develop a scatter diagram for these data.
b. What does the scatter diagram indicate about a relationship between x and y?
c. Compute and interpret the sample covariance.
d. Compute and interpret the sample correlation coefficient.

Applications

47. Ten major college football bowl games were played in January 2010, with the University of Alabama beating the University of Texas 37 to 21 to become the national champion of college football. The results of the ten bowl games are shown below (*USA Today*, January 8, 2010). The predicted winning point margin was based on Las Vegas betting odds approximately one week before the bowl games were played. For example, Auburn was predicted to beat Northwestern in the Outback Bowl by five points. The actual winning point margin for Auburn was three points. A negative predicted winning point margin means that the team that won the bowl game was an underdog and expected to lose. For example, in the Rose Bowl, Ohio State was a two-point underdog to Oregon and ended up winning by nine points.

WEB file
BowlGames

Bowl Game	Score	Predicted Point Margin	Actual Point Margin
Outback	Auburn 38 Northwestern 35	5	3
Gator	Florida State 33 West Virginia 21	1	12
Capital One	Penn State 19 LSU 17	3	2
Rose	Ohio State 26 Oregon 17	−2	9
Sugar	Florida 51 Cincinnati 24	14	27
Cotton	Mississippi State 21 Oklahoma State 7	3	14
Alamo	Texas Tech 41 Michigan State 31	9	10
Fiesta	Boise State 17 TCU 10	−4	7
Orange	Iowa 24 Georgia Tech 14	−3	10
Championship	Alabama 37 Texas 21	4	16

a. Develop a scatter diagram with predicted point margin on the horizontal axis.
b. What is the relationship between predicted and actual point margins?
c. Compute and interpret the sample covariance.
d. Compute the sample correlation coefficient. What does this value indicate about the relationship between the Las Vegas predicted point margin and the actual point margin in college football bowl games?

48. A department of transportation's study on driving speed and miles per gallon for midsize automobiles resulted in the following data:

Speed (Miles per Hour)	30	50	40	55	30	25	60	25	50	55
Miles per Gallon	28	25	25	23	30	32	21	35	26	25

Compute and interpret the sample correlation coefficient.

49. At the beginning of 2009, the economic downturn resulted in the loss of jobs and an increase in delinquent loans for housing. The national unemployment rate was 6.5% and the percentage of delinquent loans was 6.12% (*The Wall Street Journal*, January 27, 2009). In projecting where the real estate market was headed in the coming year, economists studied the relationship between the jobless rate and the percentage of delinquent loans. The expectation was that if the jobless rate continued to increase, there would also be an

increase in the percentage of delinquent loans. The data below show the jobless rate and the delinquent loan percentage for 27 major real estate markets.

WEB file Housing

Metro Area	Jobless Rate (%)	Delinquent Loan (%)	Metro Area	Jobless Rate (%)	Delinquent Loan (%)
Atlanta	7.1	7.02	New York	6.2	5.78
Boston	5.2	5.31	Orange County	6.3	6.08
Charlotte	7.8	5.38	Orlando	7.0	10.05
Chicago	7.8	5.40	Philadelphia	6.2	4.75
Dallas	5.8	5.00	Phoenix	5.5	7.22
Denver	5.8	4.07	Portland	6.5	3.79
Detroit	9.3	6.53	Raleigh	6.0	3.62
Houston	5.7	5.57	Sacramento	8.3	9.24
Jacksonville	7.3	6.99	St. Louis	7.5	4.40
Las Vegas	7.6	11.12	San Diego	7.1	6.91
Los Angeles	8.2	7.56	San Francisco	6.8	5.57
Miami	7.1	12.11	Seattle	5.5	3.87
Minneapolis	6.3	4.39	Tampa	7.5	8.42
Nashville	6.6	4.78			

a. Compute the correlation coefficient. Is there a positive correlation between the jobless rate and the percentage of delinquent housing loans? What is your interpretation?
b. Show a scatter diagram of the relationship between jobless rate and the percentage of delinquent housing loans.

50. The Dow Jones Industrial Average (DJIA) and the Standard & Poor's 500 Index (S&P 500) are both used to measure the performance of the stock market. The DJIA is based on the price of stocks for 30 large companies; the S&P 500 is based on the price of stocks for 500 companies. If both the DJIA and S&P 500 measure the performance of the stock market, how are they correlated? The following data show the daily percent increase or daily percent decrease in the DJIA and S&P 500 for a sample of nine days over a three-month period (*The Wall Street Journal,* January 15 to March 10, 2006).

WEB file StockMarket

DJIA	.20	.82	−.99	.04	−.24	1.01	.30	.55	−.25
S&P 500	.24	.19	−.91	.08	−.33	.87	.36	.83	−.16

a. Show a scatter diagram.
b. Compute the sample correlation coefficient for these data.
c. Discuss the association between the DJIA and S&P 500. Do you need to check both before having a general idea about the daily stock market performance?

51. The daily high and low temperatures for 14 cities around the world are shown (The Weather Channel, April 22, 2009).

WEB file WorldTemp

City	High	Low	City	High	Low
Athens	68	50	London	67	45
Beijing	70	49	Moscow	44	29
Berlin	65	44	Paris	69	44
Cairo	96	64	Rio de Janeiro	76	69
Dublin	57	46	Rome	69	51
Geneva	70	45	Tokyo	70	58
Hong Kong	80	73	Toronto	44	39

a. What is the sample mean high temperature?
b. What is the sample mean low temperature?
c. What is the correlation between the high and low temperatures? Discuss.

3.6 The Weighted Mean and Working with Grouped Data

In Section 3.1, we presented the mean as one of the most important measures of central location. The formula for the mean of a sample with n observations is restated as follows.

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{x_1 + x_2 + \cdots + x_n}{n} \tag{3.14}$$

In this formula, each x_i is given equal importance or weight. Although this practice is most common, in some instances, the mean is computed by giving each observation a weight that reflects its importance. A mean computed in this manner is referred to as a **weighted mean**.

Weighted Mean

The weighted mean is computed as follows:

WEIGHTED MEAN

$$\bar{x} = \frac{\Sigma w_i x_i}{\Sigma w_i} \tag{3.15}$$

where

x_i = value of observation i
w_i = weight for observation i

When the data are from a sample, equation (3.15) provides the weighted sample mean. When the data are from a population, μ replaces $\bar{x}$ and equation (3.15) provides the weighted population mean.

As an example of the need for a weighted mean, consider the following sample of five purchases of a raw material over the past three months.

Purchase	Cost per Pound ($)	Number of Pounds
1	3.00	1200
2	3.40	500
3	2.80	2750
4	2.90	1000
5	3.25	800

Note that the cost per pound varies from $2.80 to $3.40, and the quantity purchased varies from 500 to 2750 pounds. Suppose that a manager asked for information about the mean cost per pound of the raw material. Because the quantities ordered vary, we must use the formula for a weighted mean. The five cost-per-pound data values are $x_1 = 3.00$, $x_2 = 3.40$, $x_3 = 2.80$, $x_4 = 2.90$, and $x_5 = 3.25$. The weighted mean cost per pound is found by weighting each cost by its corresponding quantity. For this example, the weights are $w_1 = 1200$, $w_2 = 500$,

$w_3 = 2750$, $w_4 = 1000$, and $w_5 = 800$. Based on equation (3.15), the weighted mean is calculated as follows:

$$\bar{x} = \frac{1200(3.00) + 500(3.40) + 2750(2.80) + 1000(2.90) + 800(3.25)}{1200 + 500 + 2750 + 1000 + 800}$$

$$= \frac{18{,}500}{6250} = 2.96$$

Thus, the weighted mean computation shows that the mean cost per pound for the raw material is \$2.96. Note that using equation (3.14) rather than the weighted mean formula would have provided misleading results. In this case, the mean of the five cost-per-pound values is (3.00 + 3.40 + 2.80 + 2.90 + 3.25)/5 = 15.35/5 = \$3.07, which overstates the actual mean cost per pound purchased.

Computing a grade point average is a good example of the use of a weighted mean.

The choice of weights for a particular weighted mean computation depends upon the application. An example that is well known to college students is the computation of a grade point average (GPA). In this computation, the data values generally used are 4 for an A grade, 3 for a B grade, 2 for a C grade, 1 for a D grade, and 0 for an F grade. The weights are the number of credits hours earned for each grade. Exercise 54 at the end of this section provides an example of this weighted mean computation. In other weighted mean computations, quantities such as pounds, dollars, or volume are frequently used as weights. In any case, when observations vary in importance, the analyst must choose the weight that best reflects the importance of each observation in the determination of the mean.

Grouped Data

In most cases, measures of location and variability are computed by using the individual data values. Sometimes, however, data are available only in a grouped or frequency distribution form. In the following discussion, we show how the weighted mean formula can be used to obtain approximations of the mean, variance, and standard deviation for **grouped data**.

In Section 2.2 we provided a frequency distribution of the time in days required to complete year-end audits for the public accounting firm of Sanderson and Clifford. The frequency distribution of audit times is shown in Table 3.9. Based on this frequency distribution, what is the sample mean audit time?

To compute the mean using only the grouped data, we treat the midpoint of each class as being representative of the items in the class. Let M_i denote the midpoint for class i and let f_i denote the frequency of class i. The weighted mean formula (3.15) is then used with the data values denoted as M_i and the weights given by the frequencies f_i. In

TABLE 3.9 FREQUENCY DISTRIBUTION OF AUDIT TIMES

Audit Time (days)	Frequency
10–14	4
15–19	8
20–24	5
25–29	2
30–34	1
Total	20

this case, the denominator of equation (3.15) is the sum of the frequencies, which is the sample size n. That is, $\Sigma f_i = n$. Thus, the equation for the sample mean for grouped data is as follows.

SAMPLE MEAN FOR GROUPED DATA

$$\bar{x} = \frac{\Sigma f_i M_i}{n} \tag{3.16}$$

where

M_i = the midpoint for class i
f_i = the frequency for class i
n = the sample size

With the class midpoints, M_i, halfway between the class limits, the first class of 10–14 in Table 3.9 has a midpoint at $(10 + 14)/2 = 12$. The five class midpoints and the weighted mean computation for the audit time data are summarized in Table 3.10. As can be seen, the sample mean audit time is 19 days.

To compute the variance for grouped data, we use a slightly altered version of the formula for the variance provided in equation (3.5). In equation (3.5), the squared deviations of the data about the sample mean $\bar{x}$ were written $(x_i - \bar{x})^2$. However, with grouped data, the values are not known. In this case, we treat the class midpoint, M_i, as being representative of the x_i values in the corresponding class. Thus, the squared deviations about the sample mean, $(x_i - \bar{x})^2$, are replaced by $(M_i - \bar{x})^2$. Then, just as we did with the sample mean calculations for grouped data, we weight each value by the frequency of the class, f_i. The sum of the squared deviations about the mean for all the data is approximated by $\Sigma f_i(M_i - \bar{x})^2$. The term $n - 1$ rather than n appears in the denominator in order to make the sample variance the estimate of the population variance. Thus, the following formula is used to obtain the sample variance for grouped data.

SAMPLE VARIANCE FOR GROUPED DATA

$$s^2 = \frac{\Sigma f_i(M_i - \bar{x})^2}{n - 1} \tag{3.17}$$

TABLE 3.10 COMPUTATION OF THE SAMPLE MEAN AUDIT TIME FOR GROUPED DATA

Audit Time (days)	Class Midpoint (M_i)	Frequency (f_i)	f_iM_i
10–14	12	4	48
15–19	17	8	136
20–24	22	5	110
25–29	27	2	54
30–34	32	1	32
		20	380

Sample mean $\bar{x} = \dfrac{\Sigma f_i M_i}{n} = \dfrac{380}{20} = 19$ days

TABLE 3.11 COMPUTATION OF THE SAMPLE VARIANCE OF AUDIT TIMES FOR GROUPED DATA (SAMPLE MEAN $\bar{x} = 19$)

Audit Time (days)	Class Midpoint (M_i)	Frequency (f_i)	Deviation ($M_i - \bar{x}$)	Squared Deviation $(M_i - \bar{x})^2$	$f_i(M_i - \bar{x})^2$
10–14	12	4	−7	49	196
15–19	17	8	−2	4	32
20–24	22	5	3	9	45
25–29	27	2	8	64	128
30–34	32	1	13	169	169
		20			570
					$\Sigma f_i(M_i - \bar{x})^2$

$$\text{Sample variance } s^2 = \frac{\Sigma f_i(M_i - \bar{x})^2}{n - 1} = \frac{570}{19} = 30$$

The calculation of the sample variance for audit times based on the grouped data is shown in Table 3.11. The sample variance is 30.

The standard deviation for grouped data is simply the square root of the variance for grouped data. For the audit time data, the sample standard deviation is $s = \sqrt{30} = 5.48$.

Before closing this section on computing measures of location and dispersion for grouped data, we note that formulas (3.16) and (3.17) are for a sample. Population summary measures are computed similarly. The grouped data formulas for a population mean and variance follow.

POPULATION MEAN FOR GROUPED DATA

$$\mu = \frac{\Sigma f_i M_i}{N} \tag{3.18}$$

POPULATION VARIANCE FOR GROUPED DATA

$$\sigma^2 = \frac{\Sigma f_i(M_i - \mu)^2}{N} \tag{3.19}$$

NOTES AND COMMENTS

In computing descriptive statistics for grouped data, the class midpoints are used to approximate the data values in each class. As a result, the descriptive statistics for grouped data approximate the descriptive statistics that would result from using the original data directly. We therefore recommend computing descriptive statistics from the original data rather than from grouped data whenever possible.

Exercises

Methods

52. Consider the following data and corresponding weights.

x_i	Weight (w_i)
3.2	6
2.0	3
2.5	2
5.0	8

a. Compute the weighted mean.
b. Compute the sample mean of the four data values without weighting. Note the difference in the results provided by the two computations.

SELF test

53. Consider the sample data in the following frequency distribution.

Class	Midpoint	Frequency
3–7	5	4
8–12	10	7
13–17	15	9
18–22	20	5

a. Compute the sample mean.
b. Compute the sample variance and sample standard deviation.

Applications

SELF test

54. The grade point average for college students is based on a weighted mean computation. For most colleges, the grades are given the following data values: A (4), B (3), C (2), D (1), and F (0). After 60 credit hours of course work, a student at State University earned 9 credit hours of A, 15 credit hours of B, 33 credit hours of C, and 3 credit hours of D.
a. Compute the student's grade point average.
b. Students at State University must maintain a 2.5 grade point average for their first 60 credit hours of course work in order to be admitted to the business college. Will this student be admitted?

55. Morningstar tracks the total return for a large number of mutual funds. The following table shows the total return and the number of funds for four categories of mutual funds (*Morningstar Funds 500*, 2008).

Type of Fund	Number of Funds	Total Return (%)
Domestic Equity	9191	4.65
International Equity	2621	18.15
Specialty Stock	1419	11.36
Hybrid	2900	6.75

a. Using the number of funds as weights, compute the weighted average total return for the mutual funds covered by Morningstar.

b. Is there any difficulty associated with using the "number of funds" as the weights in computing the weighted average total return for Morningstar in part (a)? Discuss. What else might be used for weights?

c. Suppose you had invested $10,000 in mutual funds at the beginning of 2007 and diversified the investment by placing $2000 in Domestic Equity funds, $4000 in International Equity funds, $3000 in Specialty Stock funds, and $1000 in Hybrid funds. What is the expected return on the portfolio?

56. Based on a survey of 425 master's programs in business administration, the *U.S. News & World Report* ranked the Indiana University Kelley Business School as the 20th best business program in the country (*America's Best Graduate Schools,* 2009). The ranking was based in part on surveys of business school deans and corporate recruiters. Each survey respondent was asked to rate the overall academic quality of the master's program on a scale from 1 "marginal" to 5 "outstanding." Use the sample of responses shown below to compute the weighted mean score for the business school deans and the corporate recruiters. Discuss.

Quality Assessment	Business School Deans	Corporate Recruiters
5	44	31
4	66	34
3	60	43
2	10	12
1	0	0

57. The following frequency distribution shows the price per share of the 30 companies in the Dow Jones Industrial Average (*Barron's,* February 2, 2009).

Price per Share	Number of Companies
$0–9	4
$10–19	5
$20–29	7
$30–39	3
$40–49	4
$50–59	4
$60–69	0
$70–79	2
$80–89	0
$90–99	1

a. Compute the mean price per share and the standard deviation of the price per share for the Dow Jones Industrial Average companies.

b. On January 16, 2006, the mean price per share was $45.83 and the standard deviation was $18.14. Comment on the changes in the price per share over the three-year period.

Summary

In this chapter we introduced several descriptive statistics that can be used to summarize the location, variability, and shape of a data distribution. Unlike the tabular and graphical procedures introduced in Chapter 2, the measures introduced in this chapter summarize the data in terms of numerical values. When the numerical values obtained are for a sample, they are called sample

statistics. When the numerical values obtained are for a population, they are called population parameters. Some of the notation used for sample statistics and population parameters follows.

In statistical inference, the sample statistic is referred to as the point estimator of the population parameter.

	Sample Statistic	Population Parameter
Mean	$\bar{x}$	μ
Variance	s^2	σ^2
Standard deviation	s	σ
Covariance	s_{xy}	σ_{xy}
Correlation	r_{xy}	ρ_{xy}

As measures of central location, we defined the mean, median, and mode. Then the concept of percentiles was used to describe other locations in the data set. Next, we presented the range, interquartile range, variance, standard deviation, and coefficient of variation as measures of variability or dispersion. Our primary measure of the shape of a data distribution was the skewness. Negative values indicate a data distribution skewed to the left. Positive values indicate a data distribution skewed to the right. We then described how the mean and standard deviation could be used, applying Chebyshev's theorem and the empirical rule, to provide more information about the distribution of data and to identify outliers.

In Section 3.4 we showed how to develop a five-number summary and a box plot to provide simultaneous information about the location, variability, and shape of the distribution. In Section 3.5 we introduced covariance and the correlation coefficient as measures of association between two variables. In the final section, we showed how to compute a weighted mean and how to calculate a mean, variance, and standard deviation for grouped data.

Glossary

Sample statistic A numerical value used as a summary measure for a sample (e.g., the sample mean, $\bar{x}$, the sample variance, s^2, and the sample standard deviation, s).

Population parameter A numerical value used as a summary measure for a population (e.g., the population mean, μ, the population variance, σ^2, and the population standard deviation, σ).

Point estimator The sample statistic, such as $\bar{x}$, s^2, and s, when used to estimate the corresponding population parameter.

Mean A measure of central location computed by summing the data values and dividing by the number of observations.

Median A measure of central location provided by the value in the middle when the data are arranged in ascending order.

Mode A measure of location, defined as the value that occurs with greatest frequency.

Percentile A value such that at least p percent of the observations are less than or equal to this value and at least $(100 - p)$ percent of the observations are greater than or equal to this value. The 50th percentile is the median.

Quartiles The 25th, 50th, and 75th percentiles are referred to as the first quartile, the second quartile (median), and third quartile, respectively. The quartiles can be used to divide a data set into four parts, with each part containing approximately 25% of the data.

Range A measure of variability, defined to be the largest value minus the smallest value.
Interquartile range (IQR) A measure of variability, defined to be the difference between the third and first quartiles.
Variance A measure of variability based on the squared deviations of the data values about the mean.
Standard deviation A measure of variability computed by taking the positive square root of the variance.
Coefficient of variation A measure of relative variability computed by dividing the standard deviation by the mean and multiplying by 100.
Skewness A measure of the shape of a data distribution. Data skewed to the left result in negative skewness; a symmetric data distribution results in zero skewness; and data skewed to the right result in positive skewness.
***z*-score** A value computed by dividing the deviation about the mean $(x_i - \bar{x})$ by the standard deviation s. A z-score is referred to as a standardized value and denotes the number of standard deviations x_i is from the mean.
Chebyshev's theorem A theorem that can be used to make statements about the proportion of data values that must be within a specified number of standard deviations of the mean.
Empirical rule A rule that can be used to compute the percentage of data values that must be within one, two, and three standard deviations of the mean for data that exhibit a bell-shaped distribution.
Outlier An unusually small or unusually large data value.
Five-number summary An exploratory data analysis technique that uses five numbers to summarize the data: smallest value, first quartile, median, third quartile, and largest value.
Box plot A graphical summary of data based on a five-number summary.
Covariance A measure of linear association between two variables. Positive values indicate a positive relationship; negative values indicate a negative relationship.
Correlation coefficient A measure of linear association between two variables that takes on values between -1 and $+1$. Values near $+1$ indicate a strong positive linear relationship; values near -1 indicate a strong negative linear relationship; and values near zero indicate the lack of a linear relationship.
Weighted mean The mean obtained by assigning each observation a weight that reflects its importance.
Grouped data Data available in class intervals as summarized by a frequency distribution. Individual values of the original data are not available.

Key Formulas

Sample Mean

$$\bar{x} = \frac{\Sigma x_i}{n} \tag{3.1}$$

Population Mean

$$\mu = \frac{\Sigma x_i}{N} \tag{3.2}$$

Interquartile Range

$$\text{IQR} = Q_3 - Q_1 \tag{3.3}$$

Population Variance

$$\sigma^2 = \frac{\Sigma(x_i - \mu)^2}{N} \tag{3.4}$$

Sample Variance

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n - 1} \tag{3.5}$$

Standard Deviation

$$\text{Sample standard deviation} = s = \sqrt{s^2} \tag{3.6}$$

$$\text{Population standard deviation} = \sigma = \sqrt{\sigma^2} \tag{3.7}$$

Coefficient of Variation

$$\left(\frac{\text{Standard deviation}}{\text{Mean}} \times 100\right)\% \tag{3.8}$$

***z*-Score**

$$z_i = \frac{x_i - \bar{x}}{s} \tag{3.9}$$

Sample Covariance

$$s_{xy} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{n - 1} \tag{3.10}$$

Population Covariance

$$\sigma_{xy} = \frac{\Sigma(x_i - \mu_x)(y_i - \mu_y)}{N} \tag{3.11}$$

Pearson Product Moment Correlation Coefficient: Sample Data

$$r_{xy} = \frac{s_{xy}}{s_x s_y} \tag{3.12}$$

Pearson Product Moment Correlation Coefficient: Population Data

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \tag{3.13}$$

Weighted Mean

$$\bar{x} = \frac{\Sigma w_i x_i}{\Sigma w_i} \tag{3.15}$$

Sample Mean for Grouped Data

$$\bar{x} = \frac{\Sigma f_i M_i}{n} \tag{3.16}$$

Sample Variance for Grouped Data

$$s^2 = \frac{\sum f_i(M_i - \bar{x})^2}{n - 1} \tag{3.17}$$

Population Mean for Grouped Data

$$\mu = \frac{\sum f_i M_i}{N} \tag{3.18}$$

Population Variance for Grouped Data

$$\sigma^2 = \frac{\sum f_i(M_i - \mu)^2}{N} \tag{3.19}$$

Supplementary Exercises

58. According to an annual consumer spending survey, the average monthly Bank of America Visa credit card charge was $1838 (*U.S. Airways Attaché Magazine,* December 2003). A sample of monthly credit card charges provides the following data.

WEB file
Visa

236	1710	1351	825	7450
316	4135	1333	1584	387
991	3396	170	1428	1688

a. Compute the mean and median.
b. Compute the first and third quartiles.
c. Compute the range and interquartile range.
d. Compute the variance and standard deviation.
e. The skewness measure for these data is 2.12. Comment on the shape of this distribution. Is it the shape you would expect? Why or why not?
f. Do the data contain outliers?

59. The U.S. Census Bureau provides statistics on family life in the United States, including the age at the time of first marriage, current marital status, and size of household (U.S. Census Bureau website, March 20, 2006). The following data show the age at the time of first marriage for a sample of men and a sample of women.

WEB file
Ages

Men	26	23	28	25	27	30	26	35	28
	21	24	27	29	30	27	32	27	25
Women	20	28	23	30	24	29	26	25	
	22	22	25	23	27	26	19		

a. Determine the median age at the time of first marriage for men and women.
b. Compute the first and third quartiles for both men and women.
c. Twenty-five years ago the median age at the time of first marriage was 25 for men and 22 for women. What insight does this information provide about the decision of when to marry among young people today?

60. Dividend yield is the annual dividend per share a company pays divided by the current market price per share expressed as a percentage. A sample of 10 large companies provided the following dividend yield data (*The Wall Street Journal,* January 16, 2004).

Company	Yield %	Company	Yield %
Altria Group	5.0	General Motors	3.7
American Express	0.8	JPMorgan Chase	3.5
Caterpillar	1.8	McDonald's	1.6
Eastman Kodak	1.9	United Technology	1.5
ExxonMobil	2.5	Walmart Stores	0.7

a. What are the mean and median dividend yields?
b. What are the variance and standard deviation?
c. Which company provides the highest dividend yield?
d. What is the z-score for McDonald's? Interpret this z-score.
e. What is the z-score for General Motors? Interpret this z-score.
f. Based on z-scores, do the data contain any outliers?

61. The U.S. Department of Education reports that about 50% of all college students use a student loan to help cover college expenses (National Center for Educational Studies, January 2006). A sample of students who graduated with student loan debt is shown here. The data, in thousands of dollars, show typical amounts of debt upon graduation.

10.1 14.8 5.0 10.2 12.4 12.2 2.0 11.5 17.8 4.0

a. For those students who use a student loan, what is the mean loan debt upon graduation?
b. What is the variance? Standard deviation?

62. Small business owners often look to payroll service companies to handle their employee payroll. Reasons are that small business owners face complicated tax regulations and penalties for employment tax errors are costly. According to the Internal Revenue Service, 26% of all small business employment tax returns contained errors that resulted in a tax penalty to the owner (*The Wall Street Journal,* January 30, 2006). The tax penalty for a sample of 20 small business owners follows:

WEB file Penalty

820	270	450	1010	890	700	1350	350	300	1200
390	730	2040	230	640	350	420	270	370	620

a. What is the mean tax penalty for improperly filed employment tax returns?
b. What is the standard deviation?
c. Is the highest penalty, $2040, an outlier?
d. What are some of the advantages of a small business owner hiring a payroll service company to handle employee payroll services, including the employment tax returns?

63. Public transportation and the automobile are two methods an employee can use to get to work each day. Samples of times recorded for each method are shown. Times are in minutes.

Public Transportation:	28	29	32	37	33	25	29	32	41	34
Automobile:	29	31	33	32	34	30	31	32	35	33

a. Compute the sample mean time to get to work for each method.
b. Compute the sample standard deviation for each method.
c. On the basis of your results from parts (a) and (b), which method of transportation should be preferred? Explain.
d. Develop a box plot for each method. Does a comparison of the box plots support your conclusion in part (c)?

64. The National Association of Realtors reported the median home price in the United States and the increase in median home price over a five-year period (*The Wall Street Journal,*

January 16, 2006). Use the sample home prices shown here to answer the following questions.

WEB file
Homes

995.9	48.8	175.0	263.5	298.0	218.9	209.0
628.3	111.0	212.9	92.6	2325.0	958.0	212.5

a. What is the sample median home price?
b. In January 2001, the National Association of Realtors reported a median home price of $139,300 in the United States. What was the percentage increase in the median home price over the five-year period?
c. What are the first quartile and the third quartile for the sample data?
d. Provide a five-number summary for the home prices.
e. Do the data contain any outliers?
f. What is the mean home price for the sample? Why does the National Association of Realtors prefer to use the median home price in its reports?

65. The U.S. Census Bureau's American Community Survey reported the percentage of children under 18 years of age who had lived below the poverty level during the previous 12 months (U.S. Census Bureau website, August 2008). The region of the country, Northeast (NE), Southeast (SE), Midwest (MW), Southwest (SW), and West (W) and the percentage of children under 18 who had lived below the poverty level are shown for each state.

WEB file
PovertyLevel

State	Region	Poverty %	State	Region	Poverty %
Alabama	SE	23.0	Montana	W	17.3
Alaska	W	15.1	Nebraska	MW	14.4
Arizona	SW	19.5	Nevada	W	13.9
Arkansas	SE	24.3	New Hampshire	NE	9.6
California	W	18.1	New Jersey	NE	11.8
Colorado	W	15.7	New Mexico	SW	25.6
Connecticut	NE	11.0	New York	NE	20.0
Delaware	NE	15.8	North Carolina	SE	20.2
Florida	SE	17.5	North Dakota	MW	13.0
Georgia	SE	20.2	Ohio	MW	18.7
Hawaii	W	11.4	Oklahoma	SW	24.3
Idaho	W	15.1	Oregon	W	16.8
Illinois	MW	17.1	Pennsylvania	NE	16.9
Indiana	MW	17.9	Rhode Island	NE	15.1
Iowa	MW	13.7	South Carolina	SE	22.1
Kansas	MW	15.6	South Dakota	MW	16.8
Kentucky	SE	22.8	Tennessee	SE	22.7
Louisiana	SE	27.8	Texas	SW	23.9
Maine	NE	17.6	Utah	W	11.9
Maryland	NE	9.7	Vermont	NE	13.2
Massachusetts	NE	12.4	Virginia	SE	12.2
Michigan	MW	18.3	Washington	W	15.4
Minnesota	MW	12.2	West Virginia	SE	25.2
Mississippi	SE	29.5	Wisconsin	MW	14.9
Missouri	MW	18.6	Wyoming	W	12.0

a. What is the median poverty level percentage for the 50 states?
b. What are the first and third quartiles? What is your interpretation of the quartiles?
c. Show a box plot for the data. Interpret the box plot in terms of what it tells you about the level of poverty for children in the United States. Are any states considered outliers? Discuss.
d. Identify the states in the lower quartile. What is your interpretation of this group and what region or regions are represented most in the lower quartile?

66. *Travel + Leisure* magazine presented its annual list of the 500 best hotels in the world (*Travel + Leisure,* January 2009). The magazine provides a rating for each hotel along with a brief description that includes the size of the hotel, amenities, and the cost per night for a double room. A sample of 12 of the top-rated hotels in the United States follows.

WEB file Travel

Hotel	Location	Rooms	Cost/Night
Boulders Resort & Spa	Phoenix, AZ	220	499
Disney's Wilderness Lodge	Orlando, FL	727	340
Four Seasons Hotel Beverly Hills	Los Angeles, CA	285	585
Four Seasons Hotel	Boston, MA	273	495
Hay-Adams	Washington, DC	145	495
Inn on Biltmore Estate	Asheville, NC	213	279
Loews Ventana Canyon Resort	Phoenix, AZ	398	279
Mauna Lani Bay Hotel	Island of Hawaii	343	455
Montage Laguna Beach	Laguna Beach, CA	250	595
Sofitel Water Tower	Chicago, IL	414	367
St. Regis Monarch Beach	Dana Point, CA	400	675
The Broadmoor	Colorado Springs, CO	700	420

a. What is the mean number of rooms?
b. What is the mean cost per night for a double room?
c. Develop a scatter diagram with the number of rooms on the horizontal axis and the cost per night on the vertical axis. Does there appear to be a relationship between the number of rooms and the cost per night? Discuss.
d. What is the sample correlation coefficient? What does it tell you about the relationship between the number of rooms and the cost per night for a double room? Does this appear reasonable? Discuss.

67. Morningstar tracks the performance of a large number of companies and publishes an evaluation of each. Along with a variety of financial data, Morningstar includes a Fair Value estimate for the price that should be paid for a share of the company's common stock. Data for 30 companies are available in the file named FairValue. The data include the Fair Value estimate per share of common stock, the most recent price per share, and the earning per share for the company (*Morningstar Stocks 500,* 2008).

WEB file FairValue

a. Develop a scatter diagram for the Fair Value and Share Price data with Share Price on the horizontal axis. What is the sample correlation coefficient, and what can you say about the relationship between the variables?
b. Develop a scatter diagram for the Fair Value and Earnings per Share data with Earnings per Share on the horizontal axis. What is the sample correlation coefficient, and what can you say about the relationship between the variables?

68. Does a major league baseball team's record during spring training indicate how the team will play during the regular season? Over the last six years, the correlation coefficient between a team's winning percentage in spring training and its winning percentage in the regular season is .18 (*The Wall Street Journal,* March 30, 2009). Shown are the winning percentages for the 14 American League teams during the 2008 season.

WEB file SpringTraining

Team	Spring Training	Regular Season	Team	Spring Training	Regular Season
Baltimore Orioles	.407	.422	Minnesota Twins	.500	.540
Boston Red Sox	.429	.586	New York Yankees	.577	.549
Chicago White Sox	.417	.546	Oakland A's	.692	.466
Cleveland Indians	.569	.500	Seattle Mariners	.500	.377
Detroit Tigers	.569	.457	Tampa Bay Rays	.731	.599
Kansas City Royals	.533	.463	Texas Rangers	.643	.488
Los Angeles Angels	.724	.617	Toronto Blue Jays	.448	.531

a. What is the correlation coefficient between the spring training and the regular season winning percentages?
b. What is your conclusion about a team's record during spring training indicating how the team will play during the regular season? What are some of the reasons why this occurs? Discuss.

69. The days to maturity for a sample of five money market funds are shown here. The dollar amounts invested in the funds are provided. Use the weighted mean to determine the mean number of days to maturity for dollars invested in these five money market funds.

Days to Maturity	Dollar Value ($millions)
20	20
12	30
7	10
5	15
6	10

70. Automobiles traveling on a road with a posted speed limit of 55 miles per hour are checked for speed by a state police radar system. Following is a frequency distribution of speeds.

Speed (miles per hour)	Frequency
45–49	10
50–54	40
55–59	150
60–64	175
65–69	75
70–74	15
75–79	10
Total	475

a. What is the mean speed of the automobiles traveling on this road?
b. Compute the variance and the standard deviation.

Case Problem 1 Pelican Stores

Pelican Stores, a division of National Clothing, is a chain of women's apparel stores operating throughout the country. The chain recently ran a promotion in which discount coupons were sent to customers of other National Clothing stores. Data collected for a sample of 100 in-store credit card transactions at Pelican Stores during one day while the promotion was running are contained in the file named PelicanStores. Table 3.12 shows a portion of the data set. The proprietary card method of payment refers to charges made using a National Clothing charge card. Customers who made a purchase using a discount coupon are referred to as promotional customers and customers who made a purchase but did not use a discount coupon are referred to as regular customers. Because the promotional coupons were not sent to regular Pelican Stores customers, management considers the sales made to people presenting the promotional coupons as sales it would not otherwise make. Of course, Pelican also hopes that the promotional customers will continue to shop at its stores.

TABLE 3.12 SAMPLE OF 100 CREDIT CARD PURCHASES AT PELICAN STORES

WEB file
PelicanStores

Customer	Type of Customer	Items	Net Sales	Method of Payment	Gender	Marital Status	Age
1	Regular	1	39.50	Discover	Male	Married	32
2	Promotional	1	102.40	Proprietary Card	Female	Married	36
3	Regular	1	22.50	Proprietary Card	Female	Married	32
4	Promotional	5	100.40	Proprietary Card	Female	Married	28
5	Regular	2	54.00	MasterCard	Female	Married	34
6	Regular	1	44.50	MasterCard	Female	Married	44
7	Promotional	2	78.00	Proprietary Card	Female	Married	30
8	Regular	1	22.50	Visa	Female	Married	40
9	Promotional	2	56.52	Proprietary Card	Female	Married	46
10	Regular	1	44.50	Proprietary Card	Female	Married	36
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
96	Regular	1	39.50	MasterCard	Female	Married	44
97	Promotional	9	253.00	Proprietary Card	Female	Married	30
98	Promotional	10	287.59	Proprietary Card	Female	Married	52
99	Promotional	2	47.60	Proprietary Card	Female	Married	30
100	Promotional	1	28.44	Proprietary Card	Female	Married	44

Most of the variables shown in Table 3.12 are self-explanatory, but two of the variables require some clarification.

Items The total number of items purchased
Net Sales The total amount ($) charged to the credit card

Pelican's management would like to use this sample data to learn about its customer base and to evaluate the promotion involving discount coupons.

Managerial Report

Use the methods of descriptive statistics presented in this chapter to summarize the data and comment on your findings. At a minimum, your report should include the following:

1. Descriptive statistics on net sales and descriptive statistics on net sales by various classifications of customers.
2. Descriptive statistics concerning the relationship between age and net sales.

Case Problem 2 Motion Picture Industry

The motion picture industry is a competitive business. More than 50 studios produce a total of 300 to 400 new motion pictures each year, and the financial success of each motion picture varies considerably. The opening weekend gross sales, the total gross sales, the number of theaters the movie was shown in, and the number of weeks the motion picture was in the top 60 for gross sales are common variables used to measure the success of a motion picture. Data collected for a sample of 100 motion pictures produced in 2005 are

TABLE 3.13 PERFORMANCE DATA FOR 10 MOTION PICTURES

Motion Picture	Opening Gross Sales ($millions)	Total Gross Sales ($millions)	Number of Theaters	Weeks in Top 60
Coach Carter	29.17	67.25	2574	16
Ladies in Lavender	0.15	6.65	119	22
Batman Begins	48.75	205.28	3858	18
Unleashed	10.90	24.47	1962	8
Pretty Persuasion	0.06	0.23	24	4
Fever Pitch	12.40	42.01	3275	14
Harry Potter and the Goblet of Fire	102.69	287.18	3858	13
Monster-in-Law	23.11	82.89	3424	16
White Noise	24.11	55.85	2279	7
Mr. and Mrs. Smith	50.34	186.22	3451	21

WEB file Movies

contained in the file named Movies. Table 3.13 shows the data for the first 10 motion pictures in the file.

Managerial Report

Use the numerical methods of descriptive statistics presented in this chapter to learn how these variables contribute to the success of a motion picture. Include the following in your report.

1. Descriptive statistics for each of the four variables along with a discussion of what the descriptive statistics tell us about the motion picture industry.
2. What motion pictures, if any, should be considered high-performance outliers? Explain.
3. Descriptive statistics showing the relationship between total gross sales and each of the other variables. Discuss.

Case Problem 3 Heavenly Chocolates Website Transactions

Heavenly Chocolates manufactures and sells quality chocolate products at its plant and retail store located in Saratoga Springs, New York. Two years ago the company developed a website and began selling its products over the Internet. Website sales have exceeded the company's expectations, and mangement is now considering stragegies to increase sales even further. To learn more about the website customers, a sample of 50 Heavenly Chocolate transactions was selected from the previous month's sales. Data showing the day of the week each transaction was made, the type of browser the customer used, the time spent on the website, the number of website pages viewed, and the amount spent by each of the 50 customers are contained in the file named Shoppers. A portion of the data is shown in Table 3.14.

Heavenly Chocolates would like to use the sample data to determine if online shoppers who spend more time and view more pages also spend more money during their visit to the website. The company would also like to investigate the effect that the day of the week and the type of browser have on sales.

TABLE 3.14 A SAMPLE OF 50 HEAVENLY CHOCOLATES WEBSITE TRANSACTIONS

WEB file
Shoppers

Customer	Day	Browser	Time (min)	Pages Viewed	Amount Spent ($)
1	Mon	Internet Explorer	12.0	4	54.52
2	Wed	Other	19.5	6	94.90
3	Mon	Internet Explorer	8.5	4	26.68
4	Tue	Firefox	11.4	2	44.73
5	Wed	Internet Explorer	11.3	4	66.27
6	Sat	Firefox	10.5	6	67.80
7	Sun	Internet Explorer	11.4	2	36.04
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
48	Fri	Internet Explorer	9.7	5	103.15
49	Mon	Other	7.3	6	52.15
50	Fri	Internet Explorer	13.4	3	98.75

Managerial Report

Use the methods of descriptive statistics to learn about the customers who visit the Heavenly Chocolates website. Include the following in your report.

1. Graphical and numerical summaries for the length of time the shopper spends on the website, the number of pages viewed, and the mean amount spent per transaction. Discuss what you learn about Heavenly Cholcolates' online shoppers from these numerical summaries.
2. Summarize the frequency, the total dollars spent, and the mean amount spent per transaction for each day of week. What observations can you make about Heavenly Chocolates' business based on the day of the week? Discuss.
3. Summarize the frequency, the total dollars spent, and the mean amount spent per transaction for each type of browser. What observations can you make about Heavenly Chocolate's business based on the type of browser? Discuss.
4. Develop a scatter diagram and compute the sample correlation coefficient to explore the relationship between the time spent on the website and the dollar amount spent. Use the horizontal axis for the time spent on the website. Discuss.
5. Develop a scatter diagram and compute the sample correlation coefficient to explore the relationship between the the number of website pages viewed and the amount spent. Use the horizontal axis for the number of website pages viewed. Discuss.
6. Develop a scatter diagram and compute the sample correlation coefficient to explore the relationship between the time spent on the website and the number of pages viewed. Use the horizontal axis to represent the number of pages viewed. Discuss.

Appendix Descriptive Statistics Using StatTools

In this appendix, we describe how StatTools can be used to compute a variety of descriptive statistics and also display box plots. We then show how StatTools can be used to obtain covariance and correlation measures for two variables.

Descriptive Statistics

StartSalary

We use the starting salary data in Table 3.1 to illustrate. Begin by using the Data Set Manager to create a StatTools data set for these data using the procedure described in the appendix in Chapter 1. The following steps will generate a variety of descriptive statistics.

Step 1. Click the **StatTools** tab on the Ribbon
Step 2. In the **Analyses Group,** click **Summary Statistics**
Step 3. Choose the **One-Variable Summary** option
Step 4. When the StatTools-One-Variable Summary Statistics dialog box appears,
In the **Variables** section, select **Starting Salary**
Click **OK**

A variety of descriptive statistics will appear.

Box Plots

StartSalary

We use the starting salary data in Table 3.1 to illustrate. Begin by using the Data Set Manager to create a StatTools data set for the StartSalary data using the procedure described in the appendix in Chapter 1. The following steps will create the box plot for these data.

In StatTools a box plot is referred to as a box-whisker plot.

Step 1. Click the **StatTools** tab on the Ribbon
Step 2. In the **Analyses Group**, click **Summary Graphs**
Step 3. Choose the **Box-Whisker Plot** option
Step 4. When the StatTools - Box-Whisker Plot dialog box appears,
In the Variables section, select **Starting Salary**
Click **OK**

The box plot for the starting salary data created using StatTools has a slightly different appearance from the box plot shown in Figure 3.11. The procedure StatTools uses to compute quartiles is different from the procedure we showed in Section 3.1; this results in a different value for the interquartile range (IQR) and therefore different lower and upper limits for the box plot. In StatTools the whiskers are shown as solid lines, the location of the sample mean is shown using the symbol ✳, and two types of outliers are identified. Mild outliers, shown with the symbol □, are observations that are no more than 1.5(IQR) from the edges of the box, and extreme outliers, shown with the symbol ■, are observations that are greater than 3(IQR) from the edges of the box.

MajorSalary

Comparative Analysis StatTools can also be used to display box plots for several groups. We illustrate using the MajorSalary data set. Begin by using the Data Set Manager to create a StatTools data set for the MajorSalary data using the procedure described in the appendix to Chapter 1. The following steps will create the box plot for these data.

Step 1. Click the **StatTools** tab on the Ribbon
Step 2. In the **Analyses Group,** click **Summary Graphs**
Step 3. Choose the **Box-Whisker Plot** option
Step 4. When the StatTools - Box-Whisker Plot dialog box appears,
Click the **Format** button and select the **Stacked** option
In the Variables section,
In the **Cat** column select **Major**
In the **Val** column select **Monthly Salary**
Click **OK**

The box plot for the monthly starting salaries by major created using StatTools has a slightly different appearance from the box plot shown in Figure 3.12; however, the interpretations based on both plots are the same.

Covariance and Correlation

We use the stereo and sound equipment data in Table 3.7 to demonstrate the computation of the sample covariance and the sample correlation coefficient. Begin by using the Data Set Manager to create a StatTools data set for these data using the procedure described in the appendix in Chapter 1. The following steps will provide the sample covariance and sample correlation coefficient.

Stereo

Step 1. Click the **StatTools** tab on the Ribbon
Step 2. In the **Analyses Group,** click **Summary Statistics**
Step 3. Choose the **Correlation and Covariance** option
Step 4. When the StatTools - Correlation and Covariance dialog box appears,
In the **Variables** section,
Select **No. of Commercials**
Select **Sales Volume**
In the **Tables to Create** section,
Select **Table of Correlations**
Select **Table of Covariances**
In the **Table Structure** section select **Symmetric**
Click **OK**

A table showing the correlation coefficient and the covariance will appear.

CHAPTER 4

Introduction to Probability

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STATISTICS IN PRACTICE: OCEANWIDE SEAFOOD

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STATISTICS in PRACTICE

OCEANWIDE SEAFOOD*

SPRINGBORO, OHIO

Oceanwide Seafood is the leading provider of quality seafood in southwestern Ohio. The company stocks over 90 varieties of fresh and frozen seafood products from around the world and prepares specialty cuts according to customer specifications. Customers include major restaurants and retail food stores in Ohio, Kentucky, and Indiana. Established in 2005, the company has become successful by providing superior customer service and exceptional quality seafood.

Probability and statistical information are used for both operational and marketing decisions. For instance, a time series showing monthly sales is used to track the company's growth and to set future target sales levels. Statistics such as the mean customer order size and the mean number of days a customer takes to make payments help identify the firm's best customers as well as provide benchmarks for handling accounts receivable issues. In addition, data on monthly inventory levels are used in the analysis of operating profits and trends in product sales.

Probability analysis has helped Oceanwide determine reasonable and profitable prices for its products. For example, when Oceanwide receives a whole fresh fish from one of its suppliers, the fish must be processed and cut to fill individual customer orders. A fresh 100-pound whole tuna packed in ice might cost Oceanwide $500. At first glance, the company's cost for tuna appears to be $500/100 = $5 per pound. However, due to the loss in the processing and cutting operation, a 100-pound whole tuna will not provide 100 pounds of finished product. If the processing and cutting operation yields 75% of the whole tuna, the number of pounds of finished product available for sale to customers would be .75(100) = 75 pounds, not 100 pounds. In this case, the company's actual cost of tuna would be $500/75 = $6.67 per pound. Thus, Oceanwide would need to use a cost of $6.67 per pound to determine a profitable price to charge its customers.

To help determine the yield percentage that is likely for processing and cutting whole tuna, data were collected on the yields from a sample of whole tunas. Let Y denote the yield percentage for whole tuna. Using the data, Oceanwide was able to determine that 5% of the time the yield for whole tuna was at least 90%. In conditional probability notation, this probability is written $P(Y \geq 90\% \mid \text{Tuna}) = .05$; in other words, the probability that the yield will be at least 90% given that the fish is a tuna is .05. If Oceanwide established the selling price for tuna based on a 90% yield, 95% of the time the company would realize a yield less than expected. As a result, the company would be understating its cost per pound and also understating the price of tuna for its customers. Additional conditional probability information for other yield percentages helped management select a 70% yield as the basis for determining the cost of tuna and the price to charge its customers. Similar conditional probabilities for other seafood products helped management establish pricing yield percentages for each type of seafood product. In this chapter, you will learn how to compute and interpret conditional probabilities and other probabilities that are helpful in the decision-making process.

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Oceanwide Seafood uses probability analysis to help determine reasonable and profitable prices for its products.

*The authors are indebted to Dale Hartlage, president of Oceanwide Seafood Company, for providing this Statistics in Practice.

Managers often base their decisions on an analysis of uncertainties such as the following:

1. What are the chances that sales will decrease if we increase prices?
2. What is the likelihood a new assembly method will increase productivity?
3. How likely is it that the project will be finished on time?
4. What is the chance that a new investment will be profitable?

Some of the earliest work on probability originated in a series of letters between Pierre de Fermat and Blaise Pascal in the 1650s.

Probability is a numerical measure of the likelihood that an event will occur. Thus, probabilities can be used as measures of the degree of uncertainty associated with the four events previously listed. If probabilities are available, we can determine the likelihood of each event occurring.

Probability values are always assigned on a scale from 0 to 1. A probability near zero indicates an event is unlikely to occur; a probability near 1 indicates an event is almost certain to occur. Other probabilities between 0 and 1 represent degrees of likelihood that an event will occur. For example, if we consider the event "rain tomorrow," we understand that when the weather report indicates "a near-zero probability of rain," it means almost no chance of rain. However, if a .90 probability of rain is reported, we know that rain is likely to occur. A .50 probability indicates that rain is just as likely to occur as not. Figure 4.1 depicts the view of probability as a numerical measure of the likelihood of an event occurring.

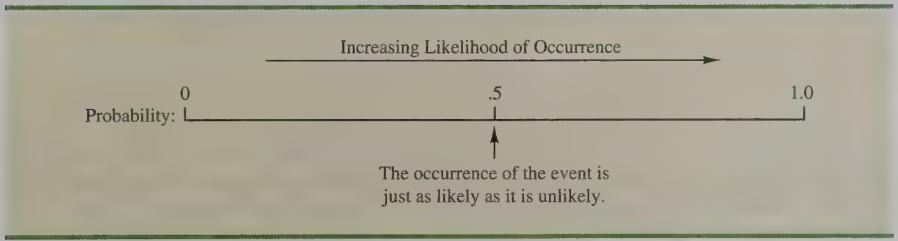
4.1 Experiments, Counting Rules, and Assigning Probabilities

In discussing probability, we define an **experiment** as a process that generates well-defined outcomes. On any single repetition of an experiment, one and only one of the possible experimental outcomes will occur. Several examples of experiments and their associated outcomes follow.

Experiment	Experimental Outcomes
Toss a coin	Head, tail
Select a part for inspection	Defective, nondefective
Conduct a sales call	Purchase, no purchase
Roll a die	1, 2, 3, 4, 5, 6
Play a football game	Win, lose, tie

By specifying all possible experimental outcomes, we identify the **sample space** for an experiment.

FIGURE 4.1 PROBABILITY AS A NUMERICAL MEASURE OF THE LIKELIHOOD OF AN EVENT OCCURRING



SAMPLE SPACE

The sample space for an experiment is the set of all experimental outcomes.

Experimental outcomes are also called sample points.

An experimental outcome is also called a **sample point** to identify it as an element of the sample space.

Consider the first experiment in the preceding table—tossing a coin. The upward face of the coin—a head or a tail—determines the experimental outcomes (sample points). If we let S denote the sample space, we can use the following notation to describe the sample space.

$$S = \{\text{Head, Tail}\}$$

The sample space for the second experiment in the table—selecting a part for inspection—can be described as follows:

$$S = \{\text{Defective, Nondefective}\}$$

Both of the experiments just described have two experimental outcomes (sample points). However, suppose we consider the fourth experiment listed in the table—rolling a die. The possible experimental outcomes, defined as the number of dots appearing on the upward face of the die, are the six points in the sample space for this experiment.

$$S = \{1, 2, 3, 4, 5, 6\}$$

Counting Rules, Combinations, and Permutations

Being able to identify and count the experimental outcomes is a necessary step in assigning probabilities. We now discuss three useful counting rules.

Multiple-step experiments The first counting rule applies to multiple-step experiments. Consider the experiment of tossing two coins. Let the experimental outcomes be defined in terms of the pattern of heads and tails appearing on the upward faces of the two coins. How many experimental outcomes are possible for this experiment? The experiment of tossing two coins can be thought of as a two-step experiment in which step 1 is the tossing of the first coin and step 2 is the tossing of the second coin. If we use H to denote a head and T to denote a tail, (H, H) indicates the experimental outcome with a head on the first coin and a head on the second coin. Continuing this notation, we can describe the sample space (S) for this coin-tossing experiment as follows:

$$S = \{(H, H), (H, T), (T, H), (T, T)\}$$

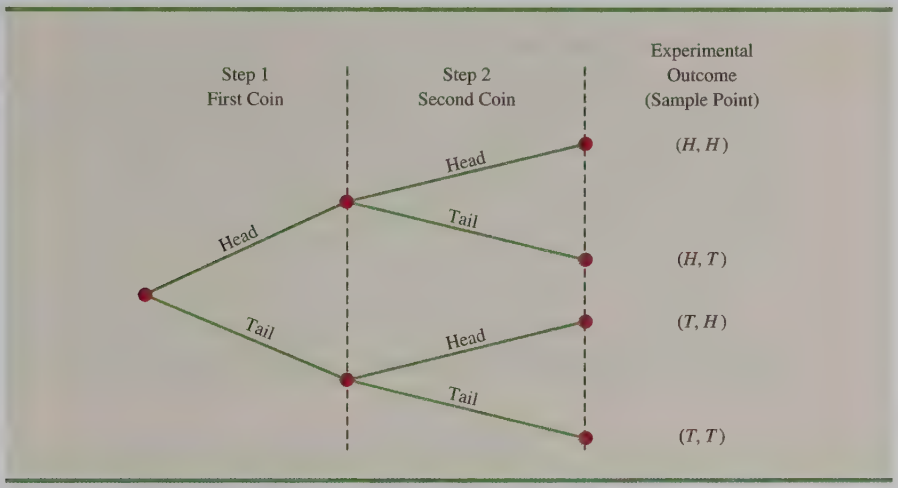
Thus, we see that four experimental outcomes are possible. In this case, we can easily list all of the experimental outcomes.

The counting rule for multiple-step experiments makes it possible to determine the number of experimental outcomes without listing them.

COUNTING RULE FOR MULTIPLE-STEP EXPERIMENTS

If an experiment can be described as a sequence of k steps with n_1 possible outcomes on the first step, n_2 possible outcomes on the second step, and so on, then the total number of experimental outcomes is given by $(n_1)(n_2) \ldots (n_k)$.

FIGURE 4.2 TREE DIAGRAM FOR THE EXPERIMENT OF TOSSING TWO COINS



Viewing the experiment of tossing two coins as a sequence of first tossing one coin ($n_1 = 2$) and then tossing the other coin ($n_2 = 2$), we can see from the counting rule that $(2)(2) = 4$ distinct experimental outcomes are possible. As shown, they are $S = \{(H, H), (H, T), (T, H), (T, T)\}$. The number of experimental outcomes in an experiment involving tossing six coins is $(2)(2)(2)(2)(2)(2) = 64$.

Without the tree diagram, one might think only three experimental outcomes are possible for two tosses of a coin: 0 heads, 1 head, and 2 heads.

A **tree diagram** is a graphical representation that helps in visualizing a multiple-step experiment. Figure 4.2 shows a tree diagram for the experiment of tossing two coins. The sequence of steps moves from left to right through the tree. Step 1 corresponds to tossing the first coin, and step 2 corresponds to tossing the second coin. For each step, the two possible outcomes are head or tail. Note that for each possible outcome at step 1 two branches correspond to the two possible outcomes at step 2. Each of the points on the right end of the tree corresponds to an experimental outcome. Each path through the tree from the leftmost node to one of the nodes at the right side of the tree corresponds to a unique sequence of outcomes.

Let us now see how the counting rule for multiple-step experiments can be used in the analysis of a capacity expansion project for the Kentucky Power & Light Company (KP&L). KP&L is starting a project designed to increase the generating capacity of one of its plants in northern Kentucky. The project is divided into two sequential stages or steps: stage 1 (design) and stage 2 (construction). Even though each stage will be scheduled and controlled as closely as possible, management cannot predict beforehand the exact time required to complete each stage of the project. An analysis of similar construction projects revealed possible completion times for the design stage of 2, 3, or 4 months and possible completion times for the construction stage of 6, 7, or 8 months. In addition, because of the critical need for additional electrical power, management set a goal of 10 months for the completion of the entire project.

Because this project has three possible completion times for the design stage (step 1) and three possible completion times for the construction stage (step 2), the counting rule for multiple-step experiments can be applied here to determine a total of $(3)(3) = 9$ experimental outcomes. To describe the experimental outcomes, we use a two-number notation; for instance, (2, 6) indicates that the design stage is completed in 2 months and the

TABLE 4.1 EXPERIMENTAL OUTCOMES (SAMPLE POINTS) FOR THE KP&L PROJECT

Completion Time (months)			
Stage 1 Design	**Stage 2 Construction**	**Notation for Experimental Outcome**	**Total Project Completion Time (months)**
2	6	(2, 6)	8
2	7	(2, 7)	9
2	8	(2, 8)	10
3	6	(3, 6)	9
3	7	(3, 7)	10
3	8	(3, 8)	11
4	6	(4, 6)	10
4	7	(4, 7)	11
4	8	(4, 8)	12

construction stage is completed in 6 months. This experimental outcome results in a total of 2 + 6 = 8 months to complete the entire project. Table 4.1 summarizes the nine experimental outcomes for the KP&L problem. The tree diagram in Figure 4.3 shows how the nine outcomes (sample points) occur.

The counting rule and tree diagram help the project manager identify the experimental outcomes and determine the possible project completion times. From the information in Figure 4.3, we see that the project will be completed in 8 to 12 months, with six of the nine experimental outcomes providing the desired completion time of 10 months or less. Even though identifying the experimental outcomes may be helpful, we need to consider how probability values can be assigned to the experimental outcomes before making an assessment of the probability that the project will be completed within the desired 10 months.

Combinations A second useful counting rule allows one to count the number of experimental outcomes when the experiment involves selecting n objects from a (usually larger) set of N objects. It is called the counting rule for combinations.

COUNTING RULE FOR COMBINATIONS

The number of combinations of N objects taken n at a time is

$$C_n^N = \binom{N}{n} = \frac{N!}{n!(N-n)!} \tag{4.1}$$

where

$$N! = N(N-1)(N-2)\cdots(2)(1)$$
$$n! = n(n-1)(n-2)\cdots(2)(1)$$

and, by definition,

$$0! = 1$$

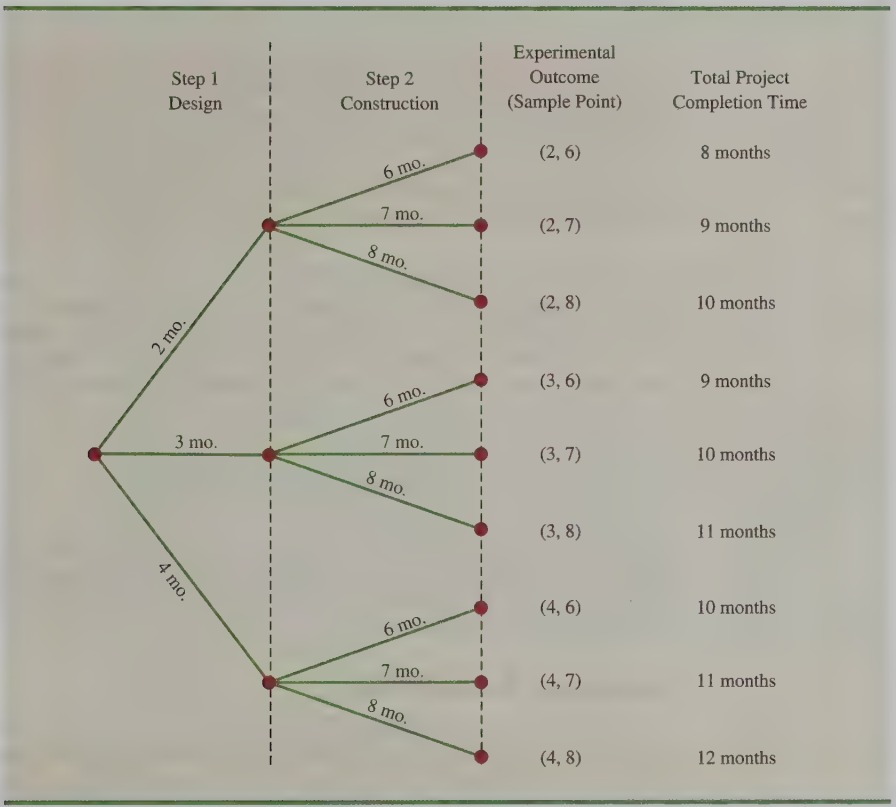
The notation ! means *factorial;* for example, 5 factorial is 5! = (5)(4)(3)(2)(1) = 120.

In sampling from a finite population of size N, the counting rule for combinations is used to find the number of different samples of size n that can be selected.

As an illustration of the counting rule for combinations, consider a quality control procedure in which an inspector randomly selects two of five parts to test for defects. In a group of five parts, how many combinations of two parts can be selected? The counting rule in equation (4.1) shows that with $N = 5$ and $n = 2$, we have

$$C_2^5 = \binom{5}{2} = \frac{5!}{2!(5-2)!} = \frac{(5)(4)(3)(2)(1)}{(2)(1)(3)(2)(1)} = \frac{120}{12} = 10$$

FIGURE 4.3 TREE DIAGRAM FOR THE KP&L PROJECT



Thus, 10 outcomes are possible for the experiment of randomly selecting two parts from a group of five. If we label the five parts as A, B, C, D, and E, the 10 combinations or experimental outcomes can be identified as AB, AC, AD, AE, BC, BD, BE, CD, CE, and DE.

As another example, consider that the Florida lottery system uses the random selection of six integers from a group of 53 to determine the weekly winner. The counting rule for combinations, equation (4.1), can be used to determine the number of ways six different integers can be selected from a group of 53.

$$\binom{53}{6} = \frac{53!}{6!(53-6)!} = \frac{53!}{6!47!} = \frac{(53)(52)(51)(50)(49)(48)}{(6)(5)(4)(3)(2)(1)} = 22{,}957{,}480$$

The counting rule for combinations shows that the chance of winning the lottery is very unlikely.

The counting rule for combinations tells us that almost 23 million experimental outcomes are possible in the lottery drawing. An individual who buys a lottery ticket has 1 chance in 22,957,480 of winning.

Permutations A third counting rule that is sometimes useful is the counting rule for permutations. It allows one to compute the number of experimental outcomes when n objects are to be selected from a set of N objects where the order of selection is

important. The same n objects selected in a different order are considered a different experimental outcome.

COUNTING RULE FOR PERMUTATIONS

The number of permutations of N objects taken n at a time is given by

$$P_n^N = n!\binom{N}{n} = \frac{N!}{(N-n)!} \tag{4.2}$$

The counting rule for permutations closely relates to the one for combinations; however, an experiment results in more permutations than combinations for the same number of objects because every selection of n objects can be ordered in $n!$ different ways.

As an example, consider again the quality control process in which an inspector selects two of five parts to inspect for defects. How many permutations may be selected? The counting rule in equation (4.2) shows that with $N = 5$ and $n = 2$, we have

$$P_2^5 = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{(5)(4)(3)(2)(1)}{(3)(2)(1)} = \frac{120}{6} = 20$$

Thus, 20 outcomes are possible for the experiment of randomly selecting two parts from a group of five when the order of selection must be taken into account. If we label the parts A, B, C, D, and E, the 20 permutations are AB, BA, AC, CA, AD, DA, AE, EA, BC, CB, BD, DB, BE, EB, CD, DC, CE, EC, DE, and ED.

Assigning Probabilities

Now let us see how probabilities can be assigned to experimental outcomes. The three approaches most frequently used are the classical, relative frequency, and subjective methods. Regardless of the method used, two **basic requirements for assigning probabilities** must be met.

BASIC REQUIREMENTS FOR ASSIGNING PROBABILITIES

1. The probability assigned to each experimental outcome must be between 0 and 1, inclusively. If we let E_i denote the ith experimental outcome and $P(E_i)$ its probability, then this requirement can be written as

$$0 \le P(E_i) \le 1 \text{ for all } i \tag{4.3}$$

2. The sum of the probabilities for all the experimental outcomes must equal 1. For n experimental outcomes, this requirement can be written as

$$P(E_1) + P(E_2) + \cdots + P(E_n) = 1 \tag{4.4}$$

The **classical method** of assigning probabilities is appropriate when all the experimental outcomes are equally likely. If n experimental outcomes are possible, a probability of $1/n$ is assigned to each experimental outcome. When using this approach, the two basic requirements for assigning probabilities are automatically satisfied.

For an example, consider the experiment of tossing a fair coin; the two experimental outcomes—head and tail—are equally likely. Because one of the two equally likely outcomes is a head, the probability of observing a head is 1/2, or .50. Similarly, the probability of observing a tail is also 1/2, or .50.

As another example, consider the experiment of rolling a die. It would seem reasonable to conclude that the six possible outcomes are equally likely, and hence each outcome is assigned a probability of 1/6. If $P(1)$ denotes the probability that one dot appears on the upward face of the die, then $P(1) = 1/6$. Similarly, $P(2) = 1/6$, $P(3) = 1/6$, $P(4) = 1/6$, $P(5) = 1/6$, and $P(6) = 1/6$. Note that these probabilities satisfy the two basic requirements of equations (4.3) and (4.4) because each of the probabilities is greater than or equal to zero and they sum to 1.

The **relative frequency method** of assigning probabilities is appropriate when data are available to estimate the proportion of the time the experimental outcome will occur if the experiment is repeated a large number of times. As an example, consider a study of waiting times in the X-ray department for a local hospital. A clerk recorded the number of patients waiting for service at 9:00 A.M. on 20 successive days and obtained the following results.

Number Waiting	Number of Days Outcome Occurred
0	2
1	5
2	6
3	4
4	3
Total	20

These data show that on 2 of the 20 days, zero patients were waiting for service; on 5 of the days, one patient was waiting for service; and so on. Using the relative frequency method, we would assign a probability of 2/20 = .10 to the experimental outcome of zero patients waiting for service, 5/20 = .25 to the experimental outcome of one patient waiting, 6/20 = .30 to two patients waiting, 4/20 = .20 to three patients waiting, and 3/20 = .15 to four patients waiting. As with the classical method, using the relative frequency method automatically satisfies the two basic requirements of equations (4.3) and (4.4).

The **subjective method** of assigning probabilities is most appropriate when one cannot realistically assume that the experimental outcomes are equally likely and when little relevant data are available. When the subjective method is used to assign probabilities to the experimental outcomes, we may use any information available, such as our experience or intuition. After considering all available information, a probability value that expresses our *degree of belief* (on a scale from 0 to 1) that the experimental outcome will occur is specified. Because subjective probability expresses a person's degree of belief, it is personal. Using the subjective method, different people can be expected to assign different probabilities to the same experimental outcome.

The subjective method requires extra care to ensure that the two basic requirements of equations (4.3) and (4.4) are satisfied. Regardless of a person's degree of belief, the probability value assigned to each experimental outcome must be between 0 and 1, inclusive, and the sum of all the probabilities for the experimental outcomes must equal 1.

Consider the case in which Tom and Judy Elsbernd make an offer to purchase a house. Two outcomes are possible:

E_1 = their offer is accepted
E_2 = their offer is rejected

Judy believes that the probability that their offer will be accepted is .8; thus, Judy would set $P(E_1) = .8$ and $P(E_2) = .2$. Tom, however, believes that the probability that their offer will be accepted is .6; hence, Tom would set $P(E_1) = .6$ and $P(E_2) = .4$. Note that Tom's probability estimate for E_1 reflects a greater pessimism that their offer will be accepted.

Bayes' theorem (see Section 4.5) provides a means for combining subjectively determined prior probabilities with probabilities obtained by other means to obtain revised, or posterior, probabilities.

Both Judy and Tom assigned probabilities that satisfy the two basic requirements. The fact that their probability estimates are different emphasizes the personal nature of the subjective method.

Even in business situations where either the classical or the relative frequency approach can be applied, managers may want to provide subjective probability estimates. In such cases, the best probability estimates often are obtained by combining the estimates from the classical or relative frequency approach with subjective probability estimates.

Probabilities for the KP&L Project

To perform further analysis on the KP&L project, we must develop probabilities for each of the nine experimental outcomes listed in Table 4.1. On the basis of experience and judgment, management concluded that the experimental outcomes were not equally likely. Hence, the classical method of assigning probabilities could not be used. Management then decided to conduct a study of the completion times for similar projects undertaken by KP&L over the past three years. The results of a study of 40 similar projects are summarized in Table 4.2.

After reviewing the results of the study, management decided to employ the relative frequency method of assigning probabilities. Management could have provided subjective probability estimates, but felt that the current project was quite similar to the 40 previous projects. Thus, the relative frequency method was judged best.

In using the data in Table 4.2 to compute probabilities, we note that outcome (2, 6)—stage 1 completed in 2 months and stage 2 completed in 6 months—occurred six times in the 40 projects. We can use the relative frequency method to assign a probability of 6/40 = .15 to this outcome. Similarly, outcome (2, 7) also occurred in six of the 40 projects, providing a 6/40 = .15 probability. Continuing in this manner, we obtain the probability assignments for the sample points of the KP&L project shown in Table 4.3. Note that $P(2, 6)$ represents the probability of the sample point (2, 6), $P(2, 7)$ represents the probability of the sample point (2, 7), and so on.

TABLE 4.2 COMPLETION RESULTS FOR 40 KP&L PROJECTS

Completion Time (months)			Number of Past Projects Having These Completion Times
Stage 1 Design	Stage 2 Construction	Sample Point	
2	6	(2, 6)	6
2	7	(2, 7)	6
2	8	(2, 8)	2
3	6	(3, 6)	4
3	7	(3, 7)	8
3	8	(3, 8)	2
4	6	(4, 6)	2
4	7	(4, 7)	4
4	8	(4, 8)	6
		Total	40

TABLE 4.3 PROBABILITY ASSIGNMENTS FOR THE KP&L PROJECT BASED ON THE RELATIVE FREQUENCY METHOD

Sample Point	Project Completion Time	Probability of Sample Point
(2, 6)	8 months	$P(2, 6) = 6/40 = .15$
(2, 7)	9 months	$P(2, 7) = 6/40 = .15$
(2, 8)	10 months	$P(2, 8) = 2/40 = .05$
(3, 6)	9 months	$P(3, 6) = 4/40 = .10$
(3, 7)	10 months	$P(3, 7) = 8/40 = .20$
(3, 8)	11 months	$P(3, 8) = 2/40 = .05$
(4, 6)	10 months	$P(4, 6) = 2/40 = .05$
(4, 7)	11 months	$P(4, 7) = 4/40 = .10$
(4, 8)	12 months	$P(4, 8) = 6/40 = .15$
		Total 1.00

NOTES AND COMMENTS

1. In statistics, the notion of an experiment differs somewhat from the notion of an experiment in the physical sciences. In the physical sciences, researchers usually conduct an experiment in a laboratory or a controlled environment in order to learn about cause and effect. In statistical experiments, probability determines outcomes. Even though the experiment is repeated in exactly the same way, an entirely different outcome may occur. Because of this influence of probability on the outcome, the experiments of statistics are sometimes called *random experiments*.
2. When drawing a random sample without replacement from a population of size N, the counting rule for combinations is used to find the number of different samples of size n that can be selected.

Exercises

Methods

1. An experiment has three steps with three outcomes possible for the first step, two outcomes possible for the second step, and four outcomes possible for the third step. How many experimental outcomes exist for the entire experiment?

SELF test

2. How many ways can three items be selected from a group of six items? Use the letters A, B, C, D, E, and F to identify the items, and list each of the different combinations of three items.
3. How many permutations of three items can be selected from a group of six? Use the letters A, B, C, D, E, and F to identify the items, and list each of the permutations of items B, D, and F.
4. Consider the experiment of tossing a coin three times.
   a. Develop a tree diagram for the experiment.
   b. List the experimental outcomes.
   c. What is the probability for each experimental outcome?
5. Suppose an experiment has five equally likely outcomes: E_1, E_2, E_3, E_4, E_5. Assign probabilities to each outcome and show that the requirements in equations (4.3) and (4.4) are satisfied. What method did you use?

SELF test

6. An experiment with three outcomes has been repeated 50 times, and it was learned that E_1 occurred 20 times, E_2 occurred 13 times, and E_3 occurred 17 times. Assign probabilities to the outcomes. What method did you use?
7. A decision maker subjectively assigned the following probabilities to the four outcomes of an experiment: $P(E_1) = .10$, $P(E_2) = .15$, $P(E_3) = .40$, and $P(E_4) = .20$. Are these probability assignments valid? Explain.

Applications

8. In the city of Milford, applications for zoning changes go through a two-step process: a review by the planning commission and a final decision by the city council. At step 1 the planning commission reviews the zoning change request and makes a positive or negative recommendation concerning the change. At step 2 the city council reviews the planning commission's recommendation and then votes to approve or to disapprove the zoning change. Suppose the developer of an apartment complex submits an application for a zoning change. Consider the application process as an experiment.
   a. How many sample points are there for this experiment? List the sample points.
   b. Construct a tree diagram for the experiment.

SELF test

9. Simple random sampling uses a sample of size n from a population of size N to obtain data that can be used to make inferences about the characteristics of a population. Suppose that, from a population of 50 bank accounts, we want to take a random sample of four accounts in order to learn about the population. How many different random samples of four accounts are possible?

SELF test

10. Many students accumulate debt by the time they graduate from college. Shown in the following table is the percentage of graduates with debt and the average amount of debt for these graduates at four universities and four liberal arts colleges (*U.S. News and World Report, America's Best Colleges,* 2008).

University	% with Debt	Amount($)	College	% with Debt	Amount($)
Pace	72	32,980	Wartburg	83	28,758
Iowa State	69	32,130	Morehouse	94	27,000
Massachusetts	55	11,227	Wellesley	55	10,206
SUNY—Albany	64	11,856	Wofford	49	11,012

   a. If you randomly choose a graduate of Morehouse College, what is the probability that this individual graduated with debt?
   b. If you randomly choose one of these eight institutions for a follow-up study on student loans, what is the probability that you will choose an institution with more than 60% of its graduates having debt?
   c. If you randomly choose one of these eight institutions for a follow-up study on student loans, what is the probability that you will choose an institution whose graduates with debts have an average debt of more than $30,000?
   d. What is the probability that a graduate of Pace University does not have debt?
   e. For graduates of Pace University with debt, the average amount of debt is $32,980. Considering all graduates from Pace University, what is the average debt per graduate?

11. The National Occupant Protection Use Survey (NOPUS) was conducted to provide probability-based data on motorcycle helmet use in the United States. The survey was conducted by sending observers to randomly selected roadway sites where they collected data on motorcycle helmet use, including the number of motorcyclists wearing a Department of Transportation (DOT)–compliant helmet (National Highway Traffic Safety Administration website, January 7, 2010). Sample data consistent with the most recent NOPUS are shown below:

	Type of Helmet	
Region	DOT-Compliant	Noncompliant
Northeast	96	62
Midwest	86	43
South	92	49
West	76	16
Total	350	170

a. Use the sample data to compute an estimate of the probability that a motorcyclist wears a DOT-compliant helmet.
b. The probability that a motorcyclist wore a DOT-compliant helmet five years ago was .48, and last year this probability was .63. Would the National Highway Traffic Safety Administration be pleased with the most recent survey results shown above?
c. What is the probability of DOT-compliant helmet use by region of the country? What region has the highest probability of DOT-compliant helmet use?

12. The Powerball lottery is played twice each week in 28 states, the Virgin Islands, and the District of Columbia. To play Powerball a participant must purchase a ticket and then select five numbers from the digits 1 through 55 and a Powerball number from the digits 1 through 42. To determine the winning numbers for each game, lottery officials draw five white balls out of a drum with 55 white balls, and one red ball out of a drum with 42 red balls. To win the jackpot, a participant's numbers must match the numbers on the five white balls in any order and the number on the red Powerball. Eight coworkers at the ConAgra Foods plant in Lincoln, Nebraska, claimed the record $365 million jackpot on February 18, 2006, by matching the numbers 15-17-43-44-49 and the Powerball number 29. A variety of other cash prizes are awarded each time the game is played. For instance, a prize of $200,000 is paid if the participant's five numbers match the numbers on the five white balls (Powerball website, March 19, 2006).
a. Compute the number of ways the first five numbers can be selected.
b. What is the probability of winning a prize of $200,000 by matching the numbers on the five white balls?
c. What is the probability of winning the Powerball jackpot?

13. A company that manufactures toothpaste is studying five different package designs. Assuming that one design is just as likely to be selected by a consumer as any other design, what selection probability would you assign to each of the package designs? In an actual experiment, 100 consumers were asked to pick the design they preferred. The following data were obtained. Do the data confirm the belief that one design is just as likely to be selected as another? Explain.

Design	Number of Times Preferred
1	5
2	15
3	30
4	40
5	10

4.2 Events and Their Probabilities

In the introduction to this chapter we used the term *event* much as it would be used in everyday language. Then, in Section 4.1 we introduced the concept of an experiment and its associated experimental outcomes or sample points. Sample points and events provide the foundation for the study of probability. As a result, we must now introduce the formal definition of an **event** as it relates to sample points. Doing so will provide the basis for determining the probability of an event.

EVENT

An event is a collection of sample points.

For an example, let us return to the KP&L project and assume that the project manager is interested in the event that the entire project can be completed in 10 months or less. Referring to Table 4.3, we see that six sample points—(2, 6), (2, 7), (2, 8), (3, 6), (3, 7), and (4, 6)—provide a project completion time of 10 months or less. Let C denote the event that the project is completed in 10 months or less; we write

$$C = \{(2, 6), (2, 7), (2, 8), (3, 6), (3, 7), (4, 6)\}$$

Event C is said to occur if *any one* of these six sample points appears as the experimental outcome.

Other events that might be of interest to KP&L management include the following.

L = The event that the project is completed in *less* than 10 months
M = The event that the project is completed in *more* than 10 months

Using the information in Table 4.3, we see that these events consist of the following sample points.

$$L = \{(2, 6), (2, 7), (3, 6)\}$$
$$M = \{(3, 8), (4, 7), (4, 8)\}$$

A variety of additional events can be defined for the KP&L project, but in each case the event must be identified as a collection of sample points for the experiment.

Given the probabilities of the sample points shown in Table 4.3, we can use the following definition to compute the probability of any event that KP&L management might want to consider.

PROBABILITY OF AN EVENT

The probability of any event is equal to the sum of the probabilities of the sample points in the event.

Using this definition, we calculate the probability of a particular event by adding the probabilities of the sample points (experimental outcomes) that make up the event. We can now compute the probability that the project will take 10 months or less to complete. Because this event is given by $C = \{(2, 6), (2, 7), (2, 8), (3, 6), (3, 7), (4, 6)\}$, the probability of event C, denoted $P(C)$, is given by

$$P(C) = P(2, 6) + P(2, 7) + P(2, 8) + P(3, 6) + P(3, 7) + P(4, 6)$$

Refer to the sample point probabilities in Table 4.3; we have

$$P(C) = .15 + .15 + .05 + .10 + .20 + .05 = .70$$

Similarly, because the event that the project is completed in less than 10 months is given by $L = \{(2, 6), (2, 7), (3, 6)\}$, the probability of this event is given by

$$\begin{aligned} P(L) &= P(2, 6) + P(2, 7) + P(3, 6) \\ &= .15 + .15 + .10 = .40 \end{aligned}$$

Finally, for the event that the project is completed in more than 10 months, we have $M = \{(3, 8), (4, 7), (4, 8)\}$ and thus

$$\begin{aligned} P(M) &= P(3, 8) + P(4, 7) + P(4, 8) \\ &= .05 + .10 + .15 = .30 \end{aligned}$$

Using these probability results, we can now tell KP&L management that there is a .70 probability that the project will be completed in 10 months or less, a .40 probability that the project will be completed in less than 10 months, and a .30 probability that the project will be completed in more than 10 months. This procedure of computing event probabilities can be repeated for any event of interest to the KP&L management.

Any time that we can identify all the sample points of an experiment and assign probabilities to each, we can compute the probability of an event using the definition. However, in many experiments the large number of sample points makes the identification of the sample points, as well as the determination of their associated probabilities, extremely cumbersome, if not impossible. In the remaining sections of this chapter, we present some basic probability relationships that can be used to compute the probability of an event without knowledge of all the sample point probabilities.

NOTES AND COMMENTS

1. The sample space, S, is an event. Because it contains all the experimental outcomes, it has a probability of 1; that is, $P(S) = 1$.
2. When the classical method is used to assign probabilities, the assumption is that the experimental outcomes are equally likely. In such cases, the probability of an event can be computed by counting the number of experimental outcomes in the event and dividing the result by the total number of experimental outcomes.

Exercises

Methods

14. An experiment has four equally likely outcomes: E_1, E_2, E_3, and E_4.
    a. What is the probability that E_2 occurs?
    b. What is the probability that any two of the outcomes occur (e.g., E_1 or E_3)?
    c. What is the probability that any three of the outcomes occur (e.g., E_1 or E_2 or E_4)?

SELF test

15. Consider the experiment of selecting a playing card from a deck of 52 playing cards. Each card corresponds to a sample point with a 1/52 probability.
    a. List the sample points in the event an ace is selected.
    b. List the sample points in the event a club is selected.
    c. List the sample points in the event a face card (jack, queen, or king) is selected.
    d. Find the probabilities associated with each of the events in parts (a), (b), and (c).

16. Consider the experiment of rolling a pair of dice. Suppose that we are interested in the sum of the face values showing on the dice.
    a. How many sample points are possible? (*Hint:* Use the counting rule for multiple-step experiments.)
    b. List the sample points.
    c. What is the probability of obtaining a value of 7?
    d. What is the probability of obtaining a value of 9 or greater?
    e. Because each roll has six possible even values (2, 4, 6, 8, 10, and 12) and only five possible odd values (3, 5, 7, 9, and 11), the dice should show even values more often than odd values. Do you agree with this statement? Explain.
    f. What method did you use to assign the probabilities requested?

Applications

SELF test

17. Refer to the KP&L sample points and sample point probabilities in Tables 4.2 and 4.3.
    a. The design stage (stage 1) will run over budget if it takes four months to complete. List the sample points in the event the design stage is over budget.
    b. What is the probability that the design stage is over budget?
    c. The construction stage (stage 2) will run over budget if it takes eight months to complete. List the sample points in the event the construction stage is over budget.
    d. What is the probability that the construction stage is over budget?
    e. What is the probability that both stages are over budget?

18. To investigate how often families eat at home, Harris Interactive surveyed 496 adults living with children under the age of 18 (*USA Today,* January 3, 2007). The survey results are shown in the following table.

Number of Family Meals per Week	Number of Survey Responses
0	11
1	11
2	30
3	36
4	36
5	119
6	114
7 or more	139

For a randomly selected family with children under the age of 18, compute the following.
    a. The probability the family eats no meals at home during the week
    b. The probability the family eats at least four meals at home during the week
    c. The probability the family eats two or fewer meals at home during the week

19. Do you think the government protects investors adequately? This question was part of an online survey of investors aged less than 65 living in the United States and Great Britain (Financial Times/Harris Poll, October 1, 2009). The numbers of investors from the United States and Great Britain who answered Yes, No, or Unsure to this question are provided below.

Response	United States	Great Britain
Yes	187	197
No	334	411
Unsure	256	213

    a. Estimate the probability that an investor in the United States thinks the government is not protecting investors adequately.
    b. Estimate the probability that an investor in Great Britain thinks the government is not protecting investors adequately or is unsure the government is protecting investors adequately.
    c. For a randomly selected investor from these two countries, estimate the probability that the investor thinks the government is not protecting investors adequately.
    d. Based on the survey results, does there appear to be much difference between the perceptions of investors in the United States and investors in Great Britain regarding the issue of the government protecting investors adequately?

20. *Fortune* magazine publishes an annual list of the 500 largest companies in the United States. The following data show the five states with the largest number of *Fortune* 500 companies (*The New York Times Almanac*, 2006).

State	Number of Companies
New York	54
California	52
Texas	48
Illinois	33
Ohio	30

Suppose a *Fortune* 500 company is chosen for a follow-up questionnaire. What are the probabilities of the following events?

a. Let N be the event the company is headquartered in New York. Find $P(N)$.
b. Let T be the event the company is headquartered in Texas. Find $P(T)$.
c. Let B be the event the company is headquartered in one of these five states. Find $P(B)$.

21. The U.S. adult population by age is as follows (*The World Almanac*, 2009). The data are in millions of people.

Age	Number
18 to 24	29.8
25 to 34	40.0
35 to 44	43.4
45 to 54	43.9
55 to 64	32.7
65 and over	37.8

Assume that a person will be randomly chosen from this population.

a. What is the probability that the person is 18 to 24 years old?
b. What is the probability that the person is 18 to 34 years old?
c. What is the probability that the person is 45 or older?

4.3 Some Basic Relationships of Probability

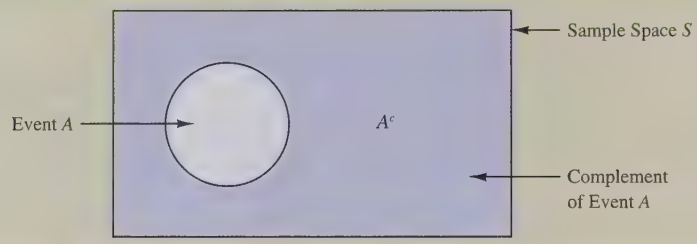
Complement of an Event

Given an event A, the **complement of A** is defined to be the event consisting of all sample points that are *not* in A. The complement of A is denoted by A^c. Figure 4.4 is a diagram, known as a **Venn diagram**, which illustrates the concept of a complement. The rectangular area represents the sample space for the experiment and as such contains all possible sample points. The circle represents event A and contains only the sample points that belong to A. The shaded region of the rectangle contains all sample points not in event A and is by definition the complement of A.

In any probability application, either event A or its complement A^c must occur. Therefore, we have

$$P(A) + P(A^c) = 1$$

FIGURE 4.4 COMPLEMENT OF EVENT A IS SHADED



Solving for $P(A)$, we obtain the following result.

COMPUTING PROBABILITY USING THE COMPLEMENT

$$P(A) = 1 - P(A^c) \qquad (4.5)$$

Equation (4.5) shows that the probability of an event A can be computed easily if the probability of its complement, $P(A^c)$, is known.

As an example, consider the case of a sales manager who, after reviewing sales reports, states that 80% of new customer contacts result in no sale. By allowing A to denote the event of a sale and A^c to denote the event of no sale, the manager is stating that $P(A^c) = .80$. Using equation (4.5), we see that

$$P(A) = 1 - P(A^c) = 1 - .80 = .20$$

We can conclude that a new customer contact has a .20 probability of resulting in a sale.

In another example, a purchasing agent states a .90 probability that a supplier will send a shipment that is free of defective parts. Using the complement, we can conclude that there is a $1 - .90 = .10$ probability that the shipment will contain defective parts.

Addition Law

The addition law is helpful when we are interested in knowing the probability that at least one of two events occurs. That is, with events A and B we are interested in knowing the probability that event A or event B or both occur.

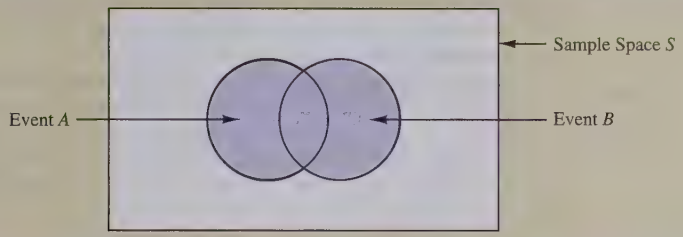
Before we present the addition law, we need to discuss two concepts related to the combination of events: the *union* of events and the *intersection* of events. Given two events A and B, the **union of A and B** is defined as follows.

UNION OF TWO EVENTS

The *union* of A and B is the event containing *all* sample points belonging to A *or* B *or both*. The union is denoted by $A \cup B$.

The Venn diagram in Figure 4.5 depicts the union of events A and B. Note that the two circles contain all the sample points in event A as well as all the sample points in event B.

FIGURE 4.5 UNION OF EVENTS A AND B IS SHADED



The fact that the circles overlap indicates that some sample points are contained in both A and B.

The definition of the **intersection of A and B** follows.

INTERSECTION OF TWO EVENTS

Given two events A and B, the *intersection* of A and B is the event containing the sample points belonging to *both A and B*. The intersection is denoted by $A \cap B$.

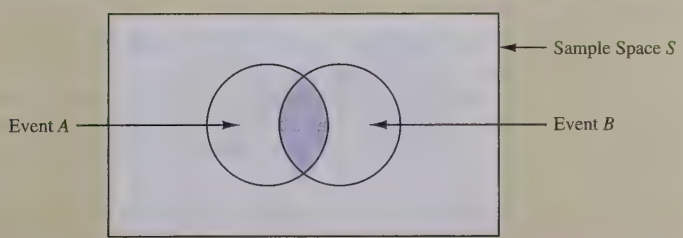
The Venn diagram depicting the intersection of events A and B is shown in Figure 4.6. The area where the two circles overlap is the intersection; it contains the sample points that are in both A and B.

Let us now continue with a discussion of the addition law. The **addition law** provides a way to compute the probability that event A or event B or both occur. In other words, the addition law is used to compute the probability of the union of two events. The addition law is written as follows.

ADDITION LAW

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{4.6}$$

FIGURE 4.6 INTERSECTION OF EVENTS A AND B IS SHADED



To understand the addition law intuitively, note that the first two terms in the addition law, $P(A) + P(B)$, account for all the sample points in $A \cup B$. However, because the sample points in the intersection $A \cap B$ are in both A and B, when we compute $P(A) + P(B)$, we are in effect counting each of the sample points in $A \cap B$ twice. We correct for this overcounting by subtracting $P(A \cap B)$.

As an example of an application of the addition law, let us consider the case of a small assembly plant with 50 employees. Each worker is expected to complete work assignments on time and in such a way that the assembled product will pass a final inspection. On occasion, some of the workers fail to meet the performance standards by completing work late or assembling a defective product. At the end of a performance evaluation period, the production manager found that 5 of the 50 workers completed work late, 6 of the 50 workers assembled a defective product, and 2 of the 50 workers both completed work late *and* assembled a defective product.

Let

L = the event that the work is completed late
D = the event that the assembled product is defective

The relative frequency information leads to the following probabilities.

$$P(L) = \frac{5}{50} = .10$$

$$P(D) = \frac{6}{50} = .12$$

$$P(L \cap D) = \frac{2}{50} = .04$$

After reviewing the performance data, the production manager decided to assign a poor performance rating to any employee whose work was either late or defective; thus the event of interest is $L \cup D$. What is the probability that the production manager assigned an employee a poor performance rating?

Note that the probability question is about the union of two events. Specifically, we want to know $P(L \cup D)$. Using equation (4.6), we have

$$P(L \cup D) = P(L) + P(D) - P(L \cap D)$$

Knowing values for the three probabilities on the right side of this expression, we can write

$$P(L \cup D) = .10 + .12 - .04 = .18$$

This calculation tells us that there is a .18 probability that a randomly selected employee received a poor performance rating.

As another example of the addition law, consider a recent study conducted by the personnel manager of a major computer software company. The study showed that 30% of the employees who left the firm within two years did so primarily because they were dissatisfied with their salary, 20% left because they were dissatisfied with their work assignments, and 12% of the former employees indicated dissatisfaction with *both* their salary and their work assignments. What is the probability that an employee who leaves within

two years does so because of dissatisfaction with salary, dissatisfaction with the work assignment, or both?

Let

S = the event that the employee leaves because of salary
W = the event that the employee leaves because of work assignment

We have $P(S) = .30$, $P(W) = .20$, and $P(S \cap W) = .12$. Using equation (4.6), the addition law, we have

$$P(S \cup W) = P(S) + P(W) - P(S \cap W) = .30 + .20 - .12 = .38$$

We find a .38 probability that an employee leaves for salary or work assignment reasons.

Before we conclude our discussion of the addition law, let us consider a special case that arises for **mutually exclusive events**.

MUTUALLY EXCLUSIVE EVENTS

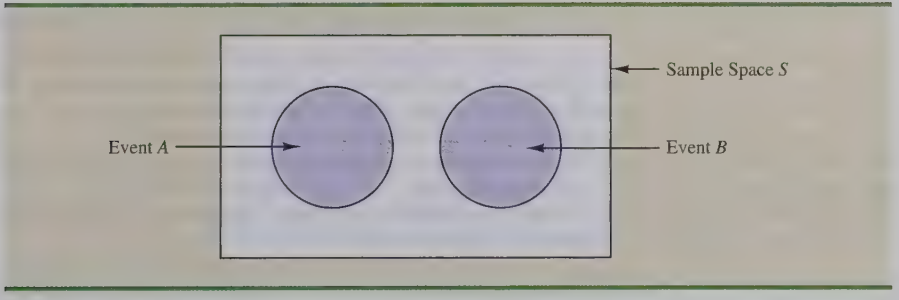
Two events are said to be mutually exclusive if the events have no sample points in common.

Events A and B are mutually exclusive if, when one event occurs, the other cannot occur. Thus, a requirement for A and B to be mutually exclusive is that their intersection must contain no sample points. The Venn diagram depicting two mutually exclusive events A and B is shown in Figure 4.7. In this case $P(A \cap B) = 0$ and the addition law can be written as follows.

ADDITION LAW FOR MUTUALLY EXCLUSIVE EVENTS

$$P(A \cup B) = P(A) + P(B)$$

FIGURE 4.7 MUTUALLY EXCLUSIVE EVENTS



Exercises

Methods

22. Suppose that we have a sample space with five equally likely experimental outcomes: E_1, E_2, E_3, E_4, E_5. Let

$$A = \{E_1, E_2\}$$
$$B = \{E_3, E_4\}$$
$$C = \{E_2, E_3, E_5\}$$

a. Find $P(A)$, $P(B)$, and $P(C)$.
b. Find $P(A \cup B)$. Are A and B mutually exclusive?
c. Find A^c, C^c, $P(A^c)$, and $P(C^c)$.
d. Find $A \cup B^c$ and $P(A \cup B^c)$.
e. Find $P(B \cup C)$.

SELF test

23. Suppose that we have a sample space $S = \{E_1, E_2, E_3, E_4, E_5, E_6, E_7\}$, where $E_1, E_2, \ldots, E_7$ denote the sample points. The following probability assignments apply: $P(E_1) = .05$, $P(E_2) = .20$, $P(E_3) = .20$, $P(E_4) = .25$, $P(E_5) = .15$, $P(E_6) = .10$, and $P(E_7) = .05$. Let

$$A = \{E_1, E_4, E_6\}$$
$$B = \{E_2, E_4, E_7\}$$
$$C = \{E_2, E_3, E_5, E_7\}$$

a. Find $P(A)$, $P(B)$, and $P(C)$.
b. Find $A \cup B$ and $P(A \cup B)$.
c. Find $A \cap B$ and $P(A \cap B)$.
d. Are events A and C mutually exclusive?
e. Find B^c and $P(B^c)$.

Applications

24. Clarkson University surveyed alumni to learn more about what they think of Clarkson. One part of the survey asked respondents to indicate whether their overall experience at Clarkson fell short of expectations, met expectations, or surpassed expectations. The results showed that 4% of the respondents did not provide a response, 26% said that their experience fell short of expectations, and 65% of the respondents said that their experience met expectations.
a. If we chose an alumnus at random, what is the probability that the alumnus would say their experience *surpassed* expectations?
b. If we chose an alumnus at random, what is the probability that the alumnus would say their experience met or surpassed expectations?

25. The U.S. Census Bureau provides data on the number of young adults, ages 18–24, who are living in their parents' home.[1] Let

M = the event a male young adult is living in his parents' home
F = the event a female young adult is living in her parents' home

If we randomly select a male young adult and a female young adult, the Census Bureau data enable us to conclude $P(M) = .56$ and $P(F) = .42$ (*The World Almanac,* 2006). The probability that both are living in their parents' home is .24.
a. What is the probability at least one of the two young adults selected is living in his or her parents' home?
b. What is the probability both young adults selected are living on their own (neither is living in their parents' home)?

[1]The data include single young adults who are living in college dormitories because it is assumed these young adults will return to their parents' home when school is not in session.

26. Information about mutual funds provided by Morningstar Investment Research includes the type of mutual fund (Domestic Equity, International Equity, or Fixed Income) and the Morningstar rating for the fund. The rating is expressed from 1-star (lowest rating) to 5-star (highest rating). A sample of 25 mutual funds was selected from *Morningstar Funds 500* (2008). The following counts were obtained:
    - Sixteen mutual funds were Domestic Equity funds.
    - Thirteen mutual funds were rated 3-star or less.
    - Seven of the Domestic Equity funds were rated 4-star.
    - Two of the Domestic Equity funds were rated 5-star.

    Assume that one of these 25 mutual funds will be randomly selected in order to learn more about the mutual fund and its investment strategy.
    a. What is the probability of selecting a Domestic Equity fund?
    b. What is the probability of selecting a fund with a 4-star or 5-star rating?
    c. What is the probability of selecting a fund that is both a Domestic Equity fund *and* a fund with a 4-star or 5-star rating?
    d. What is the probability of selecting a fund that is a Domestic Equity fund *or* a fund with a 4-star or 5-star rating?

27. What NCAA college basketball conferences have the higher probability of having a team play in college basketball's national championship game? Over the last 20 years, the Atlantic Coast Conference (ACC) ranks first by having a team in the championship game 10 times. The Southeastern Conference (SEC) ranks second by having a team in the championship game 8 times. However, these two conferences have both had teams in the championship game only one time, when Arkansas (SEC) beat Duke (ACC) 76–70 in 1994 (NCAA website, April 2009). Use these data to estimate the following probabilities.
    a. What is the probability the ACC will have a team in the championship game?
    b. What is the probability the SEC will have team in the championship game?
    c. What is the probability the ACC and SEC will both have teams in the championship game?
    d. What is the probability at least one team from these two conferences will be in the championship game? That is, what is the probability a team from the ACC or SEC will play in the championship game?
    e. What is the probability that the championship game will not a have team from one of these two conferences?

28. A survey of magazine subscribers showed that 45.8% rented a car during the past 12 months for business reasons, 54% rented a car during the past 12 months for personal reasons, and 30% rented a car during the past 12 months for both business and personal reasons.
    a. What is the probability that a subscriber rented a car during the past 12 months for business or personal reasons?
    b. What is the probability that a subscriber did not rent a car during the past 12 months for either business or personal reasons?

29. High school seniors with strong academic records apply to the nation's most selective colleges in greater numbers each year. Because the number of slots remains relatively stable, some colleges reject more early applicants. Suppose that for a recent admissions class, an Ivy League college received 2851 applications for early admission. Of this group, it admitted 1033 students early, rejected 854 outright, and deferred 964 to the regular admission pool for further consideration. In the past, this school has admitted 18% of the deferred early admission applicants during the regular admission process. Counting the students admitted early and the students admitted during the regular admission process, the total class size was 2375. Let E, R, and D represent the events that a student who applies for early admission is admitted early, rejected outright, or deferred to the regular admissions pool.
    a. Use the data to estimate $P(E)$, $P(R)$, and $P(D)$.
    b. Are events E and D mutually exclusive? Find $P(E \cap D)$.

c. For the 2375 students that were admitted, what is the probability that a randomly selected student was accepted during early admission?
d. Suppose a student applies for early admission. What is the probability that the student will be admitted for early admission or be deferred and later admitted during the regular admission process?

4.4 Conditional Probability

Often, the probability of an event is influenced by whether a related event already occurred. Suppose we have an event A with probability $P(A)$. If we obtain new information and learn that a related event, denoted by B, already occurred, we will want to take advantage of this information by calculating a new probability for event A. This new probability of event A is called a **conditional probability** and is written $P(A \mid B)$. We use the notation $\mid$ to indicate that we are considering the probability of event A *given* the condition that event B has occurred. Hence, the notation $P(A \mid B)$ reads "the probability of A given B."

As an illustration of the application of conditional probability, consider the situation of the promotion status of male and female officers of a major metropolitan police force in the eastern United States. The police force consists of 1200 officers, 960 men and 240 women. Over the past two years, 324 officers on the police force received promotions. The specific breakdown of promotions for male and female officers is shown in Table 4.4.

After reviewing the promotion record, a committee of female officers raised a discrimination case on the basis that 288 male officers had received promotions but only 36 female officers had received promotions. The police administration argued that the relatively low number of promotions for female officers was due not to discrimination, but to the fact that relatively few females are members of the police force. Let us show how conditional probability could be used to analyze the discrimination charge.

Let

M = event an officer is a man
W = event an officer is a woman
A = event an officer is promoted
A^c = event an officer is not promoted

Dividing the data values in Table 4.4 by the total of 1200 officers enables us to summarize the available information with the following probability values.

$P(M \cap A) = 288/1200 = .24$ probability that a randomly selected officer is a man *and* is promoted

$P(M \cap A^c) = 672/1200 = .56$ probability that a randomly selected officer is a man *and* is not promoted

TABLE 4.4 PROMOTION STATUS OF POLICE OFFICERS OVER THE PAST TWO YEARS

	Men	Women	Total
Promoted	288	36	324
Not Promoted	672	204	876
Total	960	240	1200

TABLE 4.5 JOINT PROBABILITY TABLE FOR PROMOTIONS

Joint probabilities appear in the body of the table.

	Men (M)	Women (W)	Total
Promoted (A)	.24	.03	.27
Not Promoted (A^c)	.56	.17	.73
Total	.80	.20	1.00

Marginal probabilities appear in the margins of the table.

$P(W \cap A) = 36/1200 = .03$ probability that a randomly selected officer is a woman *and* is promoted

$P(W \cap A^c) = 204/1200 = .17$ probability that a randomly selected officer is a woman *and* is not promoted

Because each of these values gives the probability of the intersection of two events, the probabilities are called **joint probabilities**. Table 4.5, which provides a summary of the probability information for the police officer promotion situation, is referred to as a *joint probability table.*

The values in the margins of the joint probability table provide the probabilities of each event separately. That is, $P(M) = .80$, $P(W) = .20$, $P(A) = .27$, and $P(A^c) = .73$. These probabilities are referred to as **marginal probabilities** because of their location in the margins of the joint probability table. We note that the marginal probabilities are found by summing the joint probabilities in the corresponding row or column of the joint probability table. For instance, the marginal probability of being promoted is $P(A) = P(M \cap A) + P(W \cap A) = .24 + .03 = .27$. From the marginal probabilities, we see that 80% of the force is male, 20% of the force is female, 27% of all officers received promotions, and 73% were not promoted.

Let us begin the conditional probability analysis by computing the probability that an officer is promoted given that the officer is a man. In conditional probability notation, we are attempting to determine $P(A \mid M)$. To calculate $P(A \mid M)$, we first realize that this notation simply means that we are considering the probability of the event A (promotion) given that the condition designated as event M (the officer is a man) is known to exist. Thus $P(A \mid M)$ tells us that we are now concerned only with the promotion status of the 960 male officers. Because 288 of the 960 male officers received promotions, the probability of being promoted given that the officer is a man is $288/960 = .30$. In other words, given that an officer is a man, that officer had a 30% chance of receiving a promotion over the past two years.

This procedure was easy to apply because the values in Table 4.4 show the number of officers in each category. We now want to demonstrate how conditional probabilities such as $P(A \mid M)$ can be computed directly from related event probabilities rather than the frequency data of Table 4.4.

We have shown that $P(A \mid M) = 288/960 = .30$. Let us now divide both the numerator and denominator of this fraction by 1200, the total number of officers in the study.

$$P(A \mid M) = \frac{288}{960} = \frac{288/1200}{960/1200} = \frac{.24}{.80} = .30$$

We now see that the conditional probability $P(A \mid M)$ can be computed as .24/.80. Refer to the joint probability table (Table 4.5). Note in particular that .24 is the joint probability of

A and M; that is, $P(A \cap M) = .24$. Also note that .80 is the marginal probability that a randomly selected officer is a man; that is, $P(M) = .80$. Thus, the conditional probability $P(A \mid M)$ can be computed as the ratio of the joint probability $P(A \cap M)$ to the marginal probability $P(M)$.

$$P(A \mid M) = \frac{P(A \cap M)}{P(M)} = \frac{.24}{.80} = .30$$

The fact that conditional probabilities can be computed as the ratio of a joint probability to a marginal probability provides the following general formula for conditional probability calculations for two events A and B.

CONDITIONAL PROBABILITY

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \tag{4.7}$$

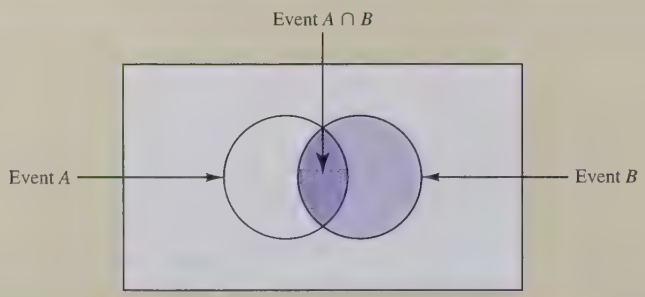
or

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \tag{4.8}$$

The Venn diagram in Figure 4.8 is helpful in obtaining an intuitive understanding of conditional probability. The circle on the right shows that event B has occurred; the portion of the circle that overlaps with event A denotes the event $(A \cap B)$. We know that once event B has occurred, the only way that we can also observe event A is for the event $(A \cap B)$ to occur. Thus, the ratio $P(A \cap B)/P(B)$ provides the conditional probability that we will observe event A given that event B has already occurred.

Let us return to the issue of discrimination against the female officers. The marginal probability in row 1 of Table 4.5 shows that the probability of promotion of an officer is $P(A) = .27$ (regardless of whether that officer is male or female). However, the critical issue in the discrimination case involves the two conditional probabilities $P(A \mid M)$ and $P(A \mid W)$. That is, what is the probability of a promotion *given* that the officer is a man, and what is the probability of a promotion *given* that the officer is a woman? If these two probabilities are equal, a discrimination argument has no basis because the chances of a promotion are the same for male and female officers. However, a difference in the two conditional probabilities will support the position that male and female officers are treated differently in promotion decisions.

FIGURE 4.8 CONDITIONAL PROBABILITY $P(A \mid B) = P(A \cap B)/P(B)$



We already determined that $P(A \mid M) = .30$. Let us now use the probability values in Table 4.5 and the basic relationship of conditional probability in equation (4.7) to compute the probability that an officer is promoted given that the officer is a woman; that is, $P(A \mid W)$. Using equation (4.7), with W replacing B, we obtain

$$P(A \mid W) = \frac{P(A \cap W)}{P(W)} = \frac{.03}{.20} = .15$$

What conclusion do you draw? The probability of a promotion given that the officer is a man is .30, twice the .15 probability of a promotion given that the officer is a woman. Although the use of conditional probability does not in itself prove that discrimination exists in this case, the conditional probability values support the argument presented by the female officers.

Independent Events

In the preceding illustration, $P(A) = .27$, $P(A \mid M) = .30$, and $P(A \mid W) = .15$. We see that the probability of a promotion (event A) is affected or influenced by whether the officer is a man or a woman. Particularly, because $P(A \mid M) \neq P(A)$, we would say that events A and M are dependent events. That is, the probability of event A (promotion) is altered or affected by knowing that event M (the officer is a man) exists. Similarly, with $P(A \mid W) \neq P(A)$, we would say that events A and W are *dependent events*. However, if the probability of event A is not changed by the existence of event M—that is, $P(A \mid M) = P(A)$—we would say that events A and M are **independent events**. This situation leads to the following definition of the independence of two events.

INDEPENDENT EVENTS

Two events A and B are independent if

$$P(A \mid B) = P(A) \tag{4.9}$$

or

$$P(B \mid A) = P(B) \tag{4.10}$$

Otherwise, the events are dependent.

Multiplication Law

Whereas the addition law of probability is used to compute the probability of a union of two events, the multiplication law is used to compute the probability of the intersection of two events. The multiplication law is based on the definition of conditional probability. Using equations (4.7) and (4.8) and solving for $P(A \cap B)$, we obtain the **multiplication law**.

MULTIPLICATION LAW

$$P(A \cap B) = P(B)P(A \mid B) \tag{4.11}$$

or

$$P(A \cap B) = P(A)P(B \mid A) \tag{4.12}$$

To illustrate the use of the multiplication law, consider a newspaper circulation department where it is known that 84% of the households in a particular neighborhood subscribe to the daily edition of the paper. If we let D denote the event that a household subscribes to the daily edition, $P(D) = .84$. In addition, it is known that the probability that a household that already holds a

daily subscription also subscribes to the Sunday edition (event S) is .75; that is, $P(S \mid D) = .75$. What is the probability that a household subscribes to both the Sunday and daily editions of the newspaper? Using the multiplication law, we compute the desired $P(S \cap D)$ as

$$P(S \cap D) = P(D)P(S \mid D) = .84(.75) = .63$$

We now know that 63% of the households subscribe to both the Sunday and daily editions.

Before concluding this section, let us consider the special case of the multiplication law when the events involved are independent. Recall that events A and B are independent whenever $P(A \mid B) = P(A)$ or $P(B \mid A) = P(B)$. Hence, using equations (4.11) and (4.12) for the special case of independent events, we obtain the following multiplication law.

MULTIPLICATION LAW FOR INDEPENDENT EVENTS

$$P(A \cap B) = P(A)P(B) \qquad (4.13)$$

To compute the probability of the intersection of two independent events, we simply multiply the corresponding probabilities. Note that the multiplication law for independent events provides another way to determine whether A and B are independent. That is, if $P(A \cap B) = P(A)P(B)$, then A and B are independent; if $P(A \cap B) \neq P(A)P(B)$, then A and B are dependent.

As an application of the multiplication law for independent events, consider the situation of a service station manager who knows from past experience that 80% of the customers use a credit card when they purchase gasoline. What is the probability that the next two customers purchasing gasoline will each use a credit card? If we let

A = the event that the first customer uses a credit card

B = the event that the second customer uses a credit card

then the event of interest is $A \cap B$. Given no other information, we can reasonably assume that A and B are independent events. Thus,

$$P(A \cap B) = P(A)P(B) = (.80)(.80) = .64$$

To summarize this section, we note that our interest in conditional probability is motivated by the fact that events are often related. In such cases, we say the events are dependent and the conditional probability formulas in equations (4.7) and (4.8) must be used to compute the event probabilities. If two events are not related, they are independent; in this case neither event's probability is affected by whether the other event occurred.

NOTES AND COMMENTS

Do not confuse the notion of mutually exclusive events with that of independent events. Two events with nonzero probabilities cannot be both mutually exclusive and independent. If one mutually exclusive event is known to occur, the other cannot occur; thus, the probability of the other event occurring is reduced to zero. They are therefore dependent.

Exercises

Methods

SELF test

30. Suppose that we have two events, A and B, with $P(A) = .50$, $P(B) = .60$, and $P(A \cap B) = .40$.
    a. Find $P(A \mid B)$.
    b. Find $P(B \mid A)$.
    c. Are A and B independent? Why or why not?

31. Assume that we have two events, A and B, that are mutually exclusive. Assume further that we know $P(A) = .30$ and $P(B) = .40$.
    a. What is $P(A \cap B)$?
    b. What is $P(A \mid B)$?
    c. A student in statistics argues that the concepts of mutually exclusive events and independent events are really the same, and that if events are mutually exclusive they must be independent. Do you agree with this statement? Use the probability information in this problem to justify your answer.
    d. What general conclusion would you make about mutually exclusive and independent events given the results of this problem?

Applications

32. The automobile industry sold 657,000 vehicles in the United States during January 2009 (*The Wall Street Journal,* February 4, 2009). This volume was down 37% from January 2008 as economic conditions continued to decline. The Big Three U.S. automakers—General Motors, Ford, and Chrysler—sold 280,500 vehicles, down 48% from January 2008. A summary of sales by automobile manufacturer and type of vehicle sold is shown in the following table. Data are in thousands of vehicles. The non-U.S. manufacturers are led by Toyota, Honda, and Nissan. The category Light Truck includes pickup, minivan, SUV, and crossover models.

		Type of Vehicle	
		Car	Light Truck
Manufacturer	U.S.	87.4	193.1
	Non-U.S.	228.5	148.0

    a. Develop a joint probability table for these data and use the table to answer the remaining questions.
    b. What are the marginal probabilities? What do they tell you about the probabilities associated with the manufacturer and the type of vehicle sold?
    c. If a vehicle was manufactured by one of the U.S. automakers, what is the probability that the vehicle was a car? What is the probability that it was a light truck?
    d. If a vehicle was not manufactured by one of the U.S. automakers, what is the probability that the vehicle was a car? What is the probability that it was a light truck?
    e. If the vehicle was a light truck, what is the probability that it was manufactured by one of the U.S. automakers?
    f. What does the probability information tell you about sales?

SELF test

33. In a survey of MBA students, the following data were obtained on "students' first reason for application to the school in which they matriculated."

		Reason for Application			
		School Quality	School Cost or Convenience	Other	Totals
Enrollment Status	Full Time	421	393	76	890
	Part Time	400	593	46	1039
	Totals	821	986	122	1929

    a. Develop a joint probability table for these data.
    b. Use the marginal probabilities of school quality, school cost or convenience, and other to comment on the most important reason for choosing a school.

c. If a student goes full time, what is the probability that school quality is the first reason for choosing a school?
d. If a student goes part time, what is the probability that school quality is the first reason for choosing a school?
e. Let A denote the event that a student is full time and let B denote the event that the student lists school quality as the first reason for applying. Are events A and B independent? Justify your answer.

34. The U.S. Department of Transportation reported that during November, 83.4% of Southwest Airlines' flights, 75.1% of US Airways' flights, and 70.1% of JetBlue's flights arrived on time (*USA Today,* January 4, 2007). Assume that this on-time performance is applicable for flights arriving at concourse A of the Rochester International Airport, and that 40% of the arrivals at concourse A are Southwest Airlines flights, 35% are US Airways flights, and 25% are JetBlue flights.
a. Develop a joint probability table with three rows (airlines) and two columns (on-time arrivals vs. late arrivals).
b. An announcement has just been made that Flight 1424 will be arriving at gate 20 in concourse A. What is the most likely airline for this arrival?
c. What is the probability that Flight 1424 will arrive on time?
d. Suppose that an announcement is made saying that Flight 1424 will be arriving late. What is the most likely airline for this arrival? What is the least likely airline?

35. According to the Ameriprise Financial Money Across Generations study, 9 out of 10 parents with adult children ages 20 to 35 have helped their adult children with some type of financial assistance ranging from college, a car, rent, utilities, credit-card debt, and/or down payments for houses (*Money,* January 2009). The following table, with sample data consistent with the study, shows the number of times parents have given their adult children financial assistance to buy a car and to pay rent.

		Pay Rent	
		Yes	**No**
Buy a Car	**Yes**	56	52
	No	14	78

a. Develop a joint probability table and use it to answer the remaining questions.
b. Using the marginal probabilities for buy a car and pay rent, are parents more likely to assist their adult children with buying a car or paying rent? What is your interpretation of the marginal probabilities?
c. If parents provided financial assistance to buy a car, what it the probability that the parents assisted with paying rent?
d. If parents did not provide financial assistance to buy a car, what is the probability that the parents assisted with paying rent?
e. Is financial assistance to buy a car independent of financial assistance to pay rent? Use probabilities to justify your answer.
f. What is the probability that parents provided financial assistance for their adult children by either helping buy a car or pay rent?

36. Jerry Stackhouse of the National Basketball Association's Dallas Mavericks is the best free-throw shooter on the team, making 89% of his shots (ESPN website, July, 2008). Assume that late in a basketball game, Jerry Stackhouse is fouled and is awarded two shots.
a. What is the probability that he will make both shots?
b. What is the probability that he will make at least one shot?
c. What is the probability that he will miss both shots?

d. Late in a basketball game, a team often intentionally fouls an opposing player in order to stop the game clock. The usual strategy is to intentionally foul the other team's worst free-throw shooter. Assume that the Dallas Mavericks' center makes 58% of his free-throw shots. Calculate the probabilities for the center as shown in parts (a), (b), and (c), and show that intentionally fouling the Dallas Mavericks' center is a better strategy than intentionally fouling Jerry Stackhouse.

37. Visa Card USA studied how frequently young consumers, ages 18 to 24, use plastic (debit and credit) cards in making purchases (Associated Press, January 16, 2006). The results of the study provided the following probabilities.
- The probability that a consumer uses a plastic card when making a purchase is .37.
- Given that the consumer uses a plastic card, there is a .19 probability that the consumer is 18 to 24 years old.
- Given that the consumer uses a plastic card, there is a .81 probability that the consumer is more than 24 years old.

U.S. Census Bureau data show that 14% of the consumer population is 18 to 24 years old.

a. Given that the consumer is 18 to 24 years old, what is the probability that the consumer uses a plastic card?
b. Given that the consumer is over 24 years old, what is the probability that the consumer uses a plastic card?
c. What is the interpretation of the probabilities shown in parts (a) and (b)?
d. Should companies such as Visa, MasterCard, and Discover make plastic cards available to the 18- to 24-year-old age group before these consumers have had time to establish a credit history? If no, why? If yes, what restrictions might the companies place on this age group?

38. Students in grades 3 through 8 in New York State are required to take a state mathematics exam. To meet the state's proficiency standards, a student must demonstrate an understanding of the mathematics expected at his or her grade level. The following data show the number of students tested in the New York City school system for grades 3 through 8 and the number who met and did not meet the proficiency standards on the exam (New York City Department of Education website, January 16, 2010).

	Met Proficiency Standards?	
Grade	**Yes**	**No**
3	47,401	23,975
4	35,020	34,740
5	36,062	33,540
6	36,361	32,929
7	40,945	29,768
8	40,720	31,931

a. Develop a joint probability table for these data.
b. What are the marginal probabilities? What do they tell about the probabilities of a student meeting or not meeting the proficiency standards on the exam?
c. If a randomly selected student is a third grader, what is the probability that the student met the proficiency standards? If the student is a fourth grader, what is the probability that the student met the proficiency standards?
d. If a randomly selected student is known to have met the proficiency standards on the exam, what is the probability that the student is a third grader? What is the probability if the student is a fourth grader?

4.5 Bayes' Theorem

In the discussion of conditional probability, we indicated that revising probabilities when new information is obtained is an important phase of probability analysis. Often, we begin the analysis with initial or **prior probability** estimates for specific events of interest. Then, from sources such as a sample, a special report, or a product test, we obtain additional information about the events. Given this new information, we update the prior probability values by calculating revised probabilities, referred to as **posterior probabilities**. **Bayes' theorem** provides a means for making these probability calculations. The steps in this probability revision process are shown in Figure 4.9.

As an application of Bayes' theorem, consider a manufacturing firm that receives shipments of parts from two different suppliers. Let A_1 denote the event that a part is from supplier 1 and A_2 denote the event that a part is from supplier 2. Currently, 65% of the parts purchased by the company are from supplier 1 and the remaining 35% are from supplier 2. Hence, if a part is selected at random, we would assign the prior probabilities $P(A_1) = .65$ and $P(A_2) = .35$.

The quality of the purchased parts varies with the source of supply. Historical data suggest that the quality ratings of the two suppliers are as shown in Table 4.6. If we let G denote the event that a part is good and B denote the event that a part is bad, the information in Table 4.6 provides the following conditional probability values.

$$P(G \mid A_1) = .98 \quad P(B \mid A_1) = .02$$
$$P(G \mid A_2) = .95 \quad P(B \mid A_2) = .05$$

The tree diagram in Figure 4.10 depicts the process of the firm receiving a part from one of the two suppliers and then discovering that the part is good or bad as a two-step experiment. We see that four experimental outcomes are possible; two correspond to the part being good and two correspond to the part being bad.

Each of the experimental outcomes is the intersection of two events, so we can use the multiplication rule to compute the probabilities. For instance,

$$P(A_1, G) = P(A_1 \cap G) = P(A_1)P(G \mid A_1)$$

The process of computing these joint probabilities can be depicted in what is called a probability tree (see Figure 4.11). From left to right through the tree, the probabilities

FIGURE 4.9 PROBABILITY REVISION USING BAYES' THEOREM

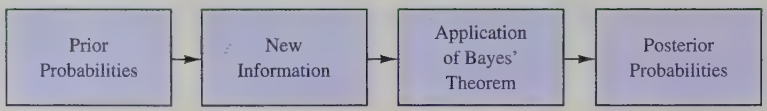
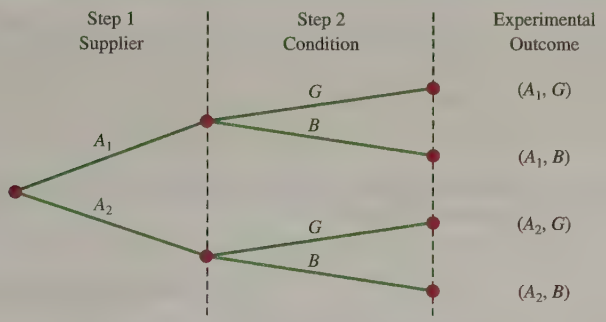


TABLE 4.6 HISTORICAL QUALITY LEVELS OF TWO SUPPLIERS

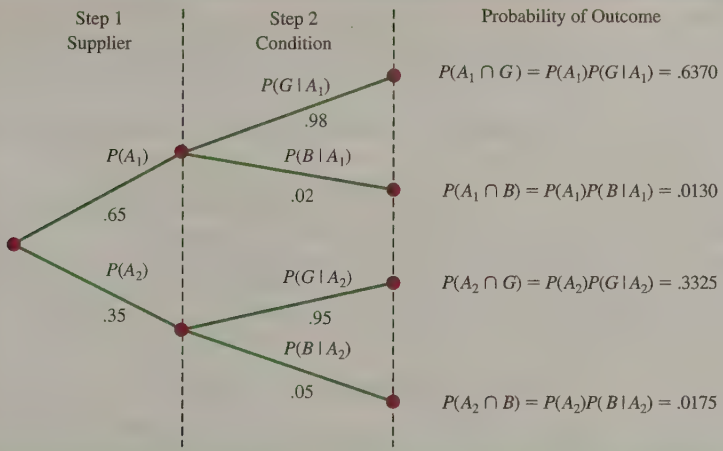
	Percentage Good Parts	Percentage Bad Parts
Supplier 1	98	2
Supplier 2	95	5

FIGURE 4.10 TREE DIAGRAM FOR TWO-SUPPLIER EXAMPLE



Note: Step 1 shows that the part comes from one of two suppliers, and step 2 shows whether the part is good or bad.

FIGURE 4.11 PROBABILITY TREE FOR TWO-SUPPLIER EXAMPLE



for each branch at step 1 are prior probabilities and the probabilities for each branch at step 2 are conditional probabilities. To find the probabilities of each experimental outcome, we simply multiply the probabilities on the branches leading to the outcome. Each of these joint probabilities is shown in Figure 4.11 along with the known probabilities for each branch.

Suppose now that the parts from the two suppliers are used in the firm's manufacturing process and that a machine breaks down because it attempts to process a bad part. Given the information that the part is bad, what is the probability that it came from supplier 1 and what is the probability that it came from supplier 2? With the information in the probability tree (Figure 4.11), Bayes' theorem can be used to answer these questions.

Because B denotes the event that a part is bad, we are looking for the posterior probabilities $P(A_1 \mid B)$ and $P(A_2 \mid B)$. From the law of conditional probability, we know that

$$P(A_1 \mid B) = \frac{P(A_1 \cap B)}{P(B)} \tag{4.14}$$

Referring to the probability tree, we see that

$$P(A_1 \cap B) = P(A_1)P(B \mid A_1) \tag{4.15}$$

To find $P(B)$, we note that event B can occur in only two ways: $(A_1 \cap B)$ and $(A_2 \cap B)$. Therefore, we have

$$\begin{aligned} P(B) &= P(A_1 \cap B) + P(A_2 \cap B) \\ &= P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2) \end{aligned} \tag{4.16}$$

Substituting from equations (4.15) and (4.16) into equation (4.14) and writing a similar result for $P(A_2 \mid B)$, we obtain Bayes' theorem for the case of two events.

The Reverend Thomas Bayes (1702–1761), a Presbyterian minister, is credited with the original work leading to the version of Bayes' theorem in use today.

BAYES' THEOREM (TWO-EVENT CASE)

$$P(A_1 \mid B) = \frac{P(A_1)P(B \mid A_1)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2)} \tag{4.17}$$

$$P(A_2 \mid B) = \frac{P(A_2)P(B \mid A_2)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2)} \tag{4.18}$$

Using equation (4.17) and the probability values provided in the example, we have

$$\begin{aligned} P(A_1 \mid B) &= \frac{P(A_1)P(B \mid A_1)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2)} \\ &= \frac{(.65)(.02)}{(.65)(.02) + (.35)(.05)} = \frac{.0130}{.0130 + .0175} \\ &= \frac{.0130}{.0305} = .4262 \end{aligned}$$

In addition, using equation (4.18), we find $P(A_2 \mid B)$.

$$\begin{aligned} P(A_2 \mid B) &= \frac{(.35)(.05)}{(.65)(.02) + (.35)(.05)} \\ &= \frac{.0175}{.0130 + .0175} = \frac{.0175}{.0305} = .5738 \end{aligned}$$

Note that in this application we started with a probability of .65 that a part selected at random was from supplier 1. However, given information that the part is bad, the probability that the part is from supplier 1 drops to .4262. In fact, if the part is bad, it has better than a 50–50 chance that it came from supplier 2; that is, $P(A_2 \mid B) = .5738$.

Bayes' theorem is applicable when the events for which we want to compute posterior probabilities are mutually exclusive and their union is the entire sample space.[2] For the case of n mutually exclusive events $A_1, A_2, \ldots, A_n$, whose union is the entire sample

[2] If the union of events is the entire sample space, the events are said to be *collectively exhaustive*.

space, Bayes' theorem can be used to compute any posterior probability $P(A_i \mid B)$ as shown here.

BAYES' THEOREM

$$P(A_i \mid B) = \frac{P(A_i)P(B \mid A_i)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2) + \cdots + P(A_n)P(B \mid A_n)} \quad (4.19)$$

With prior probabilities $P(A_1), P(A_2), \ldots, P(A_n)$ and the appropriate conditional probabilities $P(B \mid A_1), P(B \mid A_2), \ldots, P(B \mid A_n)$, equation (4.19) can be used to compute the posterior probability of the events $A_1, A_2, \ldots, A_n$.

Tabular Approach

A tabular approach is helpful in conducting the Bayes' theorem calculations. Such an approach is shown in Table 4.7 for the parts supplier problem. The computations shown there are done in the following steps.

Step 1. Prepare the following three columns:
Column 1—The mutually exclusive events A_i for which posterior probabilities are desired
Column 2—The prior probabilities $P(A_i)$ for the events
Column 3—The conditional probabilities $P(B \mid A_i)$ of the new information B given each event

Step 2. In column 4, compute the joint probabilities $P(A_i \cap B)$ for each event and the new information B by using the multiplication law. These joint probabilities are found by multiplying the prior probabilities in column 2 by the corresponding conditional probabilities in column 3; that is, $P(A_i \cap B) = P(A_i)P(B \mid A_i)$.

Step 3. Sum the joint probabilities in column 4. The sum is the probability of the new information, $P(B)$. Thus we see in Table 4.7 that there is a .0130 probability that the part came from supplier 1 and is bad and a .0175 probability that the part came from supplier 2 and is bad. Because these are the only two ways in which a bad part can be obtained, the sum .0130 + .0175 shows an overall probability of .0305 of finding a bad part from the combined shipments of the two suppliers.

Step 4. In column 5, compute the posterior probabilities using the basic relationship of conditional probability.

$$P(A_i \mid B) = \frac{P(A_i \cap B)}{P(B)}$$

Note that the joint probabilities $P(A_i \cap B)$ are in column 4 and the probability $P(B)$ is the sum of column 4.

TABLE 4.7 TABULAR APPROACH TO BAYES' THEOREM CALCULATIONS FOR THE TWO-SUPPLIER PROBLEM

(1) Events A_i	(2) Prior Probabilities $P(A_i)$	(3) Conditional Probabilities $P(B \mid A_i)$	(4) Joint Probabilities $P(A_i \cap B)$	(5) Posterior Probabilities $P(A_i \mid B)$
A_1	.65	.02	.0130	.0130/.0305 = .4262
A_2	.35	.05	.0175	.0175/.0305 = .5738
	1.00		$P(B)$ = .0305	1.0000

FIGURE 4.12 EXCEL WORKSHEET FOR COMPUTING POSTERIOR PROBABILITIES

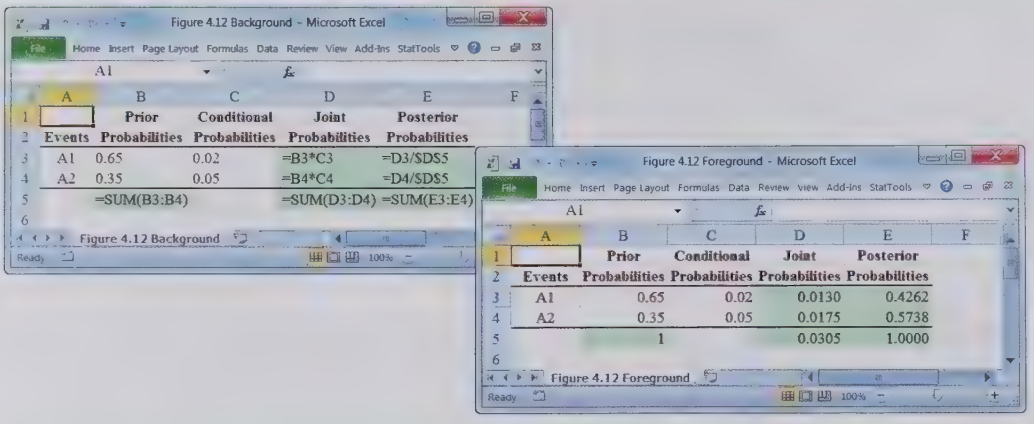


Figure 4.12 Background - Microsoft Excel
Figure 4.12 Background
Figure 4.12 Foreground - Microsoft Excel
Figure 4.12 Foreground

Figure 4.12 Background

	A	B	C	D	E	F
1		Prior	Conditional	Joint	Posterior	
2	Events	Probabilities	Probabilities	Probabilities	Probabilities	
3	A1	0.65	0.02	=B3*C3	=D3/$D$5	
4	A2	0.35	0.05	=B4*C4	=D4/$D$5	
5		=SUM(B3:B4)		=SUM(D3:D4)	=SUM(E3:E4)	
6						

Figure 4.12 Foreground

	A	B	C	D	E	F
1		Prior	Conditional	Joint	Posterior	
2	Events	Probabilities	Probabilities	Probabilities	Probabilities	
3	A1	0.65	0.02	0.0130	0.4262	
4	A2	0.35	0.05	0.0175	0.5738	
5		1		0.0305	1.0000	
6						

Using Excel to Compute Posterior Probabilities

The tabular approach to Bayes' theorem can be easily implemented within an Excel worksheet. In this subsection we show how the Bayes' theorem calculations shown in Table 4.7 can be made. Refer to Figure 4.12 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet is in the foreground.

Enter Data: Labels are placed in rows 1 and 2 and the two mutually exclusive events are identified in cells A3:A4. The prior probabilities were entered into cells B3:B4 and the conditional probabilities were entered into cells C3:C4.

Enter Functions and Formulas: The formulas in cells D3:D4 show that the joint probabilities are the product of the prior probabilities in column B and the conditional probabilities in column C. The results, shown in cells D3:D4 of the value worksheet, are the same as those shown in Table 4.7. The sum in cell D5 shows that the probability of finding a bad part is .0305. This probability is the denominator in Bayes' theorem. The formulas in cells E3:E4 provide the posterior probabilities. From the value worksheet we see that they are the same as the posterior probabilities in Table 4.7.

Note that the same worksheet could be used to find the posterior probabilities of the two suppliers given a good part is found. The prior probabilities in cells B3:B4 would not change. But we would replace the conditional probabilities in cells C3:C4 with the values .98 and .95, respectively. Doing so would provide $P(A_1 \mid G) = .6570$ and $P(A_2 \mid G) = .3430$ in cells E3:E4.

This worksheet can also be modified to handle Bayes' theorem calculations for more than two events. Just add a row for each additional event and move the sum row down.

NOTES AND COMMENTS

1. Bayes' theorem is used extensively in decision analysis. The prior probabilities are often subjective estimates provided by a decision maker. Sample information is obtained and posterior probabilities are computed for use in choosing the best decision.
2. An event and its complement are mutually exclusive, and their union is the entire sample space. Thus, Bayes' theorem is always applicable for computing posterior probabilities of an event and its complement.

Exercises

Methods

SELF test

39. The prior probabilities for events A_1 and A_2 are $P(A_1) = .40$ and $P(A_2) = .60$. It is also known that $P(A_1 \cap A_2) = 0$. Suppose $P(B \mid A_1) = .20$ and $P(B \mid A_2) = .05$.
    a. Are A_1 and A_2 mutually exclusive? Explain.
    b. Compute $P(A_1 \cap B)$ and $P(A_2 \cap B)$.
    c. Compute $P(B)$.
    d. Apply Bayes' theorem to compute $P(A_1 \mid B)$ and $P(A_2 \mid B)$.

40. The prior probabilities for events A_1, A_2, and A_3 are $P(A_1) = .20$, $P(A_2) = .50$, and $P(A_3) = .30$. The conditional probabilities of event B given A_1, A_2, and A_3 are $P(B \mid A_1) = .50$, $P(B \mid A_2) = .40$, and $P(B \mid A_3) = .30$.
    a. Compute $P(B \cap A_1)$, $P(B \cap A_2)$, and $P(B \cap A_3)$.
    b. Apply Bayes' theorem, equation (4.19), to compute the posterior probability $P(A_2 \mid B)$.
    c. Use the tabular approach to applying Bayes' theorem to compute $P(A_1 \mid B)$, $P(A_2 \mid B)$, and $P(A_3 \mid B)$.

Applications

41. A consulting firm submitted a bid for a large research project. The firm's management initially felt they had a 50–50 chance of getting the project. However, the agency to which the bid was submitted subsequently requested additional information on the bid. Past experience indicates that for 75% of the successful bids and 40% of the unsuccessful bids, the agency requested additional information.
    a. What is the prior probability of the bid being successful (that is, prior to the request for additional information)?
    b. What is the conditional probability of a request for additional information given that the bid will ultimately be successful?
    c. Compute the posterior probability that the bid will be successful given a request for additional information.

SELF test

42. A local bank reviewed its credit card policy with the intention of recalling some of its credit cards. In the past approximately 5% of cardholders defaulted, leaving the bank unable to collect the outstanding balance. Hence, management established a prior probability of .05 that any particular cardholder will default. The bank also found that the probability of missing a monthly payment is .20 for customers who do not default. Of course, the probability of missing a monthly payment for those who default is 1.
    a. Given that a customer missed one or more monthly payments, compute the posterior probability that the customer will default.
    b. The bank would like to recall its card if the probability that a customer will default is greater than .20. Should the bank recall its card if the customer misses a monthly payment? Why or why not?

43. Two Wharton professors analyzed 1,613,234 putts by golfers on the Professional Golfers Association (PGA) Tour and found that 983,764 of the putts were made and 629,470 of the putts were missed. Further analysis showed that for putts that were made, 64.0% of the time the player was attempting to make a par putt and 18.8% of the time the player was attempting to make a birdie putt. And, for putts that were missed, 20.3% of the time the player was attempting to make a par putt and 73.4% of the time the player was attempting to make a birdie putt (*Is Tiger Woods Loss Averse? Persistent Bias in the Face of Experience, Competition, and High Stakes,* Pope, D. G., and M. E. Schweitzer, June 2009, The Wharton School, University of Pennsylvania.).

a. What is the probability that a PGA Tour player makes a putt?
b. Suppose that a PGA Tour player has a putt for par. What is the probability that the player will make the putt?
c. Suppose that a PGA Tour player has a putt for birdie. What is the probability that the player will make the putt?
d. Comment on the differences in the probabilities computed in parts (b) and (c).

44. The American Council of Education reported that 47% of college freshmen earn a degree and graduate within five years (Associated Press, May 6, 2002). Assume that graduation records show that women make up 50% of the students who graduated within five years, but only 45% of the students who did not graduate within five years. The students who had not graduated within five years either dropped out or were still working on their degrees.
a. Let A_1 = the student graduated within five years
A_2 = the student did not graduate within five years
W = the student is a female student
Using the given information, what are the values for $P(A_1)$, $P(A_2)$, $P(W \mid A_1)$, and $P(W \mid A_2)$?
b. What is the probability that a female student will graduate within five years?
c. What is the probability that a male student will graduate within five years?
d. Given the preceding results, what are the percentage of women and the percentage of men in the entering freshman class?

45. In an article about investment alternatives, *Money* magazine reported that drug stocks provide a potential for long-term growth, with over 50% of the adult population of the United States taking prescription drugs on a regular basis. For adults age 65 and older, 82% take prescription drugs regularly. For adults age 18 to 64, 49% take prescription drugs regularly. The age 18–64 age group accounts for 83.5% of the adult population (*Statistical Abstract of the United States*, 2008).
a. What is the probability that a randomly selected adult is 65 or older?
b. Given an adult takes prescription drugs regularly, what is the probability that the adult is 65 or older?

Summary

In this chapter we introduced basic probability concepts and illustrated how probability analysis can be used to provide helpful information for decision making. We described how probability can be interpreted as a numerical measure of the likelihood that an event will occur. In addition, we saw that the probability of an event can be computed either by summing the probabilities of the experimental outcomes (sample points) comprising the event or by using the relationships established by the addition, conditional probability, and multiplication laws of probability. For cases in which additional information is available, we showed how Bayes' theorem can be used to obtain revised or posterior probabilities.

Glossary

Probability A numerical measure of the likelihood that an event will occur.
Experiment A process that generates well-defined outcomes.
Sample space The set of all experimental outcomes.
Sample point An element of the sample space. A sample point represents an experimental outcome.
Tree diagram A graphical representation that helps in visualizing a multiple-step experiment.
Basic requirements for assigning probabilities Two requirements that restrict the manner in which probability assignments can be made: (1) for each experimental outcome E_i,

we must have $0 \leq P(E_i) \leq 1$; (2) considering all experimental outcomes, we must have $P(E_1) + P(E_2) + \cdots + P(E_n) = 1$.

Classical method A method of assigning probabilities that is appropriate when all the experimental outcomes are equally likely.

Relative frequency method A method of assigning probabilities that is appropriate when data are available to estimate the proportion of the time the experimental outcome will occur if the experiment is repeated a large number of times.

Subjective method A method of assigning probabilities on the basis of judgment.

Event A collection of sample points.

Complement of *A* The event consisting of all sample points that are not in A.

Venn diagram A graphical representation for showing symbolically the sample space and operations involving events in which the sample space is represented by a rectangle and events are represented as circles within the sample space.

Union of *A* and *B* The event containing all sample points belonging to A or B or both. The union is denoted $A \cup B$.

Intersection of *A* and *B* The event containing the sample points belonging to both A and B. The intersection is denoted $A \cap B$.

Addition law A probability law used to compute the probability of the union of two events. It is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. For mutually exclusive events, $P(A \cap B) = 0$; in this case the addition law reduces to $P(A \cup B) = P(A) + P(B)$.

Mutually exclusive events Events that have no sample points in common; that is, $A \cap B$ is empty and $P(A \cap B) = 0$.

Conditional probability The probability of an event given that another event already occurred. The conditional probability of A given B is $P(A \mid B) = P(A \cap B)/P(B)$.

Joint probability The probability of two events both occurring; that is, the probability of the intersection of two events.

Marginal probability The values in the margins of a joint probability table that provide the probabilities of each event separately.

Independent events Two events A and B where $P(A \mid B) = P(A)$ or $P(B \mid A) = P(B)$; that is, the events have no influence on each other.

Multiplication law A probability law used to compute the probability of the intersection of two events. It is $P(A \cap B) = P(B)P(A \mid B)$ or $P(A \cap B) = P(A)P(B \mid A)$. For independent events it reduces to $P(A \cap B) = P(A)P(B)$.

Prior probabilities Initial estimates of the probabilities of events.

Posterior probabilities Revised probabilities of events based on additional information.

Bayes' theorem A method used to compute posterior probabilities.

Key Formulas

Counting Rule for Combinations

$$C_n^N = \binom{N}{n} = \frac{N!}{n!(N-n)!} \tag{4.1}$$

Counting Rule for Permutations

$$P_n^N = n!\binom{N}{n} = \frac{N!}{(N-n)!} \tag{4.2}$$

Computing Probability Using the Complement

$$P(A) = 1 - P(A^c) \tag{4.5}$$

Addition Law

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{4.6}$$

Conditional Probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \tag{4.7}$$

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \tag{4.8}$$

Multiplication Law

$$P(A \cap B) = P(B)P(A \mid B) \tag{4.11}$$

$$P(A \cap B) = P(A)P(B \mid A) \tag{4.12}$$

Multiplication Law for Independent Events

$$P(A \cap B) = P(A)P(B) \tag{4.13}$$

Bayes' Theorem

$$P(A_i \mid B) = \frac{P(A_i)P(B \mid A_i)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2) + \cdots + P(A_n)P(B \mid A_n)} \tag{4.19}$$

Supplementary Exercises

46. *The Wall Street Journal*/Harris Personal Finance poll asked 2082 adults if they owned a home (All Business website, January 23, 2008). A total of 1249 survey respondents answered Yes. Of the 450 respondents in the 18–34 age group, 117 responded Yes.
    a. What is the probability that a respondent to the poll owned a home?
    b. What is the probability that a respondent in the 18–34 age group owned a home?
    c. What is the probability that a respondent to the poll did not own a home?
    d. What is the probability that a respondent in the 18–34 age group did not own a home?

47. A financial manager made two new investments—one in the oil industry and one in municipal bonds. After a one-year period, each of the investments will be classified as either successful or unsuccessful. Consider the making of the two investments as an experiment.
    a. How many sample points exist for this experiment?
    b. Show a tree diagram and list the sample points.
    c. Let O = the event that the oil industry investment is successful and M = the event that the municipal bond investment is successful. List the sample points in O and in M.
    d. List the sample points in the union of the events ($O \cup M$).
    e. List the sample points in the intersection of the events ($O \cap M$).
    f. Are events O and M mutually exclusive? Explain.

48. Statistics from the 2009 Major League Baseball season show that there were 157 players who had at least 500 plate appearances. For this group, 42 players had a batting average of .300 or higher, 53 players hit 25 or more home runs, and 14 players had a batting average of .300 or higher and hit 25 or more home runs. Only four players had 200 or more hits (ESPN website, January 10, 2010). Use the 157 players who had at least 500 plate appearances to answer the following questions.
    a. What is the probability that a randomly selected player had a batting average of .300 or higher?
    b. What is the probability that a randomly selected player hit 25 or more home runs?
    c. Are the events "having a batting average of .300 or higher" and "hitting 25 or more home runs" mutually exclusive?

d. What is the probability that a randomly selected player had a batting average of .300 or higher or hit 25 or more home runs?
e. What is the probability that a randomly selected player had 200 or more hits? Does obtaining 200 or more hits appear to be more difficult than hitting 25 or more home runs? Explain.

49. A study of 31,000 hospital admissions in New York State found that 4% of the admissions led to treatment-caused injuries. One-seventh of these treatment-caused injuries resulted in death, and one-fourth were caused by negligence. Malpractice claims were filed in 1 out of 7.5 cases involving negligence, and payments were made in one out of every two claims.
a. What is the probability that a person admitted to the hospital will suffer a treatment-caused injury due to negligence?
b. What is the probability that a person admitted to the hospital will die from a treatment-caused injury?
c. In the case of a negligent treatment-caused injury, what is the probability that a malpractice claim will be paid?

50. A telephone survey to determine viewer response to a new television show obtained the following data.

Rating	Frequency
Poor	4
Below average	8
Average	11
Above average	14
Excellent	13

a. What is the probability that a randomly selected viewer will rate the new show as average or better?
b. What is the probability that a randomly selected viewer will rate the new show below average or worse?

51. The following crosstabulation shows household income by educational level of the head of household (*Statistical Abstract of the United States*, 2008).

	Household Income ($1000s)					
Education Level	**Under 25**	**25.0–49.9**	**50.0–74.9**	**75.0–99.9**	**100 or more**	**Total**
Not H.S. Graduate	4,207	3,459	1,389	539	367	9,961
H.S. Graduate	4,917	6,850	5,027	2,637	2,668	22,099
Some College	2,807	5,258	4,678	3,250	4,074	20,067
Bachelor's Degree	885	2,094	2,848	2,581	5,379	13,787
Beyond Bach. Deg.	290	829	1,274	1,241	4,188	7,822
Total	13,106	18,490	15,216	10,248	16,676	73,736

a. Develop a joint probability table.
b. What is the probability of a head of household not being a high school graduate?
c. What is the probability of a head of household having a bachelor's degree or more education?
d. What is the probability of a household headed by someone with a bachelor's degree earning $100,000 or more?
e. What is the probability of a household having income below $25,000?

f. What is the probability of a household headed by someone with a bachelor's degree earning less than $25,000?
g. Is household income independent of educational level?

52. An MBA new-matriculants survey provided the following data for 2018 students.

Age Group	Applied to More Than One School: Yes	Applied to More Than One School: No
23 and under	207	201
24–26	299	379
27–30	185	268
31–35	66	193
36 and over	51	169

a. For a randomly selected MBA student, prepare a joint probability table for the experiment consisting of observing the student's age and whether the student applied to one or more schools.
b. What is the probability that a randomly selected applicant is 23 or under?
c. What is the probability that a randomly selected applicant is older than 26?
d. What is the probability that a randomly selected applicant applied to more than one school?

53. Refer again to the data from the MBA new-matriculants survey in exercise 52.
a. Given that a person applied to more than one school, what is the probability that the person is 24–26 years old?
b. Given that a person is in the 36-and-over age group, what is the probability that the person applied to more than one school?
c. What is the probability that a person is 24–26 years old or applied to more than one school?
d. Suppose a person is known to have applied to only one school. What is the probability that the person is 31 or more years old?
e. Is the number of schools applied to independent of age? Explain.

54. A poll conducted to learn about attitudes toward investment and retirement asked male and female respondents how important they felt level of risk was in choosing a retirement investment. The following joint probability table was constructed from the data provided. "Important" means the respondent said level of risk was either important or very important.

	Male	Female	Total
Important	.22	.27	.49
Not Important	.28	.23	.51
Total	.50	.50	1.00

a. What is the probability that a survey respondent will say level of risk is important?
b. What is the probability that a male respondent will say level of risk is important?
c. What is the probability that a female respondent will say level of risk is important?
d. Is the level of risk independent of the gender of the respondent? Why or why not?
e. Do male and female attitudes toward risk differ?

55. A large consumer goods company ran a television advertisement for one of its soap products. On the basis of a survey that was conducted, probabilities were assigned to the following events.

B = individual purchased the product
S = individual recalls seeing the advertisement
$B \cap S$ = individual purchased the product and recalls seeing the advertisement

The probabilities assigned were $P(B) = .20$, $P(S) = .40$, and $P(B \cap S) = .12$.

a. What is the probability of an individual's purchasing the product given that the individual recalls seeing the advertisement? Does seeing the advertisement increase the probability that the individual will purchase the product? As a decision maker, would you recommend continuing the advertisement (assuming that the cost is reasonable)?
b. Assume that individuals who do not purchase the company's soap product buy from its competitors. What would be your estimate of the company's market share? Would you expect that continuing the advertisement will increase the company's market share? Why or why not?
c. The company also tested another advertisement and assigned it values of $P(S) = .30$ and $P(B \cap S) = .10$. What is $P(B \mid S)$ for this other advertisement? Which advertisement seems to have had the bigger effect on customer purchases?

56. Cooper Realty is a small real estate company located in Albany, New York, specializing primarily in residential listings. It recently became interested in determining the likelihood of one of its listings being sold within a certain number of days. An analysis of company sales of 800 homes in previous years produced the following data.

		Days Listed Until Sold			
		Under 30	31–90	Over 90	Total
Initial Asking Price	Under $150,000	50	40	10	100
	$150,000–$199,999	20	150	80	250
	$200,000–$250,000	20	280	100	400
	Over $250,000	10	30	10	50
	Total	100	500	200	800

a. If A is defined as the event that a home is listed for more than 90 days before being sold, estimate the probability of A.
b. If B is defined as the event that the initial asking price is under $150,000, estimate the probability of B.
c. What is the probability of $A \cap B$?
d. Assuming that a contract was just signed to list a home with an initial asking price of less than $150,000, what is the probability that the home will take Cooper Realty more than 90 days to sell?
e. Are events A and B independent?

57. A company studied the number of lost-time accidents occurring at its Brownsville, Texas, plant. Historical records show that 6% of the employees suffered lost-time accidents last year. Management believes that a special safety program will reduce such accidents to 5% during the current year. In addition, it estimates that 15% of employees who had lost-time accidents last year will experience a lost-time accident during the current year.

a. What percentage of the employees will experience lost-time accidents in both years?
b. What percentage of the employees will suffer at least one lost-time accident over the two-year period?

58. A survey showed that 8% of Internet users age 18 and older report keeping a blog. Referring to the 18–29 age group as young adults, the survey showed that for bloggers 54% are young adults and for nonbloggers 24% are young adults (Pew Internet & American Life Project, July 19, 2006).
    a. Develop a joint probability table for these data with two rows (bloggers vs. nonbloggers) and two columns (young adults vs. older adults).
    b. What is the probability that an Internet user is a young adult?
    c. What is the probability that an Internet user keeps a blog and is a young adult?
    d. Suppose that in a follow-up phone survey we contact someone who is 24 years old. What is the probability that this person keeps a blog?

59. An oil company purchased an option on land in Alaska. Preliminary geologic studies assigned the following prior probabilities.

$$P(\text{high-quality oil}) = .50$$
$$P(\text{medium-quality oil}) = .20$$
$$P(\text{no oil}) = .30$$

    a. What is the probability of finding oil?
    b. After 200 feet of drilling on the first well, a soil test is taken. The probabilities of finding the particular type of soil identified by the test follow.

$$P(\text{soil} \mid \text{high-quality oil}) = .20$$
$$P(\text{soil} \mid \text{medium-quality oil}) = .80$$
$$P(\text{soil} \mid \text{no oil}) = .20$$

    How should the firm interpret the soil test? What are the revised probabilities, and what is the new probability of finding oil?

60. ParFore created a website to market golf equipment and apparel. Management would like a certain offer to appear for female visitors and a different offer to appear for male visitors. From a sample of past website visits, management learned that 60% of the visitors to the website ParFore are male and 40% are female.
    a. What is the prior probability that the next visitor to the website will be female?
    b. Suppose you know that the current visitor to the website ParFore previously visited the Dillard's website, and that women are three times as likely to visit the Dillard's website as men. What is the revised probability that the current visitor to the website ParFore is female? Should you display the offer that appeals more to female visitors or the one that appeals more to male visitors?

Case Problem Hamilton County Judges

Hamilton County judges try thousands of cases per year. In an overwhelming majority of the cases disposed, the verdict stands as rendered. However, some cases are appealed, and of those appealed, some of the cases are reversed. Kristen DeIGuzzi of *The Cincinnati Enquirer* conducted a study of cases handled by Hamilton County judges over a three-year period. Shown in Table 4.8 are the results for 182,908 cases handled (disposed) by 38 judges in Common Pleas Court, Domestic Relations Court, and Municipal Court. Two of the judges (Dinkelacker and Hogan) did not serve in the same court for the entire three-year period.

The purpose of the newspaper's study was to evaluate the performance of the judges. Appeals are often the result of mistakes made by judges, and the newspaper wanted to know which judges were doing a good job and which were making too many mistakes. You are

TABLE 4.8 TOTAL CASES DISPOSED, APPEALED, AND REVERSED IN HAMILTON COUNTY COURTS

WEB file Judge

Common Pleas Court

Judge	Total Cases Disposed	Appealed Cases	Reversed Cases
Fred Cartolano	3,037	137	12
Thomas Crush	3,372	119	10
Patrick Dinkelacker	1,258	44	8
Timothy Hogan	1,954	60	7
Robert Kraft	3,138	127	7
William Mathews	2,264	91	18
William Morrissey	3,032	121	22
Norbert Nadel	2,959	131	20
Arthur Ney, Jr.	3,219	125	14
Richard Niehaus	3,353	137	16
Thomas Nurre	3,000	121	6
John O'Connor	2,969	129	12
Robert Ruehlman	3,205	145	18
J. Howard Sundermann	955	60	10
Ann Marie Tracey	3,141	127	13
Ralph Winkler	3,089	88	6
Total	43,945	1762	199

Domestic Relations Court

Judge	Total Cases Disposed	Appealed Cases	Reversed Cases
Penelope Cunningham	2,729	7	1
Patrick Dinkelacker	6,001	19	4
Deborah Gaines	8,799	48	9
Ronald Panioto	12,970	32	3
Total	30,499	106	17

Municipal Court

Judge	Total Cases Disposed	Appealed Cases	Reversed Cases
Mike Allen	6,149	43	4
Nadine Allen	7,812	34	6
Timothy Black	7,954	41	6
David Davis	7,736	43	5
Leslie Isaiah Gaines	5,282	35	13
Karla Grady	5,253	6	0
Deidra Hair	2,532	5	0
Dennis Helmick	7,900	29	5
Timothy Hogan	2,308	13	2
James Patrick Kenney	2,798	6	1
Joseph Luebbers	4,698	25	8
William Mallory	8,277	38	9
Melba Marsh	8,219	34	7
Beth Mattingly	2,971	13	1
Albert Mestemaker	4,975	28	9
Mark Painter	2,239	7	3
Jack Rosen	7,790	41	13
Mark Schweikert	5,403	33	6
David Stockdale	5,371	22	4
John A. West	2,797	4	2
Total	108,464	500	104

called in to assist in the data analysis. Use your knowledge of probability and conditional probability to help with the ranking of the judges. You also may be able to analyze the likelihood of appeal and reversal for cases handled by different courts.

Managerial Report

Prepare a report with your rankings of the judges. Also, include an analysis of the likelihood of appeal and case reversal in the three courts. At a minimum, your report should include the following:

1. The probability of cases being appealed and reversed in the three different courts.
2. The probability of a case being appealed for each judge.
3. The probability of a case being reversed for each judge.
4. The probability of reversal given an appeal for each judge.
5. Rank the judges within each court. State the criteria you used and provide a rationale for your choice.

CHAPTER 5

Discrete Probability Distributions

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STATISTICS IN PRACTICE: CITIBANK

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- Continuous Random Variables

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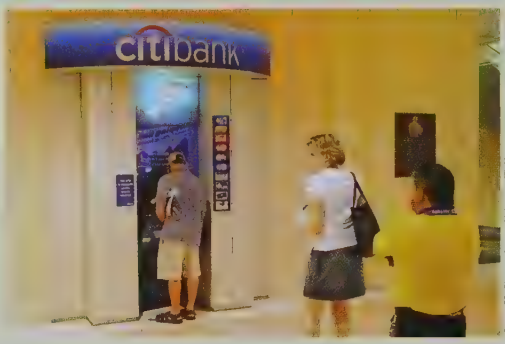
STATISTICS in PRACTICE

CITIBANK*

LONG ISLAND CITY, NEW YORK

Citibank, the retail banking division of Citigroup, offers a wide range of financial services including checking and saving accounts, loans and mortgages, insurance, and investment services. It delivers these services through a unique system referred to as Citibanking.

Citibank was one of the first banks in the United States to introduce automated teller machines (ATMs). Citibank's ATMs, located in Citicard Banking Centers (CBCs), let customers do all of their banking in one place with the touch of a finger, 24 hours a day, 7 days a week. More than 150 different banking functions—from deposits to managing investments—can be performed with ease. Citibank customers use ATMs for 80% of their transactions.



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A Citibank state-of-the-art ATM.

Each Citibank CBC operates as a waiting line system with randomly arriving customers seeking service at one of the ATMs. If all ATMs are busy, the arriving customers wait in line. Periodic CBC capacity studies are used to analyze customer waiting times and to determine whether additional ATMs are needed.

Data collected by Citibank showed that the random customer arrivals followed a probability distribution known as the Poisson distribution. Using the Poisson distribution, Citibank can compute probabilities for the number of customers arriving at a CBC during any time period and make decisions concerning the number of ATMs needed. For example, let x = the number of customers arriving during a one-minute period. Assuming that a particular CBC has a mean arrival rate of two customers per minute, the following table shows the probabilities for the number of customers arriving during a one-minute period.

x	Probability
0	.1353
1	.2707
2	.2707
3	.1804
4	.0902
5 or more	.0527

Discrete probability distributions, such as the one used by Citibank, are the topic of this chapter. In addition to the Poisson distribution, you will learn about the binomial and hypergeometric distributions and how they can be used to provide helpful probability information.

*The authors are indebted to Ms. Stacey Karter, Citibank, for providing this Statistics in Practice.

In this chapter we continue the study of probability by introducing the concepts of random variables and probability distributions. The focus of this chapter is discrete probability distributions. Three special discrete probability distributions—the binomial, Poisson, and hypergeometric—are covered.

5.1 Random Variables

In Chapter 4 we defined the concept of an experiment and its associated experimental outcomes. A random variable provides a means for describing experimental outcomes using numerical values. Random variables must assume numerical values.

RANDOM VARIABLE

A **random variable** is a numerical description of the outcome of an experiment.

Random variables must have numerical values.

In effect, a random variable associates a numerical value with each possible experimental outcome. The particular numerical value of the random variable depends on the outcome of the experiment. A random variable can be classified as being either *discrete* or *continuous* depending on the numerical values it assumes.

Discrete Random Variables

A random variable that may assume either a finite number of values or an infinite sequence of values such as 0, 1, 2, . . . is referred to as a **discrete random variable**. For example, consider the experiment of an accountant taking the certified public accountant (CPA) examination. The examination has four parts. We can define a random variable as x = the number of parts of the CPA examination passed. It is a discrete random variable because it may assume the finite number of values 0, 1, 2, 3, or 4.

As another example of a discrete random variable, consider the experiment of cars arriving at a tollbooth. The random variable of interest is x = the number of cars arriving during a one-day period. The possible values for x come from the sequence of integers 0, 1, 2, and so on. Hence, x is a discrete random variable assuming one of the values in this infinite sequence.

Although the outcomes of many experiments can naturally be described by numerical values, others cannot. For example, a survey question might ask an individual to recall the message in a recent television commercial. This experiment would have two possible outcomes: The individual cannot recall the message, and the individual can recall the message. We can still describe these experimental outcomes numerically by defining the discrete random variable x as follows: let $x = 0$ if the individual cannot recall the message and $x = 1$ if the individual can recall the message. The numerical values for this random variable are arbitrary (we could use 5 and 10), but they are acceptable in terms of the definition of a random variable—namely, x is a random variable because it provides a numerical description of the outcome of the experiment.

Table 5.1 provides some additional examples of discrete random variables. Note that in each example the discrete random variable assumes a finite number of values or an infinite sequence of values such as 0, 1, 2, ... These types of discrete random variables are discussed in detail in this chapter.

TABLE 5.1 EXAMPLES OF DISCRETE RANDOM VARIABLES

Experiment	Random Variable (x)	Possible Values for the Random Variable
Contact five customers	Number of customers who place an order	0, 1, 2, 3, 4, 5
Inspect a shipment of 50 radios	Number of defective radios	0, 1, 2, . . . , 49, 50
Operate a restaurant for one day	Number of customers	0, 1, 2, 3, . . .
Sell an automobile	Gender of the customer	0 if male; 1 if female

Continuous Random Variables

A random variable that may assume any numerical value in an interval or collection of intervals is called a **continuous random variable**. Experimental outcomes based on measurement scales such as time, weight, distance, and temperature can be described by continuous random variables. For example, consider an experiment of monitoring incoming telephone calls to the claims office of a major insurance company. Suppose the random variable of interest is x = the time between consecutive incoming calls in minutes. This random variable may assume any value in the interval $x \geq 0$. Actually, an infinite number of values are possible for x, including values such as 1.26 minutes, 2.751 minutes, 4.3333 minutes, and so on. As another example, consider a 90-mile section of interstate highway I-75 north of Atlanta, Georgia. For an emergency ambulance service located in Atlanta, we might define the random variable as x = number of miles to the location of the next traffic accident along this section of I-75. In this case, x would be a continuous random variable assuming any value in the interval $0 \leq x \leq 90$. Additional examples of continuous random variables are listed in Table 5.2. Note that each example describes a random variable that may assume any value in an interval of values. Continuous random variables and their probability distributions will be the topic of Chapter 6.

TABLE 5.2 EXAMPLES OF CONTINUOUS RANDOM VARIABLES

Experiment	Random Variable (x)	Possible Values for the Random Variable
Operate a bank	Time between customer arrivals in minutes	$x \geq 0$
Fill a soft drink can (max = 12.1 ounces)	Number of ounces	$0 \leq x \leq 12.1$
Construct a new library	Percentage of project complete after six months	$0 \leq x \leq 100$
Test a new chemical process	Temperature when the desired reaction takes place (min 150° F; max 212° F)	$150 \leq x \leq 212$

NOTES AND COMMENTS

One way to determine whether a random variable is discrete or continuous is to think of the values of the random variable as points on a line segment. Choose two points representing values of the random variable. If the entire line segment between the two points also represents possible values for the random variable, then the random variable is continuous.

Exercises

Methods

SELF test

1. Consider the experiment of tossing a coin twice.
   a. List the experimental outcomes.
   b. Define a random variable that represents the number of heads occurring on the two tosses.
   c. Show what value the random variable would assume for each of the experimental outcomes.
   d. Is this random variable discrete or continuous?

2. Consider the experiment of a worker assembling a product.
   a. Define a random variable that represents the time in minutes required to assemble the product.
   b. What values may the random variable assume?
   c. Is the random variable discrete or continuous?

Applications

SELF test

3. Three students scheduled interviews for summer employment at the Brookwood Institute. In each case the interview results in either an offer for a position or no offer. Experimental outcomes are defined in terms of the results of the three interviews.
   a. List the experimental outcomes.
   b. Define a random variable that represents the number of offers made. Is the random variable continuous?
   c. Show the value of the random variable for each of the experimental outcomes.
4. In November the U.S. unemployment rate was 8.7% (U.S. Department of Labor website, January 10, 2010). The Census Bureau includes nine states in the Northeast region. Assume that the random variable of interest is the number of Northeastern states with an unemployment rate in November that was less than 8.7%. What values may this random variable have?
5. To perform a certain type of blood analysis, lab technicians must perform two procedures. The first procedure requires either one or two separate steps, and the second procedure requires either one, two, or three steps.
   a. List the experimental outcomes associated with performing the blood analysis.
   b. If the random variable of interest is the total number of steps required to do the complete analysis (both procedures), show what value the random variable will assume for each of the experimental outcomes.
6. Listed is a series of experiments and associated random variables. In each case, identify the values that the random variable can assume and state whether the random variable is discrete or continuous.

Experiment	Random Variable (x)
a. Take a 20-question examination	Number of questions answered correctly
b. Observe cars arriving at a tollbooth for one hour	Number of cars arriving at tollbooth
c. Audit 50 tax returns	Number of returns containing errors
d. Observe an employee's work	Number of nonproductive hours in an eight-hour workday
e. Weigh a shipment of goods	Number of pounds

5.2 Discrete Probability Distributions

The **probability distribution** for a random variable describes how probabilities are distributed over the values of the random variable. For a discrete random variable x, the probability distribution is defined by a **probability function**, denoted by $f(x)$. The probability function provides the probability for each value of the random variable.

As an illustration of a discrete random variable and its probability distribution, consider the sales of automobiles at DiCarlo Motors in Saratoga, New York. Over the past 300 days of operation, sales data show 54 days with no automobiles sold, 117 days with 1 automobile sold, 72 days with 2 automobiles sold, 42 days with 3 automobiles sold, 12 days with

TABLE 5.3 PROBABILITY DISTRIBUTION FOR THE NUMBER OF AUTOMOBILES SOLD DURING A DAY AT DICARLO MOTORS

x	$f(x)$
0	.18
1	.39
2	.24
3	.14
4	.04
5	.01
Total	1.00

4 automobiles sold, and 3 days with 5 automobiles sold. Suppose we consider the experiment of selecting a day of operation at DiCarlo Motors and define the random variable of interest as x = the number of automobiles sold during a day. From historical data, we know x is a discrete random variable that can assume the values 0, 1, 2, 3, 4, or 5. In probability function notation, $f(0)$ provides the probability of 0 automobiles sold, $f(1)$ provides the probability of 1 automobile sold, and so on. Because historical data show 54 of 300 days with 0 automobiles sold, we assign the value 54/300 = .18 to $f(0)$, indicating that the probability of 0 automobiles being sold during a day is .18. Similarly, because 117 of 300 days had 1 automobile sold, we assign the value 117/300 = .39 to $f(1)$, indicating that the probability of exactly 1 automobile being sold during a day is .39. Continuing in this way for the other values of the random variable, we compute the values for $f(2)$, $f(3)$, $f(4)$, and $f(5)$ as shown in Table 5.3, the probability distribution for the number of automobiles sold during a day at DiCarlo Motors.

A primary advantage of defining a random variable and its probability distribution is that once the probability distribution is known, it is relatively easy to determine the probability of a variety of events that may be of interest to a decision maker. For example, using the probability distribution for DiCarlo Motors as shown in Table 5.3, we see that the most probable number of automobiles sold during a day is 1 with a probability of $f(1) = .39$. In addition, there is an $f(3) + f(4) + f(5) = .14 + .04 + .01 = .19$ probability of selling 3 or more automobiles during a day. These probabilities, plus others the decision maker may ask about, provide information that can help the decision maker understand the process of selling automobiles at DiCarlo Motors.

In the development of a probability function for any discrete random variable, the following two conditions must be satisfied.

These conditions are the analogs to the two basic requirements for assigning probabilities to experimental outcomes presented in Chapter 4.

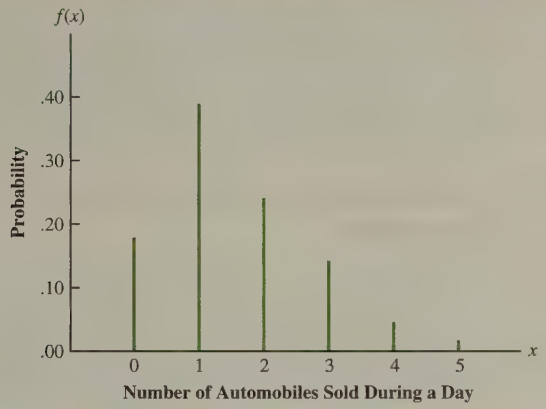
REQUIRED CONDITIONS FOR A DISCRETE PROBABILITY FUNCTION

$$f(x) \geq 0 \qquad (5.1)$$

$$\Sigma f(x) = 1 \qquad (5.2)$$

Table 5.3 shows that the probabilities for the random variable x satisfy equation (5.1); $f(x)$ is greater than or equal to 0 for all values of x. In addition, because the probabilities sum to 1, equation (5.2) is satisfied. Thus, the DiCarlo Motors probability function is a valid discrete probability function.

FIGURE 5.1 GRAPHICAL REPRESENTATION OF THE PROBABILITY DISTRIBUTION FOR THE NUMBER OF AUTOMOBILES SOLD DURING A DAY AT DICARLO MOTORS



We can also present probability distributions graphically. In Figure 5.1 the values of the random variable x for DiCarlo Motors are shown on the horizontal axis and the probability associated with these values is shown on the vertical axis.

In addition to tables and graphs, a formula that gives the probability function, $f(x)$, for every value of x is often used to describe probability distributions. The simplest example of a discrete probability distribution given by a formula is the **discrete uniform probability distribution**. Its probability function is defined by equation (5.3).

DISCRETE UNIFORM PROBABILITY FUNCTION

$$f(x) = 1/n \tag{5.3}$$

where

n = the number of values the random variable may have

For example, suppose that for the experiment of rolling a die we define the random variable x to be the number of dots on the upward face. For this experiment, $n = 6$ values are possible for the random variable; $x = 1, 2, 3, 4, 5, 6$. Thus, the probability function for this discrete uniform random variable is

$$f(x) = 1/6 \qquad x = 1, 2, 3, 4, 5, 6$$

The possible values of the random variable and the associated probabilities are shown.

x	$f(x)$
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

As another example, consider the random variable x with the following discrete probability distribution.

x	$f(x)$
1	1/10
2	2/10
3	3/10
4	4/10

This probability distribution can be defined by the formula

$$f(x) = \frac{x}{10} \qquad \text{for } x = 1, 2, 3, \text{ or } 4$$

Evaluating $f(x)$ for a given value of the random variable will provide the associated probability. For example, using the preceding probability function, we see that $f(2) = 2/10$ provides the probability that the random variable assumes a value of 2.

The more widely used discrete probability distributions generally are specified by formulas. Three important cases are the binomial, Poisson, and hypergeometric distributions; these distributions are discussed later in the chapter.

Exercises

Methods

SELF test

7. The probability distribution for the random variable x follows.

x	$f(x)$
20	.20
25	.15
30	.25
35	.40

a. Is this probability distribution valid? Explain.
b. What is the probability that $x = 30$?
c. What is the probability that x is less than or equal to 25?
d. What is the probability that x is greater than 30?

Applications

SELF test

8. The following data were collected by counting the number of operating rooms in use at Tampa General Hospital over a 20-day period: On three of the days only one operating room was used, on five of the days two were used, on eight of the days three were used, and on four days all four of the hospital's operating rooms were used.
   a. Use the relative frequency approach to construct a probability distribution for the number of operating rooms in use on any given day.
   b. Draw a graph of the probability distribution.
   c. Show that your probability distribution satisfies the required conditions for a valid discrete probability distribution.

9. For unemployed persons in the United States, the average number of months of unemployment at the end of December 2009 was approximately seven months (Bureau of Labor Statistics, January 2010). Suppose the following data are for a particular region in upstate New York. The values in the first column show the number of months unemployed, and the values in the second column show the corresponding number of unemployed persons.

Months Unemployed	Number Unemployed
1	1029
2	1686
3	2269
4	2675
5	3487
6	4652
7	4145
8	3587
9	2325
10	1120

Let x be a random variable indicating the number of months a person is unemployed.
   a. Use the data to develop a probability distribution for x.
   b. Show that your probability distribution satisfies the conditions for a valid discrete probability distribution.
   c. What is the probability that a person is unemployed for two months or less? Unemployed for more than two months?
   d. What is the probability that a person is unemployed for more than six months?

10. The percent frequency distributions of job satisfaction scores for a sample of information systems (IS) senior executives and middle managers are as follows. The scores range from a low of 1 (very dissatisfied) to a high of 5 (very satisfied).

Job Satisfaction Score	IS Senior Executives (%)	IS Middle Managers (%)
1	5	4
2	9	10
3	3	12
4	42	46
5	41	28

   a. Develop a probability distribution for the job satisfaction score of a senior executive.
   b. Develop a probability distribution for the job satisfaction score of a middle manager.
   c. What is the probability that a senior executive will report a job satisfaction score of 4 or 5?

d. What is the probability that a middle manager is very satisfied?
e. Compare the overall job satisfaction of senior executives and middle managers.

11. A technician services mailing machines at companies in the Phoenix area. Depending on the type of malfunction, the service call can take one, two, three, or four hours. The different types of malfunctions occur at about the same frequency.
    a. Develop a probability distribution for the duration of a service call.
    b. Draw a graph of the probability distribution.
    c. Show that your probability distribution satisfies the conditions required for a discrete probability function.
    d. What is the probability that a service call will take three hours?
    e. A service call has just come in, but the type of malfunction is unknown. It is 3:00 P.M. and service technicians usually get off work at 5:00 P.M. What is the probability that the service technician will have to work overtime to fix the machine today?

12. The two largest cable providers are Comcast Cable Communications, with 21.5 million subscribers, and Time Warner Cable, with 11.0 million subscribers (*The New York Times Almanac*, 2007). Suppose that the management of Time Warner Cable subjectively assesses a probability distribution for the number of new subscribers next year in the state of New York as follows.

x	$f(x)$
100,000	.10
200,000	.20
300,000	.25
400,000	.30
500,000	.10
600,000	.05

    a. Is this probability distribution valid? Explain.
    b. What is the probability that Time Warner will obtain more than 400,000 new subscribers?
    c. What is the probability that Time Warner will obtain fewer than 200,000 new subscribers?

13. A psychologist determined that the number of sessions required to obtain the trust of a new patient is either 1, 2, or 3. Let x be a random variable indicating the number of sessions required to gain the patient's trust. The following probability function has been proposed.

$$f(x) = \frac{x}{6} \qquad \text{for } x = 1, 2, \text{ or } 3$$

    a. Is this probability function valid? Explain.
    b. What is the probability that it takes exactly two sessions to gain the patient's trust?
    c. What is the probability that it takes at least two sessions to gain the patient's trust?

14. The following table is a partial probability distribution for the MRA Company's projected profits (x = profit in \$1000s) for the first year of operation (the negative value denotes a loss).

x	$f(x)$
−100	.10
0	.20
50	.30
100	.25
150	.10
200	

    a. What is the proper value for $f(200)$? What is your interpretation of this value?
    b. What is the probability that MRA will be profitable?
    c. What is the probability that MRA will make at least \$100,000?

5.3 Expected Value and Variance

Expected Value

The **expected value**, or mean, of a random variable is a measure of the central location for the random variable. The formula for the expected value of a discrete random variable x follows.

The expected value is a weighted average of the values of the random variable, where the weights are the probabilities.

EXPECTED VALUE OF A DISCRETE RANDOM VARIABLE

$$E(x) = \mu = \Sigma x f(x) \qquad (5.4)$$

Both the notations $E(x)$ and μ are used to denote the expected value of a random variable.

Equation (5.4) shows that to compute the expected value of a discrete random variable, we must multiply each value of the random variable by the corresponding probability $f(x)$ and then add the resulting products. Using the DiCarlo Motors automobile sales example from Section 5.2, we show in Table 5.4 the calculation of the expected value for the number of automobiles sold during a day. The sum of the entries in the $xf(x)$ column shows that the expected value is 1.50 automobiles per day. We therefore know that although sales of 0, 1, 2, 3, 4, or 5 automobiles are possible on any one day, over time DiCarlo can anticipate selling an average of 1.50 automobiles per day. Assuming 30 days of operation during a month, we can use the expected value of 1.50 to forecast average monthly sales of $30(1.50) = 45$ automobiles.

The expected value does not have to be a value the random variable can assume.

Variance

Even though the expected value provides the mean value for the random variable, we often need a measure of variability, or dispersion. Just as we used the variance in Chapter 3 to summarize the variability in data, we now use **variance** to summarize the variability in the values of a random variable. The formula for the variance of a discrete random variable follows.

The variance is a weighted average of the squared deviations of a random variable from its mean. The weights are the probabilities.

VARIANCE OF A DISCRETE RANDOM VARIABLE

$$\text{Var}(x) = \sigma^2 = \Sigma(x - \mu)^2 f(x) \qquad (5.5)$$

TABLE 5.4 CALCULATION OF THE EXPECTED VALUE FOR THE NUMBER OF AUTOMOBILES SOLD DURING A DAY AT DICARLO MOTORS

x	$f(x)$	$xf(x)$
0	.18	0(.18) = .00
1	.39	1(.39) = .39
2	.24	2(.24) = .48
3	.14	3(.14) = .42
4	.04	4(.04) = .16
5	.01	5(.01) = .05
		1.50

$E(x) = \mu = \Sigma x f(x)$

TABLE 5.5 CALCULATION OF THE VARIANCE FOR THE NUMBER OF AUTOMOBILES SOLD DURING A DAY AT DICARLO MOTORS

x	$x - \mu$	$(x - \mu)^2$	$f(x)$	$(x - \mu)^2 f(x)$
0	$0 - 1.50 = -1.50$	2.25	.18	$2.25(.18) = .4050$
1	$1 - 1.50 = -.50$	.25	.39	$.25(.39) = .0975$
2	$2 - 1.50 = .50$	.25	.24	$.25(.24) = .0600$
3	$3 - 1.50 = 1.50$	2.25	.14	$2.25(.14) = .3150$
4	$4 - 1.50 = 2.50$	6.25	.04	$6.25(.04) = .2500$
5	$5 - 1.50 = 3.50$	12.25	.01	$12.25(.01) = .1225$
				1.2500

$\sigma^2 = \Sigma(x - \mu)^2 f(x)$

As equation (5.5) shows, an essential part of the variance formula is the deviation, $x - \mu$, which measures how far a particular value of the random variable is from the expected value, or mean, μ. In computing the variance of a random variable, the deviations are squared and then weighted by the corresponding value of the probability function. The sum of these weighted squared deviations for all values of the random variable is referred to as the *variance*. The notations Var(x) and σ^2 are both used to denote the variance of a random variable.

The calculation of the variance for the probability distribution of the number of automobiles sold during a day at DiCarlo Motors is summarized in Table 5.5. We see that the variance is 1.25. The **standard deviation**, σ, is defined as the positive square root of the variance. Thus, the standard deviation for the number of automobiles sold during a day is

$$\sigma = \sqrt{1.25} = 1.118$$

The standard deviation is measured in the same units as the random variable ($\sigma = 1.118$ automobiles) and therefore is often preferred in describing the variability of a random variable. The variance σ^2 is measured in squared units and is thus more difficult to interpret.

Using Excel to Compute the Expected Value, Variance, and Standard Deviation

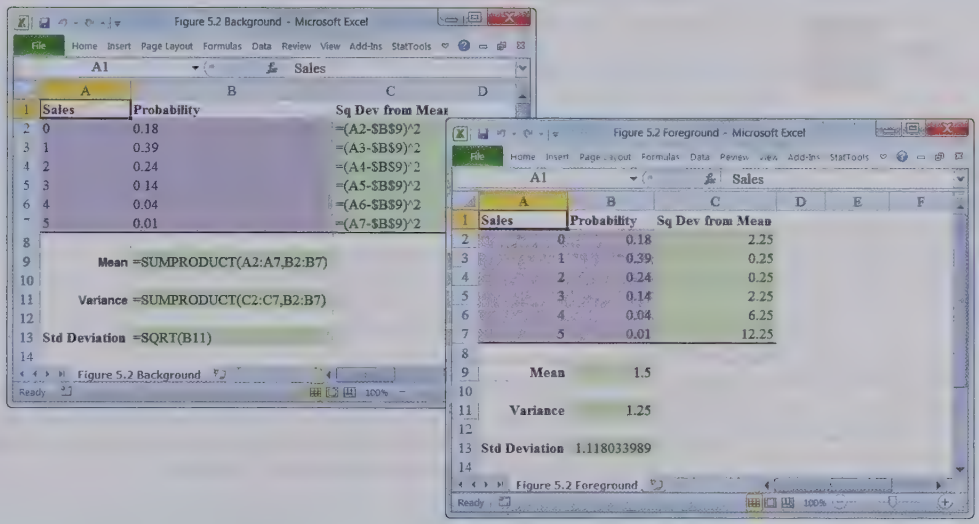
The calculations involved in computing the expected value and variance for a discrete random variable can easily be made in an Excel worksheet. One approach is to enter the formulas necessary to make the calculations in Tables 5.4 and 5.5. An easier way, however, is to make use of Excel's SUMPRODUCT function. In this subsection we show how to use the SUMPRODUCT function to compute the expected value and variance for daily automobile sales at DiCarlo Motors. Refer to Figure 5.2 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet is in the foreground.

Enter Data: The data needed are the values for the random variable and the corresponding probabilities. Labels, values for the random variable, and the corresponding probabilities are entered in cells A1:B7.

Enter Functions and Formulas: The SUMPRODUCT function multiplies each value in one range by the corresponding value in another range and sums the products. To use the SUMPRODUCT function to compute the expected value of daily automobile sales at DiCarlo Motors, we entered the following formula into cell B9:

=SUMPRODUCT(A2:A7,B2:B7)

FIGURE 5.2 EXCEL WORKSHEET FOR EXPECTED VALUE, VARIANCE, AND STANDARD DEVIATION



Note that the first range, A2:A7, contains the values for the random variable, daily automobile sales. The second range, B2:B7, contains the corresponding probabilities. Thus, the SUMPRODUCT function in cell B9 is computing A2*B2 + A3*B3 + A4*B4 + A5*B5 + A6*B6 + A7*B7; hence, it is applying the formula in equation (5.4) to compute the expected value. The result, shown in cell B9 of the value worksheet, is 1.5.

The formulas in cells C2:C7 are used to compute the squared deviations from the expected value or mean of 1.5 (the mean is in cell B9). The results, shown in the value worksheet, are the same as the results shown in Table 5.5. The formula necessary to compute the variance for daily automobile sales was entered into cell B11. It uses the SUMPRODUCT function to multiply each value in the range C2:C7 by each corresponding value in the range B2:B7 and sums the products. The result, shown in the value worksheet, is 1.25. Because the standard deviation is the square root of the variance, we entered the formula =SQRT(B11) into cell B13 to compute the standard deviation for daily automobile sales. The result, shown in the value worksheet, is 1.118034.

Exercises

Methods

15. The following table provides a probability distribution for the random variable x.

x	$f(x)$
3	.25
6	.50
9	.25

a. Compute $E(x)$, the expected value of x.
b. Compute σ^2, the variance of x.
c. Compute σ, the standard deviation of x.

SELF test

16. The following table provides a probability distribution for the random variable y.

y	$f(y)$
2	.20
4	.30
7	.40
8	.10

a. Compute $E(y)$.
b. Compute Var(y) and σ.

Applications

17. The number of students taking the SAT has risen to an all-time high of more than 1.5 million (College Board, August 26, 2008). Students are allowed to repeat the test in hopes of improving the score that is sent to college and university admission offices. The number of times the SAT was taken and the number of students are as follows.

Number of Times	Number of Students
1	721,769
2	601,325
3	166,736
4	22,299
5	6,730

a. Let x be a random variable indicating the number of times a student takes the SAT. Show the probability distribution for this random variable.
b. What is the probability that a student takes the SAT more than one time?
c. What is the probability that a student takes the SAT three or more times?
d. What is the expected value of the number of times the SAT is taken? What is your interpretation of the expected value?
e. What is the variance and standard deviation for the number of times the SAT is taken?

SELF test

18. The American Housing Survey reported the following data on the number of bedrooms in owner-occupied and renter-occupied houses in central cities (U.S. Census Bureau website, March 31, 2003).

	Number of Houses (1000s)	
Bedrooms	Renter-Occupied	Owner-Occupied
0	547	23
1	5012	541
2	6100	3832
3	2644	8690
4 or more	557	3783

a. Define a random variable x = number of bedrooms in renter-occupied houses and develop a probability distribution for the random variable. (Let $x = 4$ represent 4 or more bedrooms.)

b. Compute the expected value and variance for the number of bedrooms in renter-occupied houses.
c. Define a random variable y = number of bedrooms in owner-occupied houses and develop a probability distribution for the random variable. (Let $y = 4$ represent 4 or more bedrooms.)
d. Compute the expected value and variance for the number of bedrooms in owner-occupied houses.
e. What observations can you make from a comparison of the number of bedrooms in renter-occupied versus owner-occupied homes?

19. The National Basketball Association (NBA) records a variety of statistics for each team. Two of these statistics are the percentage of field goals made by the team and the percentage of three-point shots made by the team. For a portion of the 2004 season, the shooting records of the 29 teams in the NBA showed that the probability of scoring two points by making a field goal was .44, and the probability of scoring three points by making a three-point shot was .34 (NBA website, January 3, 2004).
a. What is the expected value of a two-point shot for these teams?
b. What is the expected value of a three-point shot for these teams?
c. If the probability of making a two-point shot is greater than the probability of making a three-point shot, why do coaches allow some players to shoot the three-point shot if they have the opportunity? Use expected value to explain your answer.

20. The probability distribution for damage claims paid by the Newton Automobile Insurance Company on collision insurance follows.

Payment ($)	Probability
0	.85
500	.04
1000	.04
3000	.03
5000	.02
8000	.01
10000	.01

a. Use the expected collision payment to determine the collision insurance premium that would enable the company to break even.
b. The insurance company charges an annual rate of $520 for the collision coverage. What is the expected value of the collision policy for a policyholder? (*Hint:* It is the expected payments from the company minus the cost of coverage.) Why does the policyholder purchase a collision policy with this expected value?

21. The following probability distributions of job satisfaction scores for a sample of information systems (IS) senior executives and middle managers range from a low of 1 (very dissatisfied) to a high of 5 (very satisfied).

	Probability	
Job Satisfaction Score	IS Senior Executives	IS Middle Managers
1	.05	.04
2	.09	.10
3	.03	.12
4	.42	.46
5	.41	.28

a. What is the expected value of the job satisfaction score for senior executives?
b. What is the expected value of the job satisfaction score for middle managers?

c. Compute the variance of job satisfaction scores for executives and middle managers.
d. Compute the standard deviation of job satisfaction scores for both probability distributions.
e. Compare the overall job satisfaction of senior executives and middle managers.

22. The demand for a product of Carolina Industries varies greatly from month to month. The probability distribution in the following table, based on the past two years of data, shows the company's monthly demand.

Unit Demand	Probability
300	.20
400	.30
500	.35
600	.15

a. If the company bases monthly orders on the expected value of the monthly demand, what should Carolina's monthly order quantity be for this product?
b. Assume that each unit demanded generates $70 in revenue and that each unit ordered costs $50. How much will the company gain or lose in a month if it places an order based on your answer to part (a) and the actual demand for the item is 300 units?

23. The New York City Housing and Vacancy Survey showed a total of 59,324 rent-controlled housing units and 236,263 rent-stabilized units built in 1947 or later. For these rental units, the probability distributions for the number of persons living in the unit are given (U.S. Census Bureau website, January 12, 2004).

Number of Persons	Rent-Controlled	Rent-Stabilized
1	.61	.41
2	.27	.30
3	.07	.14
4	.04	.11
5	.01	.03
6	.00	.01

a. What is the expected value of the number of persons living in each type of unit?
b. What is the variance of the number of persons living in each type of unit?
c. Make some comparisons between the number of persons living in rent-controlled units and the number of persons living in rent-stabilized units.

24. The J. R. Ryland Computer Company is considering a plant expansion to enable the company to begin production of a new computer product. The company's president must determine whether to make the expansion a medium- or large-scale project. Demand for the new product is uncertain, which for planning purposes may be low demand, medium demand, or high demand. The probability estimates for demand are .20, .50, and .30, respectively. Letting x and y indicate the annual profit in thousands of dollars, the firm's planners developed the following profit forecasts for the medium- and large-scale expansion projects.

		Medium-Scale Expansion Profit		Large-Scale Expansion Profit	
		x	$f(x)$	y	$f(y)$
Demand	Low	50	.20	0	.20
	Medium	150	.50	100	.50
	High	200	.30	300	.30

a. Compute the expected value for the profit associated with the two expansion alternatives. Which decision is preferred for the objective of maximizing the expected profit?
b. Compute the variance for the profit associated with the two expansion alternatives. Which decision is preferred for the objective of minimizing the risk or uncertainty?

5.4 Binomial Probability Distribution

The binomial probability distribution is a discrete probability distribution that provides many applications. It is associated with a multiple-step experiment that we call the binomial experiment.

A Binomial Experiment

A **binomial experiment** exhibits the following four properties.

PROPERTIES OF A BINOMIAL EXPERIMENT

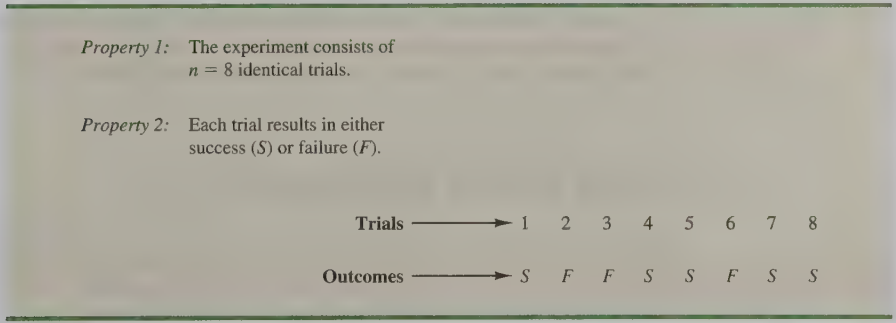
1. The experiment consists of a sequence of n identical trials.
2. Two outcomes are possible on each trial. We refer to one outcome as a *success* and the other outcome as a *failure*.
3. The probability of a success, denoted by p, does not change from trial to trial. Consequently, the probability of a failure, denoted by $1 - p$, does not change from trial to trial.
4. The trials are independent.

Jakob Bernoulli (1654–1705), the first of the Bernoulli family of Swiss mathematicians, published a treatise on probability that contained the theory of permutations and combinations, as well as the binomial theorem.

If properties 2, 3, and 4 are present, we say the trials are generated by a Bernoulli process. If, in addition, property 1 is present, we say we have a binomial experiment. Figure 5.3 depicts one possible sequence of successes and failures for a binomial experiment involving eight trials.

In a binomial experiment, our interest is in the *number of successes occurring in the n trials*. If we let x denote the number of successes occurring in the n trials, we see that x can assume the values of $0, 1, 2, 3, \ldots, n$. Because the number of values is finite, x is a *discrete* random variable. The probability distribution associated with this random variable is called the **binomial probability distribution**. For example, consider the experiment of tossing a

FIGURE 5.3 ONE POSSIBLE SEQUENCE OF SUCCESSES AND FAILURES FOR AN EIGHT-TRIAL BINOMIAL EXPERIMENT


Property 1: The experiment consists of n = 8 identical trials.
Property 2: Each trial results in either success (S) or failure (F).
Trials → 1 2 3 4 5 6 7 8
Outcomes → S F F S S F S S

coin five times and on each toss observing whether the coin lands with a head or a tail on its upward face. Suppose we want to count the number of heads appearing over the five tosses. Does this experiment show the properties of a binomial experiment? What is the random variable of interest? Note that:

1. The experiment consists of five identical trials; each trial involves the tossing of one coin.
2. Two outcomes are possible for each trial: a head or a tail. We can designate head a success and tail a failure.
3. The probability of a head and the probability of a tail are the same for each trial, with $p = .5$ and $1 - p = .5$.
4. The trials or tosses are independent because the outcome on any one trial is not affected by what happens on other trials or tosses.

Thus, the properties of a binomial experiment are satisfied. The random variable of interest is x = the number of heads appearing in the five trials. In this case, x can assume the values of 0, 1, 2, 3, 4, or 5.

As another example, consider an insurance salesperson who visits 10 randomly selected families. The outcome associated with each visit is classified as a success if the family purchases an insurance policy and a failure if the family does not. From past experience, the salesperson knows the probability that a randomly selected family will purchase an insurance policy is .10. Checking the properties of a binomial experiment, we observe that:

1. The experiment consists of 10 identical trials; each trial involves contacting one family.
2. Two outcomes are possible on each trial: the family purchases a policy (success) or the family does not purchase a policy (failure).
3. The probabilities of a purchase and a nonpurchase are assumed to be the same for each sales call, with $p = .10$ and $1 - p = .90$.
4. The trials are independent because the families are randomly selected.

Because the four assumptions are satisfied, this example is a binomial experiment. The random variable of interest is the number of sales obtained in contacting the 10 families. In this case, x can assume the values of 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10.

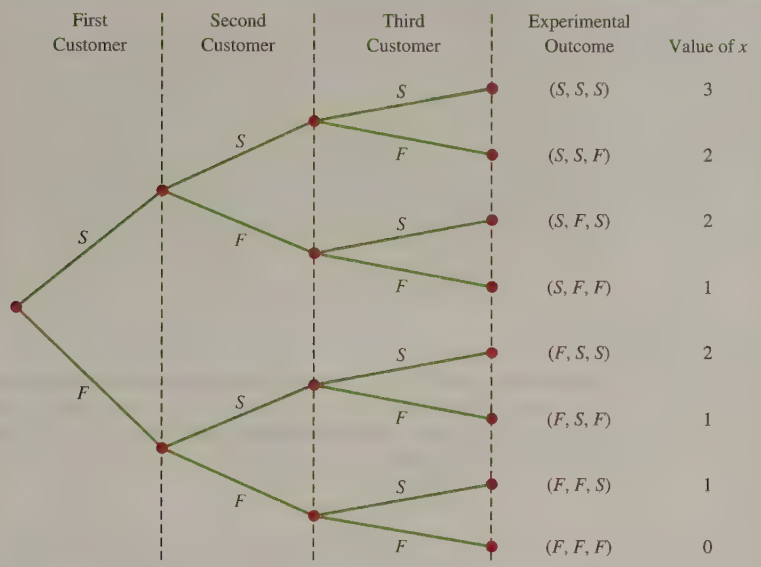
Property 3 of the binomial experiment is called the *stationarity assumption* and is sometimes confused with property 4, independence of trials. To see how they differ, consider again the case of the salesperson calling on families to sell insurance policies. If, as the day wore on, the salesperson got tired and lost enthusiasm, the probability of success (selling a policy) might drop to .05, for example, by the tenth call. In such a case, property 3 (stationarity) would not be satisfied, and we would not have a binomial experiment. Even if property 4 held—that is, the purchase decisions of each family were made independently—it would not be a binomial experiment if property 3 was not satisfied.

In applications involving binomial experiments, a special mathematical formula, called the *binomial probability function,* can be used to compute the probability of x successes in the n trials. Using probability concepts introduced in Chapter 4, we will show in the context of an illustrative problem how the formula can be developed.

Martin Clothing Store Problem

Let us consider the purchase decisions of the next three customers who enter the Martin Clothing Store. On the basis of past experience, the store manager estimates the probability that any one customer will make a purchase is .30. What is the probability that two of the next three customers will make a purchase?

FIGURE 5.4 TREE DIAGRAM FOR THE MARTIN CLOTHING STORE PROBLEM



S = Purchase
F = No purchase
x = Number of customers making a purchase

Using a tree diagram (Figure 5.4), we can see that the experiment of observing the three customers each making a purchase decision has eight possible outcomes. Using S to denote success (a purchase) and F to denote failure (no purchase), we are interested in experimental outcomes involving two successes in the three trials (purchase decisions). Next, let us verify that the experiment involving the sequence of three purchase decisions can be viewed as a binomial experiment. Checking the four requirements for a binomial experiment, we note that:

1. The experiment can be described as a sequence of three identical trials, one trial for each of the three customers who will enter the store.
2. Two outcomes—the customer makes a purchase (success) or the customer does not make a purchase (failure)—are possible for each trial.
3. The probability that the customer will make a purchase (.30) or will not make a purchase (.70) is assumed to be the same for all customers.
4. The purchase decision of each customer is independent of the decisions of the other customers.

Hence, the properties of a binomial experiment are present.

The number of experimental outcomes resulting in exactly x successes in n trials can be computed using the following formula.[1]

[1]This formula, introduced in Chapter 4, determines the number of combinations of n objects selected x at a time. For the binomial experiment, this combinatorial formula provides the number of experimental outcomes (sequences of n trials) resulting in x successes.

NUMBER OF EXPERIMENTAL OUTCOMES PROVIDING EXACTLY x SUCCESSES IN n TRIALS

$$\binom{n}{x} = \frac{n!}{x!(n-x)!} \tag{5.6}$$

where

$$n! = n(n-1)(n-2)\cdots(2)(1)$$

and, by definition,

$$0! = 1$$

Now let us return to the Martin Clothing Store experiment involving three customer purchase decisions. Equation (5.6) can be used to determine the number of experimental outcomes involving two purchases; that is, the number of ways of obtaining $x = 2$ successes in the $n = 3$ trials. From equation (5.6) we have

$$\binom{n}{x} = \binom{3}{2} = \frac{3!}{2!(3-2)!} = \frac{(3)(2)(1)}{(2)(1)(1)} = \frac{6}{2} = 3$$

Equation (5.6) shows that three of the experimental outcomes yield two successes. From Figure 5.4 we see these three outcomes are denoted by (S, S, F), (S, F, S), and (F, S, S).

Using equation (5.6) to determine how many experimental outcomes have three successes (purchases) in the three trials, we obtain

$$\binom{n}{x} = \binom{3}{3} = \frac{3!}{3!(3-3)!} = \frac{3!}{3!0!} = \frac{(3)(2)(1)}{3(2)(1)(1)} = \frac{6}{6} = 1$$

From Figure 5.4 we see that the one experimental outcome with three successes is identified by (S, S, S).

We know that equation (5.6) can be used to determine the number of experimental outcomes that result in x successes. If we are to determine the probability of x successes in n trials, however, we must also know the probability associated with each of these experimental outcomes. Because the trials of a binomial experiment are independent, we can simply multiply the probabilities associated with each trial outcome to find the probability of a particular sequence of successes and failures.

The probability of purchases by the first two customers and no purchase by the third customer, denoted (S, S, F), is given by

$$pp(1-p)$$

With a .30 probability of a purchase on any one trial, the probability of a purchase on the first two trials and no purchase on the third is given by

$$(.30)(.30)(.70) = (.30)^2(.70) = .063$$

Two other experimental outcomes also result in two successes and one failure. The probabilities for all three experimental outcomes involving two successes follow.

Trial Outcomes				
1st Customer	2nd Customer	3rd Customer	Experimental Outcome	Probability of Experimental Outcome
Purchase	Purchase	No purchase	(S, S, F)	$pp(1-p) = p^2(1-p) = (.30)^2(.70) = .063$
Purchase	No purchase	Purchase	(S, F, S)	$p(1-p)p = p^2(1-p) = (.30)^2(.70) = .063$
No purchase	Purchase	Purchase	(F, S, S)	$(1-p)pp = p^2(1-p) = (.30)^2(.70) = .063$

Observe that all three experimental outcomes with two successes have exactly the same probability. This observation holds in general. In any binomial experiment, all sequences of trial outcomes yielding x successes in n trials have the *same probability* of occurrence. The probability of each sequence of trials yielding x successes in n trials follows.

$$\begin{array}{l}\text{Probability of a particular}\\ \text{sequence of trial outcomes}\\ \text{with } x \text{ successes in } n \text{ trials}\end{array} = p^x(1-p)^{(n-x)} \qquad (5.7)$$

For the Martin Clothing Store, this formula shows that any experimental outcome with two successes has a probability of $p^2(1-p)^{(3-2)} = p^2(1-p)^1 = (.30)^2(.70)^1 = .063$.

Because equation (5.6) shows the number of outcomes in a binomial experiment with x successes and equation (5.7) gives the probability for each sequence involving x successes, we combine equations (5.6) and (5.7) to obtain the following **binomial probability function**.

BINOMIAL PROBABILITY FUNCTION

$$f(x) = \binom{n}{x} p^x(1-p)^{(n-x)} \qquad (5.8)$$

where

x = the number of successes
p = the probability of a success on one trial
n = the number of trials
$f(x)$ = the probability of x successes in n trials
$\binom{n}{x} = \dfrac{n!}{x!(n-x)!}$

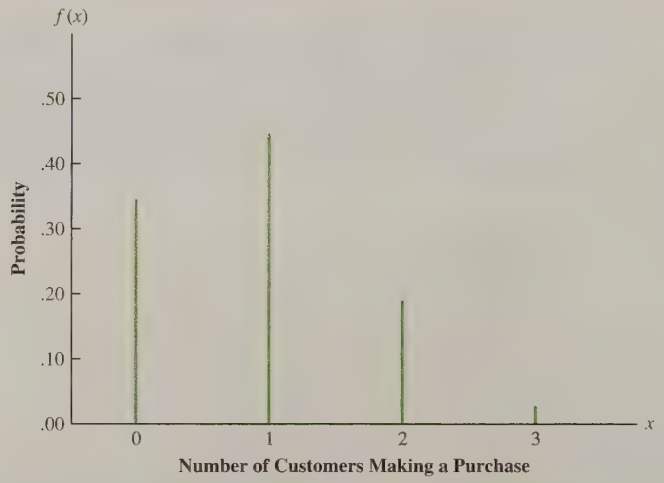
For the binomial probability distribution, x is a discrete random variable with the probability function $f(x)$ applicable for values of $x = 0, 1, 2, \ldots, n$.

In the Martin Clothing Store example, let us use equation (5.8) to compute the probability that no customer makes a purchase, exactly one customer makes a purchase, exactly two customers make a purchase, and all three customers make a purchase. The calculations are summarized in Table 5.6, which gives the probability distribution of the number of customers making a purchase. Figure 5.5 is a graph of this probability distribution.

TABLE 5.6 PROBABILITY DISTRIBUTION FOR THE NUMBER OF CUSTOMERS MAKING A PURCHASE

x	$f(x)$
0	$\frac{3!}{0!3!}(.30)^0(.70)^3 = .343$
1	$\frac{3!}{1!2!}(.30)^1(.70)^2 = .441$
2	$\frac{3!}{2!1!}(.30)^2(.70)^1 = .189$
3	$\frac{3!}{3!0!}(.30)^3(.70)^0 = .027$
	1.000

FIGURE 5.5 GRAPHICAL REPRESENTATION OF THE PROBABILITY DISTRIBUTION FOR THE NUMBER OF CUSTOMERS MAKING A PURCHASE



The binomial probability function can be applied to *any* binomial experiment. If we are satisfied that a situation demonstrates the properties of a binomial experiment and if we know the values of n and p, we can use equation (5.8) to compute the probability of x successes in the n trials.

If we consider variations of the Martin experiment, such as 10 customers rather than three entering the store, the binomial probability function given by equation (5.8) is still applicable. Suppose we have a binomial experiment with $n = 10$, $x = 4$, and $p = .30$. The probability of making exactly four sales to 10 customers entering the store is

$$f(4) = \frac{10!}{4!6!}(.30)^4(.70)^6 = .2001$$

Using Excel to Compute Binomial Probabilities

For many probability functions that can be specified as formulas, Excel provides functions for computing probabilities and cumulative probabilities. In this section, we show how Excel's BINOM.DIST function can be used to compute binomial probabilities and cumulative binomial probabilities. We begin by showing how to compute the binomial probabilities for the Martin Clothing Store example shown in Table 5.6. Refer to Figure 5.6 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet is in the foreground.

Enter Data: In order to compute a binomial probability we must know the number of trials (n), the probability of success (p), and the value of the random variable (x). For the Martin Clothing Store example, the number of trials is 3; this value has been entered into cell D1. The probability of success is .3; this value has been entered into cell D2. Because we want to compute the probability for $x = 0$, 1, 2, and 3, these values were entered into cells B5:B8.

Enter Functions and Formulas: The BINOM.DIST function has four inputs: the first is the value of x, the second is the value of n, the third is the value of p, and the fourth is FALSE or TRUE. We choose FALSE for the fourth input if a probability is desired and TRUE if a cumulative probability is desired. The formula =BINOM.DIST(B5,$D$1,$D$2,FALSE) has been entered into cell C5 to compute the probability of 0 successes in 3 trials. Note in the value worksheet that the probability computed for $f(0)$, .343, is the same as that shown in Table 5.6. The formula in cell C5 is copied to cells C6:C8 to compute the probabilities for $x = 1$, 2, and 3 successes, respectively.

FIGURE 5.6 EXCEL WORKSHEET FOR COMPUTING BINOMIAL PROBABILITIES OF NUMBER OF CUSTOMERS MAKING A PURCHASE

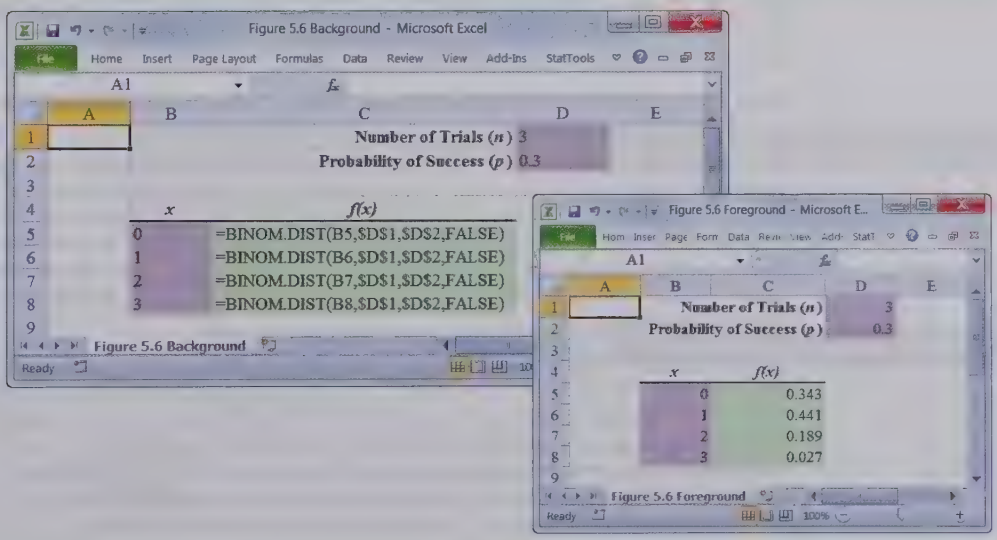
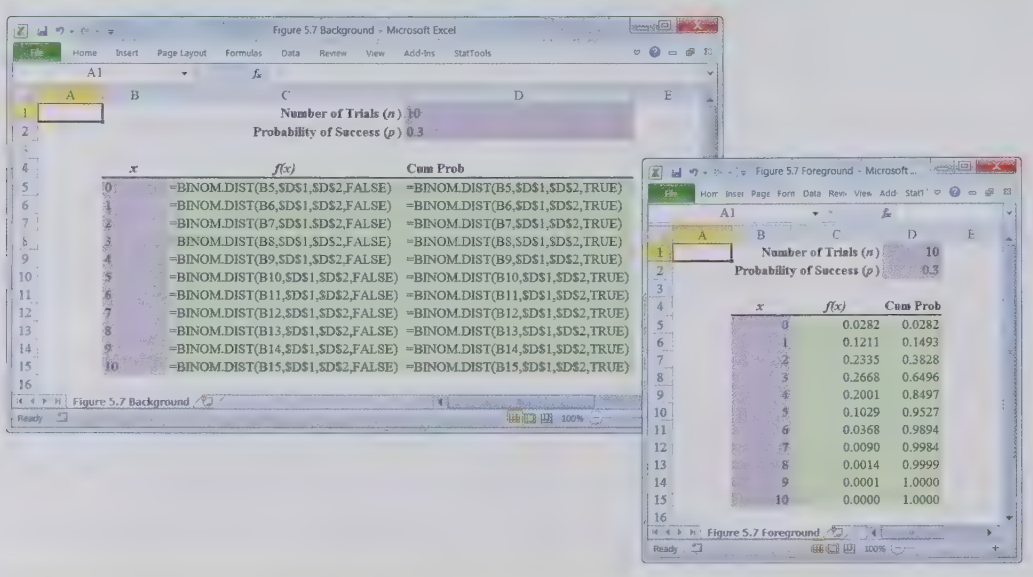


FIGURE 5.7 EXCEL WORKSHEET FOR COMPUTING PROBABILITIES AND CUMULATIVE PROBABILITIES FOR NUMBER OF PURCHASES WITH 10 CUSTOMERS



We can also compute cumulative probabilities using Excel's BINOM.DIST function. To illustrate, let us consider the case of 10 customers entering the Martin Clothing Store and compute the probabilities and cumulative probabilities for the number of customers making a purchase. Recall that the cumulative probability for $x = 1$ is the probability of 1 or fewer purchases, the cumulative probability for $x = 2$ is the probability of 2 or fewer purchases, and so on. So, the cumulative probability for $x = 10$ is 1. Refer to Figure 5.7 as we describe the tasks involved in computing these cumulative probabilities. The formula worksheet is in the background; the value worksheet is in the foreground.

Enter Data: We entered the number of trials (10) into cell D1, the probability of success (.3) into cell D2, and the values for the random variable into cells B5:B15.

Enter Functions and Formulas: The binomial probabilities for each value of the random variable are computed in column C and the cumulative probabilities are computed in column D. We entered the formula =BINOM.DIST(B5,$D$1,$D$2,FALSE) into cell C5 to compute the probability of 0 successes in 10 trials. Note that we used FALSE as the fourth input in the BINOM.DIST function. The probability (.0282) is shown in cell C5 of the value worksheet. The formula in cell C5 is simply copied to cells C6:C15 to compute the remaining probabilities.

To compute the cumulative probabilities we start by entering the formula =BINOM.DIST (B5,$D$1,$D$2,TRUE) into cell D5. Note that we used TRUE as the fourth input in the BINOM.DIST function. The formula in cell D5 is then copied to cells D6:D15 to compute the remaining cumulative probabilities. In cell D5 of the value worksheet we see that the cumulative probability for $x = 0$ is the same as the probability for $x = 0$. Each of the remaining cumulative probabilities is the sum of the previous cumulative probability and the individual probability in column C. For instance, the cumulative probability for $x = 4$ is given by .6496 + .2001 = .8497. Note also that the cumulative probability for $x = 10$ is 1.

The cumulative probability of $x = 9$ is also 1 because the probability of $x = 10$ is zero (to four decimal places of accuracy).

Expected Value and Variance for the Binomial Distribution

In Section 5.3 we provided formulas for computing the expected value and variance of a discrete random variable. In the special case where the random variable has a binomial distribution with a known number of trials n and a known probability of success p, the general formulas for the expected value and variance can be simplified. The results follow.

EXPECTED VALUE AND VARIANCE FOR THE BINOMIAL DISTRIBUTION

$$E(x) = \mu = np \tag{5.9}$$

$$\text{Var}(x) = \sigma^2 = np(1 - p) \tag{5.10}$$

For the Martin Clothing Store problem with three customers, we can use equation (5.9) to compute the expected number of customers who will make a purchase.

$$E(x) = np = 3(.30) = .9$$

Suppose that for the next month the Martin Clothing Store forecasts that 1000 customers will enter the store. What is the expected number of customers who will make a purchase? The answer is $\mu = np = (1000)(.3) = 300$. Thus, to increase the expected number of purchases, Martin's must induce more customers to enter the store and/or somehow increase the probability that any individual customer will make a purchase after entering.

For the Martin Clothing Store problem with three customers, we see that the variance and standard deviation for the number of customers who will make a purchase are

$$\sigma^2 = np(1 - p) = 3(.3)(.7) = .63$$

$$\sigma = \sqrt{.63} = .79$$

For the next 1000 customers entering the store, the variance and standard deviation for the number of customers who will make a purchase are

$$\sigma^2 = np(1 - p) = 1000(.3)(.7) = 210$$

$$\sigma = \sqrt{210} = 14.49$$

NOTES AND COMMENTS

Statisticians have developed tables that give probabilities and cumulative probabilities for a binomial random variable. These tables can be found in some statistics textbooks. With modern calculators and the capability of the BINOM.DIST function in Excel, such tables are unnecessary.

Exercises

Methods

SELF test

25. Consider a binomial experiment with two trials and $p = .4$.
    a. Draw a tree diagram for this experiment (see Figure 5.4).
    b. Compute the probability of one success, $f(1)$.
    c. Compute $f(0)$.
    d. Compute $f(2)$.

e. Compute the probability of at least one success.
f. Compute the expected value, variance, and standard deviation.

26. Consider a binomial experiment with $n = 10$ and $p = .10$.
a. Compute $f(0)$.
b. Compute $f(2)$.
c. Compute $P(x \leq 2)$.
d. Compute $P(x \geq 1)$.
e. Compute $E(x)$.
f. Compute $\text{Var}(x)$ and σ.

27. Consider a binomial experiment with $n = 20$ and $p = .70$.
a. Compute $f(12)$.
b. Compute $f(16)$.
c. Compute $P(x \geq 16)$.
d. Compute $P(x \leq 15)$.
e. Compute $E(x)$.
f. Compute $\text{Var}(x)$ and σ.

Applications

28. A Harris Interactive survey for InterContinental Hotels & Resorts asked respondents, "When traveling internationally, do you generally venture out on your own to experience culture, or stick with your tour group and itineraries?" The survey found that 23% of the respondents stick with their tour group (*USA Today,* January 21, 2004).
a. In a sample of six international travelers, what is the probability that two will stick with their tour group?
b. In a sample of six international travelers, what is the probability that at least two will stick with their tour group?
c. In a sample of 10 international travelers, what is the probability that none will stick with the tour group?

29. In San Francisco, 30% of workers take public transportation daily (*USA Today,* December 21, 2005).
a. In a sample of 10 workers, what is the probability that exactly three workers take public transportation daily?
b. In a sample of 10 workers, what is the probability that at least three workers take public transportation daily?

30. When a new machine is functioning properly, only 3% of the items produced are defective. Assume that we will randomly select two parts produced on the machine and that we are interested in the number of defective parts found.
a. Describe the conditions under which this situation would be a binomial experiment.
b. Draw a tree diagram similar to Figure 5.4 showing this problem as a two-trial experiment.
c. How many experimental outcomes result in exactly one defect being found?
d. Compute the probabilities associated with finding no defects, exactly one defect, and two defects.

31. A Randstad/Harris interactive survey reported that 25% of employees said their company is loyal to them (*USA Today,* November 11, 2009). Suppose 10 employees are selected randomly and will be interviewed about company loyalty.
a. Is the selection of 10 employees a binomial experiment? Explain.
b. What is the probability that none of the 10 employees will say their company is loyal to them?
c. What is the probability that 4 of the 10 employees will say their company is loyal to them?

d. What is the probability that at least 2 of the 10 employees will say their company is loyal to them?

32. Military radar and missile detection systems are designed to warn a country of an enemy attack. A reliability question is whether a detection system will be able to identify an attack and issue a warning. Assume that a particular detection system has a .90 probability of detecting a missile attack. Use the binomial probability distribution to answer the following questions.
    a. What is the probability that a single detection system will detect an attack?
    b. If two detection systems are installed in the same area and operate independently, what is the probability that at least one of the systems will detect the attack?
    c. If three systems are installed, what is the probability that at least one of the systems will detect the attack?
    d. Would you recommend that multiple detection systems be used? Explain.

33. Twelve of the top twenty finishers in the 2009 PGA Championship at Hazeltine National Golf Club in Chaska, Minnesota, used a Titleist brand golf ball (GolfBallTest website, November 12, 2009). Suppose these results are representative of the probability that a randomly selected PGA Tour player uses a Titleist brand golf ball. For a sample of 15 PGA Tour players, make the following calculations.
    a. Compute the probability that exactly 10 of the 15 PGA Tour players use a Titleist brand golf ball.
    b. Compute the probability that more than 10 of the 15 PGA Tour players use a Titleist brand golf ball.
    c. For a sample of 15 PGA Tour players, compute the expected number of players who use a Titleist brand golf ball.
    d. For a sample of 15 PGA Tour players, compute the variance and standard deviation of the number of players who use a Titleist brand golf ball.

34. The Census Bureau's Current Population Survey shows that 28% of individuals, ages 25 and older, have completed four years of college (*The New York Times Almanac,* 2006). For a sample of 15 individuals, ages 25 and older, answer the following questions:
    a. What is the probability that four will have completed four years of college?
    b. What is the probability that three or more will have completed four years of college?

35. A university found that 20% of its students withdraw without completing the introductory statistics course. Assume that 20 students registered for the course.
    a. Compute the probability that two or fewer will withdraw.
    b. Compute the probability that exactly four will withdraw.
    c. Compute the probability that more than three will withdraw.
    d. Compute the expected number of withdrawals.

36. According to a survey conducted by TD Ameritrade, one out of four investors has exchange-traded funds in his or her portfolio (*USA Today,* January 11, 2007). Consider a sample of 20 investors.
    a. Compute the probability that exactly four investors have exchange-traded funds in their portfolios.
    b. Compute the probability that at least two of the investors have exchange-traded funds in their portfolios.
    c. If you found that exactly 12 of the investors have exchange-traded funds in their portfolios, would you doubt the accuracy of the survey results?
    d. Compute the expected number of investors who have exchange-traded funds in their portfolios.

37. Twenty-three percent of automobiles are not covered by insurance (CNN, February 23, 2006). On a particular weekend, 35 automobiles are involved in traffic accidents.
    a. What is the expected number of these automobiles that are not covered by insurance?
    b. What are the variance and standard deviation?

5.5

Poisson Probability Distribution

The Poisson probability distribution is often used to model random arrivals in waiting line situations.

In this section we consider a discrete random variable that is often useful in estimating the number of occurrences over a specified interval of time or space. For example, the random variable of interest might be the number of arrivals at a car wash in one hour, the number of repairs needed in 10 miles of highway, or the number of leaks in 100 miles of pipeline. If the following two properties are satisfied, the number of occurrences is a random variable described by the **Poisson probability distribution**.

PROPERTIES OF A POISSON EXPERIMENT

1. The probability of an occurrence is the same for any two intervals of equal length.
2. The occurrence or nonoccurrence in any interval is independent of the occurrence or nonoccurrence in any other interval.

The **Poisson probability function** is defined by equation (5.11).

Siméon Poisson taught mathematics at the Ecole Polytechnique in Paris from 1802 to 1808. In 1837, he published a work entitled "Researches on the Probability of Criminal and Civil Verdicts," which includes a discussion of what later became known as the Poisson distribution.

POISSON PROBABILITY FUNCTION

$$f(x) = \frac{\mu^x e^{-\mu}}{x!} \tag{5.11}$$

where

$f(x)$ = the probability of x occurrences in an interval
μ = expected value or mean number of occurrences in an interval
e = 2.71828

For the Poisson probability distribution, x is a discrete random variable indicating the number of occurrences in the interval. Since there is no stated upper limit for the number of occurrences, the probability function $f(x)$ is applicable for values $x = 0, 1, 2, \ldots$ without limit. In practical applications, x will eventually become large enough so that $f(x)$ is approximately zero and the probability of any larger values of x becomes negligible.

An Example Involving Time Intervals

Bell Labs used the Poisson distribution to model the arrival of telephone calls.

Suppose that we are interested in the number of arrivals at the drive-up teller window of a bank during a 15-minute period on weekday mornings. If we can assume that the probability of a car arriving is the same for any two time periods of equal length and that the arrival or nonarrival of a car in any time period is independent of the arrival or nonarrival in any other time period, the Poisson probability function is applicable. Suppose these assumptions are satisfied and an analysis of historical data shows that the average number of cars arriving in a 15-minute period of time is 10; in this case, the following probability function applies.

$$f(x) = \frac{10^x e^{-10}}{x!}$$

The random variable here is x = number of cars arriving in any 15-minute period.

If management wanted to know the probability of exactly five arrivals in 15 minutes, we would set $x = 5$ and thus obtain

$$\text{Probability of exactly 5 arrivals in 15 minutes} = f(5) = \frac{10^5 e^{-10}}{5!} = .0378$$

The probability of five arrivals in 15 minutes was obtained by using a calculator to evaluate the probability function. Excel also provides a function called POISSON.DIST for computing Poisson probabilities and cumulative probabilities. This function is easier to use when numerous probabilities and cumulative probabilities are desired. At the end of this section, we show how to compute these probabilities with Excel.

A property of the Poisson distribution is that the mean and variance are equal.

In the preceding example, the mean of the Poisson distribution is $\mu = 10$ arrivals per 15-minute period. A property of the Poisson distribution is that the mean of the distribution and the variance of the distribution are *equal*. Thus, the variance for the number of arrivals during 15-minute periods is $\sigma^2 = 10$. The standard deviation is $\sigma = \sqrt{10} = 3.16$.

Our illustration involves a 15-minute period, but other time periods can be used. Suppose we want to compute the probability of one arrival in a 3-minute period. Because 10 is the expected number of arrivals in a 15-minute period, we see that $10/15 = 2/3$ is the expected number of arrivals in a 1-minute period and that $(2/3)(3 \text{ minutes}) = 2$ is the expected number of arrivals in a 3-minute period. Thus, the probability of x arrivals in a 3-minute time period with $\mu = 2$ is given by the following Poisson probability function.

$$f(x) = \frac{2^x e^{-2}}{x!}$$

The probability of one arrival in a 3-minute period is calculated as follows:

$$\text{Probability of exactly 1 arrival in 3 minutes} = f(1) = \frac{2^1 e^{-2}}{1!} = .2707$$

Earlier we computed the probability of five arrivals in a 15-minute period; it was .0378. Note that the probability of one arrival in a three-minute period (.2707) is not the same. When computing a Poisson probability for a different time interval, we must first convert the mean arrival rate to the time period of interest and then compute the probability.

An Example Involving Length or Distance Intervals

Let us illustrate an application not involving time intervals in which the Poisson distribution is useful. Suppose we are concerned with the occurrence of major defects in a highway one month after resurfacing. We will assume that the probability of a defect is the same for any two highway intervals of equal length and that the occurrence or nonoccurrence of a defect in any one interval is independent of the occurrence or nonoccurrence of a defect in any other interval. Hence, the Poisson distribution can be applied.

Suppose we learn that major defects one month after resurfacing occur at the average rate of two per mile. Let us find the probability of no major defects in a particular 3-mile section of the highway. Because we are interested in an interval with a length of 3 miles, $\mu = (2 \text{ defects/mile})(3 \text{ miles}) = 6$ represents the expected number of major defects over the 3-mile section of highway. Using equation (5.11), the probability of no major defects is $f(0) = 6^0 e^{-6}/0! = .0025$. Thus, it is unlikely that no major defects will occur in the 3-mile section. In fact, this example indicates a $1 - .0025 = .9975$ probability of at least one major defect in the 3-mile highway section.

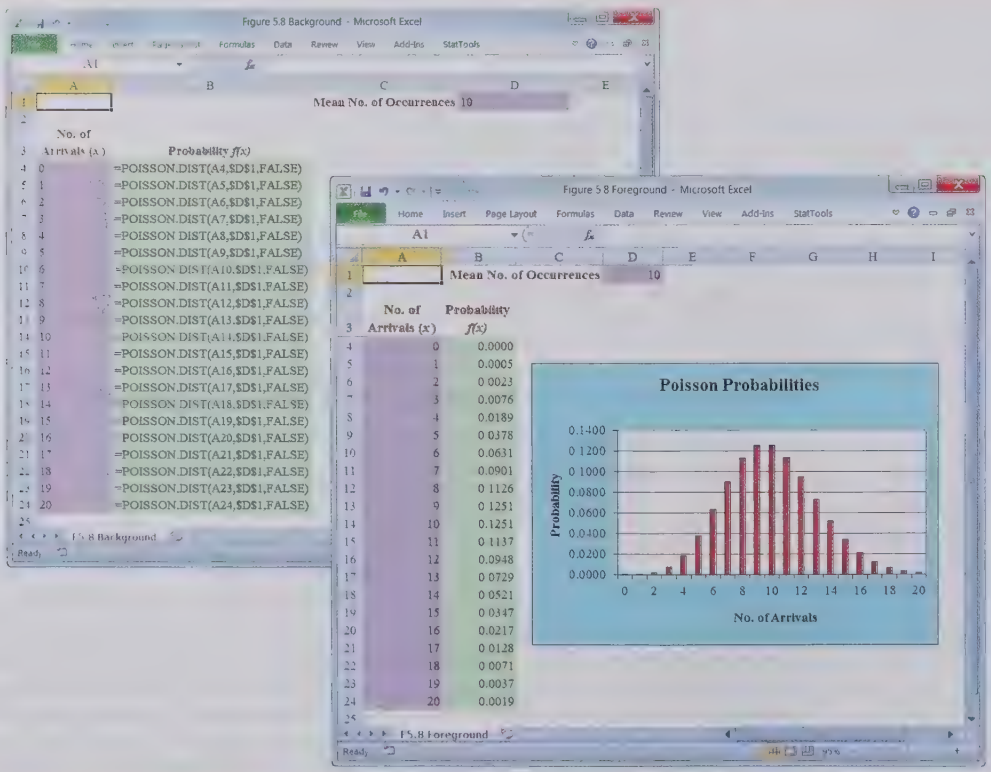
Using Excel to Compute Poisson Probabilities

The Excel function for computing Poisson probabilities and cumulative probabilities is called POISSON.DIST. It works in much the same way as the Excel function for computing binomial probabilities. Here we show how to use it to compute Poisson probabilities and cumulative probabilities. To illustrate, we use the example introduced earlier in this section; cars arrive at a bank drive-up teller window at the mean rate of 10 per 15-minute time interval. Refer to Figure 5.8 as we describe the tasks involved.

Enter Data: In order to compute a Poisson probability, we must know the mean number of occurrences (μ) per time period and the number of occurrences for which we want to compute the probability (x). For the drive-up teller window example, the occurrences of interest are the arrivals of cars. The mean arrival rate is 10, which has been entered into cell D1. Earlier in this section, we computed the probability of 5 arrivals. But suppose we now want to compute the probability of 0 up through 20 arrivals. To do so, we enter the values 0, 1, 2, . . . , 20 into cells A4:A24.

Enter Functions and Formulas: The POISSON.DIST function has three inputs: the first is the value of x, the second is the value of μ, and the third is FALSE or TRUE. We choose FALSE for the third input if a probability is desired and TRUE if a cumulative probability is

FIGURE 5.8 EXCEL WORKSHEET FOR COMPUTING POISSON PROBABILITIES

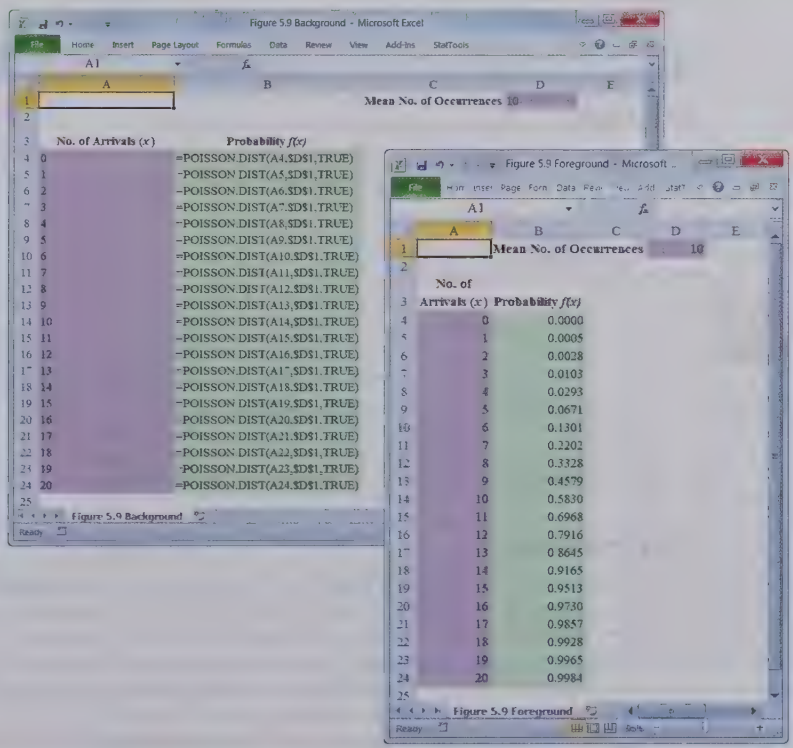


desired. The formula =POISSON.DIST(A4,$D$1,FALSE) has been entered into cell B4 to compute the probability of 0 arrivals in a 15-minute period. The value worksheet in the foreground shows that the probability of 0 arrivals is 0.0000. The formula in cell B4 is copied to cells B5:B24 to compute the probabilities for 1 through 20 arrivals. Note, in cell B9 of the value worksheet, that the probability of 5 arrivals is .0378. This result is the same as we calculated earlier in the text.

Notice how easy it was to compute all the probabilities for 0 through 20 arrivals using the POISSON.DIST function. These calculations would take quite a bit of work using a calculator. We have also used Excel's chart tools to develop a graph of the Poisson probability distribution of arrivals. See the value worksheet in Figure 5.8. This chart gives a nice graphical presentation of the probabilities for the various number of arrival possibilities in a 15-minute interval. We can quickly see that the most likely number of arrivals is 9 or 10 and that the probabilities fall off rather smoothly for smaller and larger values.

Let us now see how cumulative probabilities are generated using Excel's POISSON.DIST function. It is really a simple extension of what we have already done. We again use the example of arrivals at a drive-up teller window. Refer to Figure 5.9 as we describe the tasks involved.

FIGURE 5.9 EXCEL WORKSHEET FOR COMPUTING CUMULATIVE POISSON PROBABILITIES



Enter Data: To compute cumulative Poisson probabilities we must provide the mean number of occurrences (μ) per time period and the values of x that we are interested in. The mean arrival rate (10) has been entered into cell D1. Suppose we want to compute the cumulative probabilities for a number of arrivals ranging from zero up through 20. To do so, we enter the values 0, 1, 2, . . . , 20 into cells A4:A24.

Enter Functions and Formulas: Refer to the formula worksheet in the background of Figure 5.8. The formulas we enter into cells B4:B24 of Figure 5.9 are the same as in Figure 5.8 with one exception. Instead of FALSE for the third input, we enter the word TRUE to obtain cumulative probabilities. After entering these formulas into cells B4:B24 of the worksheet in Figure 5.9, the cumulative probabilities shown were obtained.

Note, in Figure 5.9, that the probability of 5 or fewer arrivals is .0671 and that the probability of 4 or fewer arrivals is .0293. Thus, the probability of exactly 5 arrivals is the difference in these two numbers: $f(5) = .0671 - .0293 = .0378$. We computed this probability earlier in this section and in Figure 5.8. Using these cumulative probabilities, it is easy to compute the probability that a random variable lies within a certain interval. For instance, suppose we wanted to know the probability of more than 5 and fewer than 16 arrivals. We would just find the cumulative probability of 15 arrivals and subtract from that the cumulative probability for 5 arrivals. Referring to Figure 5.9 to obtain the appropriate probabilities, we obtain $.9513 - .0671 = .8842$. With such a high probability, we could conclude that 6 to 15 cars will arrive in most 15-minute intervals. Using the cumulative probability for 20 arrivals, we can also conclude that the probability of more than 20 arrivals in a 15-minute period is $1 - .9984 = .0016$; thus, there is almost no chance of more than 20 cars arriving.

Exercises

Methods

38. Consider a Poisson distribution with $\mu = 3$.
    a. Write the appropriate Poisson probability function.
    b. Compute $f(2)$.
    c. Compute $f(1)$.
    d. Compute $P(x \geq 2)$.

SELF test

39. Consider a Poisson distribution with a mean of two occurrences per time period.
    a. Write the appropriate Poisson probability function.
    b. What is the expected number of occurrences in three time periods?
    c. Write the appropriate Poisson probability function to determine the probability of x occurrences in three time periods.
    d. Compute the probability of two occurrences in one time period.
    e. Compute the probability of six occurrences in three time periods.
    f. Compute the probability of five occurrences in two time periods.

Applications

40. Phone calls arrive at the rate of 48 per hour at the reservation desk for Regional Airways.
    a. Compute the probability of receiving three calls in a 5-minute interval of time.
    b. Compute the probability of receiving exactly 10 calls in 15 minutes.
    c. Suppose no calls are currently on hold. If the agent takes 5 minutes to complete the current call, how many callers do you expect to be waiting by that time? What is the probability that none will be waiting?
    d. If no calls are currently being processed, what is the probability that the agent can take 3 minutes for personal time without being interrupted by a call?

41. During the period of time that a local university takes phone-in registrations, calls come in at the rate of one every two minutes.
    a. What is the expected number of calls in one hour?
    b. What is the probability of three calls in five minutes?
    c. What is the probability of no calls in a five-minute period?

SELF test

42. More than 50 million guests stay at bed and breakfasts (B&Bs) each year. The website for the Bed and Breakfast Inns of North America, which averages seven visitors per minute, enables many B&Bs to attract guests (*Time,* September 2001).
    a. Compute the probability of no website visitors in a one-minute period.
    b. Compute the probability of two or more website visitors in a one-minute period.
    c. Compute the probability of one or more website visitors in a 30-second period.
    d. Compute the probability of five or more website visitors in a one-minute period.

43. Airline passengers arrive randomly and independently at the passenger-screening facility at a major international airport. The mean arrival rate is 10 passengers per minute.
    a. Compute the probability of no arrivals in a one-minute period.
    b. Compute the probability that three or fewer passengers arrive in a one-minute period.
    c. Compute the probability of no arrivals in a 15-second period.
    d. Compute the probability of at least one arrival in a 15-second period.

44. An average of 15 aircraft accidents occur each year (*The World Almanac and Book of Facts,* 2004).
    a. Compute the mean number of aircraft accidents per month.
    b. Compute the probability of no accidents during a month.
    c. Compute the probability of exactly one accident during a month.
    d. Compute the probability of more than one accident during a month.

45. The National Safety Council (NSC) estimates that off-the-job accidents cost U.S. businesses almost $200 billion annually in lost productivity (National Safety Council, March 2006). Based on NSC estimates, companies with 50 employees are expected to average three employee off-the-job accidents per year. Answer the following questions for companies with 50 employees.
    a. What is the probability of no off-the-job accidents during a one-year period?
    b. What is the probability of at least two off-the-job accidents during a one-year period?
    c. What is the expected number of off-the-job accidents during six months?
    d. What is the probability of no off-the-job accidents during the next six months?

5.6 Hypergeometric Probability Distribution

The **hypergeometric probability distribution** is closely related to the binomial distribution. The two probability distributions differ in two key ways. With the hypergeometric distribution, the trials are not independent, and the probability of success changes from trial to trial.

In the usual notation for the hypergeometric distribution, r denotes the number of elements in the population of size N labeled success, and $N - r$ denotes the number of elements in the population labeled failure. The **hypergeometric probability function** is used to compute the probability that in a random selection of n elements, selected without replacement, we obtain x elements labeled success and $n - x$ elements labeled failure. For this outcome to occur, we must obtain x successes from the r successes in the population and $n - x$ failures from the $N - r$ failures. The following hypergeometric probability function provides $f(x)$, the probability of obtaining x successes in n trials.

HYPERGEOMETRIC PROBABILITY FUNCTION

$$f(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}} \tag{5.12}$$

where

x = the number of successes
n = the number of trials
$f(x)$ = the probability of x successes in n trials
N = the number of elements in the population
r = the number of elements in the population labeled success

Note that $\binom{N}{n}$ represents the number of ways n elements can be selected from a population of size N; $\binom{r}{x}$ represents the number of ways that x successes can be selected from a total of r successes in the population; and $\binom{N-r}{n-x}$ represents the number of ways that $n - x$ failures can be selected from a total of $N - r$ failures in the population.

For the hypergeometric probability distribution, x is a discrete random variable and the probability function $f(x)$ given by equation (5.12) is usually applicable for values of $x = 0, 1, 2, \ldots, n$. However, only values of x where the number of observed successes is *less than or equal to* the number of successes in the population ($x \leq r$) and where the number of observed failures is *less than or equal to* the number of failures in the population ($n - x \leq N - r$) are valid. If these two conditions do not hold for one or more values of x, the corresponding $f(x) = 0$, indicating that the probability of this value of x is zero.

To illustrate the computations involved in using equation (5.12), let us consider the following quality control application. Electric fuses produced by Ontario Electric are packaged in boxes of 12 units each. Suppose an inspector randomly selects 3 of the 12 fuses in a box for testing. If the box contains exactly five defective fuses, what is the probability that the inspector will find exactly one of the three fuses defective? In this application, $n = 3$ and $N = 12$. With $r = 5$ defective fuses in the box, the probability of finding $x = 1$ defective fuse is

$$f(1) = \frac{\binom{5}{1}\binom{7}{2}}{\binom{12}{3}} = \frac{\left(\frac{5!}{1!4!}\right)\left(\frac{7!}{2!5!}\right)}{\left(\frac{12!}{3!9!}\right)} = \frac{(5)(21)}{220} = .4773$$

Now suppose that we wanted to know the probability of finding *at least* one defective fuse. The easiest way to answer this question is first to compute the probability that the inspector does not find any defective fuses. The probability of $x = 0$ is

$$f(0) = \frac{\binom{5}{0}\binom{7}{3}}{\binom{12}{3}} = \frac{\left(\frac{5!}{0!5!}\right)\left(\frac{7!}{3!4!}\right)}{\left(\frac{12!}{3!9!}\right)} = \frac{(1)(35)}{220} = .1591$$

With a probability of zero defective fuses $f(0) = .1591$, we conclude that the probability of finding at least one defective fuse must be $1 - .1591 = .8409$. Thus, there is a reasonably high probability that the inspector will find at least one defective fuse.

The mean and variance of a hypergeometric distribution are as follows.

$$E(x) = \mu = n\left(\frac{r}{N}\right) \tag{5.13}$$

$$\text{Var}(x) = \sigma^2 = n\left(\frac{r}{N}\right)\left(1 - \frac{r}{N}\right)\left(\frac{N-n}{N-1}\right) \tag{5.14}$$

In the preceding example $n = 3$, $r = 5$, and $N = 12$. Thus, the mean and variance for the number of defective fuses are

$$\mu = n\left(\frac{r}{N}\right) = 3\left(\frac{5}{12}\right) = 1.25$$

$$\sigma^2 = n\left(\frac{r}{N}\right)\left(1 - \frac{r}{N}\right)\left(\frac{N-n}{N-1}\right) = 3\left(\frac{5}{12}\right)\left(1 - \frac{5}{12}\right)\left(\frac{12-3}{12-1}\right) = .60$$

The standard deviation is $\sigma = \sqrt{.60} = .77$.

Using Excel to Compute Hypergeometric Probabilities

The Excel function for computing hypergeometric probabilities is HYPGEOM.DIST. It has five inputs: the first is the value of x, the second is the value of n, the third is the value of r, the fourth is the value of N, and the fifth is FALSE or TRUE. We choose FALSE if a probability is desired and TRUE if a cumulative probability is desired. This function's usage is similar to that of BINOM.DIST for the binomial distribution and POISSON.DIST for the Poisson distribution, so we dispense with showing a worksheet figure and just explain how to use the function.

Let us reconsider the example of selecting 3 fuses for inspection from a fuse box containing 12 fuses, 5 of which are defective. We want to find the probability that 1 of the 3 fuses selected is defective. In this case, the five inputs are $x = 1$, $n = 3$, $r = 5$, $N = 12$, and FALSE. So, the appropriate formula to place in a cell of an Excel worksheet is =HYPGEOM.DIST(1,3,5,12,FALSE). Placing this formula in a cell of an Excel worksheet provides a hypergeometric probability of .4773.

If we want to know the probability that none of the 3 fuses selected is defective, the five function inputs are $x = 0$, $n = 3$, $r = 5$, $N = 12$, and FALSE. So, using the HYPGEOM.DIST function to compute the probability of randomly selecting 3 fuses without any being defective, we would enter the following formula into an Excel worksheet: =HYPGEOM.DIST(0,3,5,12,FALSE). The probability is .1591.

Cumulative probabilities can be obtained in a similar fashion by using TRUE for the fifth input. For instance, to compute the probability of finding at most 1 defective fuse, the appropriate formula is =HYPGEOM.DIST(1,3,5,12,TRUE). Placing this formula in a cell of an Excel worksheet provides a hypergeometric cumulative probability of .6364.

NOTES AND COMMENTS

Consider a hypergeometric distribution with n trials. Let $p = (r/N)$ denote the probability of a success on the first trial. If the population size is large, the term $(N - n)/(N - 1)$ in equation (5.14) approaches 1. As a result, the expected value and variance can be written $E(x) = np$ and $\text{Var}(x) = np(1 - p)$. Note that these expressions are the same as the expressions used to compute the expected value and variance of a binomial distribution, as in equations (5.9) and (5.10). When the population size is large, a hypergeometric distribution can be approximated by a binomial distribution with n trials and a probability of success $p = (r/N)$

Exercises

Methods

SELF test

46. Suppose $N = 10$ and $r = 3$. Compute the hypergeometric probabilities for the following values of n and x.
    a. $n = 4, x = 1$
    b. $n = 2, x = 2$
    c. $n = 2, x = 0$
    d. $n = 4, x = 2$
    e. $n = 4, x = 4$

47. Suppose $N = 15$ and $r = 4$. What is the probability of $x = 3$ for $n = 10$?

Applications

48. In a survey conducted by the Gallup Organization, respondents were asked, "What is your favorite sport to watch?" Football and basketball ranked number 1 and 2 in terms of preference (Gallup website, January 3, 2004). Assume that in a group of 10 individuals, seven prefer football and three prefer basketball. A random sample of three of these individuals is selected.
    a. What is the probability that exactly two prefer football?
    b. What is the probability that the majority (either two or three) prefer football?

49. Blackjack, or twenty-one as it is frequently called, is a popular gambling game played in Las Vegas casinos. A player is dealt two cards. Face cards (jacks, queens, and kings) and tens have a point value of 10. Aces have a point value of 1 or 11. A 52-card deck contains 16 cards with a point value of 10 (jacks, queens, kings, and tens) and four aces.
    a. What is the probability that both cards dealt are aces or 10-point cards?
    b. What is the probability that both of the cards are aces?
    c. What is the probability that both of the cards have a point value of 10?
    d. A blackjack is a 10-point card and an ace for a value of 21. Use your answers to parts (a), (b), and (c) to determine the probability that a player is dealt blackjack. (*Hint:* Part (d) is not a hypergeometric problem. Develop your own logical relationship as to how the hypergeometric probabilities from parts (a), (b), and (c) can be combined to answer this question.)

SELF test

50. Axline Computers manufactures personal computers at two plants, one in Texas and the other in Hawaii. The Texas plant has 40 employees; the Hawaii plant has 20. A random sample of 10 employees is to be asked to fill out a benefits questionnaire.
    a. What is the probability that none of the employees in the sample works at the plant in Hawaii?
    b. What is the probability that one of the employees in the sample works at the plant in Hawaii?
    c. What is the probability that two or more of the employees in the sample work at the plant in Hawaii?
    d. What is the probability that nine of the employees in the sample work at the plant in Texas?

51. The Zagat Restaurant Survey provides food, decor, and service ratings for some of the top restaurants across the United States. For 15 restaurants located in Boston, the average price of a dinner, including one drink and tip, was $48.60. You are leaving for a business trip to Boston and will eat dinner at three of these restaurants. Your company will reimburse you for a maximum of $50 per dinner. Business associates familiar with these restaurants have told you that the meal cost at one-third of these restaurants will exceed $50. Suppose that you randomly select three of these restaurants for dinner.

a. What is the probability that none of the meals will exceed the cost covered by your company?
b. What is the probability that one of the meals will exceed the cost covered by your company?
c. What is the probability that two of the meals will exceed the cost covered by your company?
d. What is the probability that all three of the meals will exceed the cost covered by your company?

52. The Troubled Asset Relief Program (TARP), passed by the U.S. Congress in October 2008, provided $700 billion in assistance for the struggling U.S. economy. Over $200 billion was given to troubled financial institutions with the hope that there would be an increase in lending to help jump-start the economy. But three months later, a Federal Reserve survey found that two-thirds of the banks that had received TARP funds had tightened terms for business loans (*The Wall Street Journal,* February 3, 2009). Of the 10 banks that were the biggest recipients of TARP funds, only 3 had actually increased lending during this period.

Increased Lending	Decreased Lending
BB&T	Bank of America
Sun Trust Banks	Capital One
U.S. Bancorp	Citigroup
	Fifth Third Bancorp
	JPMorgan Chase
	Regions Financial
	Wells Fargo

For the purposes of this exercise, assume that you will randomly select 3 of these 10 banks for a study that will continue to monitor bank lending practices. Let x be a random variable indicating the number of banks in the study that had increased lending.

a. What is $f(0)$? What is your interpretation of this value?
b. What is $f(3)$? What is your interpretation of this value?
c. Compute $f(1)$ and $f(2)$. Show the probability distribution for the number of banks in the study that had increased lending. What value of x has the highest probability?
d. What is the probability that the study will have at least one bank that had increased lending?
e. Compute the expected value, variance, and standard deviation for the random variable.

Summary

A random variable provides a numerical description of the outcome of an experiment. The probability distribution for a random variable describes how the probabilities are distributed over the values the random variable can assume. For any discrete random variable x, the probability distribution is defined by a probability function, denoted by $f(x)$, which provides the probability associated with each value of the random variable. Once the probability function is defined, we can compute the expected value, variance, and standard deviation for the random variable.

The binomial distribution can be used to determine the probability of x successes in n trials whenever the experiment has the following properties:

1. The experiment consists of a sequence of n identical trials.
2. Two outcomes are possible on each trial, one called success and the other failure.

3. The probability of a success p does not change from trial to trial. Consequently, the probability of failure, $1 - p$, does not change from trial to trial.
4. The trials are independent.

When the four properties hold, the binomial probability function can be used to determine the probability of obtaining x successes in n trials. Formulas were also presented for the mean and variance of the binomial distribution.

The Poisson distribution is used when it is desirable to determine the probability of obtaining x occurrences over an interval of time or space. The following assumptions are necessary for the Poisson distribution to be applicable.

1. The probability of an occurrence of the event is the same for any two intervals of equal length.
2. The occurrence or nonoccurrence of the event in any interval is independent of the occurrence or nonoccurrence of the event in any other interval.

A third discrete probability distribution, the hypergeometric, was introduced in Section 5.6. Like the binomial, it is used to compute the probability of x successes in n trials. But, in contrast to the binomial, the probability of success changes from trial to trial.

Glossary

Random variable A numerical description of the outcome of an experiment.
Discrete random variable A random variable that may assume either a finite number of values or an infinite sequence of values.
Continuous random variable A random variable that may assume any numerical value in an interval or collection of intervals.
Probability distribution A description of how the probabilities are distributed over the values of the random variable.
Probability function A function, denoted by $f(x)$, that provides the probability that x assumes a particular value for a discrete random variable.
Discrete uniform probability distribution A probability distribution for which each possible value of the random variable has the same probability.
Expected value A measure of the central location of a random variable.
Variance A measure of the variability, or dispersion, of a random variable.
Standard deviation The positive square root of the variance.
Binomial experiment An experiment having the four properties stated at the beginning of Section 5.4.
Binomial probability distribution A probability distribution showing the probability of x successes in n trials of a binomial experiment.
Binomial probability function The function used to compute binomial probabilities.
Poisson probability distribution A probability distribution showing the probability of x occurrences of an event over a specified interval of time or space.
Poisson probability function The function used to compute Poisson probabilities.
Hypergeometric probability distribution A probability distribution showing the probability of x successes in n trials from a population with r successes and $N - r$ failures.
Hypergeometric probability function The function used to compute hypergeometric probabilities.

Key Formulas

Discrete Uniform Probability Function

$$f(x) = 1/n \tag{5.3}$$

Expected Value of a Discrete Random Variable

$$E(x) = \mu = \Sigma x f(x) \tag{5.4}$$

Variance of a Discrete Random Variable

$$\text{Var}(x) = \sigma^2 = \Sigma(x - \mu)^2 f(x) \tag{5.5}$$

Number of Experimental Outcomes Providing Exactly x Successes in n Trials

$$\binom{n}{x} = \frac{n!}{x!(n - x)!} \tag{5.6}$$

Binomial Probability Function

$$f(x) = \binom{n}{x} p^x (1 - p)^{(n-x)} \tag{5.8}$$

Expected Value for the Binomial Distribution

$$E(x) = \mu = np \tag{5.9}$$

Variance for the Binomial Distribution

$$\text{Var}(x) = \sigma^2 = np(1 - p) \tag{5.10}$$

Poisson Probability Function

$$f(x) = \frac{\mu^x e^{-\mu}}{x!} \tag{5.11}$$

Hypergeometric Probability Function

$$f(x) = \frac{\binom{r}{x}\binom{N - r}{n - x}}{\binom{N}{n}} \tag{5.12}$$

Expected Value for the Hypergeometric Distribution

$$E(x) = \mu = n\left(\frac{r}{N}\right) \tag{5.13}$$

Variance for the Hypergeometric Distribution

$$\text{Var}(x) = \sigma^2 = n\left(\frac{r}{N}\right)\left(1 - \frac{r}{N}\right)\left(\frac{N - n}{N - 1}\right) \tag{5.14}$$

Supplementary Exercises

53. The *Barron's* Big Money Poll asked 131 investment managers across the United States about their short-term investment outlook (*Barron's,* October 28, 2002). Their responses showed that 4% were very bullish, 39% were bullish, 29% were neutral, 21% were bearish, and 7% were very bearish. Let x be the random variable reflecting the level of optimism about the market. Set $x = 5$ for very bullish down through $x = 1$ for very bearish.
    a. Develop a probability distribution for the level of optimism of investment managers.
    b. Compute the expected value for the level of optimism.
    c. Compute the variance and standard deviation for the level of optimism.
    d. Comment on what your results imply about the level of optimism and its variability.

54. The American Association of Individual Investors publishes an annual guide to the top mutual funds (*The Individual Investor's Guide to the Top Mutual Funds,* 22e, American Association of Individual Investors, 2003). The total risk ratings for 29 categories of mutual funds are as follows.

Total Risk	Number of Fund Categories
Low	7
Below Average	6
Average	3
Above Average	6
High	7

    a. Let $x = 1$ for low risk up through $x = 5$ for high risk, and develop a probability distribution for level of risk.
    b. What are the expected value and variance for total risk?
    c. It turns out that 11 of the fund categories were bond funds. For the bond funds, seven categories were rated low and four were rated below average. Compare the total risk of the bond funds with the 18 categories of stock funds.

55. The budgeting process for a midwestern college resulted in expense forecasts for the coming year (in $ millions) of $9, $10, $11, $12, and $13. Because the actual expenses are unknown, the following respective probabilities are assigned: .3, .2, .25, .05, and .2.
    a. Show the probability distribution for the expense forecast.
    b. What is the expected value of the expense forecast for the coming year?
    c. What is the variance of the expense forecast for the coming year?
    d. If income projections for the year are estimated at $12 million, comment on the financial position of the college.

56. A survey showed that the average commuter spends about 26 minutes on a one-way door-to-door trip from home to work. In addition, 5% of commuters reported a one-way commute of more than one hour (Bureau of Transportation Statistics website, January 12, 2004).
    a. If 20 commuters are surveyed on a particular day, what is the probability that three will report a one-way commute of more than one hour?
    b. If 20 commuters are surveyed on a particular day, what is the probability that none will report a one-way commute of more than one hour?
    c. If a company has 2000 employees, what is the expected number of employees that have a one-way commute of more than one hour?
    d. If a company has 2000 employees, what are the variance and standard deviation of the number of employees that have a one-way commute of more than one hour?

57. A political action group is planning to interview home owners to assess the impact caused by a recent slump in housing prices. According to a *Wall Street Journal*/Harris Interactive Personal Finance poll, 26% of individuals aged 18–34, 50% of individuals aged 35–44, and 88% of individuals aged 55 and over are home owners (All Business website, January 23, 2008).
    a. How many people from the 18–34 age group must be sampled to find an expected number of at least 20 home owners?
    b. How many people from the 35–44 age group must be sampled to find an expected number of at least 20 home owners?
    c. How many people from the 55 and over age group must be sampled to find an expected number of at least 20 home owners?
    d. If the number of 18–34-year-olds sampled is equal to the value identified in part (a), what is the standard deviation of the number who will be home owners?
    e. If the number of 35–44-year-olds sampled is equal to the value identified in part (b), what is the standard deviation of the number who will be home owners?

58. Many companies use a quality control technique called acceptance sampling to monitor incoming shipments of parts, raw materials, and so on. In the electronics industry, component parts are commonly shipped from suppliers in large lots. Inspection of a sample of n components can be viewed as the n trials of a binomial experiment. The outcome for each component tested (trial) will be that the component is classified as good or defective. Reynolds Electronics accepts a lot from a particular supplier if the defective components in the lot do not exceed 1%. Suppose a random sample of five items from a recent shipment is tested.
    a. Assume that 1% of the shipment is defective. Compute the probability that no items in the sample are defective.
    b. Assume that 1% of the shipment is defective. Compute the probability that exactly one item in the sample is defective.
    c. What is the probability of observing one or more defective items in the sample if 1% of the shipment is defective?
    d. Would you feel comfortable accepting the shipment if one item was found to be defective? Why or why not?

59. The unemployment rate in the state of Arizona is 4.1% (CNN Money website, May 2, 2007). Assume that 100 employable people in Arizona are selected randomly.
    a. What is the expected number of people who are unemployed?
    b. What are the variance and standard deviation of the number of people who are unemployed?

60. A poll conducted by Zogby International showed that of those Americans who said music plays a "very important" role in their lives, 30% said their local radio stations "always" play the kind of music they like (Zogby website, January 12, 2004). Suppose a sample is taken of 800 people who say music plays an important role in their lives.
    a. How many would you expect to say that their local radio stations always play the kind of music they like?
    b. What is the standard deviation of the number of respondents who think their local radio stations always play the kind of music they like?
    c. What is the standard deviation of the number of respondents who do not think their local radio stations always play the kind of music they like?

61. Cars arrive at a car wash randomly and independently; the probability of an arrival is the same for any two time intervals of equal length. The mean arrival rate is 15 cars per hour. What is the probability that 20 or more cars will arrive during any given hour of operation?

62. A new automated production process averages 1.5 breakdowns per day. Because of the cost associated with a breakdown, management is concerned about the possibility of having

three or more breakdowns during a day. Assume that breakdowns occur randomly, that the probability of a breakdown is the same for any two time intervals of equal length, and that breakdowns in one period are independent of breakdowns in other periods. What is the probability of having three or more breakdowns during a day?

63. A regional director responsible for business development in the state of Pennsylvania is concerned about the number of small business failures. If the mean number of small business failures per month is 10, what is the probability that exactly four small businesses will fail during a given month? Assume that the probability of a failure is the same for any two months and that the occurrence or nonoccurrence of a failure in any month is independent of failures in any other month.

64. Customer arrivals at a bank are random and independent; the probability of an arrival in any one-minute period is the same as the probability of an arrival in any other one-minute period. Answer the following questions, assuming a mean arrival rate of three customers per minute.
    a. What is the probability of exactly three arrivals in a one-minute period?
    b. What is the probability of at least three arrivals in a one-minute period?

65. A deck of playing cards contains 52 cards, four of which are aces. What is the probability that the deal of a five-card hand provides:
    a. A pair of aces?
    b. Exactly one ace?
    c. No aces?
    d. At least one ace?

66. *U.S. News & World Report*'s ranking of America's best graduate schools of business showed Harvard University and Stanford University in a tie for first place. In addition, 7 of the top 10 graduate schools of business showed students with an average undergraduate grade point average (GPA) of 3.50 or higher (*America's Best Graduate Schools, 2009 Edition, U.S. News & World Report*). Suppose that we randomly select 2 of the top 10 graduate schools of business.
    a. What is the probability that exactly one school has students with an average undergraduate GPA of 3.50 or higher?
    b. What is the probability that both schools have students with an average undergraduate GPA of 3.50 or higher?
    c. What is the probability that neither school has students with an average undergraduate GPA of 3.50 or higher?

CHAPTER 6

Continuous Probability Distributions

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STATISTICS IN PRACTICE

PROCTER & GAMBLE*

CINCINNATI, OHIO

Procter & Gamble (P&G) produces and markets such products as detergents, disposable diapers, over-the-counter pharmaceuticals, dentifrices, bar soaps, mouthwashes, and paper towels. Worldwide, it has the leading brand in more categories than any other consumer products company. Since its merger with Gillette, P&G also produces and markets razors, blades, and many other personal care products.

As a leader in the application of statistical methods in decision making, P&G employs people with diverse academic backgrounds: engineering, statistics, operations research, and business. The major quantitative technologies for which these people provide support are probabilistic decision and risk analysis, advanced simulation, quality improvement, and quantitative methods (e.g., linear programming, regression analysis, probability analysis).

The Industrial Chemicals Division of P&G is a major supplier of fatty alcohols derived from natural substances such as coconut oil and from petroleum-based derivatives. The division wanted to know the economic risks and opportunities of expanding its fatty-alcohol production facilities, so it called in P&G's experts in probabilistic decision and risk analysis to help. After structuring and modeling the problem, they determined that the key to profitability was the cost difference between the petroleum- and coconut-based raw materials. Future costs were unknown, but the analysts were able to approximate them with the following continuous random variables.

x = the coconut oil price per pound of fatty alcohol

and

y = the petroleum raw material price per pound of fatty alcohol

Because the key to profitability was the difference between these two random variables, a third random variable, $d = x - y$, was used in the analysis. Experts were interviewed to determine the probability distributions for x and y. In turn, this information was used to develop a probability distribution for the difference in prices d. This continuous probability distribution showed a .90 probability that the price difference would be \$.0655 or less and a .50 probability that the price difference would be \$.035 or less. In addition, there was only a .10 probability that the price difference would be \$.0045 or less.†

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Some of Procter & Gamble's many well-known products.

The Industrial Chemicals Division thought that being able to quantify the impact of raw material price differences was key to reaching a consensus. The probabilities obtained were used in a sensitivity analysis of the raw material price difference. The analysis yielded sufficient insight to form the basis for a recommendation to management.

The use of continuous random variables and their probability distributions was helpful to P&G in analyzing the economic risks associated with its fatty-alcohol production. In this chapter, you will gain an understanding of continuous random variables and their probability distributions, including one of the most important probability distributions in statistics, the normal distribution.

*The authors are indebted to Joel Kahn of Procter & Gamble for providing this Statistics in Practice.

†The price differences stated here have been modified to protect proprietary data.

In the preceding chapter we discussed discrete random variables and their probability distributions. In this chapter we turn to the study of continuous random variables. Specifically, we discuss three continuous probability distributions: the uniform, the normal, and the exponential.

A fundamental difference separates discrete and continuous random variables in terms of how probabilities are computed. For a discrete random variable, the probability function $f(x)$ provides the probability that the random variable assumes a particular value. With continuous random variables, the counterpart of the probability function is the **probability density function**, also denoted by $f(x)$. The difference is that the probability density function does not directly provide probabilities. However, the area under the graph of $f(x)$ corresponding to a given interval does provide the probability that the continuous random variable x assumes a value in that interval. So when we compute probabilities for continuous random variables we are computing the probability that the random variable assumes any value in an interval.

Because the area under the graph of $f(x)$ at any particular point is zero, one of the implications of the definition of probability for continuous random variables is that the probability of any particular value of the random variable is zero. In Section 6.1 we demonstrate these concepts for a continuous random variable that has a uniform distribution.

Much of the chapter is devoted to describing and showing applications of the normal distribution. The normal distribution is of major importance because of its wide applicability and its extensive use in statistical inference. The chapter closes with a discussion of the exponential distribution. The exponential distribution is useful in applications involving such factors as waiting times and service times.

6.1 Uniform Probability Distribution

Whenever the probability is proportional to the length of the interval, the random variable is uniformly distributed.

Consider the random variable x representing the flight time of an airplane traveling from Chicago to New York. Suppose the flight time can be any value in the interval from 120 minutes to 140 minutes. Because the random variable x can assume any value in that interval, x is a continuous rather than a discrete random variable. Let us assume that sufficient actual flight data are available to conclude that the probability of a flight time within any 1-minute interval is the same as the probability of a flight time within any other 1-minute interval contained in the larger interval from 120 to 140 minutes. With every 1-minute interval being equally likely, the random variable x is said to have a **uniform probability distribution**. The probability density function, which defines the uniform distribution for the flight-time random variable, is

$$f(x) = \begin{cases} 1/20 & \text{for } 120 \le x \le 140 \\ 0 & \text{elsewhere} \end{cases}$$

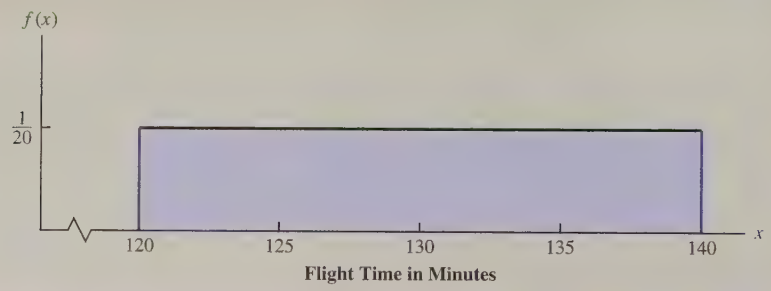
Figure 6.1 is a graph of this probability density function. In general, the uniform probability density function for a random variable x is defined by the following formula.

UNIFORM PROBABILITY DENSITY FUNCTION

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{for } a \le x \le b \\ 0 & \text{elsewhere} \end{cases} \qquad (6.1)$$

For the flight-time random variable, $a = 120$ and $b = 140$.

FIGURE 6.1 UNIFORM PROBABILITY DISTRIBUTION FOR FLIGHT TIME

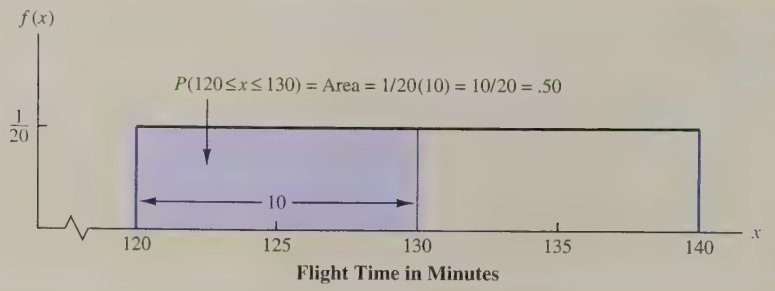


As noted in the introduction, for a continuous random variable, we consider probability only in terms of the likelihood that a random variable assumes a value within a specified interval. In the flight time example, an acceptable probability question is: What is the probability that the flight time is between 120 and 130 minutes? That is, what is $P(120 \leq x \leq 130)$? Because the flight time must be between 120 and 140 minutes and because the probability is described as being uniform over this interval, we feel comfortable saying that $P(120 \leq x \leq 130) = .50$. In the following subsection we show that this probability can be computed as the area under the graph of $f(x)$ from 120 to 130 (see Figure 6.2).

Area as a Measure of Probability

Let us make an observation about the graph in Figure 6.2. Consider the area under the graph of $f(x)$ in the interval from 120 to 130. The area is rectangular, and the area of a rectangle is simply the width multiplied by the height. With the width of the interval equal to $130 - 120 = 10$ and the height equal to the value of the probability density function $f(x) = 1/20$, we have area = width × height = $10(1/20) = 10/20 = .50$.

FIGURE 6.2 AREA PROVIDES PROBABILITY OF A FLIGHT TIME BETWEEN 120 AND 130 MINUTES



What observation can you make about the area under the graph of $f(x)$ and probability? They are identical! Indeed, this observation is valid for all continuous random variables. Once a probability density function $f(x)$ is identified, the probability that x takes a value between some lower value x_1 and some higher value x_2 can be found by computing the area under the graph of $f(x)$ over the interval from x_1 to x_2.

Given the uniform distribution for flight time and using the interpretation of area as probability, we can answer any number of probability questions about flight times. For example, what is the probability of a flight time between 128 and 136 minutes? The width of the interval is $136 - 128 = 8$. With the uniform height of $f(x) = 1/20$, we see that $P(128 \leq x \leq 136) = 8(1/20) = .40$.

Note that $P(120 \leq x \leq 140) = 20(1/20) = 1$; that is, the total area under the graph of $f(x)$ is equal to 1. This property holds for all continuous probability distributions and is the analog of the condition that the sum of the probabilities must equal 1 for a discrete probability function. For a continuous probability density function, we must also require that $f(x) \geq 0$ for all values of x. This requirement is the analog of the requirement that $f(x) \geq 0$ for discrete probability functions.

Two major differences stand out between the treatment of continuous random variables and the treatment of their discrete counterparts.

1. We no longer talk about the probability of the random variable assuming a particular value. Instead, we talk about the probability of the random variable assuming a value within some given interval.
2. The probability of a continuous random variable assuming a value within some given interval from x_1 to x_2 is defined to be the area under the graph of the probability density function between x_1 and x_2. Because a single point is an interval of zero width, this implies that the probability of a continuous random variable assuming any particular value exactly is zero. It also means that the probability of a continuous random variable assuming a value in any interval is the same whether or not the endpoints are included.

To see that the probability of any single point is 0, refer to Figure 6.2 and compute the probability of a single point, say, $x = 125$. $P(x = 125) = P(125 \leq x \leq 125) = 0(1/20) = 0$.

The calculation of the expected value and variance for a continuous random variable is analogous to that for a discrete random variable. However, because the computational procedure involves integral calculus, we leave the derivation of the appropriate formulas to more advanced texts.

For the uniform continuous probability distribution introduced in this section, the formulas for the expected value and variance are

$$E(x) = \frac{a + b}{2}$$

$$\text{Var}(x) = \frac{(b - a)^2}{12}$$

In these formulas, a is the smallest value and b is the largest value that the random variable may assume.

Applying these formulas to the uniform distribution for flight times from Chicago to New York, we obtain

$$E(x) = \frac{(120 + 140)}{2} = 130$$

$$\text{Var}(x) = \frac{(140 - 120)^2}{12} = 33.33$$

The standard deviation of flight times can be found by taking the square root of the variance. Thus, $\sigma = 5.77$ minutes.

NOTES AND COMMENTS

To see more clearly why the height of a probability density function is not a probability, think about a random variable with the following uniform probability distribution.

$$f(x) = \begin{cases} 2 & \text{for } 0 \le x \le .5 \\ 0 & \text{elsewhere} \end{cases}$$

The height of the probability density function, $f(x)$, is 2 for values of x between 0 and .5. However, we know probabilities can never be greater than 1. Thus, we see that $f(x)$ cannot be interpreted as the probability of x.

Exercises

Methods

SELF test

1. The random variable x is known to be uniformly distributed between 1.0 and 1.5.
   a. Show the graph of the probability density function.
   b. Compute $P(x = 1.25)$.
   c. Compute $P(1.0 \le x \le 1.25)$.
   d. Compute $P(1.20 < x < 1.5)$.
2. The random variable x is known to be uniformly distributed between 10 and 20.
   a. Show the graph of the probability density function.
   b. Compute $P(x < 15)$.
   c. Compute $P(12 \le x \le 18)$.
   d. Compute $E(x)$.
   e. Compute Var(x).

Applications

3. Delta Airlines quotes a flight time of 2 hours, 5 minutes for its flights from Cincinnati to Tampa. Suppose we believe that actual flight times are uniformly distributed between 2 hours and 2 hours, 20 minutes.
   a. Show the graph of the probability density function for flight time.
   b. What is the probability that the flight will be no more than 5 minutes late?
   c. What is the probability that the flight will be more than 10 minutes late?
   d. What is the expected flight time?

SELF test

4. Most computer languages include a function that can be used to generate random numbers. In Excel, the RAND function can be used to generate random numbers between 0 and 1. If we let x denote a random number generated using RAND, then x is a continuous random variable with the following probability density function.

$$f(x) = \begin{cases} 1 & \text{for } 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

   a. Graph the probability density function.
   b. What is the probability of generating a random number between .25 and .75?
   c. What is the probability of generating a random number with a value less than or equal to .30?
   d. What is the probability of generating a random number with a value greater than .60?
   e. Generate 50 random numbers by entering =RAND() into 50 cells of an Excel worksheet.
   f. Compute the mean and standard deviation for the random numbers in part (e).
5. The driving distance for the top 100 golfers on the PGA tour is between 284.7 and 310.6 yards (*Golfweek*, March 29, 2003). Assume that the driving distance for these golfers is uniformly distributed over this interval.

a. Give a mathematical expression for the probability density function of driving distance.
b. What is the probability the driving distance for one of these golfers is less than 290 yards?
c. What is the probability the driving distance for one of these golfers is at least 300 yards?
d. What is the probability the driving distance for one of these golfers is between 290 and 305 yards?
e. How many of these golfers drive the ball at least 290 yards?

6. On average, 30-minute television sitcoms have 22 minutes of programming (CNBC, February 23, 2006). Assume that the probability distribution for minutes of programming can be approximated by a uniform distribution from 18 minutes to 26 minutes.
   a. What is the probability that a sitcom will have 25 or more minutes of programming?
   b. What is the probability that a sitcom will have between 21 and 25 minutes of programming?
   c. What is the probability that a sitcom will have more than 10 minutes of commercials or other nonprogramming interruptions?

7. Suppose we are interested in bidding on a piece of land and we know one other bidder is interested.[1] The seller announced that the highest bid in excess of $10,000 will be accepted. Assume that the competitor's bid x is a random variable that is uniformly distributed between $10,000 and $15,000.
   a. Suppose you bid $12,000. What is the probability that your bid will be accepted?
   b. Suppose you bid $14,000. What is the probability that your bid will be accepted?
   c. What amount should you bid to maximize the probability that you get the property?
   d. Suppose you know someone who is willing to pay you $16,000 for the property. Would you consider bidding less than the amount in part (c)? Why or why not?

6.2

Normal Probability Distribution

Abraham de Moivre, a French mathematician, published The Doctrine of Chances *in 1733. He derived the normal distribution.*

The most important probability distribution for describing a continuous random variable is the **normal probability distribution**. The normal distribution has been used in a wide variety of practical applications in which the random variables are heights and weights of people, test scores, scientific measurements, amounts of rainfall, and other similar values. It is also widely used in statistical inference, which is the major topic of the remainder of this book. In such applications, the normal distribution provides a description of the likely results obtained through sampling.

Normal Curve

The form, or shape, of the normal distribution is illustrated by the bell-shaped normal curve in Figure 6.3. The probability density function that defines the bell-shaped curve of the normal distribution follows.

NORMAL PROBABILITY DENSITY FUNCTION

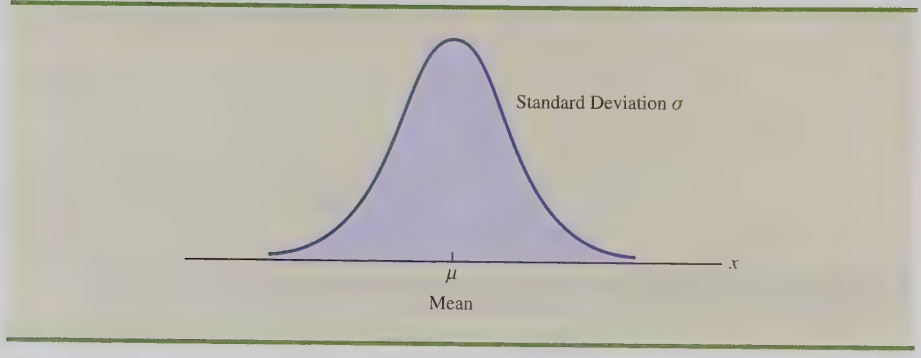
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \qquad (6.2)$$

where

μ = mean
σ = standard deviation
π = 3.14159
e = 2.71828

[1] This exercise is based on a problem suggested to us by Professor Roger Myerson of Northwestern University.

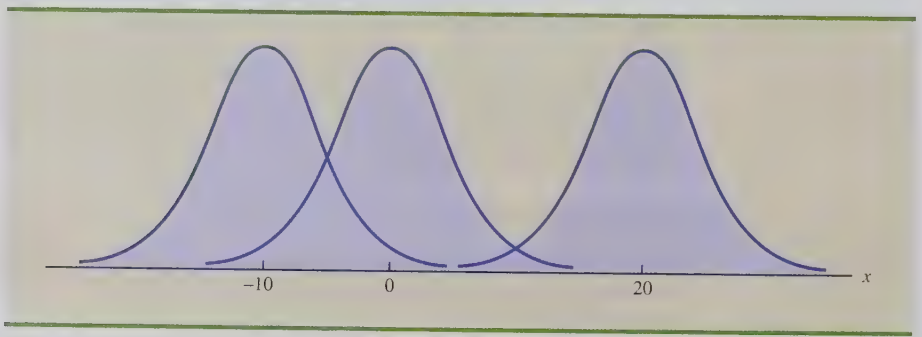
FIGURE 6.3 BELL-SHAPED CURVE FOR THE NORMAL DISTRIBUTION



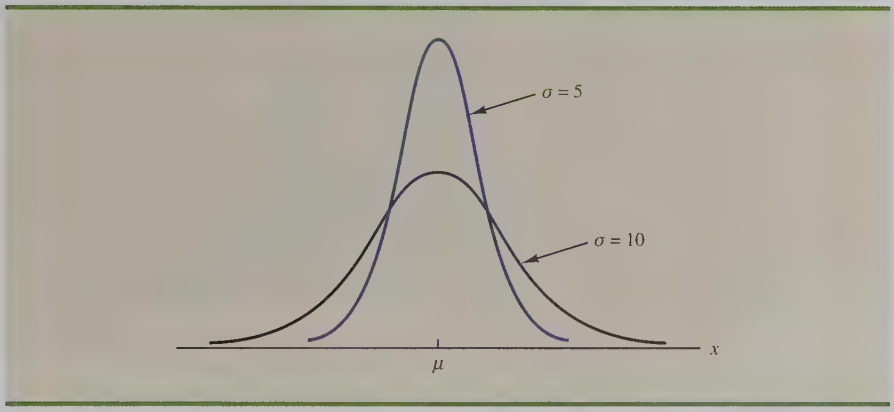
We make several observations about the characteristics of the normal distribution.

The normal curve has two parameters, μ and σ. They determine the location and shape of the normal distribution.

1. The entire family of normal distributions is differentiated by two parameters: the mean μ and the standard deviation σ.
2. The highest point on the normal curve is at the mean, which is also the median and mode of the distribution.
3. The mean of the distribution can be any numerical value: negative, zero, or positive. Three normal distributions with the same standard deviation but three different means (−10, 0, and 20) are shown here.



4. The normal distribution is symmetric, with the shape of the normal curve to the left of the mean a mirror image of the shape of the normal curve to the right of the mean. The tails of the normal curve extend to infinity in both directions and theoretically never touch the horizontal axis. Because it is symmetric, the normal distribution is not skewed; its skewness measure is zero.
5. The standard deviation determines how flat and wide the normal curve is. Larger values of the standard deviation result in wider, flatter curves, showing more variability in the data. Two normal distributions with the same mean but with different standard deviations are shown here.



6. Probabilities for the normal random variable are given by areas under the normal curve. The total area under the curve for the normal distribution is 1. Because the distribution is symmetric, the area under the curve to the left of the mean is .50 and the area under the curve to the right of the mean is .50.
7. The percentage of values in some commonly used intervals are
   a. 68.3% of the values of a normal random variable are within plus or minus one standard deviation of its mean.
   b. 95.4% of the values of a normal random variable are within plus or minus two standard deviations of its mean.
   c. 99.7% of the values of a normal random variable are within plus or minus three standard deviations of its mean.

These percentages are the basis for the empirical rule introduced in Section 3.3.

Figure 6.4 shows properties (a), (b), and (c) graphically.

FIGURE 6.4 AREAS UNDER THE CURVE FOR ANY NORMAL DISTRIBUTION

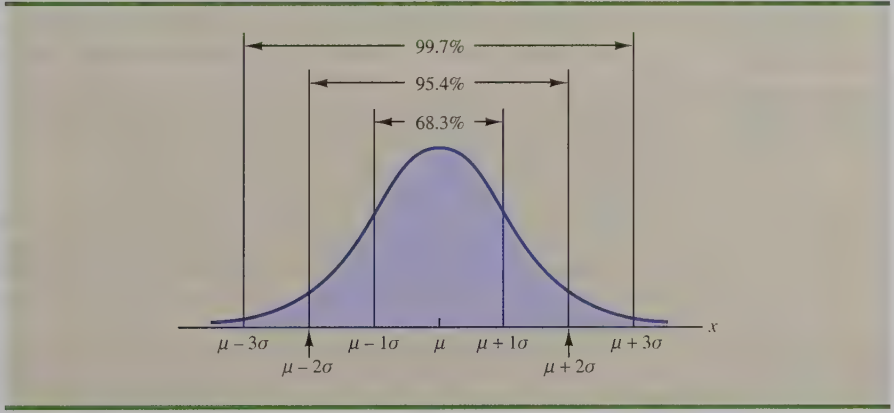
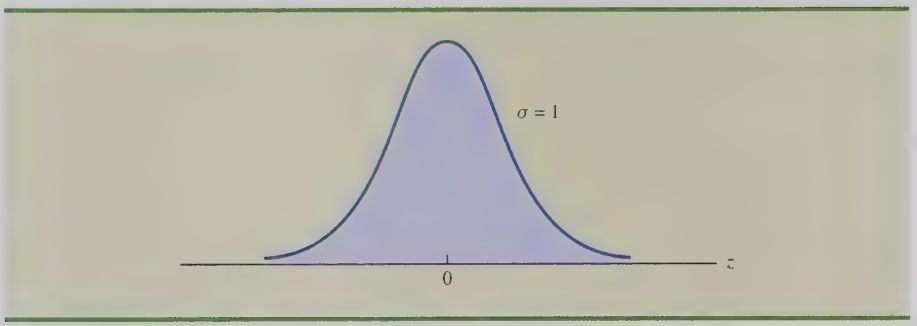


FIGURE 6.5 THE STANDARD NORMAL DISTRIBUTION



Standard Normal Probability Distribution

A random variable that has a normal distribution with a mean of zero and a standard deviation of one is said to have a **standard normal probability distribution**. The letter z is commonly used to designate this particular normal random variable. Figure 6.5 is the graph of the standard normal distribution. It has the same general appearance as other normal distributions, but with the special properties of $\mu = 0$ and $\sigma = 1$.

Because $\mu = 0$ and $\sigma = 1$, the formula for the standard normal probability density function is a simpler version of equation (6.2).

STANDARD NORMAL DENSITY FUNCTION

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

For the normal probability density function, the height of the normal curve varies and more advanced mathematics is required to compute the areas that represent probability.

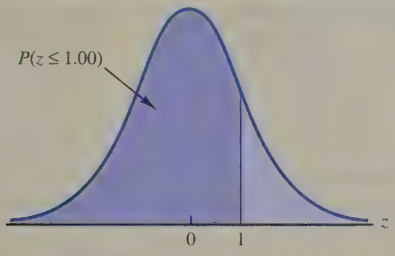
As with other continuous random variables, probability calculations with any normal distribution are made by computing areas under the graph of the probability density function. Thus, to find the probability that a normal random variable is within any specific interval, we must compute the area under the normal curve over that interval.

For the standard normal distribution, areas under the normal curve have been computed and are available in tables that can be used to compute probabilities. Such a table appears on the two pages inside the front cover of the text. The table on the left-hand page contains areas, or cumulative probabilities, for z values less than or equal to the mean of zero. The table on the right-hand page contains areas, or cumulative probabilities, for z values greater than or equal to the mean of zero.

The three types of probabilities we need to compute include (1) the probability that the standard normal random variable z will be less than or equal to a given value; (2) the probability that z will be between two given values; and (3) the probability that z will be greater than or equal to a given value. To see how the cumulative probability table for the standard normal distribution can be used to compute these three types of probabilities, let us consider some examples.

Because the standard normal random variable is continuous, $P(z \leq 1.00) = P(z < 1.00)$.

We start by showing how to compute the probability that z is less than or equal to 1.00; that is, $P(z \leq 1.00)$. This cumulative probability is the area under the normal curve to the left of $z = 1.00$ in the following graph.

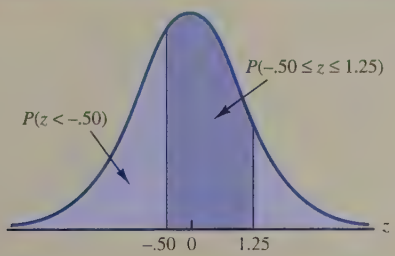


Refer to the right-hand page of the standard normal probability table inside the front cover of the text. The cumulative probability corresponding to $z = 1.00$ is the table value located at the intersection of the row labeled 1.0 and the column labeled .00. First we find 1.0 in the left column of the table and then find .00 in the top row of the table. By looking in the body of the table, we find that the 1.0 row and the .00 column intersect at the value of .8413; thus, $P(z \le 1.00) = .8413$. The following excerpt from the probability table shows these steps.

z	.00	.01	.02
.			
.			
.			
.9	.8159	.8186	.8212
1.0	.8413	.8438	.8461
1.1	.8643	.8665	.8686
1.2	.8849	.8869	.8888
.			
.			
.			

$P(z \le 1.00)$

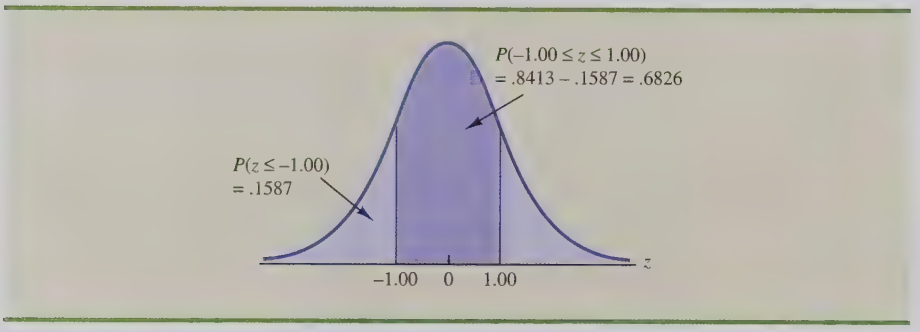
To illustrate the second type of probability calculation, we show how to compute the probability that z is in the interval between $-.50$ and 1.25; that is, $P(-.50 \le z \le 1.25)$. The following graph shows this area, or probability.



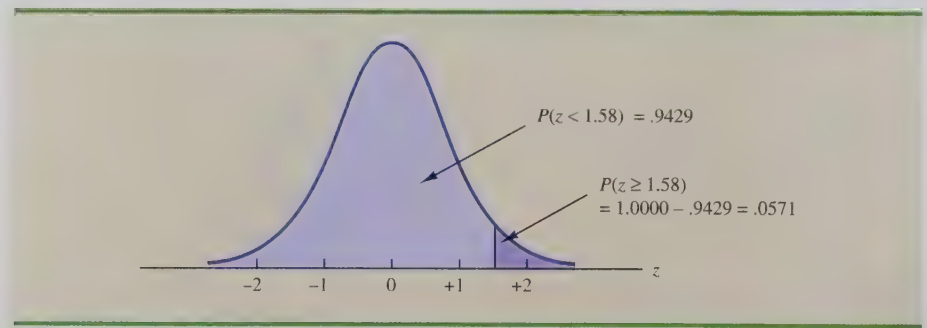
Three steps are required to compute this probability. First, we find the area under the normal curve to the left of $z = 1.25$. Second, we find the area under the normal curve to the left of $z = -.50$. Finally, we subtract the area to the left of $z = -.50$ from the area to the left of $z = 1.25$ to find $P(-.50 \le z \le 1.25)$.

To find the area under the normal curve to the left of $z = 1.25$, we first locate the 1.2 row in the standard normal probability table and then move across to the .05 column. Because the table value in the 1.2 row and the .05 column is .8944, $P(z \le 1.25) = .8944$. Similarly, to find the area under the curve to the left of $z = -.50$, we use the left-hand page of the table to locate the table value in the $-.5$ row and the .00 column; with a table value of .3085, $P(z \le -.50) = .3085$. Thus, $P(-.50 \le z \le 1.25) = P(z \le 1.25) - P(z \le -.50) = .8944 - .3085 = .5859$.

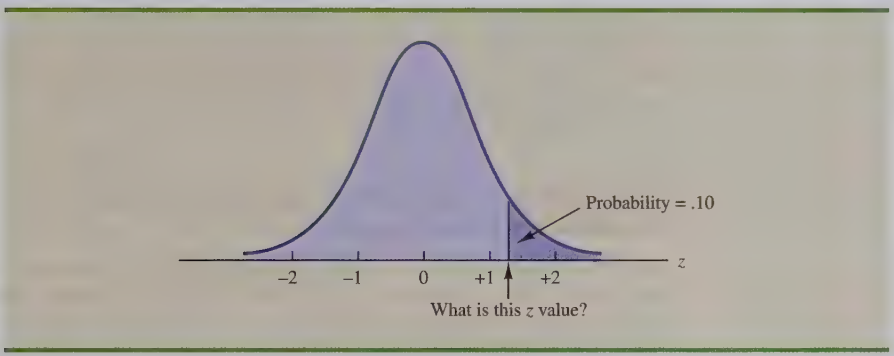
Let us consider another example of computing the probability that z is in the interval between two given values. Often it is of interest to compute the probability that a normal random variable assumes a value within a certain number of standard deviations of the mean. Suppose we want to compute the probability that the standard normal random variable is within one standard deviation of the mean; that is, $P(-1.00 \le z \le 1.00)$. To compute this probability we must find the area under the curve between -1.00 and 1.00. Earlier we found that $P(z \le 1.00) = .8413$. Referring again to the table inside the front cover of the book, we find that the area under the curve to the left of $z = -1.00$ is .1587, so $P(z \le -1.00) = .1587$. Therefore, $P(-1.00 \le z \le 1.00) = P(z \le 1.00) - P(z \le -1.00) = .8413 - .1587 = .6826$. This probability is shown graphically in the following figure.



To illustrate how to make the third type of probability computation, suppose we want to compute the probability of obtaining a z value of at least 1.58; that is, $P(z \ge 1.58)$. The value in the $z = 1.5$ row and the .08 column of the cumulative normal table is .9429; thus, $P(z < 1.58) = .9429$. However, because the total area under the normal curve is 1, $P(z \ge 1.58) = 1 - .9429 = .0571$. This probability is shown in the following figure.



In the preceding illustrations, we showed how to compute probabilities given specified z values. In some situations, we are given a probability and are interested in working backward to find the corresponding z value. Suppose we want to find a z value such that the probability of obtaining a larger z value is .10. The following figure shows this situation graphically.



Given a probability, we can use the standard normal table in an inverse fashion to find the corresponding z value.

This problem is the inverse of those in the preceding examples. Previously, we specified the z value of interest and then found the corresponding probability, or area. In this example, we are given the probability, or area, and asked to find the corresponding z value. To do so, we use the standard normal probability table somewhat differently.

Recall that the standard normal probability table gives the area under the curve to the left of a particular z value. We have been given the information that the area in the upper tail of the curve is .10. Hence, the area under the curve to the left of the unknown z value must equal .9000. Scanning the body of the table, we find .8997 is the cumulative probability value closest to .9000. The section of the table providing this result follows.

z	.06	.07	.08	.09
.				
.				
.				
1.0	.8554	.8577	.8599	.8621
1.1	.8770	.8790	.8810	.8830
1.2	.8962	.8980	.8997	.9015
1.3	.9131	.9147	.9162	.9177
1.4	.9279	.9292	.9306	.9319
.				
.				
.				

Cumulative probability value closest to .9000

Reading the z value from the leftmost column and the top row of the table, we find that the corresponding z value is 1.28. Thus, an area of approximately .9000 (actually .8997)

will be to the left of $z = 1.28$.[2] In terms of the question originally asked, there is an approximately .10 probability of a z value larger than 1.28.

The examples illustrate that the table of cumulative probabilities for the standard normal probability distribution can be used to find probabilities associated with values of the standard normal random variable z. Two types of questions can be asked. The first type of question specifies a value, or values, for z and asks us to use the table to determine the corresponding areas or probabilities. The second type of question provides an area, or probability, and asks us to use the table to determine the corresponding z value. Thus, we need to be flexible in using the standard normal probability table to answer the desired probability question. In most cases, sketching a graph of the standard normal probability distribution and shading the appropriate area will help to visualize the situation and aid in determining the correct answer.

Computing Probabilities for Any Normal Probability Distribution

The reason for discussing the standard normal distribution so extensively is that probabilities for all normal distributions are computed by using the standard normal distribution. That is, when we have a normal distribution with any mean μ and any standard deviation σ, we answer probability questions about the distribution by first converting to the standard normal distribution. Then we can use the standard normal probability table and the appropriate z values to find the desired probabilities. The formula used to convert any normal random variable x with mean μ and standard deviation σ to the standard normal random variable z follows.

The formula for the standard normal random variable is similar to the formula we introduced in Chapter 3 for computing z-scores for a data set.

CONVERTING TO THE STANDARD NORMAL RANDOM VARIABLE

$$z = \frac{x - \mu}{\sigma} \tag{6.3}$$

A value of x equal to its mean μ results in $z = (\mu - \mu)/\sigma = 0$. Thus, we see that a value of x equal to its mean μ corresponds to $z = 0$. Now suppose that x is one standard deviation above its mean; that is, $x = \mu + \sigma$. Applying equation (6.3), we see that the corresponding z value is $z = [(\mu + \sigma) - \mu]/\sigma = \sigma/\sigma = 1$. Thus, an x value that is one standard deviation above its mean corresponds to $z = 1$. In other words, *we can interpret z as the number of standard deviations that the normal random variable x is from its mean μ.*

To see how this conversion enables us to compute probabilities for any normal distribution, suppose we have a normal distribution with $\mu = 10$ and $\sigma = 2$. What is the probability that the random variable x is between 10 and 14? Using equation (6.3), we see that at $x = 10$, $z = (x - \mu)/\sigma = (10 - 10)/2 = 0$ and that at $x = 14$, $z = (14 - 10)/2 = 4/2 = 2$. Thus, the answer to our question about the probability of x being between 10 and 14 is given by the equivalent probability that z is between 0 and 2 for the standard normal distribution. In other words, the probability that we are seeking is the probability that the random variable x is between its mean and two standard deviations above the mean. Using $z = 2.00$ and the standard normal probability table inside the front cover of the text, we see that $P(z \leq 2) = .9772$.

[2] We could use interpolation in the body of the table to get a better approximation of the z value that corresponds to an area of .9000. Doing so to provide one more decimal place of accuracy would yield a z value of 1.282. However, in most practical situations, sufficient accuracy is obtained by simply using the table value closest to the desired probability.

Because $P(z \leq 0) = .5000$, we can compute $P(.00 \leq z \leq 2.00) = P(z \leq 2) - P(z \leq 0) = .9772 - .5000 = .4772$. Hence the probability that x is between 10 and 14 is .4772.

Grear Tire Company Problem

We turn now to an application of the normal probability distribution. Suppose the Grear Tire Company developed a new steel-belted radial tire to be sold through a national chain of discount stores. Because the tire is a new product, Grear's managers believe that the mileage guarantee offered with the tire will be an important factor in the acceptance of the product. Before finalizing the tire mileage guarantee policy, Grear's managers want probability information about x = number of miles the tires will last.

From actual road tests with the tires, Grear's engineering group estimated that the mean tire mileage is $\mu = 36{,}500$ miles and that the standard deviation is $\sigma = 5000$. In addition, the data collected indicate that a normal distribution is a reasonable assumption. What percentage of the tires can be expected to last more than 40,000 miles? In other words, what is the probability that the tire mileage, x, will exceed 40,000? This question can be answered by finding the area of the darkly shaded region in Figure 6.6.

At $x = 40{,}000$, we have

$$z = \frac{x - \mu}{\sigma} = \frac{40{,}000 - 36{,}500}{5000} = \frac{3500}{5000} = .70$$

Refer now to the bottom of Figure 6.6. We see that a value of $x = 40{,}000$ on the Grear Tire normal distribution corresponds to a value of $z = .70$ on the standard normal distribution. Using the standard normal probability table, we see that the area under the standard normal curve to the left of $z = .70$ is .7580. Thus, $1.000 - .7580 = .2420$ is the probability that z will exceed .70 and hence x will exceed 40,000. We can conclude that about 24.2% of the tires will exceed 40,000 in mileage.

Let us now assume that Grear is considering a guarantee that will provide a discount on replacement tires if the original tires do not provide the guaranteed mileage. What should

FIGURE 6.6 GREAR TIRE COMPANY MILEAGE DISTRIBUTION

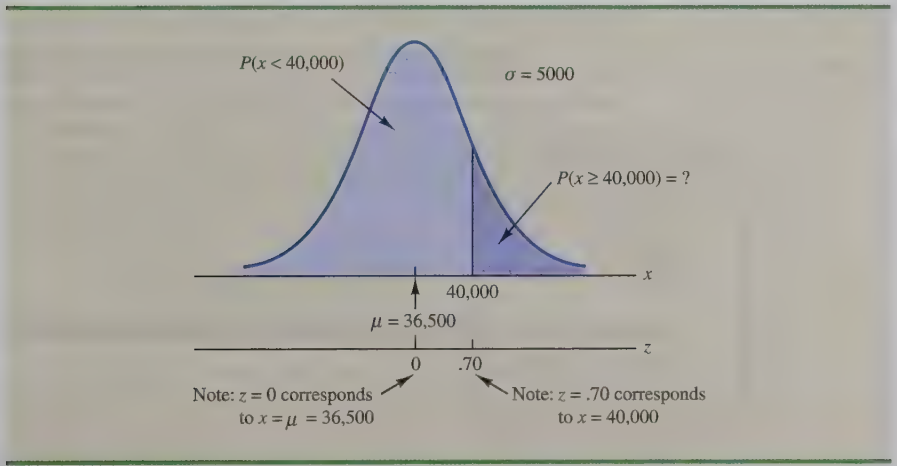
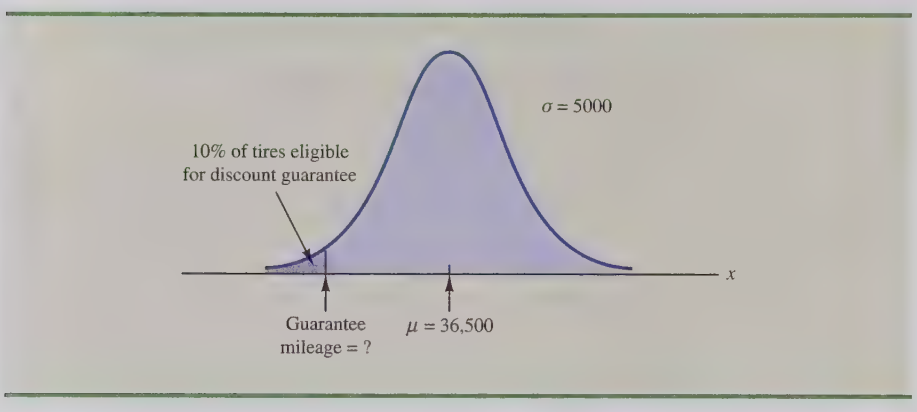


FIGURE 6.7 GREAR'S DISCOUNT GUARANTEE



the guarantee mileage be if Grear wants no more than 10% of the tires to be eligible for the discount guarantee? This question is interpreted graphically in Figure 6.7.

According to Figure 6.7, the area under the curve to the left of the unknown guarantee mileage must be .10. So, we must first find the z value that cuts off an area of .10 in the left tail of a standard normal distribution. Using the standard normal probability table, we see that $z = -1.28$ cuts off an area of .10 in the lower tail. Hence, $z = -1.28$ is the value of the standard normal random variable corresponding to the desired mileage guarantee on the Grear Tire normal distribution. To find the value of x corresponding to $z = -1.28$, we have

The guarantee mileage we need to find is 1.28 standard deviations below the mean. Thus, $x = \mu - 1.28\sigma$.

$$z = \frac{x - \mu}{\sigma} = -1.28$$

$$x - \mu = -1.28\sigma$$

$$x = \mu - 1.28\sigma$$

With $\mu = 36{,}500$ and $\sigma = 5000$,

$$x = 36{,}500 - 1.28(5000) = 30{,}100$$

With the guarantee set at 30,000 miles, the actual percentage eligible for the guarantee will be 9.68%.

Thus, a guarantee of 30,100 miles will meet the requirement that approximately 10% of the tires will be eligible for the guarantee. Perhaps, with this information, the firm will set its tire mileage guarantee at 30,000 miles.

Again, we see the important role that probability distributions play in providing decision-making information. Namely, once a probability distribution is established for a particular application, it can be used to obtain probability information about the problem. Probability does not make a decision recommendation directly, but it provides information that helps the decision maker better understand the risks and uncertainties associated with the problem. Ultimately, this information may assist the decision maker in reaching a good decision.

Using Excel to Compute Normal Probabilities

Excel provides two functions for computing probabilities and z values for a standard normal probability distribution: NORM.S.DIST and NORM.S.INV. The NORM.S.DIST function computes the cumulative probability given a z value, and the NORM.S.INV function computes the z value given a cumulative probability. Two similar functions, NORM.DIST and

NORM.INV, are available for computing the cumulative probability and the x value for any normal distribution. We begin by showing how to use the NORM.S.DIST and NORM.S.INV functions.

The letter S that appears in the name of the NORM.S.DIST and NORM.S.INV functions reminds us that these functions relate to the standard normal probability distribution.

The NORM.S.DIST function provides the area under the standard normal curve to the left of a given z value; thus, it provides the same cumulative probability we would obtain if we used the standard normal probability table inside the front cover of the text. Using the NORM.S.DIST function is just like having Excel look up cumulative normal probabilities for you. The NORM.S.INV function is the inverse of the NORM.S.DIST function; it takes a cumulative probability as input and provides the z value corresponding to that cumulative probability.

Let's see how both of these functions work by computing the probabilities and z values obtained earlier in this section using the standard normal probability table. Refer to Figure 6.8 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet is in the foreground.

Enter Data: No data are entered in the worksheet. We will simply enter the appropriate z values and probabilities directly into the formulas as needed.

Enter Functions and Formulas: The NORM.S.DIST function has two inputs: the z value and a value of TRUE or FALSE. For the second input we enter TRUE if a cumulative probability is desired, and we enter FALSE if the height of the standard normal curve is desired. Because we will always be using NORM.S.DIST to compute cumulative probabilities, we always choose TRUE for the second input. To illustrate the use of the NORM.S.DIST function, we compute the four probabilities shown in cells D3:D6 of Figure 6.8.

To compute the cumulative probability to the left of a given z value (area in lower tail), we simply evaluate NORM.S.DIST at the z value. For instance, to compute $P(z \leq 1)$ we entered the formula =NORM.S.DIST(1,TRUE) into cell D3. The result, .8413, is the same as obtained using the standard normal probability table.

The probabilities in cells D4, 0.5858, and D5, 0.6827, differ from what we computed earlier due to rounding.

To compute the probability of z being in an interval we compute the value of NORM.S.DIST at the upper endpoint of the interval and subtract the value of NORM.S.DIST at the lower endpoint of the interval. For instance, to find $P(-.50 \leq z \leq 1.25)$, we entered

FIGURE 6.8 EXCEL WORKSHEET FOR COMPUTING PROBABILITIES AND z VALUES FOR THE STANDARD NORMAL DISTRIBUTION

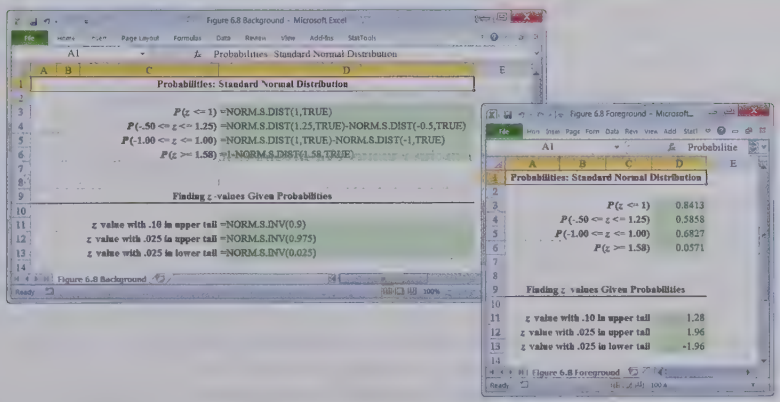


Figure 6.8 Background - Microsoft Excel
Probabilities: Standard Normal Distribution
P(z <= 1) =NORM.S.DIST(1,TRUE)
P(-.50 <= z <= 1.25) =NORM.S.DIST(1.25,TRUE)-NORM.S.DIST(-0.5,TRUE)
P(-1.00 <= z <= 1.00) =NORM.S.DIST(1,TRUE)-NORM.S.DIST(-1,TRUE)
P(z >= 1.58) =1-NORM.S.DIST(1.58,TRUE)
Finding z -values Given Probabilities
z value with .10 in upper tail =NORM.S.INV(0.9)
z value with .025 in upper tail =NORM.S.INV(0.975)
z value with .025 in lower tail =NORM.S.INV(0.025)
Figure 6.8 Foreground - Microsoft...
Probabilities: Standard Normal Distribution
P(z <= 1) 0.8413
P(-.50 <= z <= 1.25) 0.5858
P(-1.00 <= z <= 1.00) 0.6827
P(z >= 1.58) 0.0571
Finding z -values Given Probabilities
z value with .10 in upper tail 1.28
z value with .025 in upper tail 1.96
z value with .025 in lower tail -1.96

the formula =NORM.S.DIST(1.25,TRUE)-NORM.S.DIST(-.50,TRUE) into cell D4. The interval probability in cell D5 is computed in a similar fashion.

To compute the probability to the right of a given z value (upper tail area), we must subtract the cumulative probability represented by the area under the curve below the z value (lower tail area) from 1. For example, to compute $P(z \geq 1.58)$ we entered the formula =1-NORM.S.DIST(1.58,TRUE) into cell D6.

To compute the z value for a given cumulative probability (lower tail area), we use the NORM.S.INV function. To find the z value corresponding to an upper tail probability of .10, we note that the corresponding lower tail area is .90 and enter the formula =NORM.S.INV(0.9) into cell D11. Actually, NORM.S.INV(0.9) gives us the z value providing a cumulative probability (lower tail area) of .9. But it is also the z value associated with an upper tail area of .10.

Two other z values are computed in Figure 6.8. These z values will be used extensively in succeeding chapters. To compute the z value corresponding to an upper tail probability of .025, we entered the formula =NORM.S.INV(0.975) into cell D12. To compute the z value corresponding to a lower tail probability of .025, we entered the formula =NORM.S.INV(0.025) into cell D13. We see that $z = 1.96$ corresponds to an upper tail probability of .025, and $z = -1.96$ corresponds to a lower tail probability of .025.

Let us now turn to the Excel functions for computing cumulative probabilities and x values for any normal distribution. The NORM.DIST function provides the area under the normal curve to the left of a given value of the random variable x; thus it provides cumulative probabilities. The NORM.INV function is the inverse of the NORM.DIST function; it takes a cumulative probability as input and provides the value of x corresponding to that cumulative probability. The NORM.DIST and NORM.INV functions do the same thing for any normal distribution that the NORM.S.DIST and NORM.S.INV functions do for the standard normal distribution.

Let's see how both of these functions work by computing probabilities and x values for the Grear Tire Company example introduced earlier in this section. Recall that the lifetime of a Grear tire has a mean of 36,500 miles and a standard deviation of 5000 miles. Refer to Figure 6.9 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet is in the foreground.

Enter Data: No data are entered in the worksheet. We simply enter the appropriate x values and probabilities directly into the formulas as needed.

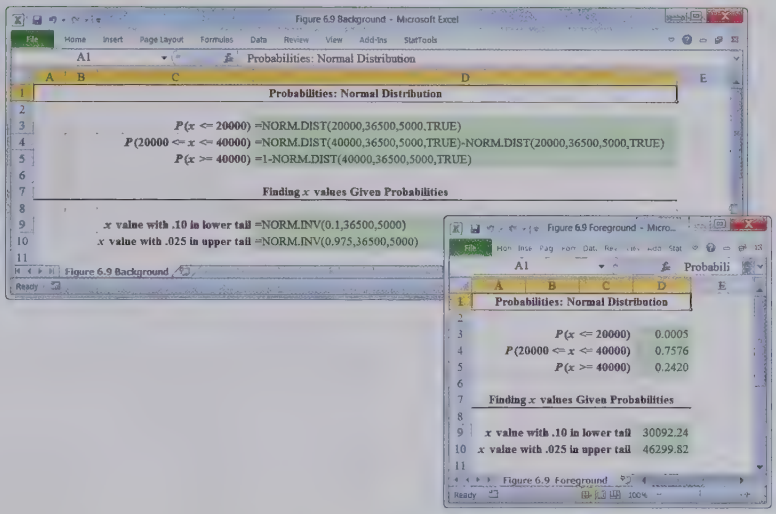
Enter Functions and Formulas: The NORM.DIST function has four inputs: (1) the x value we want to compute the cumulative probability for, (2) the mean, (3) the standard deviation, and (4) a value of TRUE or FALSE. For the fourth input, we enter TRUE if a cumulative probability is desired, and we enter FALSE if the height of the curve is desired. Because we will always be using NORM.DIST to compute cumulative probabilities, we will always choose TRUE for the fourth input.

To compute the cumulative probability to the left of a given x value (lower tail area), we simply evaluate NORM.DIST at the x value. For instance, to compute the probability that a Grear tire will last 20,000 miles or less, we entered the formula =NORM.DIST(20000,36500,5000,TRUE) into cell D3. The value worksheet shows that this cumulative probability is .0005. So, we can conclude that almost all Grear tires will last at least 20,000 miles.

To compute the probability of x being in an interval we compute the value of NORM.DIST at the upper endpoint of the interval and subtract the value of NORM.DIST at the lower endpoint of the interval. The formula in cell D4 provides the probability that a tire's lifetime is between 20,000 and 40,000 miles, $P(20{,}000 \leq x \leq 40{,}000)$. In the value worksheet, we see that this probability is .7576.

To compute the probability to the right of a given x value (upper tail area), we must subtract the cumulative probability represented by the area under the curve below the x value

FIGURE 6.9 EXCEL WORKSHEET FOR COMPUTING PROBABILITIES AND x VALUES FOR THE NORMAL DISTRIBUTION



(lower tail area) from 1. The formula in cell D5 computes the probability that a Grear tire will last for at least 40,000 miles. We see that this probability is .2420.

To compute the x value for a given cumulative probability, we use the NORM.INV function. The NORM.INV function has only three inputs. The first input is the cumulative probability; the second and third inputs are the mean and standard deviation. For instance, to compute the tire mileage corresponding to a lower tail area of .1 for Grear Tire, we enter the formula =NORM.INV(0.1,36500,5000) into cell D9. From the value worksheet, we see that 10% of the Grear tires will last for 30,092.24 miles or less.

To compute the minimum tire mileage for the top 2.5% of Grear tires, we want to find the value of x corresponding to an area of .025 in the upper tail. This calculation is the same as finding the x value that provides a cumulative probability of .975. Thus we entered the formula =NORM.INV(0.975,36500,5000) into cell D10 to compute this tire mileage. From the value worksheet, we see that 2.5% of the Grear tires will last at least 46,299.82 miles.

Exercises

Methods

8. Using Figure 6.4 as a guide, sketch a normal curve for a random variable x that has a mean of $\mu = 100$ and a standard deviation of $\sigma = 10$. Label the horizontal axis with values of 70, 80, 90, 100, 110, 120, and 130.
9. A random variable is normally distributed with a mean of $\mu = 50$ and a standard deviation of $\sigma = 5$.
   a. Sketch a normal curve for the probability density function. Label the horizontal axis with values of 35, 40, 45, 50, 55, 60, and 65. Figure 6.4 shows that the normal curve almost touches the horizontal axis at three standard deviations below and at three standard deviations above the mean (in this case at 35 and 65).

b. What is the probability that the random variable will assume a value between 45 and 55?
c. What is the probability that the random variable will assume a value between 40 and 60?

10. Draw a graph for the standard normal distribution. Label the horizontal axis at values of $-3, -2, -1, 0, 1, 2$, and 3. Then compute the following probabilities.
a. $P(z \leq 1.5)$
b. $P(z \leq 1)$
c. $P(1 \leq z \leq 1.5)$
d. $P(0 < z < 2.5)$

11. Given that z is a standard normal random variable, compute the following probabilities.
a. $P(z \leq -1.0)$
b. $P(z \geq -1)$
c. $P(z \geq -1.5)$
d. $P(-2.5 \leq z)$
e. $P(-3 < z \leq 0)$

12. Given that z is a standard normal random variable, compute the following probabilities.
a. $P(0 \leq z \leq .83)$
b. $P(-1.57 \leq z \leq 0)$
c. $P(z > .44)$
d. $P(z \geq -.23)$
e. $P(z < 1.20)$
f. $P(z \leq -.71)$

SELF test

13. Given that z is a standard normal random variable, compute the following probabilities.
a. $P(-1.98 \leq z \leq .49)$
b. $P(.52 \leq z \leq 1.22)$
c. $P(-1.75 \leq z \leq -1.04)$

14. Given that z is a standard normal random variable, find z for each situation.
a. The area to the left of z is .9750.
b. The area between 0 and z is .4750.
c. The area to the left of z is .7291.
d. The area to the right of z is .1314.
e. The area to the left of z is .6700.
f. The area to the right of z is .3300.

SELF test

15. Given that z is a standard normal random variable, find z for each situation.
a. The area to the left of z is .2119.
b. The area between $-z$ and z is .9030.
c. The area between $-z$ and z is .2052.
d. The area to the left of z is .9948.
e. The area to the right of z is .6915.

16. Given that z is a standard normal random variable, find z for each situation.
a. The area to the right of z is .01.
b. The area to the right of z is .025.
c. The area to the right of z is .05.
d. The area to the right of z is .10.

Applications

17. For borrowers with good credit scores, the mean debt for revolving and installment accounts is $15,015 (*BusinessWeek*, March 20, 2006). Assume the standard deviation is $3540 and that debt amounts are normally distributed.

a. What is the probability that the debt for a borrower with good credit is more than $18,000?
b. What is the probability that the debt for a borrower with good credit is less than $10,000?
c. What is the probability that the debt for a borrower with good credit is between $12,000 and $18,000?
d. What is the probability that the debt for a borrower with good credit is no more than $14,000?

SELF test

18. The average stock price for companies making up the S&P 500 is $30, and the standard deviation is $8.20 (*BusinessWeek,* Special Annual Issue, Spring 2003). Assume the stock prices are normally distributed.
a. What is the probability that a company will have a stock price of at least $40?
b. What is the probability that a company will have a stock price no higher than $20?
c. How high does a stock price have to be to put a company in the top 10%?

19. In an article about the cost of health care, *Money* magazine reported that a visit to a hospital emergency room for something as simple as a sore throat has a mean cost of $328 (*Money,* January 2009). Assume that the cost for this type of hospital emergency room visit is normally distributed with a standard deviation of $92. Answer the following questions about the cost of a hospital emergency room visit for this medical service.
a. What is the probability that the cost will be more than $500?
b. What is the probability that the cost will be less than $250?
c. What is the probability that the cost will be between $300 and $400?
d. If the cost to a patient is in the lower 8% of charges for this medical service, what was the cost of this patient's emergency room visit?

20. In January 2003, the American worker spent an average of 77 hours logged on to the Internet while at work (CNBC, March 15, 2003). Assume that the population mean is 77 hours, the times are normally distributed, and the standard deviation is 20 hours.
a. What is the probability that in January 2003 a randomly selected worker spent fewer than 50 hours logged on to the Internet?
b. What percentage of workers spent more than 100 hours in January 2003 logged on to the Internet?
c. A person is classified as a heavy user if he or she is in the upper 20% of usage. In January 2003, how many hours did a worker have to be logged on to the Internet to be considered a heavy user?

21. A person must score in the upper 2% of the population on an IQ test to qualify for membership in Mensa, the international high-IQ society. If IQ scores are normally distributed with a mean of 100 and a standard deviation of 15, what score must a person have to qualify for Mensa?

22. Television viewing reached a new high when the Nielsen Company reported a mean daily viewing time of 8.35 hours per household (*USA Today,* November 11, 2009). Use a normal probability distribution with a standard deviation of 2.5 hours to answer the following questions about daily television viewing per household.
a. What is the probability that a household views television between 5 and 10 hours a day?
b. How many hours of television viewing must a household have in order to be in the top 3% of all television viewing households?
c. What is the probability that a household views television more than 3 hours a day?

23. The time needed to complete a final examination in a particular college course is normally distributed with a mean of 80 minutes and a standard deviation of 10 minutes. Answer the following questions.
a. What is the probability of completing the exam in one hour or less?
b. What is the probability that a student will complete the exam in more than 60 minutes but less than 75 minutes?

c. Assume that the class has 60 students and that the examination period is 90 minutes in length. How many students do you expect will be unable to complete the exam in the allotted time?

24. Trading volume on the New York Stock Exchange is heaviest during the first half hour (early morning) and last half hour (late afternoon) of the trading day. The early morning trading volumes (millions of shares) for 13 days in January and February are shown here (*Barron's*, January 23, 2006; February 13, 2006; and February 27, 2006).

214	163	265	194	180
202	198	212	201	
174	171	211	211	

The probability distribution of trading volume is approximately normal.

a. Compute the mean and standard deviation to use as estimates of the population mean and standard deviation.
b. What is the probability that, on a randomly selected day, the early morning trading volume will be less than 180 million shares?
c. What is the probability that, on a randomly selected day, the early morning trading volume will exceed 230 million shares?
d. How many shares would have to be traded for the early morning trading volume on a particular day to be among the busiest 5% of days?

25. According to the Sleep Foundation, the average night's sleep is 6.8 hours (*Fortune*, March 20, 2006). Assume the standard deviation is .6 hours and that the probability distribution is normal.

a. What is the probability that a randomly selected person sleeps more than 8 hours?
b. What is the probability that a randomly selected person sleeps 6 hours or less?
c. Doctors suggest getting between 7 and 9 hours of sleep each night. What percentage of the population gets this much sleep?

6.3 Exponential Probability Distribution

The **exponential probability distribution** may be used for random variables such as the time between arrivals at a car wash, the time required to load a truck, the distance between major defects in a highway, and so on. The exponential probability density function follows.

EXPONENTIAL PROBABILITY DENSITY FUNCTION

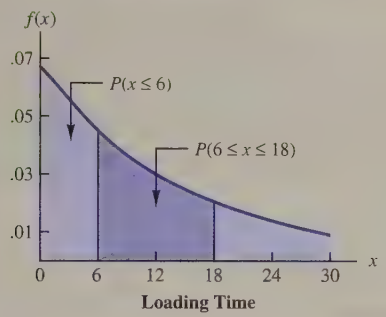
$$f(x) = \frac{1}{\mu} e^{-x/\mu} \qquad \text{for } x \geq 0 \tag{6.4}$$

where

μ = expected value or mean
$e = 2.71828$

As an example of the exponential distribution, suppose that x represents the loading time for a truck at the Schips loading dock and follows such a distribution. If the mean, or

FIGURE 6.10 EXPONENTIAL DISTRIBUTION FOR THE SCHIPS LOADING DOCK EXAMPLE



average, loading time is 15 minutes ($\mu = 15$), the appropriate probability density function for x is

$$f(x) = \frac{1}{15} e^{-x/15}$$

Figure 6.10 is the graph of this probability density function.

Computing Probabilities for the Exponential Distribution

In waiting line applications, the exponential distribution is often used for service time.

As with any continuous probability distribution, the area under the curve corresponding to an interval provides the probability that the random variable assumes a value in that interval. In the Schips loading dock example, the probability that loading a truck will take 6 minutes or less $P(x \leq 6)$ is defined to be the area under the curve in Figure 6.10 from $x = 0$ to $x = 6$. Similarly, the probability that the loading time will be 18 minutes or less $P(x \leq 18)$ is the area under the curve from $x = 0$ to $x = 18$. Note also that the probability that the loading time will be between 6 minutes and 18 minutes $P(6 \leq x \leq 18)$ is given by the area under the curve from $x = 6$ to $x = 18$.

To compute exponential probabilities such as those just described, we use the following formula. It provides the cumulative probability of obtaining a value for the exponential random variable of less than or equal to some specific value denoted by x_0.

EXPONENTIAL DISTRIBUTION: CUMULATIVE PROBABILITIES

$$P(x \leq x_0) = 1 - e^{-x_0/\mu} \qquad (6.5)$$

For the Schips loading dock example, x = loading time in minutes and $\mu = 15$ minutes. Using equation (6.5),

$$P(x \leq x_0) = 1 - e^{-x_0/15}$$

Hence, the probability that loading a truck will take 6 minutes or less is

$$P(x \leq 6) = 1 - e^{-6/15} = .3297$$

Using equation (6.5), we calculate the probability of loading a truck in 18 minutes or less.

$$P(x \leq 18) = 1 - e^{-18/15} = .6988$$

Thus, the probability that loading a truck will take between 6 minutes and 18 minutes is equal to .6988 − .3297 = .3691. Probabilities for any other interval can be computed similarly.

A property of the exponential distribution is that the mean and standard deviation are equal.

In the preceding example, the mean time it takes to load a truck is $\mu = 15$ minutes. A property of the exponential distribution is that the mean of the distribution and the standard deviation of the distribution are *equal.* Thus, the standard deviation for the time it takes to load a truck is $\sigma = 15$ minutes. The variance is $\sigma^2 = (15)^2 = 225$.

Relationship Between the Poisson and Exponential Distributions

In Section 5.5 we introduced the Poisson distribution as a discrete probability distribution that is often useful in examining the number of occurrences of an event over a specified interval of time or space. Recall that the Poisson probability function is

$$f(x) = \frac{\mu^x e^{-\mu}}{x!}$$

where

μ = expected value or mean number of occurrences over a specified interval

If arrivals follow a Poisson distribution, the time between arrivals must follow an exponential distribution.

The continuous exponential probability distribution is related to the discrete Poisson distribution. If the Poisson distribution provides an appropriate description of the number of occurrences per interval, the exponential distribution provides a description of the length of the interval between occurrences.

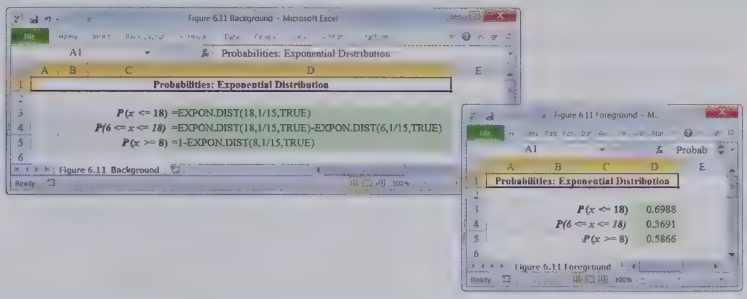
To illustrate this relationship, suppose the number of cars that arrive at a car wash during one hour is described by a Poisson probability distribution with a mean of 10 cars per hour. The Poisson probability function that gives the probability of x arrivals per hour is

$$f(x) = \frac{10^x e^{-10}}{x!}$$

Because the average number of arrivals is 10 cars per hour, the average time between cars arriving is

$$\frac{1 \text{ hour}}{10 \text{ cars}} = .1 \text{ hour/car}$$

FIGURE 6.11 EXCEL WORKSHEET FOR COMPUTING PROBABILITIES FOR THE EXPONENTIAL PROBABILITY DISTRIBUTION



Thus, the corresponding exponential distribution that describes the time between the arrivals has a mean of $\mu = .1$ hour per car; as a result, the appropriate exponential probability density function is

$$f(x) = \frac{1}{.1} e^{-x/.1} = 10e^{-10x}$$

Using Excel to Compute Exponential Probabilities

Excel's EXPON.DIST function can be used to compute exponential probabilities. We will illustrate by computing probabilities associated with the time it takes to load a truck at the Schips loading dock. This example was introduced at the beginning of the section. Refer to Figure 6.11 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet is in the foreground.

Enter Data: No data are entered in the worksheet. We simply enter the appropriate values for the exponential random variable into the formulas as needed. The random variable is x = loading time.

Enter Functions and Formulas: The EXPON.DIST function has three inputs: the first is the value of x, the second is $1/\mu$, and the third is TRUE or FALSE. We choose TRUE for the third input if a cumulative probability is desired and FALSE if the height of the probability density function is desired. We will always use TRUE because we will be computing cumulative probabilities.

The first probability we compute is the probability that the loading time is 18 minutes or less. For the Schips problem, $1/\mu = 1/15$, so we enter the formula =EXPON.DIST(18,1/15,TRUE) into cell D3 to compute the desired cumulative probability. From the value worksheet, we see that the probability of loading a truck in 18 minutes or less is .6988.

The second probability we compute is the probability that the loading time is between 6 and 18 minutes. To find this probability we first compute the cumulative probability for the upper endpoint of the time interval and subtract the cumulative probability for the lower endpoint of the interval. The formula we have entered into cell D4 calculates this probability. The value worksheet shows that this probability is .3691.

The last probability we calculate is the probability that the loading time is at least 8 minutes. Because the EXPON.DIST function only computes cumulative (lower tail) probabilities, we compute this probability by entering the formula =1-EXPON.DIST(8,1/15,TRUE) into cell D5. The value worksheet shows that the probability of a loading time of 8 minutes or more is .5866.

NOTES AND COMMENTS

As we can see in Figure 6.10, the exponential distribution is skewed to the right. Indeed, the skewness measure for the exponential distributions is 2. The exponential distribution gives us a good idea what a skewed distribution looks like.

Exercises

Methods

26. Consider the following exponential probability density function.

$$f(x) = \frac{1}{8}e^{-x/8} \qquad \text{for } x \geq 0$$

a. Find $P(x \leq 6)$.
b. Find $P(x \leq 4)$.
c. Find $P(x \geq 6)$.
d. Find $P(4 \leq x \leq 6)$.

SELF test

27. Consider the following exponential probability density function.

$$f(x) = \frac{1}{3}e^{-x/3} \qquad \text{for } x \geq 0$$

a. Write the formula for $P(x \leq x_0)$.
b. Find $P(x \leq 2)$.
c. Find $P(x \geq 3)$.
d. Find $P(x \leq 5)$.
e. Find $P(2 \leq x \leq 5)$.

Applications

28. The time required to pass through security screening at the airport can be annoying to travelers. The mean wait time during peak periods at Cincinnati/Northern Kentucky International Airport is 12.1 minutes (*The Cincinnati Enquirer,* February 2, 2006). Assume the time to pass through security screening follows an exponential distribution.
a. What is the probability that it will take less than 10 minutes to pass through security screening during a peak period?
b. What is the probability that it will take more than 20 minutes to pass through security screening during a peak period?
c. What is the probability that it will take between 10 and 20 minutes to pass through security screening during a peak period?
d. It is 8:00 A.M. (a peak period) and you just entered the security line. To catch your plane you must be at the gate within 30 minutes. If it takes 12 minutes from the time you clear security until you reach your gate, what is the probability that you will miss your flight?

SELF test

29. The time between arrivals of vehicles at a particular intersection follows an exponential probability distribution with a mean of 12 seconds.
a. Sketch this exponential probability distribution.
b. What is the probability that the arrival time between vehicles is 12 seconds or less?
c. What is the probability that the arrival time between vehicles is 6 seconds or less?
d. What is the probability of 30 or more seconds between vehicle arrivals?

30. Comcast Corporation is the largest cable television company, the second largest Internet service provider, and the fourth largest telephone service provider in the United States.

Generally known for quality and reliable service, the company periodically experiences unexpected service interruptions. On January 14, 2009, such an interruption occurred for the Comcast customers living in southwest Florida. When customers called the Comcast office, a recorded message told them that the company was aware of the service outage and that it was anticipated that service would be restored in two hours. Assume that two hours is the mean time to do the repair and that the repair time has an exponential probability distribution.

a. What is the probability that the cable service will be repaired in one hour or less?
b. What is the probability that the repair will take between one hour and two hours?
c. For a customer who calls the Comcast office at 1:00 P.M., what is the probability that the cable service will not be repaired by 5:00 P.M.?

31. Collina's Italian Café in Houston, Texas, advertises that carryout orders take about 25 minutes (Collina's website, February 27, 2008). Assume that the time required for a carryout order to be ready for customer pickup has an exponential distribution with a mean of 25 minutes.
    a. What is the probability than a carryout order will be ready within 20 minutes?
    b. If a customer arrives 30 minutes after placing an order, what is the probability that the order will not be ready?
    c. A particular customer lives 15 minutes from Collina's Italian Café. If the customer places a telephone order at 5:20 P.M., what is the probability that the customer can drive to the café, pick up the order, and return home by 6:00 P.M.?
32. Do interruptions while you are working reduce your productivity? According to a University of California–Irvine study, businesspeople are interrupted at the rate of approximately 5½ times per hour (*Fortune*, March 20, 2006). Suppose the number of interruptions follows a Poisson probability distribution.
    a. Show the probability distribution for the time between interruptions.
    b. What is the probability that a businessperson will have no interruptions during a 15-minute period?
    c. What is the probability that the next interruption will occur within 10 minutes for a particular businessperson?

Summary

This chapter extended the discussion of probability distributions to the case of continuous random variables. The major conceptual difference between discrete and continuous probability distributions involves the method of computing probabilities. With discrete distributions, the probability function $f(x)$ provides the probability that the random variable x assumes various values. With continuous distributions, the probability density function $f(x)$ does not provide probability values directly. Instead, probabilities are given by areas under the curve or graph of the probability density function $f(x)$. Because the area under the curve above a single point is zero, we observe that the probability of any particular value is zero for a continuous random variable.

Three continuous probability distributions—the uniform, normal, and exponential distributions—were treated in detail. The normal distribution is used widely in statistical inference and will be used extensively throughout the remainder of the text.

Glossary

Probability density function A function used to compute probabilities for a continuous random variable. The area under the graph of a probability density function over an interval represents probability.

Uniform probability distribution A continuous probability distribution for which the probability that the random variable will assume a value in any interval is the same for each interval of equal length.

Normal probability distribution A continuous probability distribution. Its probability density function is bell-shaped and determined by its mean μ and standard deviation σ.

Standard normal probability distribution A normal distribution with a mean of zero and a standard deviation of one.

Continuity correction factor A value of .5 that is added to or subtracted from a value of x when the continuous normal distribution is used to approximate the discrete binomial distribution.

Exponential probability distribution A continuous probability distribution that is useful in computing probabilities for the time it takes to complete a task.

Key Formulas

Uniform Probability Density Function

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{for } a \le x \le b \\ 0 & \text{elsewhere} \end{cases} \tag{6.1}$$

Normal Probability Density Function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \tag{6.2}$$

Converting to the Standard Normal Random Variable

$$z = \frac{x-\mu}{\sigma} \tag{6.3}$$

Exponential Probability Density Function

$$f(x) = \frac{1}{\mu} e^{-x/\mu} \qquad \text{for } x \ge 0 \tag{6.4}$$

Exponential Distribution: Cumulative Probabilities

$$P(x \le x_0) = 1 - e^{-x_0/\mu} \tag{6.5}$$

Supplementary Exercises

33. A business executive, transferred from Chicago to Atlanta, needs to sell her house in Chicago quickly. The executive's employer has offered to buy the house for $210,000, but the offer expires at the end of the week. The executive does not currently have a better offer but can afford to leave the house on the market for another month. From conversations with her realtor, the executive believes the price she will get by leaving the house on the market for another month is uniformly distributed between $200,000 and $225,000.
    a. If she leaves the house on the market for another month, what is the mathematical expression for the probability density function of the sales price?

b. If she leaves it on the market for another month, what is the probability that she will get at least $215,000 for the house?
c. If she leaves it on the market for another month, what is the probability that she will get less than $210,000?
d. Should the executive leave the house on the market for another month? Why or why not?

34. The U.S. Bureau of Labor Statistics reports that the average annual expenditure on food and drink for all families is $5700 (*Money,* December 2003). Assume that annual expenditure on food and drink is normally distributed and that the standard deviation is $1500.
a. What is the range of expenditures of the 10% of families with the lowest annual spending on food and drink?
b. What percentage of families spend more than $7000 annually on food and drink?
c. What is the range of expenditures for the 5% of families with the highest annual spending on food and drink?

35. Motorola used the normal distribution to determine the probability of defects and the number of defects expected in a production process. Assume a production process produces items with a mean weight of 10 ounces. Calculate the probability of a defect and the expected number of defects for a 1000-unit production run in the following situations.
a. The process standard deviation is .15, and the process control is set at plus or minus one standard deviation. Units with weights less than 9.85 or greater than 10.15 ounces will be classified as defects.
b. Through process design improvements, the process standard deviation can be reduced to .05. Assume the process control remains the same, with weights less than 9.85 or greater than 10.15 ounces being classified as defects.
c. What is the advantage of reducing process variation, thereby causing process control limits to be at a greater number of standard deviations from the mean?

36. The average annual amount American households spend for daily transportation is $6312 (*Money,* August 2001). Assume that the amount spent is normally distributed.
a. Suppose you learn that 5% of American households spend less than $1000 for daily transportation. What is the standard deviation of the amount spent?
b. What is the probability that a household spends between $4000 and $6000?
c. What is the range of spending for the 3% of households with the highest daily transportation cost?

37. *Condé Nast Traveler* publishes a Gold List of the top hotels all over the world. The Broadmoor Hotel in Colorado Springs contains 700 rooms and is on the 2004 Gold List (*Condé Nast Traveler,* January 2004). Suppose Broadmoor's marketing group forecasts a mean demand of 670 rooms for the coming weekend. Assume that demand for the upcoming weekend is normally distributed with a standard deviation of 30.
a. What is the probability that all the hotel's rooms will be rented?
b. What is the probability that 50 or more rooms will not be rented?
c. Would you recommend the hotel consider offering a promotion to increase demand? What considerations would be important?

38. Ward Doering Auto Sales is considering offering a special service contract that will cover the total cost of any service work required on leased vehicles. From experience, the company manager estimates that yearly service costs are approximately normally distributed, with a mean of $150 and a standard deviation of $25.
a. If the company offers the service contract to customers for a yearly charge of $200, what is the probability that any one customer's service costs will exceed the contract price of $200?
b. What is Ward's expected profit per service contract?

39. Is lack of sleep causing traffic fatalities? A study conducted under the auspices of the National Highway Traffic Safety Administration found that the average number of fatal crashes caused

by drowsy drivers each year was 1550 (*BusinessWeek,* January 26, 2004). Assume the annual number of fatal crashes per year is normally distributed with a standard deviation of 300.

a. What is the probability of fewer than 1000 fatal crashes in a year?
b. What is the probability that the number of fatal crashes will be between 1000 and 2000 for a year?
c. For a year to be in the upper 5% with respect to the number of fatal crashes, how many fatal crashes would have to occur?

40. Assume that the test scores from a college admissions test are normally distributed, with a mean of 450 and a standard deviation of 100.
    a. What percentage of the people taking the test score between 400 and 500?
    b. Suppose someone receives a score of 630. What percentage of the people taking the test score better? What percentage score worse?
    c. If a particular university will not admit anyone scoring below 480, what percentage of the persons taking the test would be acceptable to the university?

41. According to Salary Wizard, the average base salary for a brand manager in Houston, Texas, is $88,592 and the average base salary for a brand manager in Los Angeles, California, is $97,417 (Salary Wizard website, February 27, 2008). Assume that salaries are normally distributed, the standard deviation for brand managers in Houston is $19,900, and the standard deviation for brand managers in Los Angeles is $21,800.
    a. What is the probability that a brand manager in Houston has a base salary in excess of $100,000?
    b. What is the probability that a brand manager in Los Angeles has a base salary in excess of $100,000?
    c. What is the probability that a brand manager in Los Angeles has a base salary of less than $75,000?
    d. How much would a brand manager in Los Angeles have to make in order to have a higher salary than 99% of the brand managers in Houston?

42. A machine fills containers with a particular product. The standard deviation of filling weights is known from past data to be .6 ounce. If only 2% of the containers hold less than 18 ounces, what is the mean filling weight for the machine? That is, what must μ equal? Assume the filling weights have a normal distribution.

43. The Information Systems Audit and Control Association surveyed office workers to learn about the anticipated usage of office computers for personal holiday shopping (*USA Today,* November 11, 2009). Assume that the number of hours a worker spends doing holiday shopping on an office computer follows an exponential distribution.
    a. The study reported that there is a .53 probability that a worker uses the office computer for holiday shopping 5 hours or less. Is the mean time spent using an office computer for holiday shopping closest to 5.8, 6.2, 6.6, or 7 hours?
    b. Using the mean time from part (a), what is the probability that a worker uses the office computer for holiday shopping more than 10 hours?
    c. What is the probability that a worker uses the office computer for holiday shopping between four and eight hours?

44. A website for bed and breakfast inns gets approximately seven visitors per minute. Suppose the number of website visitors per minute follows a Poisson probability distribution.
    a. What is the mean time between visits to the website?
    b. Show the exponential probability density function for the time between website visits.
    c. What is the probability that no one will access the website in a 1-minute period?
    d. What is the probability that no one will access the website in a 12-second period?

45. The American Community Survey showed that residents of New York City have the longest travel times to get to work compared to residents of other cities in the United States (U.S. Census Bureau website, August 2008). According to the latest statistics available, the average travel time to work for residents of New York City is 38.3 minutes.

a. Assume the exponential probability distribution is applicable and show the probability density function for the travel time to work for a resident of this city.
b. What is the probability that it will take a resident of this city between 20 and 40 minutes to travel to work?
c. What is the probability that it will take a resident of this city more than one hour to travel to work?

46. The time (in minutes) between telephone calls at an insurance claims office has the following exponential probability distribution.

$$f(x) = .50e^{-.50x} \qquad \text{for } x \geq 0$$

a. What is the mean time between telephone calls?
b. What is the probability of having 30 seconds or less between telephone calls?
c. What is the probability of having 1 minute or less between telephone calls?
d. What is the probability of having 5 or more minutes without a telephone call?

Case Problem Specialty Toys

Specialty Toys, Inc., sells a variety of new and innovative children's toys. Management learned that the preholiday season is the best time to introduce a new toy, because many families use this time to look for new ideas for December holiday gifts. When Specialty discovers a new toy with good market potential, it chooses an October market entry date.

In order to get toys in its stores by October, Specialty places one-time orders with its manufacturers in June or July of each year. Demand for children's toys can be highly volatile. If a new toy catches on, a sense of shortage in the marketplace often increases the demand to high levels and large profits can be realized. However, new toys can also flop, leaving Specialty stuck with high levels of inventory that must be sold at reduced prices. The most important question the company faces is deciding how many units of a new toy should be purchased to meet anticipated sales demand. If too few are purchased, sales will be lost; if too many are purchased, profits will be reduced because of low prices realized in clearance sales.

For the coming season, Specialty plans to introduce a new product called Weather Teddy. This variation of a talking teddy bear is made by a company in Taiwan. When a child presses Teddy's hand, the bear begins to talk. A built-in barometer selects one of five responses that predict the weather conditions. The responses range from "It looks to be a very nice day! Have fun" to "I think it may rain today. Don't forget your umbrella." Tests with the product show that, even though it is not a perfect weather predictor, its predictions are surprisingly good. Several of Specialty's managers claimed Teddy gave predictions of the weather that were as good as many local television weather forecasters.

As with other products, Specialty faces the decision of how many Weather Teddy units to order for the coming holiday season. Members of the management team suggested order quantities of 15,000, 18,000, 24,000, or 28,000 units. The wide range of order quantities suggested indicates considerable disagreement concerning the market potential. The product management team asks you for an analysis of the stock-out probabilities for various order quantities, an estimate of the profit potential, and for help with making an order quantity recommendation. Specialty expects to sell Weather Teddy for $24 based on a cost of $16 per unit. If inventory remains after the holiday season, Specialty will sell all surplus inventory for $5 per unit. After reviewing the sales history of similar products, Specialty's senior sales forecaster predicted an expected demand of 20,000 units with a .95 probability that demand would be between 10,000 units and 30,000 units.

Managerial Report

Prepare a managerial report that addresses the following issues and recommends an order quantity for the Weather Teddy product.

1. Use the sales forecaster's prediction to describe a normal probability distribution that can be used to approximate the demand distribution. Sketch the distribution and show its mean and standard deviation.
2. Compute the probability of a stock-out for the order quantities suggested by members of the management team.
3. Compute the projected profit for the order quantities suggested by the management team under three scenarios: worst case in which sales = 10,000 units, most likely case in which sales = 20,000 units, and best case in which sales = 30,000 units.
4. One of Specialty's managers felt that the profit potential was so great that the order quantity should have a 70% chance of meeting demand and only a 30% chance of any stock-outs. What quantity would be ordered under this policy, and what is the projected profit under the three sales scenarios?
5. Provide your own recommendation for an order quantity and note the associated profit projections. Provide a rationale for your recommendation.

CHAPTER 7

Sampling and Sampling Distributions

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STATISTICS in PRACTICE

MEADWESTVACO CORPORATION*
STAMFORD, CONNECTICUT

MeadWestvaco Corporation, a leading producer of packaging, coated and specialty papers, consumer and office products, and specialty chemicals, employs more than 30,000 people. It operates worldwide in 29 countries and serves customers located in approximately 100 countries. MeadWestvaco holds a leading position in paper production, with an annual capacity of 1.8 million tons. The company's products include textbook paper, glossy magazine paper, beverage packaging systems, and office products. MeadWestvaco's internal consulting group uses sampling to provide a variety of information that enables the company to obtain significant productivity benefits and remain competitive.

For example, MeadWestvaco maintains large woodland holdings, which supply the trees, or raw material, for many of the company's products. Managers need reliable and accurate information about the timberlands and forests to evaluate the company's ability to meet its future raw material needs. What is the present volume in the forests? What is the past growth of the forests? What is the projected future growth of the forests? With answers to these important questions, MeadWestvaco's managers can develop plans for the future, including long-term planting and harvesting schedules for the trees.

How does MeadWestvaco obtain the information it needs about its vast forest holdings? Data collected from sample plots throughout the forests are the basis for learning about the population of trees owned by the company. To identify the sample plots, the timberland holdings are first divided into three sections based on location and types of trees. Using maps and random numbers, MeadWestvaco analysts identify random samples of 1/5- to 1/7-acre plots in each section of the forest.

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Random sampling of its forest holdings enables MeadWestvaco Corporation to meet future raw material needs.

MeadWestvaco foresters collect data from these sample plots to learn about the forest population.

Foresters throughout the organization participate in the field data collection process. Periodically, two-person teams gather information on each tree in every sample plot. The sample data are entered into the company's continuous forest inventory (CFI) computer system. Reports from the CFI system include a number of frequency distribution summaries containing statistics on types of trees, present forest volume, past forest growth rates, and projected future forest growth and volume. Sampling and the associated statistical summaries of the sample data provide the reports essential for the effective management of MeadWestvaco's forests and timberlands.

In this chapter you will learn about simple random sampling and the sample selection process. In addition, you will learn how statistics such as the sample mean and sample proportion are used to estimate the population mean and population proportion. The important concept of a sampling distribution is also introduced.

*The authors are indebted to Dr. Edward P. Winkofsky for providing this Statistics in Practice.

In Chapter 1 we presented the following definitions of an element, a population, and a sample.

- An *element* is the entity on which data are collected.
- A *population* is the collection of all the elements of interest.
- A *sample* is a subset of the population.

The reason we select a sample is to collect data to make an inference and answer a research question about a population.

Let us begin by citing two examples in which sampling was used to answer a research question about a population.

1. Members of a political party in Texas were considering supporting a particular candidate for election to the U.S. Senate, and party leaders wanted to estimate the proportion of registered voters in the state favoring the candidate. A sample of 400 registered voters in Texas was selected and 160 of the 400 voters indicated a preference for the candidate. Thus, an estimate of the proportion of the population of registered voters favoring the candidate is 160/400 = .40.
2. A tire manufacturer is considering producing a new tire designed to provide an increase in mileage over the firm's current line of tires. To estimate the mean useful life of the new tires, the manufacturer produced a sample of 120 tires for testing. The test results provided a sample mean of 36,500 miles. Hence, an estimate of the mean useful life for the population of new tires was 36,500 miles.

A sample mean provides an estimate of a population mean, and a sample proportion provides an estimate of a population proportion. With estimates such as these, some estimation error can be expected. This chapter provides the basis for determining how large that error might be.

It is important to realize that sample results provide only *estimates* of the values of the corresponding population characteristics. We do not expect exactly .40, or 40%, of the population of registered voters to favor the candidate, nor do we expect the sample mean of 36,500 miles to exactly equal the mean mileage for the population of all new tires produced. The reason is simply that the sample contains only a portion of the population. Some sampling error is to be expected. With proper sampling methods, the sample results will provide "good" estimates of the population parameters. But how good can we expect the sample results to be? Fortunately, statistical procedures are available for answering this question.

Let us define some of the terms used in sampling. The **sampled population** is the population from which the sample is drawn, and a **frame** is a list of the elements that the sample will be selected from. In the first example, the sampled population is all registered voters in Texas, and the frame is a list of all the registered voters. Because the number of registered voters in Texas is a finite number, the first example is an illustration of sampling from a finite population. In Section 7.2, we discuss how a simple random sample can be selected when sampling from a finite population.

The sampled population for the tire mileage example is more difficult to define because the sample of 120 tires was obtained from a production process at a particular point in time. We can think of the sampled population as the conceptual population of all the tires that could have been made by the production process at that particular point in time. In this sense the sampled population is considered infinite, making it impossible to construct a frame to draw the sample from. In Section 7.2, we discuss how to select a random sample in such a situation.

In this chapter, we show how simple random sampling can be used to select a sample from a finite population and describe how a random sample can be taken from an infinite population that is generated by an ongoing process. We then show how data obtained from a sample can be used to compute estimates of a population mean, a population standard deviation, and a population proportion. In addition, we introduce the important concept of a sampling distribution. As we will show, knowledge of the appropriate sampling distribution enables us to make statements about how close the sample estimates are to the corresponding population parameters. The last section discusses some alternatives to simple random sampling that are often employed in practice.

7.1 The Electronics Associates Sampling Problem

The director of personnel for Electronics Associates, Inc. (EAI), has been assigned the task of developing a profile of the company's 2500 managers. The characteristics to be identified include the mean annual salary for the managers and the proportion of managers having completed the company's management training program.

WEB file
EAI

Using the 2500 managers as the population for this study, we can find the annual salary and the training program status for each individual by referring to the firm's personnel records. The data set containing this information for all 2500 managers in the population is in the file named EAI.

Using the EAI data and the formulas presented in Chapter 3, we compute the population mean and the population standard deviation for the annual salary data.

$$\text{Population mean: } \mu = \$51{,}800$$
$$\text{Population standard deviation: } \sigma = \$4000$$

The data for the training program status show that 1500 of the 2500 managers completed the training program.

Numerical characteristics of a population are called **parameters**. Letting p denote the proportion of the population that completed the training program, we see that $p = 1500/2500 = .60$. The population mean annual salary ($\mu = \$51{,}800$), the population standard deviation of annual salary ($\sigma = \$4000$), and the population proportion that completed the training program ($p = .60$) are parameters of the population of EAI managers.

Often the cost of collecting information from a sample is substantially less than from a population, especially when personal interviews must be conducted to collect the information.

Now, suppose that the necessary information on all the EAI managers was not readily available in the company's database. The question we now consider is how the firm's director of personnel can obtain estimates of the population parameters by using a sample of managers rather than all 2500 managers in the population. Suppose that a sample of 30 managers will be used. Clearly, the time and the cost of developing a profile would be substantially less for 30 managers than for the entire population. If the personnel director could be assured that a sample of 30 managers would provide adequate information about the population of 2500 managers, working with a sample would be preferable to working with the entire population. Let us explore the possibility of using a sample for the EAI study by first considering how we can identify a sample of 30 managers.

7.2 Selecting a Sample

In this section we describe how to select a sample. We first describe how to sample from a finite population and then describe how to select a sample from an infinite population.

Sampling from a Finite Population

Other methods of probability sampling are described in Section 7.7.

Statisticians recommend selecting a probability sample when sampling from a finite population because a probability sample allows them to make valid statistical inferences about the population. The simplest type of probability sample is one in which each sample of size n has the same probability of being selected. It is called a simple random sample. A simple random sample of size n from a finite population of size N is defined as follows.

SIMPLE RANDOM SAMPLE (FINITE POPULATION)

A **simple random sample** of size n from a finite population of size N is a sample selected such that each possible sample of size n has the same probability of being selected.

The random numbers generated using Excel's RAND function follow a uniform probability distribution between 0 and 1.

The procedures used to select a simple random sample from a finite population are based upon the use of random numbers. We can use Excel's RAND function to generate a random number between 0 and 1 by entering the formula =RAND() into any cell in a worksheet. The number generated is called a random number because the mathematical procedure used

by the RAND function guarantees that every number between 0 and 1 has the same probability of being selected. Let us see how these random numbers can be used to select a simple random sample.

Our procedure for selecting a simple random sample of size n from a population of size N involves two steps.

Step 1. Assign a random number to each element of the population.
Step 2. Select the n elements corresponding to the n smallest random numbers.

Because each set of n elements in the population has the same probability of being assigned the n smallest random numbers, each set of n elements has the same probability of being selected for the sample. If we select the sample using this two-step procedure, every sample of size n has the same probability of being selected; thus, the sample selected satisfies the definition of a simple random sample.

Let us consider an example involving selecting a simple random sample of size $n = 5$ from a population of size $N = 16$. Table 7.1 contains a list of the 16 teams in the National Baseball League. Suppose we want to select a simple random sample of 5 teams to conduct in-depth interviews about how they manage their minor league franchises.

Step 1 of our simple random sampling procedure requires that we assign a random number to each of the 16 teams in the population. Figure 7.1 shows a worksheet used to generate a random number corresponding to each of the 16 teams in the population. The names of the baseball teams are in column A, and the random numbers generated are in column B. From the formula worksheet in the background we see that the formula =RAND() has been entered into cells B2:B17 to generate the random numbers between 0 and 1. From the value worksheet in the foreground we see that Arizona is assigned the random number .850862, Atlanta has been assigned the random number .706245, and so on.

The second step is to select the five teams corresponding to the five smallest random numbers as our sample. Looking through the random numbers in Figure 7.1, we see that the team corresponding to the smallest random number (.066942) is St. Louis, and that the four teams corresponding to the next four smallest random numbers are Washington, Houston, San Diego, and San Francisco. Thus, these five teams make up the simple random sample.

Searching through the list of random numbers in Figure 7.1 to find the five smallest random numbers is tedious, and it is easy to make mistakes. Excel's Sort procedure simplifies this step. We illustrate by sorting the list of baseball teams in Figure 7.1 to find the five teams corresponding to the five smallest random numbers. Refer to the foreground worksheet in Figure 7.1 as we describe the steps involved.

Step 1. Select any cell in the range B2:B17
Step 2. Click the **Home** tab on the Ribbon
Step 3. In the **Editing** group, click **Sort & Filter**
Step 4. Choose **Sort Smallest to Largest**

TABLE 7.1 NATIONAL BASEBALL LEAGUE TEAMS

Arizona	Milwaukee
Atlanta	New York
Chicago	Philadelphia
Cincinnati	Pittsburgh
Colorado	San Diego
Florida	San Francisco
Houston	St. Louis
Los Angeles	Washington

FIGURE 7.1 WORKSHEET USED TO GENERATE A RANDOM NUMBER CORRESPONDING TO EACH TEAM

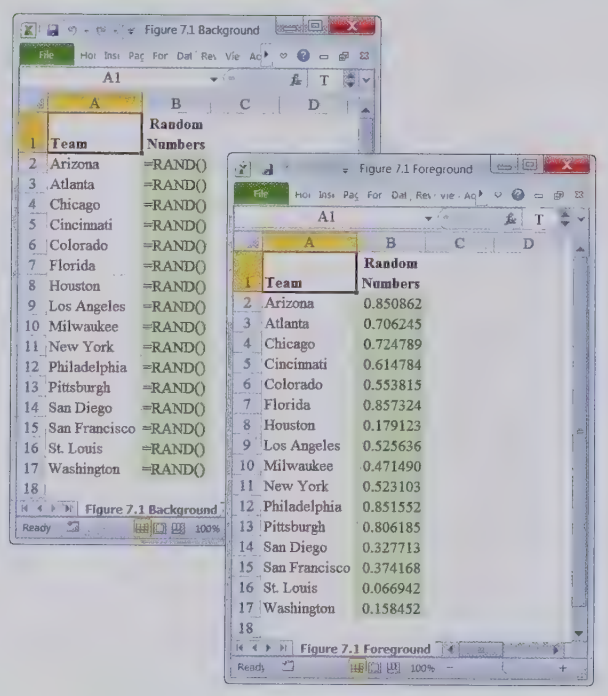


Figure 7.1 Background

	A	B	C	D
1	Team	Random Numbers		
2	Arizona	=RAND()		
3	Atlanta	=RAND()		
4	Chicago	=RAND()		
5	Cincinnati	=RAND()		
6	Colorado	=RAND()		
7	Florida	=RAND()		
8	Houston	=RAND()		
9	Los Angeles	=RAND()		
10	Milwaukee	=RAND()		
11	New York	=RAND()		
12	Philadelphia	=RAND()		
13	Pittsburgh	=RAND()		
14	San Diego	=RAND()		
15	San Francisco	=RAND()		
16	St. Louis	=RAND()		
17	Washington	=RAND()		
18				

Figure 7.1 Foreground

	A	B	C	D
1	Team	Random Numbers		
2	Arizona	0.850862		
3	Atlanta	0.706245		
4	Chicago	0.724789		
5	Cincinnati	0.614784		
6	Colorado	0.553815		
7	Florida	0.857324		
8	Houston	0.179123		
9	Los Angeles	0.525636		
10	Milwaukee	0.471490		
11	New York	0.523103		
12	Philadelphia	0.851552		
13	Pittsburgh	0.806185		
14	San Diego	0.327713		
15	San Francisco	0.374168		
16	St. Louis	0.066942		
17	Washington	0.158452		
18				

WEB file
National League

After completing these steps we obtain the worksheet shown in Figure 7.2.[1] The teams listed in rows 2–6 are the ones corresponding to the smallest five random numbers; they are our simple random sample. Note that the random numbers shown in Figure 7.2 are in ascending order, and that the teams are not in their original order. For instance, St. Louis is the next to last team listed in Figure 7.1, but it is the first team selected in the simple random sample. Washington, the second team in our sample, is the sixteenth team in the original list, and so on.

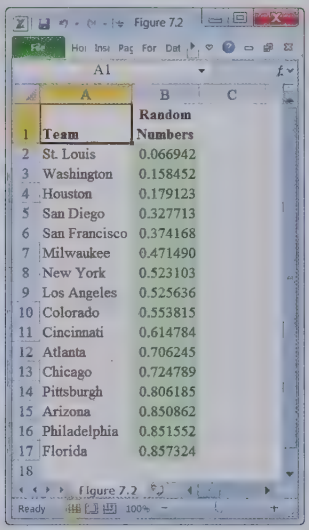
The Excel Sort procedure for identifying the managers associated with the 30 smallest random numbers is especially valuable with such a large population.

We now use this simple random sampling procedure to select a simple random sample of 30 EAI managers from the population of 2500 EAI managers. We begin by generating 2500 random numbers, one for each manager in the population. Then we select 30 managers corresponding to the 30 smallest random numbers as our sample. Refer to Figure 7.3 as we describe the steps involved.

Enter Data: The first three columns of the worksheet in the background show the annual salary data and training program status for the first 30 managers in the population of 2500 EAI managers. (The complete worksheet contains all 2500 managers.)

[1] In order to show the random numbers from Figure 7.1 in ascending order in this worksheet, we turned off the automatic recalculation option prior to sorting for illustrative purposes. If the recalculation option were not turned off, a new set of random numbers would have been generated when the sort was completed. But the same five teams would be selected.

FIGURE 7.2 USING EXCEL'S SORT PROCEDURE TO SELECT THE SIMPLE RANDOM SAMPLE OF FIVE TEAMS



	A	B	C
1	Team	Random Numbers	
2	St. Louis	0.066942	
3	Washington	0.158452	
4	Houston	0.179123	
5	San Diego	0.327713	
6	San Francisco	0.374168	
7	Milwaukee	0.471490	
8	New York	0.523103	
9	Los Angeles	0.525636	
10	Colorado	0.553815	
11	Cincinnati	0.614784	
12	Atlanta	0.706245	
13	Chicago	0.724789	
14	Pittsburgh	0.806185	
15	Arizona	0.850862	
16	Philadelphia	0.851552	
17	Florida	0.857324	
18			

Enter Functions and Formulas: In the background worksheet, the label **Random Numbers** has been entered into cell D1 and the formula =RAND() has been entered into cells D2:D2501 to generate a random number between 0 and 1 for each of the 2500 EAI managers. The random number generated for the first manager is 0.613872, the random number generated for the second manager is 0.473204, and so on.

Apply Tools: All that remains is to find the managers associated with the 30 smallest random numbers. To do so, we sort the data in columns A through D into ascending order by the random numbers in column D.

Step 1. Select any cell in the range D2:D2501
Step 2. Click the **Home** tab on the Ribbon
Step 3. In the **Editing** group, click **Sort & Filter**
Step 4. Choose **Sort Smallest to Largest**

After completing these steps we obtain the worksheet shown in the foreground of Figure 7.3. The managers listed in rows 2–31 are the ones corresponding to the smallest 30 random numbers that were generated. Hence, this group of 30 managers is a simple random sample. Note that the random numbers shown in the foreground of Figure 7.3 are in ascending order, and that the managers are not in their original order. For instance, manager 812 in the population is associated with the smallest random number and is the first element in the sample, and manager 13 in the population (see row 14 of the background worksheet) has been included as the 22nd observation in the sample (row 23 of the foreground worksheet).

Sampling from an Infinite Population

Sometimes we want to select a sample from a population, but the population is infinitely large or the elements of the population are being generated by an ongoing process for

FIGURE 7.3 USING EXCEL TO SELECT A SIMPLE RANDOM SAMPLE

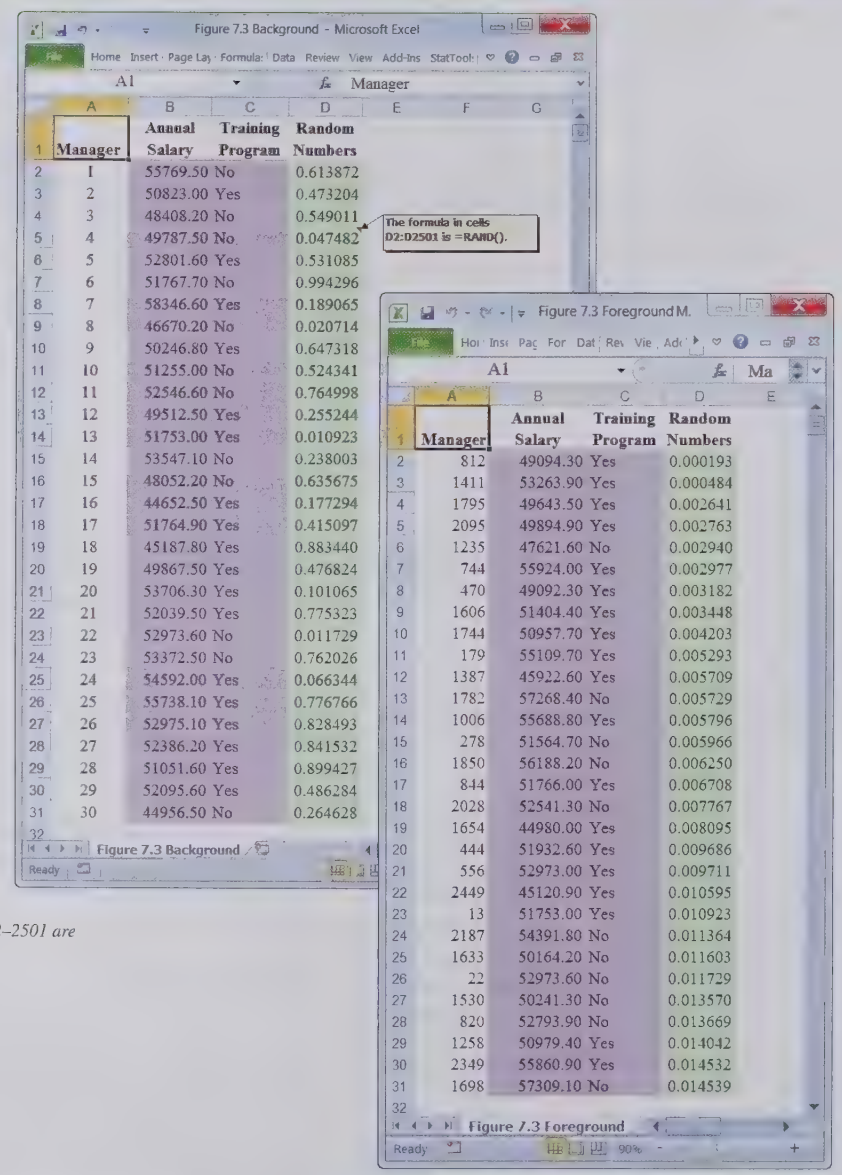


Figure 7.3 Background - Microsoft Excel

Manager	Annual Salary	Training Program	Random Numbers
1	55769.50	No	0.613872
2	50823.00	Yes	0.473204
3	48408.20	No	0.549011
4	49787.50	No	0.047482
5	52801.60	Yes	0.531085
6	51767.70	No	0.994296
7	58346.60	Yes	0.189065
8	46670.20	No	0.020714
9	50246.80	Yes	0.647318
10	51255.00	No	0.524341
11	52546.60	No	0.764998
12	49512.50	Yes	0.255244
13	51753.00	Yes	0.010923
14	53547.10	No	0.238003
15	48052.20	No	0.635675
16	44652.50	Yes	0.177294
17	51764.90	Yes	0.415097
18	45187.80	Yes	0.883440
19	49867.50	Yes	0.476824
20	53706.30	Yes	0.101065
21	52039.50	Yes	0.775323
22	52973.60	No	0.011729
23	53372.50	No	0.762026
24	54592.00	Yes	0.066344
25	55738.10	Yes	0.776766
26	52975.10	Yes	0.828493
27	52386.20	Yes	0.841532
28	51051.60	Yes	0.899427
29	52095.60	Yes	0.486284
30	44956.50	No	0.264628

The formula in cells D2:D2501 is =RAND().

Figure 7.3 Foreground M.

Manager	Annual Salary	Training Program	Random Numbers
812	49094.30	Yes	0.000193
1411	53263.90	Yes	0.000484
1795	49643.50	Yes	0.002641
2095	49894.90	Yes	0.002763
1235	47621.60	No	0.002940
744	55924.00	Yes	0.002977
470	49092.30	Yes	0.003182
1606	51404.40	Yes	0.003448
1744	50957.70	Yes	0.004203
179	55109.70	Yes	0.005293
1387	45922.60	Yes	0.005709
1782	57268.40	No	0.005729
1006	55688.80	Yes	0.005796
278	51564.70	No	0.005966
1850	56188.20	No	0.006250
844	51766.00	Yes	0.006708
2028	52541.30	No	0.007767
1654	44980.00	Yes	0.008095
444	51932.60	Yes	0.009686
556	52973.00	Yes	0.009711
2449	45120.90	Yes	0.010595
13	51753.00	Yes	0.010923
2187	54391.80	No	0.011364
1633	50164.20	No	0.011603
22	52973.60	No	0.011729
1530	50241.30	No	0.013570
820	52793.90	No	0.013669
1258	50979.40	Yes	0.014042
2349	55860.90	Yes	0.014532
1698	57309.10	No	0.014539

Note: Rows 32–2501 are not shown.

which there is no limit on the number of elements that can be generated. Thus, it is not possible to develop a list of all the elements in the population. This is considered the infinite population case. With an infinite population, we cannot select a simple random sample because we cannot construct a frame consisting of all the elements. In the infinite population case, statisticians recommend selecting what is called a random sample.

RANDOM SAMPLE (INFINITE POPULATION)

A **random sample** of size n from an infinite population is a sample selected such that the following conditions are satisfied.

1. Each element selected comes from the same population.
2. Each element is selected independently.

Care and judgment must be exercised in implementing the selection process for obtaining a random sample from an infinite population. Each case may require a different selection procedure. Let us consider two examples to see what we mean by the conditions (1) each element selected comes from the same population and (2) each element is selected independently.

A common quality control application involves a production process where there is no limit on the number of elements that can be produced. The conceptual population we are sampling from is all the elements that could be produced (not just the ones that are produced) by the ongoing production process. Because we cannot develop a list of all the elements that could be produced, the population is considered infinite. To be more specific, let us consider a production line designed to fill boxes of a breakfast cereal with a mean weight of 24 ounces of breakfast cereal per box. Samples of 12 boxes filled by this process are periodically selected by a quality control inspector to determine if the process is operating properly or if, perhaps, a machine malfunction has caused the process to begin underfilling or overfilling the boxes.

With a production operation such as this, the biggest concern in selecting a random sample is to make sure that condition 1, the sampled elements are selected from the same population, is satisfied. To ensure that this condition is satisfied, the boxes must be selected at approximately the same point in time. This way the inspector avoids the possibility of selecting some boxes when the process is operating properly and other boxes when the process is not operating properly and is underfilling or overfilling the boxes. With a production process such as this, the second condition, each element is selected independently, is satisfied by designing the production process so that each box of cereal is filled independently. With this assumption, the quality control inspector only needs to worry about satisfying the same population condition.

As another example of selecting a random sample from an infinite population, consider the population of customers arriving at a fast-food restaurant. Suppose an employee is asked to select and interview a sample of customers in order to develop a profile of customers who visit the restaurant. The customer arrival process is ongoing and there is no way to obtain a list of all customers in the population. So, for practical purposes, the population for this ongoing process is considered infinite. As long as a sampling procedure is designed so that all the elements in the sample are customers of the restaurant and they are selected independently, a random sample will be obtained. In this case, the employee collecting the sample needs to select the sample from people who come into the restaurant and make a purchase to ensure that the same population condition is satisfied. If, for instance, the employee selected someone for the sample who came into the restaurant just to use the restroom, that person would not be a customer and the same population condition would be violated. So, as long as the interviewer selects the sample from people making a purchase at the restaurant, condition 1 is satisfied. Ensuring that the customers are selected independently can be more difficult.

The purpose of the second condition of the random sample selection procedure (each element is selected independently) is to prevent selection bias. In this case, selection bias would occur if the interviewer were free to select customers for the sample arbitrarily. The interviewer might feel more comfortable selecting customers in a particular age group and

might avoid customers in other age groups. Selection bias would also occur if the interviewer selected a group of five customers who entered the restaurant together and asked all of them to participate in the sample. Such a group of customers would be likely to exhibit similar characteristics, which might provide misleading information about the population of customers. Selection bias such as this can be avoided by ensuring that the selection of a particular customer does not influence the selection of any other customer. In other words, the elements (customers) are selected independently.

McDonald's, the fast-food restaurant leader, implemented a random sampling procedure for this situation. The sampling procedure was based on the fact that some customers presented discount coupons. Whenever a customer presented a discount coupon, the next customer served was asked to complete a customer profile questionnaire. Because arriving customers presented discount coupons randomly and independently of other customers, this sampling procedure ensured that customers were selected independently. As a result, the sample satisfied the requirements of a random sample from an infinite population.

Situations involving sampling from an infinite population are usually associated with a process that operates over time. Examples include parts being manufactured on a production line, repeated experimental trials in a laboratory, transactions occurring at a bank, telephone calls arriving at a technical support center, and customers entering a retail store. In each case, the situation may be viewed as a process that generates elements from an infinite population. As long as the sampled elements are selected from the same population and are selected independently, the sample is considered a random sample from an infinite population.

NOTES AND COMMENTS

1. In this section we have been careful to define two types of samples: a simple random sample from a finite population and a random sample from an infinite population. In the remainder of the text, we will generally refer to both of these as either a *random sample* or simply a *sample*. We will not make a distinction of the sample being a "simple" random sample unless it is necessary for the exercise or discussion.
2. Statisticians who specialize in sample surveys from finite populations use sampling methods that provide probability samples. With a probability sample, each possible sample has a known probability of selection and a random process is used to select the elements for the sample. Simple random sampling is one of these methods. In Section 7.7, we describe some other probability sampling methods: stratified random sampling, cluster sampling, and systematic sampling. We use the term *simple* in simple random sampling to clarify that this is the probability sampling method that assures each sample of size n has the same probability of being selected.
3. The number of different simple random samples of size n that can be selected from a finite population of size N is

$$\frac{N!}{n!(N-n)!}$$

In this formula, $N!$ and $n!$ are the factorial formulas discussed in Chapter 4. For the EAI problem with $N = 2500$ and $n = 30$, this expression can be used to show that approximately 2.75×10^{69} different simple random samples of 30 EAI managers can be obtained.

Exercises

Methods

SELF test

1. Consider a finite population with five elements labeled A, B, C, D, and E. Ten possible simple random samples of size 2 can be selected.
   a. List the 10 samples beginning with AB, AC, and so on.
   b. Using simple random sampling, what is the probability that each sample of size 2 is selected?

c. Suppose we use Excel's RAND function to assign random numbers to the five elements: A (.7266), B (.0476), C (.2459), D (.0957), E (.9408). List the simple random sample of size 2 that will be selected by using these random numbers.

2. Assume a finite population has 10 elements. Number the elements from 1 to 10 and use the following 10 random numbers to select a sample of size 4.

.7545 .0936 .0341 .3242 .1449 .9060 .2420 .9773 .5428 .0729

SELF test

3. The American League consists of 14 baseball teams. Suppose a sample of 5 teams is to be selected to conduct player interviews. The following table lists the 14 teams and the random numbers assigned by Excel's RAND function. Use these random numbers to select a sample of size 5.

WEB file
American League

Team	Random Number	Team	Random Number
New York	0.178624	Boston	0.290197
Baltimore	0.578370	Tampa Bay	0.867778
Toronto	0.965807	Minnesota	0.811810
Chicago	0.562178	Cleveland	0.960271
Detroit	0.253574	Kansas City	0.326836
Oakland	0.288287	Anaheim	0.895267
Texas	0.500879	Seattle	0.839071

4. The 10 most active stocks on the New York Stock Exchange on March 6, 2006, are listed here (*The Wall Street Journal,* March 7, 2006).

AT&T	Lucent	Nortel	Qwest	Bell South
Pfizer	Texas Instruments	Gen. Elect.	iShrMsJpn	LSI Logic

Exchange authorities decided to sample three of these stocks to investigate trading practices. Select a simple random sample of three stocks for this investigation.

WEB file
EAI

5. In this section we used a two-step procedure to select a simple random sample of 30 EAI managers. Use this procedure to select a simple random sample of 50 EAI managers.

6. Indicate which of the following situations involve sampling from a finite population and which involve sampling from an infinite population. In cases where the sampled population is finite, describe how you would construct a frame.
   a. Select a sample of licensed drivers in the state of New York.
   b. Select a sample of boxes of cereal off the production line for the Breakfast Choice Company.
   c. Select a sample of cars crossing the Golden Gate Bridge on a typical weekday.
   d. Select a sample of students in a statistics course at Indiana University.
   e. Select a sample of the orders being processed by a mail-order firm.

7.3 Point Estimation

Now that we have described how to select a simple random sample, let us return to the EAI problem. A simple random sample of 30 managers and the corresponding data on annual salary and management training program participation are as shown in Table 7.2. The notation x_1, x_2, and so on is used to denote the annual salary of the first manager in the sample, the annual salary of the second manager in the sample, and so on. Participation in the management training program is indicated by Yes in the management training program column.

TABLE 7.2 ANNUAL SALARY AND TRAINING PROGRAM STATUS FOR A SIMPLE RANDOM SAMPLE OF 30 EAI MANAGERS

Annual Salary ($)	Management Training Program	Annual Salary ($)	Management Training Program
$x_1 = 49{,}094.30$	Yes	$x_{16} = 51{,}766.00$	Yes
$x_2 = 53{,}263.90$	Yes	$x_{17} = 52{,}541.30$	No
$x_3 = 49{,}643.50$	Yes	$x_{18} = 44{,}980.00$	Yes
$x_4 = 49{,}894.90$	Yes	$x_{19} = 51{,}932.60$	Yes
$x_5 = 47{,}621.60$	No	$x_{20} = 52{,}973.00$	Yes
$x_6 = 55{,}924.00$	Yes	$x_{21} = 45{,}120.90$	Yes
$x_7 = 49{,}092.30$	Yes	$x_{22} = 51{,}753.00$	Yes
$x_8 = 51{,}404.40$	Yes	$x_{23} = 54{,}391.80$	No
$x_9 = 50{,}957.70$	Yes	$x_{24} = 50{,}164.20$	No
$x_{10} = 55{,}109.70$	Yes	$x_{25} = 52{,}973.60$	No
$x_{11} = 45{,}922.60$	Yes	$x_{26} = 50{,}241.30$	No
$x_{12} = 57{,}268.40$	No	$x_{27} = 52{,}793.90$	No
$x_{13} = 55{,}688.80$	Yes	$x_{28} = 50{,}979.40$	Yes
$x_{14} = 51{,}564.70$	No	$x_{29} = 55{,}860.90$	Yes
$x_{15} = 56{,}188.20$	No	$x_{30} = 57{,}309.10$	No

To estimate the value of a population parameter, we compute a corresponding characteristic of the sample, referred to as a **sample statistic**. For example, to estimate the population mean μ and the population standard deviation σ for the annual salary of EAI managers, we use the data in Table 7.2 to calculate the corresponding sample statistics: the sample mean and the sample standard deviation s. Using the formulas for a sample mean and a sample standard deviation presented in Chapter 3, the sample mean is

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{1{,}554{,}420}{30} = \$51{,}814$$

and the sample standard deviation is

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n - 1}} = \sqrt{\frac{325{,}009{,}260}{29}} = \$3348$$

To estimate p, the proportion of managers in the population who completed the management training program, we use the corresponding sample proportion $\bar{p}$. Let x denote the number of managers in the sample who completed the management training program. The data in Table 7.2 show that $x = 19$. Thus, with a sample size of $n = 30$, the sample proportion is

$$\bar{p} = \frac{x}{n} = \frac{19}{30} = .63$$

By making the preceding computations, we perform the statistical procedure called *point estimation*. We refer to the sample mean $\bar{x}$ as the **point estimator** of the population mean μ, the sample standard deviation s as the point estimator of the population standard deviation σ, and the sample proportion $\bar{p}$ as the point estimator of the population proportion p. The numerical value obtained for $\bar{x}$, s, or $\bar{p}$ is called the **point estimate**. Thus, for the simple

TABLE 7.3 SUMMARY OF POINT ESTIMATES OBTAINED FROM A SIMPLE RANDOM SAMPLE OF 30 EAI MANAGERS

Population Parameter	Parameter Value	Point Estimator	Point Estimate
μ = Population mean annual salary	$51,800	$\bar{x}$ = Sample mean annual salary	$51,814
σ = Population standard deviation for annual salary	$4000	s = Sample standard deviation for annual salary	$3348
p = Population proportion having completed the management training program	.60	$\bar{p}$ = Sample proportion having completed the management training program	.63

random sample of 30 EAI managers shown in Table 7.2, $51,814 is the point estimate of μ, $3348 is the point estimate of σ, and .63 is the point estimate of p. Table 7.3 summarizes the sample results and compares the point estimates to the actual values of the population parameters.

As is evident from Table 7.3, the point estimates differ somewhat from the corresponding population parameters. This difference is to be expected because a sample, and not a census of the entire population, is being used to develop the point estimates. In the next chapter, we will show how to construct an interval estimate in order to provide information about how close the point estimate is to the population parameter.

Practical Advice

The subject matter of most of the rest of the book is concerned with statistical inference. Point estimation is a form of statistical inference. We use a sample statistic to make an inference about a population parameter. When making inferences about a population based on a sample, it is important to have a close correspondence between the sampled population and the target population. The **target population** is the population we want to make inferences about, while the sampled population is the population from which the sample is actually taken. In this section, we have described the process of drawing a simple random sample from the population of EAI managers and making point estimates of characteristics of that same population. So the sampled population and the target population are identical, which is the desired situation. But in other cases, it is not as easy to obtain a close correspondence between the sampled and target populations.

Consider the case of an amusement park selecting a sample of its customers to learn about characteristics such as age and time spent at the park. Suppose all the sample elements were selected on a day when park attendance was restricted to employees of a large company. Then the sampled population would be composed of employees of that company and members of their families. If the target population we wanted to make inferences about were typical park customers over a typical summer, then we might encounter a significant difference between the sampled population and the target population. In such a case, we would question the validity of the point estimates being made. Park management would be in the best position to know whether a sample taken on a particular day was likely to be representative of the target population.

In summary, whenever a sample is used to make inferences about a population, we should make sure that the study is designed so that the sampled population and the target population are in close agreement. Good judgment is a necessary ingredient of sound statistical practice.

Exercises

Methods

SELF test

7. The following data are from a simple random sample.

   5 8 10 7 10 14

   a. What is the point estimate of the population mean?
   b. What is the point estimate of the population standard deviation?

8. A survey question for a sample of 150 individuals yielded 75 Yes responses, 55 No responses, and 20 No Opinions.
   a. What is the point estimate of the proportion in the population who respond Yes?
   b. What is the point estimate of the proportion in the population who respond No?

Applications

SELF test

9. A simple random sample of 5 months of sales data provided the following information:

Month:	1	2	3	4	5
Units Sold:	94	100	85	94	92

   a. Develop a point estimate of the population mean number of units sold per month.
   b. Develop a point estimate of the population standard deviation.

WEB file
MutualFund

10. *BusinessWeek* published information on 283 equity mutual funds (*BusinessWeek,* January 26, 2004). A sample of 40 of those funds is contained in the data set MutualFund. Use the data set to answer the following questions.
    a. Develop a point estimate of the proportion of the *BusinessWeek* equity funds that are load funds.
    b. Develop a point estimate of the proportion of funds that are classified as high risk.
    c. Develop a point estimate of the proportion of funds that have a below-average risk rating.

11. Many drugs used to treat cancer are expensive. *BusinessWeek* reported on the cost per treatment of Herceptin, a drug used to treat breast cancer (*BusinessWeek,* January 30, 2006). Typical treatment costs (in dollars) for Herceptin are provided by a simple random sample of 10 patients.

4376	5578	2717	4920	4495
4798	6446	4119	4237	3814

    a. Develop a point estimate of the mean cost per treatment with Herceptin.
    b. Develop a point estimate of the standard deviation of the cost per treatment with Herceptin.

12. A sample of 50 *Fortune* 500 companies (*Fortune,* April 14, 2003) showed that 5 were based in New York, 6 in California, 2 in Minnesota, and 1 in Wisconsin.
    a. Develop an estimate of the proportion of *Fortune* 500 companies based in New York.
    b. Develop an estimate of the number of *Fortune* 500 companies based in Minnesota.
    c. Develop an estimate of the proportion of *Fortune* 500 companies that are not based in these four states.

13. The American Association of Individual Investors (AAII) polls its subscribers on a weekly basis to determine the number who are bullish, bearish, or neutral on the short-term prospects for the stock market. Its findings for the week ending March 2, 2006, are consistent with the following sample results (AAII website, March 7, 2006).

    Bullish 409 Neutral 299 Bearish 291

Develop a point estimate of the following population parameters.

a. The proportion of all AAII subscribers who are bullish on the stock market.
b. The proportion of all AAII subscribers who are neutral on the stock market.
c. The proportion of all AAII subscribers who are bearish on the stock market.

WEB file EAI

14. In this section we showed how a simple random sample of 30 EAI managers can be used to develop point estimates of the population mean annual salary, the population standard deviation for annual salary, and the population proportion having completed the management training program.
    a. Use Excel to select a simple random sample of 50 EAI managers.
    b. Develop a point estimate of the mean annual salary.
    c. Develop a point estimate of the population standard deviation for annual salary.
    d. Develop a point estimate of the population proportion having completed the management training program.

7.4 Introduction to Sampling Distributions

In the preceding section we said that the sample mean $\bar{x}$ is the point estimator of the population mean μ, and the sample proportion $\bar{p}$ is the point estimator of the population proportion p. For the simple random sample of 30 EAI managers shown in Table 7.2, the point estimate of μ is $\bar{x} = \$51{,}814$ and the point estimate of p is $\bar{p} = .63$. Suppose we select another simple random sample of 30 EAI managers and obtain the following point estimates:

$$\text{Sample mean: } \bar{x} = \$52{,}670$$
$$\text{Sample proportion: } \bar{p} = .70$$

Note that different values of $\bar{x}$ and $\bar{p}$ were obtained. Indeed, a second simple random sample of 30 EAI managers cannot be expected to provide the same point estimates as the first sample.

Now, suppose we repeat the process of selecting a simple random sample of 30 EAI managers over and over again, each time computing the values of $\bar{x}$ and $\bar{p}$. Table 7.4 contains a portion of the results obtained for 500 simple random samples, and Table 7.5 shows the frequency and relative frequency distributions for the 500 $\bar{x}$ values. Figure 7.4 shows the relative frequency histogram for the $\bar{x}$ values.

In Chapter 5 we defined a random variable as a numerical description of the outcome of an experiment. If we consider the process of selecting a simple random sample as an

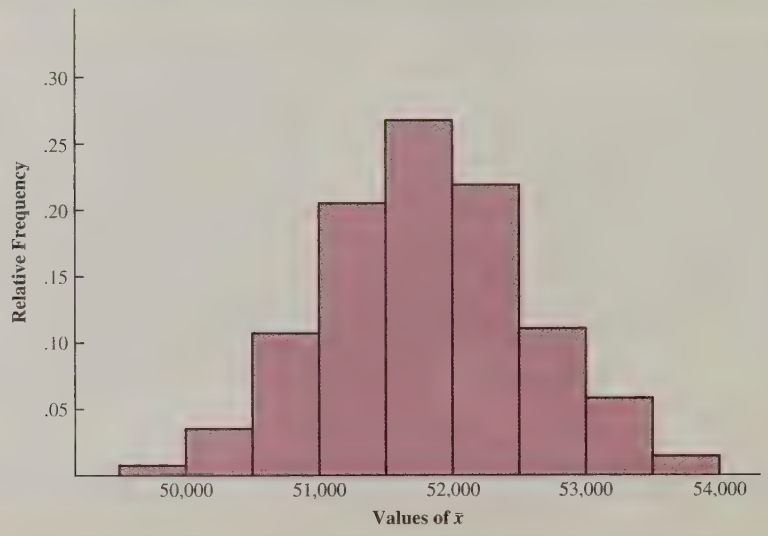
TABLE 7.4 VALUES OF $\bar{x}$ AND $\bar{p}$ FROM 500 SIMPLE RANDOM SAMPLES OF 30 EAI MANAGERS

Sample Number	Sample Mean ($\bar{x}$)	Sample Proportion ($\bar{p}$)
1	51,814	.63
2	52,670	.70
3	51,780	.67
4	51,588	.53
.	.	.
.	.	.
.	.	.
500	51,752	.50

TABLE 7.5 FREQUENCY AND RELATIVE FREQUENCY DISTRIBUTIONS OF $\bar{x}$ FROM 500 SIMPLE RANDOM SAMPLES OF 30 EAI MANAGERS

Mean Annual Salary ($)	Frequency	Relative Frequency
49,500.00–49,999.99	2	.004
50,000.00–50,499.99	16	.032
50,500.00–50,999.99	52	.104
51,000.00–51,499.99	101	.202
51,500.00–51,999.99	133	.266
52,000.00–52,499.99	110	.220
52,500.00–52,999.99	54	.108
53,000.00–53,499.99	26	.052
53,500.00–53,999.99	6	.012
	Totals 500	1.000

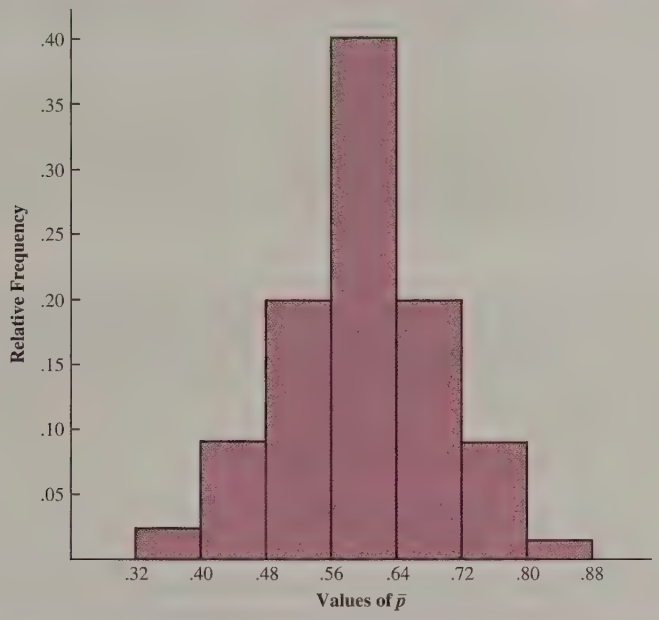
FIGURE 7.4 RELATIVE FREQUENCY HISTOGRAM OF $\bar{x}$ VALUES FROM 500 SIMPLE RANDOM SAMPLES OF SIZE 30 EACH



The ability to understand the material in subsequent chapters depends heavily on the ability to understand and use the sampling distributions presented in this chapter.

experiment, the sample mean $\bar{x}$ is the numerical description of the outcome of the experiment. Thus, the sample mean $\bar{x}$ is a random variable. As a result, just like other random variables, $\bar{x}$ has a mean or expected value, a standard deviation, and a probability distribution. Because the various possible values of $\bar{x}$ are the result of different simple random samples, the probability distribution of $\bar{x}$ is called the **sampling distribution** of $\bar{x}$. Knowledge of this sampling distribution and its properties will enable us to make probability statements about how close the sample mean $\bar{x}$ is to the population mean μ.

FIGURE 7.5 RELATIVE FREQUENCY HISTOGRAM OF $\bar{p}$ VALUES FROM 500 SIMPLE RANDOM SAMPLES OF SIZE 30 EACH



Let us return to Figure 7.4. We would need to enumerate every possible sample of 30 managers and compute each sample mean to completely determine the sampling distribution of $\bar{x}$. However, the histogram of 500 $\bar{x}$ values gives an approximation of this sampling distribution. From the approximation we observe the bell-shaped appearance of the distribution. We note that the largest concentration of the $\bar{x}$ values and the mean of the 500 $\bar{x}$ values is near the population mean $\mu = \$51{,}800$. We will describe the properties of the sampling distribution of $\bar{x}$ more fully in the next section.

The 500 values of the sample proportion $\bar{p}$ are summarized by the relative frequency histogram in Figure 7.5. As in the case of $\bar{x}$, $\bar{p}$ is a random variable. If every possible sample of size 30 were selected from the population and if a value of $\bar{p}$ were computed for each sample, the resulting probability distribution would be the sampling distribution of $\bar{p}$. The relative frequency histogram of the 500 sample values in Figure 7.5 provides a general idea of the appearance of the sampling distribution of $\bar{p}$.

In practice, we select only one simple random sample from the population. We repeated the sampling process 500 times in this section simply to illustrate that many different samples are possible and that the different samples generate a variety of values for the sample statistics $\bar{x}$ and $\bar{p}$. The probability distribution of any particular sample statistic is called the sampling distribution of the statistic. In Section 7.5 we discuss the characteristics of the sampling distribution of $\bar{x}$. In Section 7.6 we discuss the characteristics of the sampling distribution of $\bar{p}$.

7.5 Sampling Distribution of $\bar{x}$

In the previous section we said that the sample mean $\bar{x}$ is a random variable and its probability distribution is called the sampling distribution of $\bar{x}$.

SAMPLING DISTRIBUTION OF $\bar{x}$

The sampling distribution of $\bar{x}$ is the probability distribution of all possible values of the sample mean $\bar{x}$.

This section describes the properties of the sampling distribution of $\bar{x}$. Just as with other probability distributions we studied, the sampling distribution of $\bar{x}$ has an expected value or mean, a standard deviation, and a characteristic shape or form. Let us begin by considering the mean of all possible $\bar{x}$ values, which is referred to as the expected value of $\bar{x}$.

Expected Value of $\bar{x}$

In the EAI sampling problem we saw that different simple random samples result in a variety of values for the sample mean $\bar{x}$. Because many different values of the random variable $\bar{x}$ are possible, we are often interested in the mean of all possible values of $\bar{x}$ that can be generated by the various simple random samples. The mean of the $\bar{x}$ random variable is the expected value of $\bar{x}$. Let $E(\bar{x})$ represent the expected value of $\bar{x}$ and μ represent the mean of the population from which we are selecting a simple random sample. It can be shown that with simple random sampling, $E(\bar{x})$ and μ are equal.

The expected value of $\bar{x}$ equals the mean of the population from which the sample is selected.

EXPECTED VALUE OF $\bar{x}$

$$E(\bar{x}) = \mu \tag{7.1}$$

where

$E(\bar{x})$ = the expected value of $\bar{x}$
μ = the population mean

This result shows that with simple random sampling, the expected value or mean of the sampling distribution of $\bar{x}$ is equal to the mean of the population. In Section 7.1 we saw that the mean annual salary for the population of EAI managers is $\mu = \$51{,}800$. Thus, according to equation (7.1), the mean of all possible sample means for the EAI study is also $51,800.

When the expected value of a point estimator equals the population parameter, we say the point estimator is **unbiased**. Thus, equation (7.1) shows that $\bar{x}$ is an unbiased estimator of the population mean μ.

Standard Deviation of $\bar{x}$

Let us define the standard deviation of the sampling distribution of $\bar{x}$. We will use the following notation.

$\sigma_{\bar{x}}$ = the standard deviation of $\bar{x}$
σ = the standard deviation of the population
n = the sample size
N = the population size

It can be shown that the formula for the standard deviation of $\bar{x}$ depends on whether the population is finite or infinite. The two formulas for the standard deviation of $\bar{x}$ follow.

STANDARD DEVIATION OF $\bar{x}$

Finite Population	*Infinite Population*	
$\sigma_{\bar{x}} = \sqrt{\frac{N-n}{N-1}}\left(\frac{\sigma}{\sqrt{n}}\right)$	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$	**(7.2)**

In comparing the two formulas in (7.2), we see that the factor $\sqrt{(N-n)/(N-1)}$ is required for the finite population case but not for the infinite population case. This factor is commonly referred to as the **finite population correction factor**. In many practical sampling situations, we find that the population involved, although finite, is "large," whereas the sample size is relatively "small." In such cases the finite population correction factor $\sqrt{(N-n)/(N-1)}$ is close to 1. As a result, the difference between the values of the standard deviation of $\bar{x}$ for the finite and infinite population cases becomes negligible. Then, $\sigma_{\bar{x}} = \sigma/\sqrt{n}$ becomes a good approximation to the standard deviation of $\bar{x}$ even though the population is finite. This observation leads to the following general guideline, or rule of thumb, for computing the standard deviation of $\bar{x}$.

USE THE FOLLOWING EXPRESSION TO COMPUTE THE STANDARD DEVIATION OF $\bar{x}$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \qquad \textbf{(7.3)}$$

whenever

1. The population is infinite; or
2. The population is finite *and* the sample size is less than or equal to 5% of the population size; that is, $n/N \leq .05$.

Problem 17 shows that when $n/N \leq .05$, the finite population correction factor has little effect on the value of $\sigma_{\bar{x}}$.

In cases where $n/N > .05$, the finite population version of formula (7.2) should be used in the computation of $\sigma_{\bar{x}}$. Unless otherwise noted, throughout the text we will assume that the population size is "large," $n/N \leq .05$, and expression (7.3) can be used to compute $\sigma_{\bar{x}}$.

The term standard error *is used throughout statistical inference to refer to the standard deviation of a point estimator.*

To compute $\sigma_{\bar{x}}$, we need to know σ, the standard deviation of the population. To further emphasize the difference between $\sigma_{\bar{x}}$ and σ, we refer to the standard deviation of $\bar{x}$, $\sigma_{\bar{x}}$, as the **standard error** of the mean. In general, the term *standard error* refers to the standard deviation of a point estimator. Later we will see that the value of the standard error of the mean is helpful in determining how far the sample mean may be from the population mean. Let us now return to the EAI example and compute the standard error of the mean associated with simple random samples of 30 EAI managers.

In Section 7.1 we saw that the standard deviation of annual salary for the population of 2500 EAI managers is $\sigma = 4000$. In this case, the population is finite, with $N = 2500$. However, with a sample size of 30, we have $n/N = 30/2500 = .012$. Because the sample size is less than 5% of the population size, we can ignore the finite population correction factor and use equation (7.3) to compute the standard error.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4000}{\sqrt{30}} = 730.3$$

Form of the Sampling Distribution of $\bar{x}$

The preceding results concerning the expected value and standard deviation for the sampling distribution of $\bar{x}$ are applicable for any population. The final step in identifying the characteristics of the sampling distribution of $\bar{x}$ is to determine the form or shape of the sampling distribution. We will consider two cases: (1) The population has a normal distribution; and (2) the population does not have a normal distribution.

Population has a normal distribution In many situations it is reasonable to assume that the population from which we are selecting a random sample has a normal, or nearly normal, distribution. When the population has a normal distribution, the sampling distribution of $\bar{x}$ is normally distributed for any sample size.

Population does not have a normal distribution When the population from which we are selecting a random sample does not have a normal distribution, the **central limit theorem** is helpful in identifying the shape of the sampling distribution of $\bar{x}$. A statement of the central limit theorem as it applies to the sampling distribution of $\bar{x}$ follows.

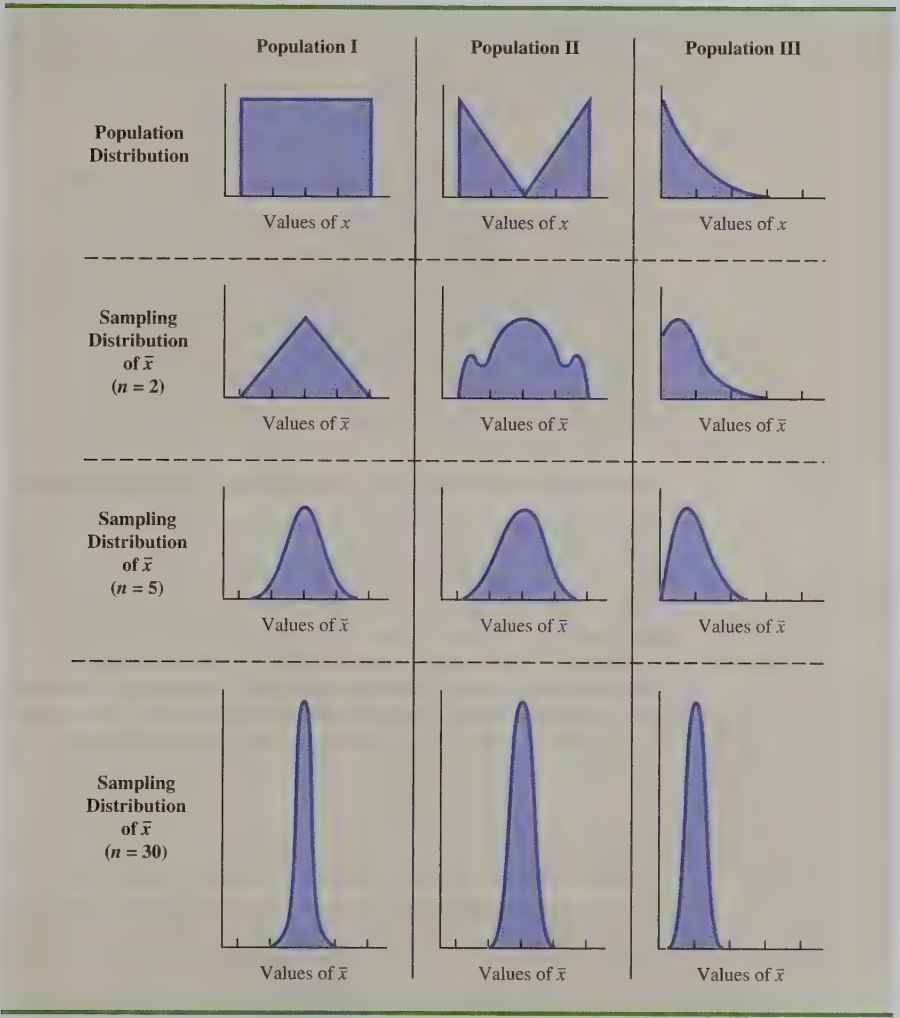
CENTRAL LIMIT THEOREM

In selecting random samples of size n from a population, the sampling distribution of the sample mean $\bar{x}$ can be approximated by a *normal distribution* as the sample size becomes large.

Figure 7.6 shows how the central limit theorem works for three different populations; each column refers to one of the populations. The top panel of the figure shows that none of the populations are normally distributed. Population I follows a uniform distribution. Population II is often called the rabbit-eared distribution. It is symmetric, but the more likely values fall in the tails of the distribution. Population III is shaped like the exponential distribution; it is skewed to the right.

The bottom three panels of Figure 7.6 show the shape of the sampling distribution for samples of size $n = 2$, $n = 5$, and $n = 30$. When the sample size is 2, we see that the shape of each sampling distribution is different from the shape of the corresponding population

FIGURE 7.6 ILLUSTRATION OF THE CENTRAL LIMIT THEOREM FOR THREE POPULATIONS



distribution. For samples of size 5, we see that the shapes of the sampling distributions for populations I and II begin to look similar to the shape of a normal distribution. Even though the shape of the sampling distribution for population III begins to look similar to the shape of a normal distribution, some skewness to the right is still present. Finally, for samples of size 30, the shapes of each of the three sampling distributions are approximately normal.

From a practitioner standpoint, we often want to know how large the sample size needs to be before the central limit theorem applies and we can assume that the shape of the sampling distribution is approximately normal. Statistical researchers have investigated this question

by studying the sampling distribution of $\bar{x}$ for a variety of populations and a variety of sample sizes. General statistical practice is to assume that, for most applications, the sampling distribution of $\bar{x}$ can be approximated by a normal distribution whenever the sample is size 30 or more. In cases where the population is highly skewed or outliers are present, samples of size 50 may be needed. Finally, if the population is discrete, the sample size needed for a normal approximation often depends on the population proportion. We say more about this issue when we discuss the sampling distribution of $\bar{p}$ in Section 7.6.

Sampling Distribution of $\bar{x}$ for the EAI Problem

Let us return to the EAI problem where we previously showed that $E(\bar{x}) = \$51{,}800$ and $\sigma_{\bar{x}} = 730.3$. At this point, we do not have any information about the population distribution; it may or may not be normally distributed. If the population has a normal distribution, the sampling distribution of $\bar{x}$ is normally distributed. If the population does not have a normal distribution, the simple random sample of 30 managers and the central limit theorem enable us to conclude that the sampling distribution of $\bar{x}$ can be approximated by a normal distribution. In either case, we are comfortable proceeding with the conclusion that the sampling distribution of $\bar{x}$ can be described by the normal distribution shown in Figure 7.7.

Practical Value of the Sampling Distribution of $\bar{x}$

Whenever a simple random sample is selected and the value of the sample mean is used to estimate the value of the population mean μ, we cannot expect the sample mean to exactly equal the population mean. The practical reason we are interested in the sampling distribution of $\bar{x}$ is that it can be used to provide probability information about the difference between the sample mean and the population mean. To demonstrate this use, let us return to the EAI problem.

Suppose the personnel director believes the sample mean will be an acceptable estimate of the population mean if the sample mean is within \$500 of the population mean. However, it is not possible to guarantee that the sample mean will be within \$500 of the population mean. Indeed, Table 7.5 and Figure 7.4 show that some of the 500 sample means

FIGURE 7.7 SAMPLING DISTRIBUTION OF $\bar{x}$ FOR THE MEAN ANNUAL SALARY OF A SIMPLE RANDOM SAMPLE OF 30 EAI MANAGERS

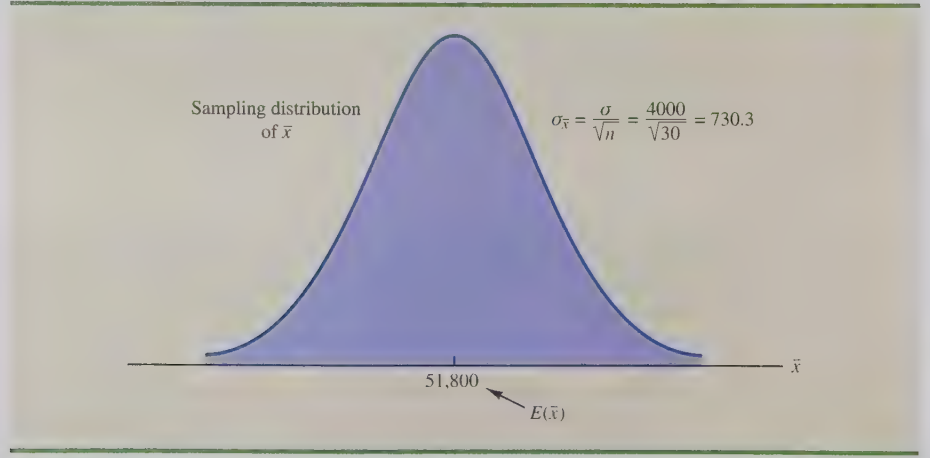
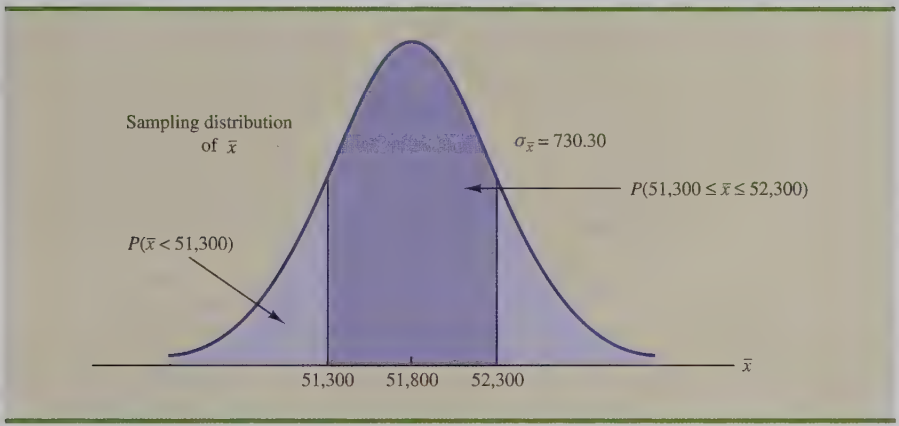


FIGURE 7.8 PROBABILITY OF A SAMPLE MEAN BEING WITHIN $500 OF THE POPULATION MEAN FOR A SIMPLE RANDOM SAMPLE OF 30 EAI MANAGERS



differed by more than $2000 from the population mean. So we must think of the personnel director's request in probability terms. That is, the personnel director is concerned with the following question: What is the probability that the sample mean computed using a simple random sample of 30 EAI managers will be within $500 of the population mean?

Because we have identified the properties of the sampling distribution of $\bar{x}$ (see Figure 7.7), we will use this distribution to answer the probability question. Refer to the sampling distribution of $\bar{x}$ shown again in Figure 7.8. With a population mean of $51,800, the personnel director wants to know the probability that $\bar{x}$ is between $51,300 and $52,300. This probability is given by the darkly shaded area of the sampling distribution shown in Figure 7.8. Because the sampling distribution is normally distributed, with mean 51,800 and standard error of the mean 730.3, we can use the standard normal probability table to find the area or probability.

We first calculate the z value at the upper endpoint of the interval (52,300) and use the table to find the cumulative probability at that point (left tail area). Then we compute the z value at the lower endpoint of the interval (51,300) and use the table to find the area under the curve to the left of that point (another left tail area). Subtracting the second tail area from the first gives us the desired probability.

At $\bar{x} = 52{,}300$, we have

$$z = \frac{52{,}300 - 51{,}800}{730.30} = .68$$

Referring to the standard normal probability table, we find a cumulative probability (area to the left of $z = .68$) of .7517.

At $\bar{x} = 51{,}300$, we have

$$z = \frac{51{,}300 - 51{,}800}{730.30} = -.68$$

The area under the curve to the left of $z = -.68$ is .2483. Therefore, $P(51{,}300 \le \bar{x} \le 52{,}300) = P(z \le .68) - P(z < -.68) = .7517 - .2483 = .5034$.

Using Excel's NORM.DIST function is easier and provides more accurate results than using the tables with rounded values for z.

The desired probability can also be computed using Excel's NORM.DIST function. The advantage of using the NORM.DIST function is that we do not have to make a separate computation of the z value. Evaluating the NORM.DIST function at the upper endpoint of the interval provides the cumulative probability at 52,300. Entering the formula =NORM.DIST(52300,51800,730.30,TRUE) into a cell of an Excel worksheet provides .7532 for this cumulative probability. Evaluating the NORM.DIST function at the lower endpoint of the interval provides the area under the curve to the left of 51,300. Entering the formula =NORM.DIST(51300,51800,730.30,TRUE) into a cell of an Excel worksheet provides .2468 for this cumulative probability. The probability of $\bar{x}$ being in the interval from 51,300 to 52,300 is then given by .7532 − .2468 = .5064. We note that this result is slightly different from the probability obtained using the table, because in using the normal table we rounded to two decimal places of accuracy when computing the z value. The result obtained using NORM.DIST is thus more accurate.

The sampling distribution of $\bar{x}$ can be used to provide probability information about how close the sample mean $\bar{x}$ is to the population mean μ.

The preceding computations show that a simple random sample of 30 EAI managers has a .5064 probability of providing a sample mean $\bar{x}$ that is within \$500 of the population mean. Thus, there is a 1 − .5064 = .4936 probability that the sampling error will be more than \$500. In other words, a simple random sample of 30 EAI managers has roughly a 50/50 chance of providing a sample mean within the allowable \$500. Perhaps a larger sample size should be considered. Let us explore this possibility by considering the relationship between the sample size and the sampling distribution of $\bar{x}$.

Relationship Between the Sample Size and the Sampling Distribution of $\bar{x}$

Suppose that in the EAI sampling problem we select a simple random sample of 100 EAI managers instead of the 30 originally considered. Intuitively, it would seem that with more data provided by the larger sample size, the sample mean based on $n = 100$ should provide a better estimate of the population mean than the sample mean based on $n = 30$. To see how much better, let us consider the relationship between the sample size and the sampling distribution of $\bar{x}$.

First note that $E(\bar{x}) = \mu$ regardless of the sample size. Thus, the mean of all possible values of $\bar{x}$ is equal to the population mean μ regardless of the sample size n. However, note that the standard error of the mean, $\sigma_{\bar{x}} = \sigma/\sqrt{n}$, is related to the square root of the sample size. Whenever the sample size is increased, the standard error of the mean $\sigma_{\bar{x}}$ decreases. With $n = 30$, the standard error of the mean for the EAI problem is 730.3. However, with the increase in the sample size to $n = 100$, the standard error of the mean is decreased to

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4000}{\sqrt{100}} = 400$$

The sampling distributions of $\bar{x}$ with $n = 30$ and $n = 100$ are shown in Figure 7.9. Because the sampling distribution with $n = 100$ has a smaller standard error, the values of $\bar{x}$ have less variation and tend to be closer to the population mean than the values of $\bar{x}$ with $n = 30$.

We can use the sampling distribution of $\bar{x}$ for the case with $n = 100$ to compute the probability that a simple random sample of 100 EAI managers will provide a sample mean that is within \$500 of the population mean. In this case the sampling distribution is normal with a mean of 51,800 and a standard deviation of 400 (see Figure 7.10). Again, we could compute the appropriate z values and use the standard normal probability distribution table to make this probability calculation. However, Excel's NORM.DIST function is easier to use and provides more accurate results. Entering the formula =NORM.DIST(52300,51800,400,TRUE) into a cell of an Excel worksheet provides the cumulative probability corresponding to $\bar{x} = 52{,}300$. The value provided by Excel is .8944. Entering the formula =NORM.DIST

FIGURE 7.9 A COMPARISON OF THE SAMPLING DISTRIBUTIONS OF $\bar{x}$ FOR SIMPLE RANDOM SAMPLES OF $n = 30$ AND $n = 100$ EAI MANAGERS

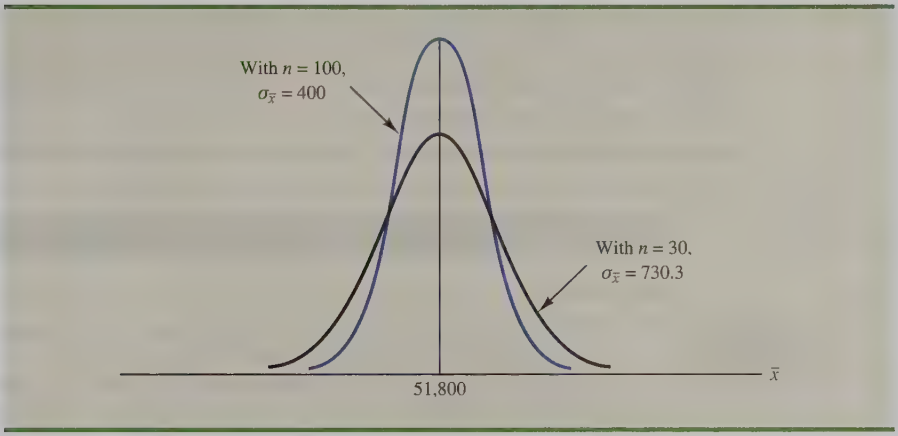
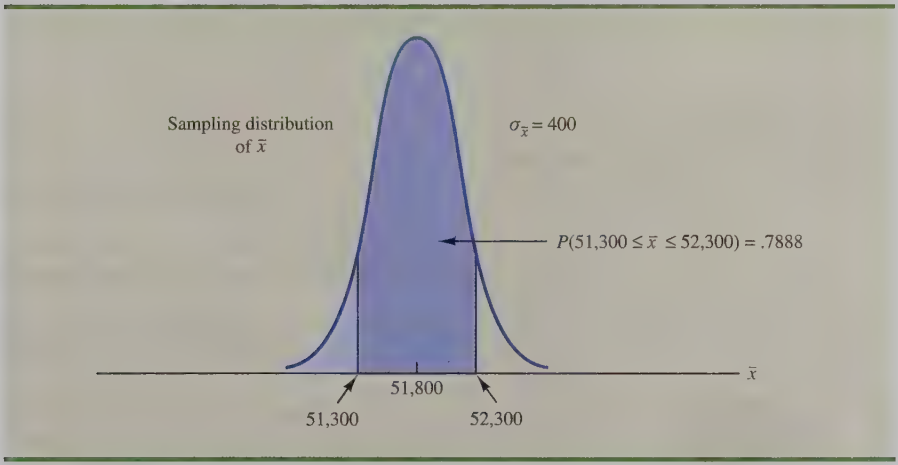


FIGURE 7.10 PROBABILITY OF A SAMPLE MEAN BEING WITHIN $500 OF THE POPULATION MEAN FOR A SIMPLE RANDOM SAMPLE OF 100 EAI MANAGERS



(51300,51800,400,TRUE) into a cell of an Excel worksheet provides the cumulative probability corresponding to $\bar{x} = 51{,}300$. The value provided by Excel is .1056. Thus, the probability of $\bar{x}$ being in the interval from 51,300 to 52,300 is given by .8944 − .1056 = .7888. By increasing the sample size from 30 to 100 EAI managers, we increase the probability that the sampling error will be $500 or less; that is, the probability of obtaining a sample mean within $500 of the population mean increases from .5064 to .7888.

The important point in this discussion is that as the sample size increases, the standard error of the mean decreases. As a result, a larger sample size will provide a higher probability that the sample mean falls within a specified distance of the population mean.

NOTES AND COMMENTS

In presenting the sampling distribution of $\bar{x}$ for the EAI problem, we took advantage of the fact that the population mean $\mu = 51{,}800$ and the population standard deviation $\sigma = 4000$ were known. However, usually the values of the population mean μ and the population standard deviation σ that are needed to determine the sampling distribution of $\bar{x}$ will be unknown. In Chapter 8 we show how the sample mean $\bar{x}$ and the sample standard deviation s are used when μ and σ are unknown.

Exercises

Methods

SELF test

15. A population has a mean of 200 and a standard deviation of 50. Suppose a simple random sample of size 100 is selected and $\bar{x}$ is used to estimate μ.
    a. What is the probability that the sample mean will be within ± 5 of the population mean?
    b. What is the probability that the sample mean will be within ± 10 of the population mean?

16. Assume the population standard deviation is $\sigma = 25$. Compute the standard error of the mean, $\sigma_{\bar{x}}$, for sample sizes of 50, 100, 150, and 200. What can you say about the size of the standard error of the mean as the sample size is increased?

17. Suppose a simple random sample of size 50 is selected from a population with $\sigma = 10$. Find the value of the standard error of the mean in each of the following cases (use the finite population correction factor if appropriate).
    a. The population size is infinite.
    b. The population size is $N = 50{,}000$.
    c. The population size is $N = 5000$.
    d. The population size is $N = 500$.

Applications

18. Refer to the EAI sampling problem. Suppose a simple random sample of 60 managers is used.
    a. Sketch the sampling distribution of $\bar{x}$ when simple random samples of size 60 are used.
    b. What happens to the sampling distribution of $\bar{x}$ if simple random samples of size 120 are used?
    c. What general statement can you make about what happens to the sampling distribution of $\bar{x}$ as the sample size is increased? Does this generalization seem logical? Explain.

SELF test

19. In the EAI sampling problem (see Figure 7.8), we showed that for $n = 30$, there was .5064 probability of obtaining a sample mean within $\pm \$500$ of the population mean.
    a. What is the probability that $\bar{x}$ is within \$500 of the population mean if a sample of size 60 is used?
    b. Answer part (a) for a sample of size 120.

20. *Barron's* reported that the average number of weeks an individual is unemployed is 17.5 weeks (*Barron's*, February 18, 2008). Assume that for the population of all unemployed individuals the population mean length of unemployment is 17.5 weeks and that the population standard deviation is 4 weeks. Suppose you would like to select a random sample of 50 unemployed individuals for a follow-up study.
    a. Show the sampling distribution of $\bar{x}$, the sample mean average for a sample of 50 unemployed individuals.
    b. What is the probability that a simple random sample of 50 unemployed individuals will provide a sample mean within 1 week of the population mean?

c. What is the probability that a simple random sample of 50 unemployed individuals will provide a sample mean within 1/2 week of the population mean?

21. The College Board reported the following mean scores for the three parts of the Scholastic Aptitude Test (SAT) (*The World Almanac*, 2009):

Critical Reading	502
Mathematics	515
Writing	494

Assume that the population standard deviation on each part of the test is $\sigma = 100$.

a. What is the probability that a random sample of 90 test takers will provide a sample mean test score within 10 points of the population mean of 502 on the Critical Reading part of the test?
b. What is the probability that a random sample of 90 test takers will provide a sample mean test score within 10 points of the population mean of 515 on the Mathematics part of the test? Compare this probability to the value computed in part (a).
c. What is the probability that a random sample of 100 test takers will provide a sample mean test score within 10 of the population mean of 494 on the writing part of the test? Comment on the differences between this probability and the values computed in parts (a) and (b).

22. The mean annual cost of automobile insurance is $939 (*CNBC,* February 23, 2006). Assume that the standard deviation is $\sigma = \$245$.
a. What is the probability that a simple random sample of automobile insurance policies will have a sample mean within $25 of the population mean for each of the following sample sizes: 30, 50, 100, and 400?
b. What is the advantage of a larger sample size when attempting to estimate the population mean?

23. *BusinessWeek* conducted a survey of graduates from 30 top MBA programs (*BusinessWeek,* September 22, 2003). On the basis of the survey, assume that the mean annual salary for male and female graduates 10 years after graduation is $168,000 and $117,000, respectively. Assume the standard deviation for the male graduates is $40,000, and for the female graduates it is $25,000.
a. What is the probability that a simple random sample of 40 male graduates will provide a sample mean within $10,000 of the population mean, $168,000?
b. What is the probability that a simple random sample of 40 female graduates will provide a sample mean within $10,000 of the population mean, $117,000?
c. In which of the preceding two cases, part (a) or part (b), do we have a higher probability of obtaining a sample estimate within $10,000 of the population mean? Why?
d. What is the probability that a simple random sample of 100 male graduates will provide a sample mean more than $4000 below the population mean?

24. The average score for male golfers is 95 and the average score for female golfers is 106 (*Golf Digest,* April 2006). Use these values as the population means for men and women and assume that the population standard deviation is $\sigma = 14$ strokes for both. A simple random sample of 30 male golfers and another simple random sample of 45 female golfers will be taken.
a. Show the sampling distribution of $\bar{x}$ for male golfers.
b. What is the probability that the sample mean is within three strokes of the population mean for the sample of male golfers?
c. What is the probability that the sample mean is within three strokes of the population mean for the sample of female golfers?
d. In which case, part (b) or part (c), is the probability of obtaining a sample mean within three strokes of the population mean higher? Why?

25. The average price of a gallon of unleaded regular gasoline was reported to be $2.34 in northern Kentucky (*The Cincinnati Enquirer,* January 21, 2006). Use this price as the population mean, and assume the population standard deviation is $.20.
    a. What is the probability that the mean price for a sample of 30 service stations is within $.03 of the population mean?
    b. What is the probability that the mean price for a sample of 50 service stations is within $.03 of the population mean?
    c. What is the probability that the mean price for a sample of 100 service stations is within $.03 of the population mean?
    d. Which, if any, of the sample sizes in parts (a), (b), and (c) would you recommend to have at least a .95 probability that the sample mean is within $.03 of the population mean?
26. To estimate the mean age for a population of 4000 employees, a simple random sample of 40 employees is selected.
    a. Would you use the finite population correction factor in calculating the standard error of the mean? Explain.
    b. If the population standard deviation is $\sigma = 8.2$ years, compute the standard error both with and without the finite population correction factor. What is the rationale for ignoring the finite population correction factor whenever $n/N \leq .05$?
    c. What is the probability that the sample mean age of the employees will be within ± 2 years of the population mean age?

7.6 Sampling Distribution of $\bar{p}$

The sample proportion $\bar{p}$ is the point estimator of the population proportion p. The formula for computing the sample proportion is

$$\bar{p} = \frac{x}{n}$$

where

x = the number of elements in the sample that possess the characteristic of interest

n = sample size

As noted in Section 7.4, the sample proportion $\bar{p}$ is a random variable and its probability distribution is called the sampling distribution of $\bar{p}$.

SAMPLING DISTRIBUTION OF $\bar{p}$

The sampling distribution of $\bar{p}$ is the probability distribution of all possible values of the sample proportion $\bar{p}$.

To determine how close the sample proportion $\bar{p}$ is to the population proportion p, we need to understand the properties of the sampling distribution of $\bar{p}$: the expected value of $\bar{p}$, the standard deviation of $\bar{p}$, and the shape or form of the sampling distribution of $\bar{p}$.

Expected Value of $\bar{p}$

The expected value of $\bar{p}$, the mean of all possible values of $\bar{p}$, is equal to the population proportion p.

EXPECTED VALUE OF $\bar{p}$

$$E(\bar{p}) = p \qquad (7.4)$$

where

$E(\bar{p})$ = the expected value of $\bar{p}$
p = the population proportion

Because $E(\bar{p}) = p$, $\bar{p}$ is an unbiased estimator of p. Recall from Section 7.1 we noted that $p = .60$ for the EAI population, where p is the proportion of the population of managers who participated in the company's management training program. Thus, the expected value of $\bar{p}$ for the EAI sampling problem is .60.

Standard Deviation of $\bar{p}$

Just as we found for the standard deviation of $\bar{x}$, the standard deviation of $\bar{p}$ depends on whether the population is finite or infinite. The two formulas for computing the standard deviation of $\bar{p}$ follow.

STANDARD DEVIATION OF $\bar{p}$

Finite Population

$$\sigma_{\bar{p}} = \sqrt{\frac{N-n}{N-1}}\sqrt{\frac{p(1-p)}{n}}$$

Infinite Population

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} \qquad (7.5)$$

Comparing the two formulas in (7.5), we see that the only difference is the use of the finite population correction factor $\sqrt{(N-n)/(N-1)}$.

As was the case with the sample mean $\bar{x}$, the difference between the expressions for the finite population and the infinite population becomes negligible if the size of the finite population is large in comparison to the sample size. We follow the same rule of thumb that we recommended for the sample mean. That is, if the population is finite with $n/N \leq .05$, we will use $\sigma_{\bar{p}} = \sqrt{p(1-p)/n}$. However, if the population is finite with $n/N > .05$, the finite population correction factor should be used. Again, unless specifically noted, throughout the text we will assume that the population size is large in relation to the sample size and thus the finite population correction factor is unnecessary.

In Section 7.5 we used the term *standard error of the mean* to refer to the standard deviation of $\bar{x}$. We stated that in general the term *standard error* refers to the standard deviation of a point estimator. Thus, for proportions we use *standard error of the proportion* to refer to the standard deviation of $\bar{p}$. Let us now return to the EAI example and compute the standard error of the proportion associated with simple random samples of 30 EAI managers.

For the EAI study we know that the population proportion of managers who participated in the management training program is $p = .60$. With $n/N = 30/2500 = .012$, we can ignore the finite population correction factor when we compute the standard error of the proportion. For the simple random sample of 30 managers, $\sigma_{\bar{p}}$ is

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.60(1-.60)}{30}} = .0894$$

Form of the Sampling Distribution of $\bar{p}$

Now that we know the mean and standard deviation of the sampling distribution of $\bar{p}$, the final step is to determine the form or shape of the sampling distribution. The sample proportion is $\bar{p} = x/n$. For a simple random sample from a large population, the value of x is a binomial random variable indicating the number of elements in the sample with the characteristic of interest. Because n is a constant, the probability of x/n is the same as the binomial probability of x, which means that the sampling distribution of $\bar{p}$ is also a discrete probability distribution and that the probability for each value of x/n is the same as the probability of x.

In Chapter 6 we also showed that a binomial distribution can be approximated by a normal distribution whenever the sample size is large enough to satisfy the following two conditions:

$$np \geq 5 \quad \text{and} \quad n(1 - p) \geq 5$$

Assuming these two conditions are satisfied, the probability distribution of x in the sample proportion, $\bar{p} = x/n$, can be approximated by a normal distribution. And because n is a constant, the sampling distribution of $\bar{p}$ can also be approximated by a normal distribution. This approximation is stated as follows:

> The sampling distribution of $\bar{p}$ can be approximated by a normal distribution whenever $np \geq 5$ and $n(1 - p) \geq 5$.

In practical applications, when an estimate of a population proportion is desired, we find that sample sizes are almost always large enough to permit the use of a normal approximation for the sampling distribution of $\bar{p}$.

Recall that for the EAI sampling problem we know that the population proportion of managers who participated in the training program is $p = .60$. With a simple random sample of size 30, we have $np = 30(.60) = 18$ and $n(1 - p) = 30(.40) = 12$. Thus, the sampling distribution of $\bar{p}$ can be approximated by a normal distribution shown in Figure 7.11.

FIGURE 7.11 SAMPLING DISTRIBUTION OF $\bar{p}$ FOR THE PROPORTION OF EAI MANAGERS WHO PARTICIPATED IN THE MANAGEMENT TRAINING PROGRAM

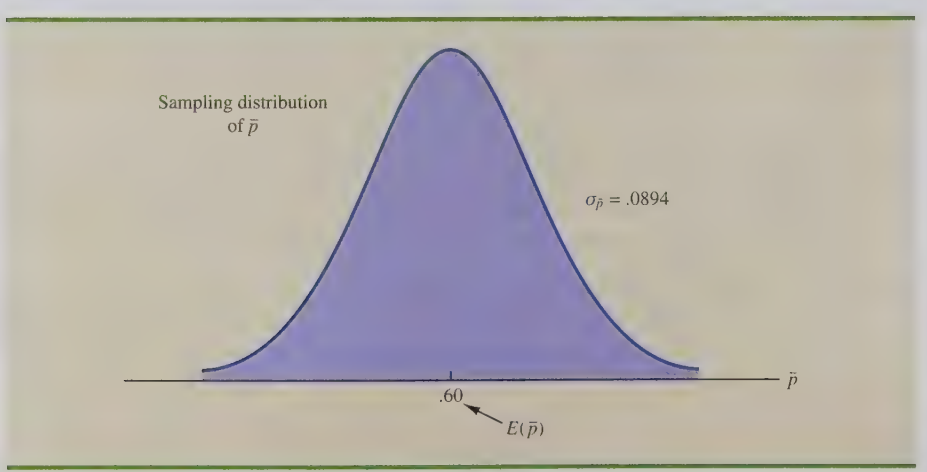
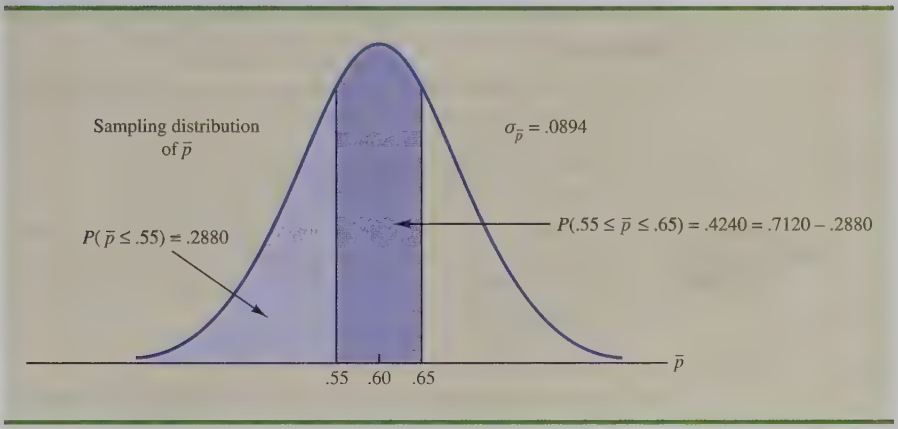


FIGURE 7.12 PROBABILITY OF OBTAINING $\bar{p}$ BETWEEN .55 AND .65



Practical Value of the Sampling Distribution of $\bar{p}$

The practical value of the sampling distribution of $\bar{p}$ is that it can be used to provide probability information about the difference between the sample proportion and the population proportion. For instance, suppose that in the EAI problem the personnel director wants to know the probability of obtaining a value of $\bar{p}$ that is within .05 of the population proportion of EAI managers who participated in the training program. That is, what is the probability of obtaining a sample with a sample proportion $\bar{p}$ between .55 and .65? The darkly shaded area in Figure 7.12 shows this probability. Using the fact that the sampling distribution of $\bar{p}$ can be approximated by a normal probability distribution with a mean of .60 and a standard error of $\sigma_{\bar{p}} = .0894$, we can use Excel's NORM.DIST function to make this calculation. Entering the formula =NORM.DIST(.65,.60,.0894,TRUE) into a cell of an Excel worksheet provides the cumulative probability corresponding to $\bar{p} = .65$. The value calculated by Excel is .7120. Entering the formula =NORM.DIST(.55,.60,.0894,TRUE) into a cell of an Excel worksheet provides the cumulative probability corresponding to $\bar{p} = .55$. The value calculated by Excel is .2880. Thus, the probability of $\bar{p}$ being in the interval from .55 to .65 is given by .7120 − .2880 = .4240.

If we consider increasing the sample size to $n = 100$, the standard error of the proportion becomes

$$\sigma_{\bar{p}} = \sqrt{\frac{.60(1 - .60)}{100}} = .0490$$

With a sample size of 100 EAI managers, the probability of the sample proportion having a value within .05 of the population proportion can now be computed. Because the sampling distribution is approximately normal, with mean .60 and standard deviation .0490, we can use Excel's NORM.DIST function to make this calculation. Entering the formula =NORM.DIST(.65,.60,.0490,TRUE) into a cell of an Excel worksheet provides the cumulative probability corresponding to $\bar{p} = .65$. The value calculated by Excel is .8462. Entering the formula =NORM.DIST(.55,.60,.0490,TRUE) into a cell of an Excel worksheet provides the cumulative probability corresponding to $\bar{p} = .55$. The value calculated by Excel is .1538. Thus, the probability of $\bar{p}$ being in the interval from .55 to .65 is given by .8462 − .1538 = .6924. Increasing the sample size increases the probability that the sampling error will be less than or equal to .05 by .2684 (from .4240 to .6924).

Exercises

Methods

27. A simple random sample of size 100 is selected from a population with $p = .40$.
    a. What is the expected value of $\bar{p}$?
    b. What is the standard error of $\bar{p}$?
    c. Show the sampling distribution of $\bar{p}$.
    d. What does the sampling distribution of $\bar{p}$ show?

SELF test

28. A population proportion is .40. A simple random sample of size 200 will be taken and the sample proportion $\bar{p}$ will be used to estimate the population proportion.
    a. What is the probability that the sample proportion will be within $\pm .03$ of the population proportion?
    b. What is the probability that the sample proportion will be within $\pm .05$ of the population proportion?

29. Assume that the population proportion is .55. Compute the standard error of the proportion, $\sigma_{\bar{p}}$, for sample sizes of 100, 200, 500, and 1000. What can you say about the size of the standard error of the proportion as the sample size is increased?

30. The population proportion is .30. What is the probability that a sample proportion will be within $\pm .04$ of the population proportion for each of the following sample sizes?
    a. $n = 100$
    b. $n = 200$
    c. $n = 500$
    d. $n = 1000$
    e. What is the advantage of a larger sample size?

Applications

SELF test

31. The president of Doerman Distributors, Inc., believes that 30% of the firm's orders come from first-time customers. A random sample of 100 orders will be used to estimate the proportion of first-time customers.
    a. Assume that the president is correct and $p = .30$. What is the sampling distribution of $\bar{p}$ for this study?
    b. What is the probability that the sample proportion $\bar{p}$ will be between .20 and .40?
    c. What is the probability that the sample proportion will be between .25 and .35?

32. *The Cincinnati Enquirer* reported that, in the United States, 66% of adults and 87% of youths ages 12 to 17 use the Internet (*The Cincinnati Enquirer,* February 7, 2006). Use the reported numbers as the population proportions and assume that samples of 300 adults and 300 youths will be used to learn about attitudes toward Internet security.
    a. Show the sampling distribution of $\bar{p}$, where $\bar{p}$ is the sample proportion of adults using the Internet.
    b. What is the probability that the sample proportion of adults using the Internet will be within $\pm .04$ of the population proportion?
    c. What is the probability that the sample proportion of youths using the Internet will be within $\pm .04$ of the population proportion?
    d. Is the probability different in parts (b) and (c)? If so, why?
    e. Answer part (b) for a sample of size 600. Is the probability smaller? Why?

33. People end up tossing 12% of what they buy at the grocery store (*Reader's Digest*, March, 2009). Assume this is the true population proportion and that you plan to take a sample survey of 540 grocery shoppers to further investigate their behavior.

a. Show the sampling distribution of $\bar{p}$, the proportion of groceries thrown out by your sample respondents.
b. What is the probability that your survey will provide a sample proportion within $\pm .03$ of the population proportion?
c. What is the probability that your survey will provide a sample proportion within $\pm .015$ of the population proportion?

34. Roper ASW conducted a survey to learn about American adults' attitudes toward money and happiness (*Money,* October 2003). Fifty-six percent of the respondents said they balance their checkbook at least once a month.
a. Suppose a sample of 400 American adults were taken. Show the sampling distribution of the proportion of adults who balance their checkbook at least once a month.
b. What is the probability that the sample proportion will be within $\pm .02$ of the population proportion?
c. What is the probability that the sample proportion will be within $\pm .04$ of the population proportion?

35. In 2008 the Better Business Bureau settled 75% of complaints it received (*USA Today*, March 2, 2009). Suppose you have been hired by the Better Business Bureau to investigate the complaints it received this year involving new car dealers. You plan to select a sample of new car dealer complaints to estimate the proportion of complaints the Better Business Bureau is able to settle. Assume the population proportion of complaints settled for new car dealers is .75, the same as the overall proportion of complaints settled in 2008.
a. Suppose you select a sample of 450 complaints involving new car dealers. Show the sampling distribution of $\bar{p}$.
b. Based upon a sample of 450 complaints, what is the probability that the sample proportion will be within .04 of the population proportion?
c. Suppose you select a sample of 200 complaints involving new car dealers. Show the sampling distribution of $\bar{p}$.
d. Based upon the smaller sample of only 200 complaints, what is the probability that the sample proportion will be within .04 of the population proportion?
e. As measured by the increase in probability, how much do you gain in precision by taking the larger sample in part (b)?

36. The Grocery Manufacturers of America reported that 76% of consumers read the ingredients listed on a product's label. Assume the population proportion is $p = .76$ and a sample of 400 consumers is selected from the population.
a. Show the sampling distribution of the sample proportion $\bar{p}$, where $\bar{p}$ is the proportion of the sampled consumers who read the ingredients listed on a product's label.
b. What is the probability that the sample proportion will be within $\pm .03$ of the population proportion?
c. Answer part (b) for a sample of 750 consumers.

37. The Food Marketing Institute shows that 17% of households spend more than $100 per week on groceries. Assume the population proportion is $p = .17$ and a simple random sample of 800 households will be selected from the population.
a. Show the sampling distribution of $\bar{p}$, the sample proportion of households spending more than $100 per week on groceries.
b. What is the probability that the sample proportion will be within $\pm .02$ of the population proportion?
c. Answer part (b) for a sample of 1600 households.

7.7 Other Sampling Methods

This section provides a brief introduction to survey sampling methods other than simple random sampling.

We described simple random sampling as a procedure for sampling from a finite population and discussed the properties of the sampling distributions of $\bar{x}$ and $\bar{p}$ when simple random sampling is used. Other methods such as stratified random sampling, cluster sampling, and systematic sampling provide advantages over simple random sampling in some of these situations. In this section we briefly introduce these alternative sampling methods.

Stratified Random Sampling

Stratified random sampling works best when the variance among elements in each stratum is relatively small.

In **stratified random sampling**, the elements in the population are first divided into groups called *strata*, such that each element in the population belongs to one and only one stratum. The basis for forming the strata, such as department, location, age, industry type, and so on, is at the discretion of the designer of the sample. However, the best results are obtained when the elements within each stratum are as much alike as possible. Figure 7.13 is a diagram of a population divided into H strata.

After the strata are formed, a simple random sample is taken from each stratum. Formulas are available for combining the results for the individual stratum samples into one estimate of the population parameter of interest. The value of stratified random sampling depends on how homogeneous the elements are within the strata. If elements within strata are alike, the strata will have low variances. Thus relatively small sample sizes can be used to obtain good estimates of the strata characteristics. If strata are homogeneous, the stratified random sampling procedure provides results just as precise as those of simple random sampling by using a smaller total sample size.

Cluster Sampling

Cluster sampling works best when each cluster provides a small-scale representation of the population.

In **cluster sampling**, the elements in the population are first divided into separate groups called *clusters*. Each element of the population belongs to one and only one cluster (see Figure 7.14). A simple random sample of the clusters is then taken. All elements within each sampled cluster form the sample. Cluster sampling tends to provide the best results when the elements within the clusters are not alike. In the ideal case, each cluster is a representative small-scale version of the entire population. The value of cluster sampling depends on how representative each cluster is of the entire population. If all clusters are alike in this regard, sampling a small number of clusters will provide good estimates of the population parameters.

FIGURE 7.13 DIAGRAM FOR STRATIFIED RANDOM SAMPLING

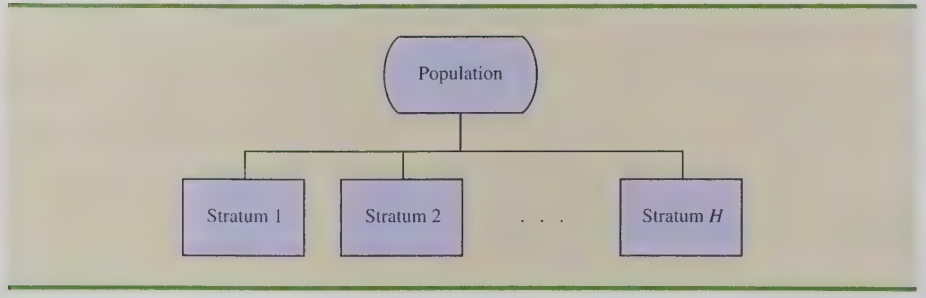
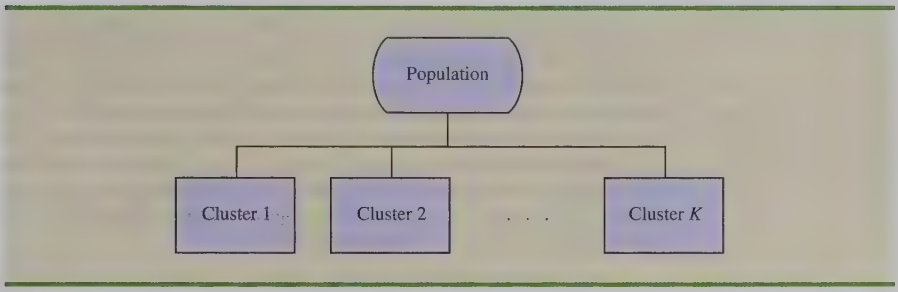


FIGURE 7.14 DIAGRAM FOR CLUSTER SAMPLING



One of the primary applications of cluster sampling is area sampling, where clusters are city blocks or other well-defined areas. Cluster sampling generally requires a larger total sample size than either simple random sampling or stratified random sampling. However, it can result in cost savings because of the fact that when an interviewer is sent to a sampled cluster (e.g., a city-block location), many sample observations can be obtained in a relatively short time. Hence, a larger sample size may be obtainable with a significantly lower total cost.

Systematic Sampling

In some sampling situations, especially those with large populations, it is time-consuming to select a simple random sample by first finding a random number and then counting or searching through the list of the population until the corresponding element is found. An alternative to simple random sampling is **systematic sampling**. For example, if a sample size of 50 is desired from a population containing 5000 elements, we will sample one element for every 5000/50 = 100 elements in the population. A systematic sample for this case involves selecting randomly one of the first 100 elements from the population list. Other sample elements are identified by starting with the first sampled element and then selecting every 100th element that follows in the population list. In effect, the sample of 50 is identified by moving systematically through the population and identifying every 100th element after the first randomly selected element. The sample of 50 usually will be easier to identify in this way than it would be if simple random sampling were used. Because the first element selected is a random choice, a systematic sample is usually assumed to have the properties of a simple random sample. This assumption is especially applicable when the list of elements in the population is a random ordering of the elements.

Convenience Sampling

The sampling methods discussed thus far are referred to as *probability sampling* techniques. Elements selected from the population have a known probability of being included in the sample. The advantage of probability sampling is that the sampling distribution of the appropriate sample statistic generally can be identified. Formulas such as the ones for simple random sampling presented in this chapter can be used to determine the properties of the sampling distribution. Then the sampling distribution can be used to make probability

statements about the error associated with using the sample results to make inferences about the population.

Convenience sampling is a *nonprobability sampling* technique. As the name implies, the sample is identified primarily by convenience. Elements are included in the sample without prespecified or known probabilities of being selected. For example, a professor conducting research at a university may use student volunteers to constitute a sample simply because they are readily available and will participate as subjects for little or no cost. Similarly, an inspector may sample a shipment of oranges by selecting oranges haphazardly from among several crates. Labeling each orange and using a probability method of sampling would be impractical. Samples such as wildlife captures and volunteer panels for consumer research are also convenience samples.

Convenience samples have the advantage of relatively easy sample selection and data collection; however, it is impossible to evaluate the "goodness" of the sample in terms of its representativeness of the population. A convenience sample may provide good results or it may not; no statistically justified procedure allows a probability analysis and inference about the quality of the sample results. Sometimes researchers apply statistical methods designed for probability samples to a convenience sample, arguing that the convenience sample can be treated as though it were a probability sample. However, this argument cannot be supported, and we should be cautious in interpreting the results of convenience samples that are used to make inferences about populations.

Judgment Sampling

One additional nonprobability sampling technique is **judgment sampling**. In this approach, the person most knowledgeable on the subject of the study selects elements of the population that he or she feels are most representative of the population. Often this method is a relatively easy way of selecting a sample. For example, a reporter may sample two or three senators, judging that those senators reflect the general opinion of all senators. However, the quality of the sample results depends on the judgment of the person selecting the sample. Again, great caution is warranted in drawing conclusions based on judgment samples used to make inferences about populations.

NOTES AND COMMENTS

We recommend using probability sampling methods when sampling from finite populations: simple random sampling, stratified random sampling, cluster sampling, or systematic sampling. For these methods, formulas are available for evaluating the "goodness" of the sample results in terms of the closeness of the results to the population parameters being estimated. An evaluation of the goodness cannot be made with convenience or judgment sampling. Thus, great care should be used in interpreting the results based on nonprobability sampling methods.

Summary

In this chapter we presented the concepts of sampling and sampling distributions. We demonstrated how a simple random sample can be selected from a finite population and how a random sample can be selected from an infinite population. The data collected from such samples can be used to develop point estimates of population parameters. Because

different samples provide different values for the point estimators, point estimators such as $\bar{x}$ and $\bar{p}$ are random variables. The probability distribution of such a random variable is called a sampling distribution. In particular, we described in detail the sampling distributions of the sample mean $\bar{x}$ and the sample proportion $\bar{p}$.

In considering the characteristics of the sampling distributions of $\bar{x}$ and $\bar{p}$, we stated that $E(\bar{x}) = \mu$ and $E(\bar{p}) = p$. After developing the standard deviation or standard error formulas for these estimators, we described the conditions necessary for the sampling distributions of $\bar{x}$ and $\bar{p}$ to follow a normal distribution. Other sampling methods including stratified random sampling, cluster sampling, systematic sampling, convenience sampling, and judgment sampling were discussed.

Glossary

Sampled population The population from which the sample is taken.
Frame A listing of the elements the sample will be selected from.
Parameter A numerical characteristic of a population, such as a population mean μ, a population standard deviation σ, a population proportion p, and so on.
Simple random sample A simple random sample of size n from a finite population of size N is a sample selected such that each possible sample of size n has the same probability of being selected.
Random sample A random sample from an infinite population is a sample selected such that the following conditions are satisfied: (1) Each element selected comes from the same population; (2) each element is selected independently.
Sample statistic A sample characteristic, such as a sample mean $\bar{x}$, a sample standard deviation s, a sample proportion $\bar{p}$, and so on. The value of the sample statistic is used to estimate the value of the corresponding population parameter.
Point estimator The sample statistic, such as $\bar{x}$, s, or $\bar{p}$, that provides the point estimate of the population parameter.
Point estimate The value of a point estimator used in a particular instance as an estimate of a population parameter.
Target population The population for which statistical inference such as point estimates are made. It is important for the target population to correspond as closely as possible to the sampled population.
Sampling distribution A probability distribution consisting of all possible values of a sample statistic.
Unbiased A property of a point estimator that is present when the expected value of the point estimator is equal to the population parameter it estimates.
Finite population correction factor The term $\sqrt{(N-n)/(N-1)}$ that is used in the formulas for $\sigma_{\bar{x}}$ and $\sigma_{\bar{p}}$ whenever a finite population, rather than an infinite population, is being sampled. The generally accepted rule of thumb is to ignore the finite population correction factor whenever $n/N \leq .05$.
Standard error The standard deviation of a point estimator.
Central limit theorem A theorem that enables one to use the normal probability distribution to approximate the sampling distribution of $\bar{x}$ whenever the sample size is large.
Stratified random sampling A probability sampling method in which the population is first divided into strata and a simple random sample is then taken from each stratum.
Cluster sampling A probability sampling method in which the population is first divided into clusters and then a simple random sample of the clusters is taken.

Systematic sampling A probability sampling method in which we randomly select one of the first k elements and then select every kth element thereafter.

Convenience sampling A nonprobability method of sampling whereby elements are selected for the sample on the basis of convenience.

Judgment sampling A nonprobability method of sampling whereby elements are selected for the sample based on the judgment of the person doing the study.

Key Formulas

Expected Value of $\bar{x}$

$$E(\bar{x}) = \mu \tag{7.1}$$

Standard Deviation of $\bar{x}$ (Standard Error)

Finite Population

$$\sigma_{\bar{x}} = \sqrt{\frac{N-n}{N-1}}\left(\frac{\sigma}{\sqrt{n}}\right)$$

Infinite Population

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \tag{7.2}$$

Expected Value of $\bar{p}$

$$E(\bar{p}) = p \tag{7.4}$$

Standard Deviation of $\bar{p}$ (Standard Error)

Finite Population

$$\sigma_{\bar{p}} = \sqrt{\frac{N-n}{N-1}}\sqrt{\frac{p(1-p)}{n}}$$

Infinite Population

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} \tag{7.5}$$

Supplementary Exercises

WEB file
Dining

38. Bob Miller, a food critic, wants to prepare an article on the quality of food at locally owned restaurants in Myrtle Beach, South Carolina. A list of 44 locally owned restaurants is contained in the data file named Dining that is on the website that accompanies this text (*Coastal Carolina Dining*, Fall 2004). Bob only has time to sample the food at 5 of these restaurants.
    a. In the data file the restaurants are listed in column A of an Excel worksheet. In column B we generated a random number for each of the restaurants in column A. Use these random numbers to select a simple random sample of five restaurants for Bob.
    b. Generate a new set of random numbers and use them to select a new simple random sample. Did you select the same restaurants?
39. Americans have become increasingly concerned about the rising cost of Medicare. In 1990, the average annual Medicare spending per enrollee was $3267; in 2003, the average

annual Medicare spending per enrollee was $6883 (*Money,* Fall 2003). Suppose you hired a consulting firm to take a sample of fifty 2003 Medicare enrollees to further investigate the nature of expenditures. Assume the population standard deviation for 2003 was $2000.

a. Show the sampling distribution of the mean amount of Medicare spending for a sample of fifty 2003 enrollees.
b. What is the probability that the sample mean will be within ±$300 of the population mean?
c. What is the probability that the sample mean will be greater than $7500? If the consulting firm tells you the sample mean for the Medicare enrollees it interviewed was $7500, would you question whether the firm followed correct simple random sampling procedures? Why or why not?

40. *BusinessWeek* surveyed MBA alumni 10 years after graduation (*BusinessWeek,* September 22, 2003). One finding was that alumni spend an average of $115.50 per week eating out socially. You have been asked to conduct a follow-up study by taking a sample of 40 of these MBA alumni. Assume the population standard deviation is $35.
    a. Show the sampling distribution of $\bar{x}$, the sample mean weekly expenditure for the 40 MBA alumni.
    b. What is the probability that the sample mean will be within $10 of the population mean?
    c. Suppose you find a sample mean of $100. What is the probability of finding a sample mean of $100 or less? Would you consider this sample to be an unusually low spending group of alumni? Why or why not?

41. The mean television viewing time for Americans is 15 hours per week (*Money,* November 2003). Suppose a sample of 60 Americans is taken to further investigate viewing habits. Assume the population standard deviation for weekly viewing time is $\sigma = 4$ hours.
    a. What is the probability that the sample mean will be within 1 hour of the population mean?
    b. What is the probability that the sample mean will be within 45 minutes of the population mean?

42. After deducting grants based on need, the average cost to attend the University of Southern California (USC) is $27,175 (*U.S. News & World Report, America's Best Colleges,* 2009 ed.). Assume the population standard deviation is $7400. Suppose that a random sample of 60 USC students will be taken from this population.
    a. What is the value of the standard error of the mean?
    b. What is the probability that the sample mean will be more than $27,175?
    c. What is the probability that the sample mean will be within $1000 of the population mean?
    d. How would the probability in part (c) change if the sample size were increased to 100?

43. Three firms carry inventories that differ in size. Firm A's inventory contains 2000 items, firm B's inventory contains 5000 items, and firm C's inventory contains 10,000 items. The population standard deviation for the cost of the items in each firm's inventory is $\sigma = 144$. A statistical consultant recommends that each firm take a sample of 50 items from its inventory to provide statistically valid estimates of the average cost per item. Managers of the small firm state that because it has the smallest population, it should be able to make the estimate from a much smaller sample than that required by the larger firms. However, the consultant states that to obtain the same standard error and thus the same precision in the sample results, all firms should use the same sample size regardless of population size.
    a. Using the finite population correction factor, compute the standard error for each of the three firms given a sample of size 50.
    b. What is the probability that for each firm the sample mean $\bar{x}$ will be within ±25 of the population mean μ?

44. A researcher reports survey results by stating that the standard error of the mean is 20. The population standard deviation is 500.
    a. How large was the sample used in this survey?
    b. What is the probability that the point estimate was within ±25 of the population mean?

45. A production process is checked periodically by a quality control inspector. The inspector selects simple random samples of 30 finished products and computes the sample mean product weights $\bar{x}$. If test results over a long period of time show that 5% of the $\bar{x}$ values are over 2.1 pounds and 5% are under 1.9 pounds, what are the mean and the standard deviation for the population of products produced with this process?

46. About 28% of private companies are owned by women (*The Cincinnati Enquirer,* January 26, 2006). Answer the following questions based on a sample of 240 private companies.
    a. Show the sampling distribution of $\bar{p}$, the sample proportion of companies that are owned by women.
    b. What is the probability the sample proportion will be within ±.04 of the population proportion?
    c. What is the probability the sample proportion will be within ±.02 of the population proportion?

47. A market research firm conducts telephone surveys with a 40% historical response rate. What is the probability that in a new sample of 400 telephone numbers, at least 150 individuals will cooperate and respond to the questions? In other words, what is the probability that the sample proportion will be at least 150/400 = .375?

48. Advertisers contract with Internet service providers and search engines to place ads on websites. They pay a fee based on the number of potential customers who click on their ad. Unfortunately, click fraud—the practice of someone clicking on an ad solely for the purpose of driving up advertising revenue—has become a problem. Forty percent of advertisers claim they have been a victim of click fraud (*BusinessWeek,* March 13, 2006). Suppose a simple random sample of 380 advertisers will be taken to learn more about how they are affected by this practice.
    a. What is the probability that the sample proportion will be within ±.04 of the population proportion experiencing click fraud?
    b. What is the probability that the sample proportion will be greater than .45?

49. The proportion of individuals insured by the All-Driver Automobile Insurance Company who received at least one traffic ticket during a five-year period is .15.
    a. Show the sampling distribution of $\bar{p}$ if a random sample of 150 insured individuals is used to estimate the proportion having received at least one ticket.
    b. What is the probability that the sample proportion will be within ±.03 of the population proportion?

50. Lori Jeffrey is a successful sales representative for a major publisher of college textbooks. Historically, Lori obtains a book adoption on 25% of her sales calls. Viewing her sales calls for one month as a sample of all possible sales calls, assume that a statistical analysis of the data yields a standard error of the proportion of .0625.
    a. How large was the sample used in this analysis? That is, how many sales calls did Lori make during the month?
    b. Let $\bar{p}$ indicate the sample proportion of book adoptions obtained during the month. Show the sampling distribution of $\bar{p}$.
    c. Using the sampling distribution of $\bar{p}$, compute the probability that Lori will obtain book adoptions on 30% or more of her sales calls during a one-month period.

Appendix Random Sampling with StatTools

MetAreas

If a list of the elements in a population is available in an Excel file, StatTools Random Sample Utility can be used to select a simple random sample. For example, a list of the top 100 metropolitan areas in the United States and Canada is provided in column A of the data set MetAreas (*Places Rated Almanac—The Millennium Edition 2000*). Column B contains the overall rating of each metropolitan area. Assume that you would like to select a simple random sample of 30 metropolitan areas in order to do an in-depth study of the cost of living in the United States and Canada.

Begin by using the Data Set Manager to create a StatTools data set for these data using the procedure described in the appendix to Chapter 1. The following steps will generate a simple random sample of 30 metropolitan areas.

Step 1. Click the **StatTools** tab on the Ribbon
Step 2. In the **Data Group** click **Data Utilities**
Step 3. Choose the **Random Sample** option
Step 4. When the StatTools - Random Sample Utility dialog box appears,
In the **Variables** section,
Select **Metropolitan Area**
Select **Rating**
In the **Options** section,
Enter 1 in the **Number of Samples** box
Enter 30 in the **Sample Size** box
Click **OK**

The random sample of 30 metropolitan areas will appear in columns A and B of the worksheet entitled Random Sample.

CHAPTER 8

Interval Estimation

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STATISTICS IN PRACTICE: FOOD LION

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8.2 POPULATION MEAN: σ UNKNOWN
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STATISTICS in PRACTICE

FOOD LION*

SALISBURY, NORTH CAROLINA

Founded in 1957 as Food Town, Food Lion is one of the largest supermarket chains in the United States, with 1300 stores in 11 Southeastern and Mid-Atlantic states. The company sells more than 24,000 different products and offers nationally and regionally advertised brand-name merchandise, as well as a growing number of high-quality private label products manufactured especially for Food Lion. The company maintains its low price leadership and quality assurance through operating efficiencies such as standard store formats, innovative warehouse design, energy-efficient facilities, and data synchronization with suppliers. Food Lion looks to a future of continued innovation, growth, price leadership, and service to its customers.

Being in an inventory-intense business, Food Lion made the decision to adopt the LIFO (last-in, first-out) method of inventory valuation. This method matches current costs against current revenues, which minimizes the effect of radical price changes on profit and loss results. In addition, the LIFO method reduces net income, thereby reducing income taxes during periods of inflation.

Food Lion establishes a LIFO index for each of seven inventory pools: Grocery, Paper/Household, Pet Supplies, Health & Beauty Aids, Dairy, Cigarette/Tobacco, and Beer/Wine. For example, a LIFO index of 1.008 for the Grocery pool would indicate that the company's grocery inventory value at current costs reflects a 0.8% increase due to inflation over the most recent one-year period.

A LIFO index for each inventory pool requires that the year-end inventory count for each product be valued at the current year-end cost and at the preceding year-end cost. To avoid excessive time and expense associated with counting the inventory in all 1200 store locations, Food Lion selects a random sample of 50 stores. Year-end physical inventories are taken in each of the sample stores. The current-year and preceding-year costs for each item are then used to construct the required LIFO indexes for each inventory pool.

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Fresh bread arriving at a Food Lion Store.

For a recent year, the sample estimate of the LIFO index for the Health & Beauty Aids inventory pool was 1.015. Using a 95% confidence level, Food Lion computed a margin of error of .006 for the sample estimate. Thus, the interval from 1.009 to 1.021 provided a 95% confidence interval estimate of the population LIFO index. This level of precision was judged to be very good.

In this chapter you will learn how to compute the margin of error associated with sample estimates. You will also learn how to use this information to construct and interpret interval estimates of a population mean and a population proportion.

*The authors are indebted to Keith Cunningham, Tax Director, and Bobby Harkey, Staff Tax Accountant, at Food Lion for providing this Statistics in Practice.

In Chapter 7, we stated that a point estimator is a sample statistic used to estimate a population parameter. For instance, the sample mean $\bar{x}$ is a point estimator of the population mean μ and the sample proportion $\bar{p}$ is a point estimator of the population proportion p. Because a point estimator cannot be expected to provide the exact value of the population parameter, an **interval estimate** is often computed by adding and subtracting a value, called the **margin of error**, to the point estimate. The general form of an interval estimate is as follows:

Point estimate $\pm$ Margin of error

The purpose of an interval estimate is to provide information about how close the point estimate, provided by the sample, is to the value of the population parameter.

In this chapter we show how to compute interval estimates of a population mean μ and a population proportion p. The general form of an interval estimate of a population mean is

$$\bar{x} \pm \text{Margin of error}$$

Similarly, the general form of an interval estimate of a population proportion is

$$\bar{p} \pm \text{Margin of error}$$

The sampling distributions of $\bar{x}$ and $\bar{p}$ play key roles in computing these interval estimates.

8.1 Population Mean: σ Known

In order to develop an interval estimate of a population mean, either the population standard deviation σ or the sample standard deviation s must be used to compute the margin of error. In most applications σ is not known, and s is used to compute the margin of error. In some applications, however, large amounts of relevant historical data are available and can be used to estimate the population standard deviation prior to sampling. Also, in quality control applications where a process is assumed to be operating correctly, or "in control," it is appropriate to treat the population standard deviation as known. We refer to such cases as **σ known** cases. In this section we introduce an example in which it is reasonable to treat σ as known and show how to construct an interval estimate for this case.

Each week Lloyd's Department Store selects a simple random sample of 100 customers in order to learn about the amount spent per shopping trip. With x representing the amount spent per shopping trip, the sample mean $\bar{x}$ provides a point estimate of μ, the mean amount spent per shopping trip for the population of all Lloyd's customers. Lloyd's has been using the weekly survey for several years. Based on the historical data, Lloyd's now assumes a known value of $\sigma = \$20$ for the population standard deviation. The historical data also indicate that the population follows a normal distribution.

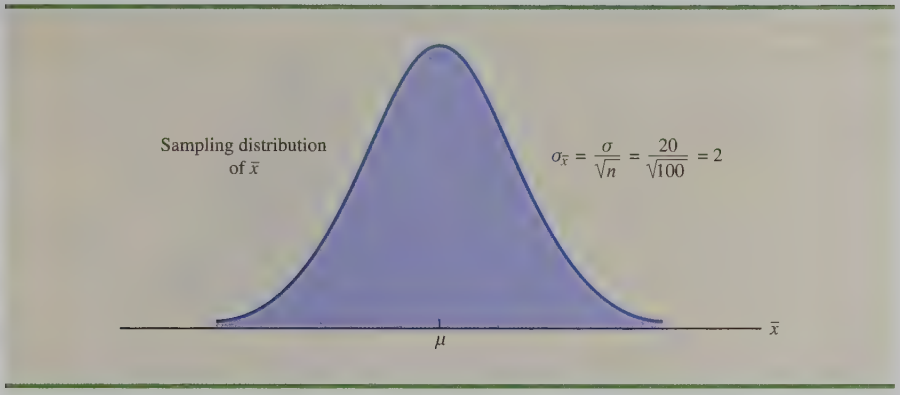
During the most recent week, Lloyd's surveyed 100 customers ($n = 100$) and obtained a sample mean of $\bar{x} = \$82$. The sample mean amount spent provides a point estimate of the population mean amount spent per shopping trip, μ. In the discussion that follows, we show how to compute the margin of error for this estimate and develop an interval estimate of the population mean.

Margin of Error and the Interval Estimate

In Chapter 7 we showed that the sampling distribution of $\bar{x}$ can be used to compute the probability that $\bar{x}$ will be within a given distance of μ. In the Lloyd's example, the historical data show that the population of amounts spent is normally distributed with a standard deviation of $\sigma = 20$. So, using what we learned in Chapter 7, we can conclude that the sampling distribution of $\bar{x}$ follows a normal distribution with a standard error of $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 20/\sqrt{100} = 2$. This sampling distribution is shown in Figure 8.1.[1] Because the sampling distribution shows how values of $\bar{x}$ are distributed around the population mean μ, the sampling distribution of $\bar{x}$ provides information about the possible differences between $\bar{x}$ and μ.

[1]We use the fact that the population of amounts spent has a normal distribution to conclude that the sampling distribution of $\bar{x}$ has a normal distribution. If the population did not have a normal distribution, we could rely on the central limit theorem and the sample size of $n = 100$ to conclude that the sampling distribution of $\bar{x}$ is approximately normal. In either case, the sampling distribution of $\bar{x}$ would appear as shown in Figure 8.1.

FIGURE 8.1 SAMPLING DISTRIBUTION OF THE SAMPLE MEAN AMOUNT SPENT FROM SIMPLE RANDOM SAMPLES OF 100 CUSTOMERS



Using the standard normal probability table, we find that 95% of the values of any normally distributed random variable are within ± 1.96 standard deviations of the mean. Thus, when the sampling distribution of $\bar{x}$ is normally distributed, 95% of the $\bar{x}$ values must be within $\pm 1.96\sigma_{\bar{x}}$ of the mean μ. In the Lloyd's example we know that the sampling distribution of $\bar{x}$ is normally distributed with a standard error of $\sigma_{\bar{x}} = 2$. Because $\pm 1.96\sigma_{\bar{x}} = 1.96(2) = 3.92$, we can conclude that 95% of all $\bar{x}$ values obtained using a sample size of $n = 100$ will be within ± 3.92 of the population mean μ. See Figure 8.2.

In the introduction to this chapter we said that the general form of an interval estimate of the population mean μ is $\bar{x} \pm$ margin of error. For the Lloyd's example, suppose we set

FIGURE 8.2 SAMPLING DISTRIBUTION OF $\bar{x}$ SHOWING THE LOCATION OF SAMPLE MEANS THAT ARE WITHIN 3.92 OF μ

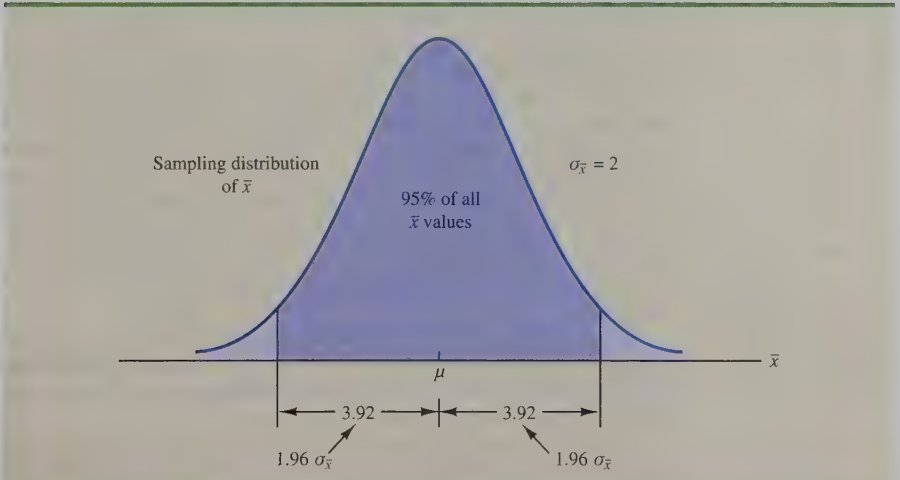
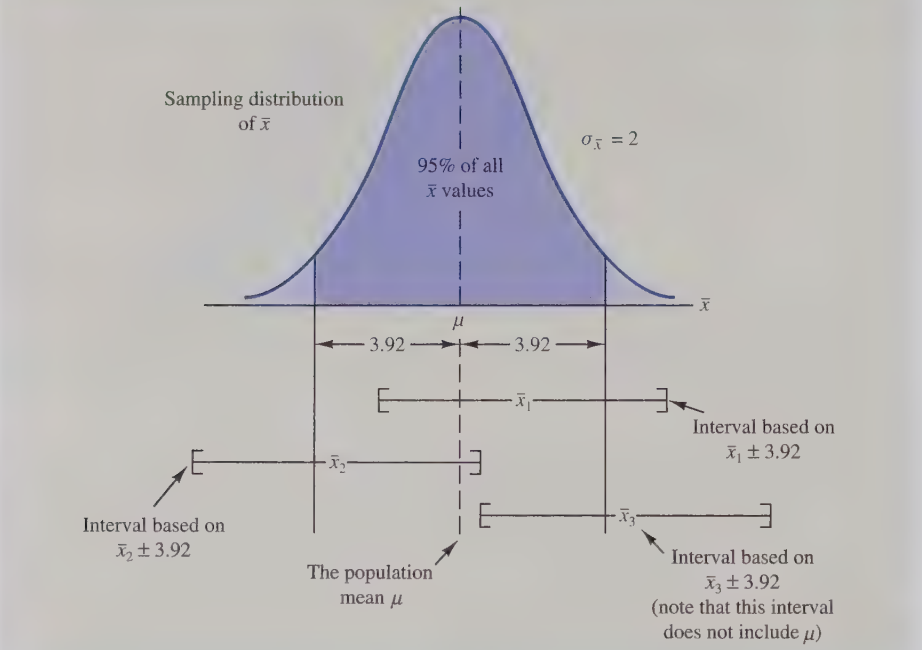


FIGURE 8.3 INTERVALS FORMED FROM SELECTED SAMPLE MEANS AT LOCATIONS $\bar{x}_1$, $\bar{x}_2$, AND $\bar{x}_3$



the margin of error equal to 3.92 and compute the interval estimate of μ using $\bar{x} \pm 3.92$. To provide an interpretation for this interval estimate, let us consider the values of $\bar{x}$ that could be obtained if we took three *different* simple random samples, each consisting of 100 Lloyd's customers. The first sample mean might turn out to have the value shown as $\bar{x}_1$ in Figure 8.3. In this case, Figure 8.3 shows that the interval formed by subtracting 3.92 from $\bar{x}_1$ and adding 3.92 to $\bar{x}_1$ includes the population mean μ. Now consider what happens if the second sample mean turns out to have the value shown as $\bar{x}_2$ in Figure 8.3. Although this sample mean differs from the first sample mean, we see that the interval formed by subtracting 3.92 from $\bar{x}_2$ and adding 3.92 to $\bar{x}_2$ also includes the population mean μ. However, consider what happens if the third sample mean turns out to have the value shown as $\bar{x}_3$ in Figure 8.3. In this case, the interval formed by subtracting 3.92 from $\bar{x}_3$ and adding 3.92 to $\bar{x}_3$ does not include the population mean μ. Because $\bar{x}_3$ falls in the upper tail of the sampling distribution and is farther than 3.92 from μ, subtracting and adding 3.92 to $\bar{x}_3$ forms an interval that does not include μ.

Any sample mean $\bar{x}$ that is within the darkly shaded region of Figure 8.3 will provide an interval that contains the population mean μ. Because 95% of all possible sample means are in the darkly shaded region, 95% of all intervals formed by subtracting 3.92 from $\bar{x}$ and adding 3.92 to $\bar{x}$ will include the population mean μ.

Recall that during the most recent week, the quality assurance team at Lloyd's surveyed 100 customers and obtained a sample mean amount spent of $\bar{x} = 82$. Using $\bar{x} \pm 3.92$ to construct the interval estimate, we obtain 82 ± 3.92. Thus, the specific interval estimate of

This discussion provides insight as to why the interval is called a 95% confidence interval.

μ based on the data from the most recent week is $82 - 3.92 = 78.08$ to $82 + 3.92 = 85.92$. Because 95% of all the intervals constructed using $\bar{x} \pm 3.92$ will contain the population mean, we say that we are 95% confident that the interval 78.08 to 85.92 includes the population mean μ. We say that this interval has been established at the 95% **confidence level**. The value .95 is referred to as the **confidence coefficient**, and the interval 78.08 to 85.92 is called the 95% **confidence interval**.

Another term sometimes associated with an interval estimate is the **level of significance**. The level of significance associated with an interval estimate is denoted by the Greek letter α. The level of significance and the confidence coefficient are related as follows:

$$\alpha = \text{Level of Significance} = 1 - \text{Confidence Coefficient}$$

The level of significance is the probability that the interval estimation procedure will generate an interval that does not contain μ. For example, the level of significance corresponding to a .95 confidence coefficient is $\alpha = 1 - .95 = .05$. In Lloyd's case, the level of significance ($\alpha = .05$) is the probability of drawing a sample, computing the sample mean, and finding that $\bar{x}$ lies in one of the tails of the sampling distribution (see $\bar{x}_3$ in Figure 8.3). When the sample mean happens to fall in the tail of the sampling distribution (and it will 5% of the time), the confidence interval generated will not contain μ.

With the margin of error given by $z_{\alpha/2}(\sigma/\sqrt{n})$, the general form of an interval estimate of a population mean for the σ known case follows.

INTERVAL ESTIMATE OF A POPULATION MEAN: σ KNOWN

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \tag{8.1}$$

where $(1 - \alpha)$ is the confidence coefficient and $z_{\alpha/2}$ is the z value providing an area of $\alpha/2$ in the upper tail of the standard normal probability distribution.

Let us use expression (8.1) to construct a 95% confidence interval for the Lloyd's example. For a 95% confidence interval, the confidence coefficient is $(1 - \alpha) = .95$ and thus, $\alpha = .05$. Using the standard normal probability table, an area of $\alpha/2 = .05/2 = .025$ in the upper tail provides $z_{.025} = 1.96$. With the Lloyd's sample mean $\bar{x} = 82$, $\sigma = 20$, and a sample size $n = 100$, we obtain

$$82 \pm 1.96\frac{20}{\sqrt{100}}$$
$$82 \pm 3.92$$

Thus, using expression (8.1), the margin of error is 3.92 and the 95% confidence interval is $82 - 3.92 = 78.08$ to $82 + 3.92 = 85.92$.

Although a 95% confidence level is frequently used, other confidence levels such as 90% and 99% may be considered. Values of $z_{\alpha/2}$ for the most commonly used confidence levels are shown in Table 8.1. Using these values and expression (8.1), the 90% confidence interval for the Lloyd's example is

$$82 \pm 1.645\frac{20}{\sqrt{100}}$$
$$82 \pm 3.29$$

TABLE 8.1 VALUES OF $z_{\alpha/2}$ FOR THE MOST COMMONLY USED CONFIDENCE LEVELS

Confidence Level	α	$\alpha/2$	$z_{\alpha/2}$
90%	.10	.05	1.645
95%	.05	.025	1.960
99%	.01	.005	2.576

Thus, at 90% confidence, the margin of error is 3.29 and the confidence interval is $82 - 3.29 = 78.71$ to $82 + 3.29 = 85.29$. Similarly, the 99% confidence interval is

$$82 \pm 2.576 \frac{20}{\sqrt{100}}$$
$$82 \pm 5.15$$

Thus, at 99% confidence, the margin of error is 5.15 and the confidence interval is $82 - 5.15 = 76.85$ to $82 + 5.15 = 87.15$.

Comparing the results for the 90%, 95%, and 99% confidence levels, we see that in order to have a higher degree of confidence, the margin of error and thus the width of the confidence interval must be larger.

Using Excel

We will use the Lloyd's Department Store data to illustrate how Excel can be used to construct an interval estimate of the population mean for the σ known case. Refer to Figure 8.4 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet appears in the foreground.

Enter Data: A label and the sales data are entered into cells A1:A101.

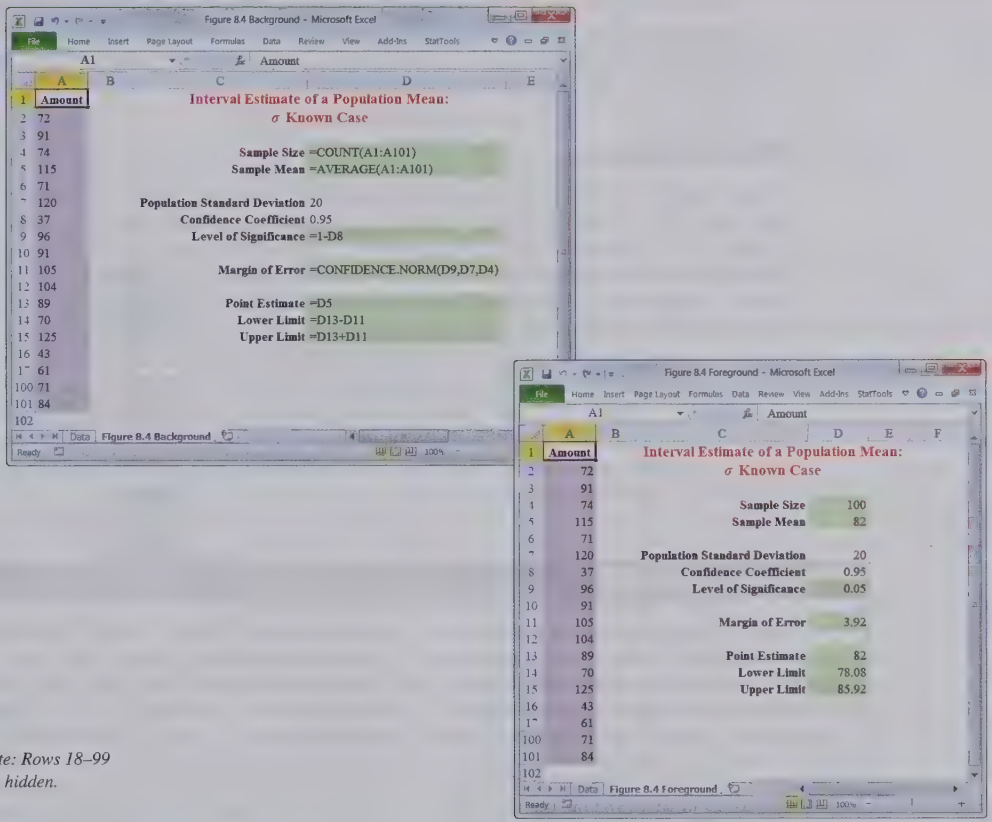
Enter Functions and Formulas: The sample size and sample mean are computed in cells D4:D5 using Excel's COUNT and AVERAGE functions, respectively. The value worksheet shows that the sample size is 100 and the sample mean is 82. The value of the known population standard deviation (20) is entered into cell D7 and the desired confidence coefficient (.95) is entered into cell D8. The level of significance is computed in cell D9 by entering the formula =1−D8; the value worksheet shows that the level of significance associated with a confidence coefficient of .95 is .05. The margin of error is computed in cell D11 using Excel's CONFIDENCE.NORM function. The CONFIDENCE.NORM function has three inputs: the level of significance (cell D9); the population standard deviation (cell D7); and the sample size (cell D4). Thus, to compute the margin of error associated with a 95% confidence interval, the following formula is entered into cell D11:

=CONFIDENCE.NORM(D9,D7,D4)

The resulting value of 3.92 is the margin of error associated with the interval estimate of the population mean amount spent per week.

Cells D13:D15 provide the point estimate and the lower and upper limits for the confidence interval. Because the point estimate is just the sample mean, the formula =D5 is entered into cell D13. To compute the lower limit of the 95% confidence interval, $\bar{x}$ − (margin of error), we enter the formula =D13−D11 into cell D14. To compute the upper limit of the 95% confidence interval, $\bar{x}$ + (margin of error), we enter the formula =D13+D11 into cell D15. The value worksheet shows a lower limit of 78.08 and an upper limit of 85.92. In other words, the 95% confidence interval for the population mean is from 78.08 to 85.92.

FIGURE 8.4 EXCEL WORKSHEET: CONSTRUCTING A 95% CONFIDENCE INTERVAL FOR LLOYD'S DEPARTMENT STORE



Note: Rows 18–99 are hidden.

A template for other problems To use this worksheet as a template for another problem of this type, we must first enter the new problem data in column A. Then, the cell formulas in cells D4 and D5 must be updated with the new data range and the known population standard deviation must be entered into cell D7. After doing so, the point estimate and a 95% confidence interval will be displayed in cells D13:D15. If a confidence interval with a different confidence coefficient is desired, we simply change the value in cell D8.

We can further simplify the use of Figure 8.4 as a template for other problems by eliminating the need to enter new data ranges in cells D4 and D5. To do so we rewrite the cell formulas as follows:

The Lloyd's data set includes a worksheet entitled Template that uses the A:A method for entering the data ranges.

Cell D4: =COUNT(A:A)
Cell D5: =AVERAGE(A:A)

With the A:A method of specifying data ranges, Excel's COUNT function will count the number of numerical values in column A and Excel's AVERAGE function will compute the

average of the numerical values in column A. Thus, to solve a new problem it is only necessary to enter the new data into column A and enter the value of the known population standard deviation into cell D7.

This worksheet can also be used as a template for text exercises in which the sample size, sample mean, and the population standard deviation are given. In this type of situation we simply replace the values in cells D4, D5, and D7 with the given values of the sample size, sample mean, and the population standard deviation.

Practical Advice

If the population follows a normal distribution, the confidence interval provided by expression (8.1) is exact. In other words, if expression (8.1) were used repeatedly to generate 95% confidence intervals, exactly 95% of the intervals generated would contain the population mean. If the population does not follow a normal distribution, the confidence interval provided by expression (8.1) will be approximate. In this case, the quality of the approximation depends on both the distribution of the population and the sample size.

In most applications, a sample size of $n \geq 30$ is adequate when using expression (8.1) to develop an interval estimate of a population mean. If the population is not normally distributed, but is roughly symmetric, sample sizes as small as 15 can be expected to provide good approximate confidence intervals. With smaller sample sizes, expression (8.1) should only be used if the analyst believes, or is willing to assume, that the population distribution is at least approximately normal.

NOTES AND COMMENTS

1. The interval estimation procedure discussed in this section is based on the assumption that the population standard deviation σ is known. By σ known we mean that historical data or other information are available that permit us to obtain a good estimate of the population standard deviation prior to taking the sample that will be used to develop an estimate of the population mean. So technically we don't mean that σ is actually known with certainty. We just mean that we obtained a good estimate of the standard deviation prior to sampling and thus we won't be using the same sample to estimate both the population mean and the population standard deviation.
2. The sample size n appears in the denominator of the interval estimation expression (8.1). Thus, if a particular sample size provides too wide an interval to be of any practical use, we may want to consider increasing the sample size. With n in the denominator, a larger sample size will provide a smaller margin of error, a narrower interval, and greater precision. The procedure for determining the size of a simple random sample necessary to obtain a desired precision is discussed in Section 8.3.

Exercises

Methods

1. A simple random sample of 40 items resulted in a sample mean of 25. The population standard deviation is $\sigma = 5$.
   a. What is the standard error of the mean, $\sigma_{\bar{x}}$?
   b. At 95% confidence, what is the margin of error?

SELF test

2. A simple random sample of 50 items from a population with $\sigma = 6$ resulted in a sample mean of 32.
   a. Provide a 90% confidence interval for the population mean.

b. Provide a 95% confidence interval for the population mean.
c. Provide a 99% confidence interval for the population mean.

3. A simple random sample of 60 items resulted in a sample mean of 80. The population standard deviation is $\sigma = 15$.
   a. Compute the 95% confidence interval for the population mean.
   b. Assume that the same sample mean was obtained from a sample of 120 items. Provide a 95% confidence interval for the population mean.
   c. What is the effect of a larger sample size on the interval estimate?

4. A 95% confidence interval for a population mean was reported to be 152 to 160. If $\sigma = 15$, what sample size was used in this study?

Applications

Restaurant

5. In an effort to estimate the mean amount spent per customer for dinner at an Atlanta restaurant, data were collected for a sample of 49 customers. The data collected are shown in the website file named Restaurant. Based upon past studies the population standard deviation is assumed known with $\sigma = \$5$.
   a. At 95% confidence, what is the margin of error?
   b. Develop a 95% confidence interval estimate of the mean amount spent for dinner.

Nielsen

6. Nielsen Media Research conducted a study of household television viewing times during the 8 P.M. to 11 P.M. time period. The data contained in the file named Nielsen are consistent with the findings reported (*The World Almanac,* 2003). Based upon past studies the population standard deviation is assumed known with $\sigma = 3.5$ hours. Develop a 95% confidence interval estimate of the mean television viewing time per week during the 8 P.M. to 11 P.M. time period.

7. *The Wall Street Journal* reported that automobile crashes cost the United States $162 billion annually (*The Wall Street Journal,* March 5, 2008). The average cost per person for crashes in the Tampa, Florida, area was reported to be $1599. Suppose this average cost was based on a sample of 50 persons who had been involved in car crashes and that the population standard deviation is $\sigma = \$600$. What is the margin of error for a 95% confidence interval? What would you recommend if the study required a margin of error of $150 or less?

8. The National Quality Research Center at the University of Michigan provides a quarterly measure of consumer opinions about products and services (*The Wall Street Journal,* February 18, 2003). A survey of 10 restaurants in the Fast Food/Pizza group showed a sample mean customer satisfaction index of 71. Past data indicate that the population standard deviation of the index has been relatively stable with $\sigma = 5$.
   a. What assumption should the researcher be willing to make if a margin of error is desired?
   b. Using 95% confidence, what is the margin of error?
   c. What is the margin of error if 99% confidence is desired?

TaxReturn

9. AARP reported on a study conducted to learn how long it takes individuals to prepare their federal income tax return (*AARP Bulletin,* April 2008). The data contained in the file named TaxReturn are consistent with the study results. These data provide the time in hours required for 40 individuals to complete their federal income tax returns. Using past years' data, the population standard deviation can be assumed known with $\sigma = 9$ hours. What is the 95% confidence interval estimate of the mean time it takes an individual to complete a federal income tax return?

10. *Playbill* magazine reported that the mean annual household income of its readers is $119,155 (*Playbill,* January 2006). Assume this estimate of the mean annual household income is based on a sample of 80 households, and based on past studies, the population standard deviation is known to be $\sigma = \$30{,}000$.
    a. Develop a 90% confidence interval estimate of the population mean.
    b. Develop a 95% confidence interval estimate of the population mean.

c. Develop a 99% confidence interval estimate of the population mean.
d. Discuss what happens to the width of the confidence interval as the confidence level is increased. Does this seem reasonable? Explain.

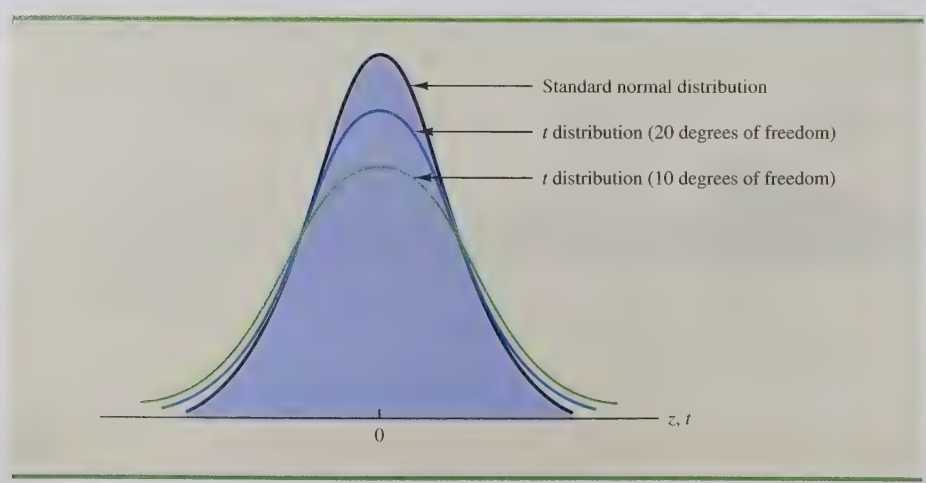
8.2 Population Mean: σ Unknown

When developing an interval estimate of a population mean we usually do not have a good estimate of the population standard deviation either. In these cases, we must use the same sample to estimate both μ and σ. This situation represents the **σ unknown** case. When s is used to estimate σ, the margin of error and the interval estimate for the population mean are based on a probability distribution known as the ***t* distribution**. Although the mathematical development of the t distribution is based on the assumption of a normal distribution for the population we are sampling from, research shows that the t distribution can be successfully applied in many situations where the population deviates significantly from normal. Later in this section we provide guidelines for using the t distribution if the population is not normally distributed.

William Sealy Gosset, writing under the name "Student," is the founder of the t distribution. Gosset, an Oxford graduate in mathematics, worked for the Guinness Brewery in Dublin, Ireland. He developed the t distribution while working on small-scale materials and temperature experiments.

The t distribution is a family of similar probability distributions, with a specific t distribution depending on a parameter known as the **degrees of freedom**. The t distribution with one degree of freedom is unique, as is the t distribution with two degrees of freedom, with three degrees of freedom, and so on. As the number of degrees of freedom increases, the difference between the t distribution and the standard normal distribution becomes smaller and smaller. Figure 8.5 shows t distributions with 10 and 20 degrees of freedom and their relationship to the standard normal probability distribution. Note that a t distribution with more degrees of freedom exhibits less variability and more

FIGURE 8.5 COMPARISON OF THE STANDARD NORMAL DISTRIBUTION WITH t DISTRIBUTIONS HAVING 10 AND 20 DEGREES OF FREEDOM



closely resembles the standard normal distribution. Note also that the mean of the t distribution is zero.

We place a subscript on t to indicate the area in the upper tail of the t distribution. For example, just as we used $z_{.025}$ to indicate the z value providing a .025 area in the upper tail of a standard normal distribution, we will use $t_{.025}$ to indicate a .025 area in the upper tail of a t distribution. In general, we will use the notation $t_{\alpha/2}$ to represent a t value with an area of $\alpha/2$ in the upper tail of the t distribution. See Figure 8.6.

Table 2 in Appendix B contains a table for the t distribution. A portion of this table is shown in Table 8.2. Each row in the table corresponds to a separate t distribution with the degrees of freedom shown. For example, for a t distribution with 9 degrees of freedom, $t_{.025} = 2.262$. Similarly, for a t distribution with 60 degrees of freedom, $t_{.025} = 2.000$. As the degrees of freedom continue to increase, $t_{.025}$ approaches $z_{.025} = 1.96$. In fact, the standard normal distribution z values can be found in the infinite degrees of freedom row (labeled ∞) of the t distribution table. If the degrees of freedom exceed 100, the infinite degrees of freedom row can be used to approximate the actual t value; in other words, for more than 100 degrees of freedom, the standard normal z value provides a good approximation to the t value.

As the degrees of freedom increase, the t distribution approaches the standard normal distribution.

Margin of Error and the Interval Estimate

In Section 8.1 we showed that an interval estimate of a population mean for the σ known case is

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

To compute an interval estimate of μ for the σ unknown case, the sample standard deviation s is used to estimate σ, and $z_{\alpha/2}$ is replaced by the t distribution value $t_{\alpha/2}$. The margin

FIGURE 8.6 t DISTRIBUTION WITH $\alpha/2$ AREA OR PROBABILITY IN THE UPPER TAIL

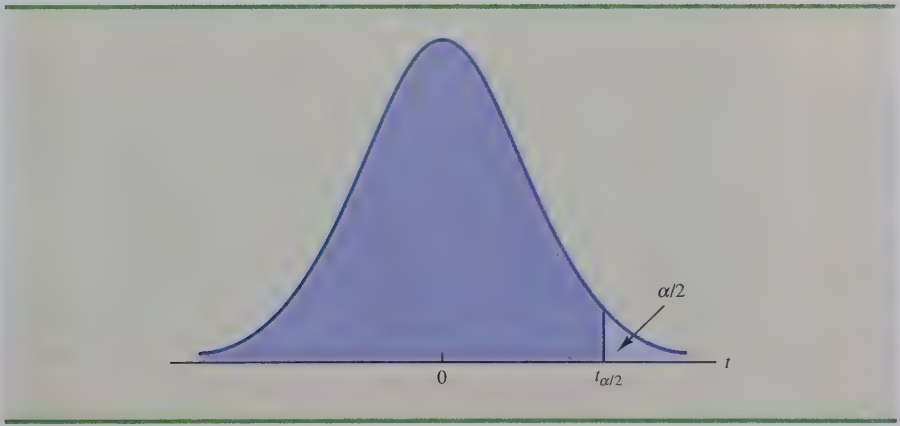
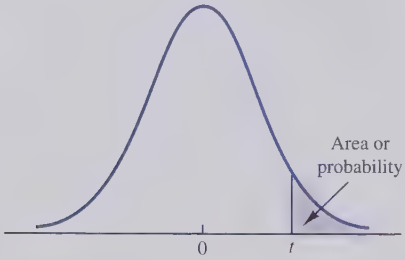


TABLE 8.2 SELECTED VALUES FROM THE t DISTRIBUTION TABLE*



Degrees of Freedom	Area in Upper Tail					
	.20	.10	.05	.025	.01	.005
1	1.376	3.078	6.314	12.706	31.821	63.656
2	1.061	1.886	2.920	4.303	6.965	9.925
3	.978	1.638	2.353	3.182	4.541	5.841
4	.941	1.533	2.132	2.776	3.747	4.604
5	.920	1.476	2.015	2.571	3.365	4.032
6	.906	1.440	1.943	2.447	3.143	3.707
7	.896	1.415	1.895	2.365	2.998	3.499
8	.889	1.397	1.860	2.306	2.896	3.355
9	.883	1.383	1.833	2.262	2.821	3.250
⋮	⋮	⋮	⋮	⋮	⋮	⋮
60	.848	1.296	1.671	2.000	2.390	2.660
61	.848	1.296	1.670	2.000	2.389	2.659
62	.847	1.295	1.670	1.999	2.388	2.657
63	.847	1.295	1.669	1.998	2.387	2.656
64	.847	1.295	1.669	1.998	2.386	2.655
65	.847	1.295	1.669	1.997	2.385	2.654
66	.847	1.295	1.668	1.997	2.384	2.652
67	.847	1.294	1.668	1.996	2.383	2.651
68	.847	1.294	1.668	1.995	2.382	2.650
69	.847	1.294	1.667	1.995	2.382	2.649
⋮	⋮	⋮	⋮	⋮	⋮	⋮
90	.846	1.291	1.662	1.987	2.368	2.632
91	.846	1.291	1.662	1.986	2.368	2.631
92	.846	1.291	1.662	1.986	2.368	2.630
93	.846	1.291	1.661	1.986	2.367	2.630
94	.845	1.291	1.661	1.986	2.367	2.629
95	.845	1.291	1.661	1.985	2.366	2.629
96	.845	1.290	1.661	1.985	2.366	2.628
97	.845	1.290	1.661	1.985	2.365	2.627
98	.845	1.290	1.661	1.984	2.365	2.627
99	.845	1.290	1.660	1.984	2.364	2.626
100	.845	1.290	1.660	1.984	2.364	2.626
∞	.842	1.282	1.645	1.960	2.326	2.576

**Note:* A more extensive table is provided as Table 2 of Appendix B.

of error is then given by $t_{\alpha/2}\, s/\sqrt{n}$. With this margin of error, the general expression for an interval estimate of a population mean when σ is unknown follows.

INTERVAL ESTIMATE OF A POPULATION MEAN: σ UNKNOWN

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} \tag{8.2}$$

where s is the sample standard deviation, $(1 - \alpha)$ is the confidence coefficient, and $t_{\alpha/2}$ is the t value providing an area of $\alpha/2$ in the upper tail of the t distribution with $n - 1$ degrees of freedom.

The reason the number of degrees of freedom associated with the t value in expression (8.2) is $n - 1$ concerns the use of s as an estimate of the population standard deviation σ. The expression for the sample standard deviation is

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n - 1}}$$

Degrees of freedom refer to the number of independent pieces of information that go into the computation of $\Sigma(x_i - \bar{x})^2$. The n pieces of information involved in computing $\Sigma(x_i - \bar{x})^2$ are as follows: $x_1 - \bar{x}, x_2 - \bar{x}, \ldots, x_n - \bar{x}$. In Section 3.2 we indicated that $\Sigma(x_i - \bar{x}) = 0$ for any data set. Thus, only $n - 1$ of the $x_i - \bar{x}$ values are independent; that is, if we know $n - 1$ of the values, the remaining value can be determined exactly by using the condition that the sum of the $x_i - \bar{x}$ values must be 0. Thus, $n - 1$ is the number of degrees of freedom associated with $\Sigma(x_i - \bar{x})^2$ and hence the number of degrees of freedom for the t distribution in expression (8.2).

To illustrate the interval estimation procedure for the σ unknown case, we will consider a study designed to estimate the mean credit card debt for the population of U.S. households. A sample of $n = 70$ households provided the credit card balances shown in Table 8.3. For this situation, no previous estimate of the population standard deviation σ is available. Thus, the sample data must be used to estimate both the population mean and the population standard deviation. Using the data in Table 8.3, we compute the sample mean $\bar{x} = \$9312$ and the

TABLE 8.3 CREDIT CARD BALANCES FOR A SAMPLE OF 70 HOUSEHOLDS

WEB file NewBalance

9430	14661	7159	9071	9691	11032
7535	12195	8137	3603	11448	6525
4078	10544	9467	16804	8279	5239
5604	13659	12595	13479	5649	6195
5179	7061	7917	14044	11298	12584
4416	6245	11346	6817	4353	15415
10676	13021	12806	6845	3467	15917
1627	9719	4972	10493	6191	12591
10112	2200	11356	615	12851	9743
6567	10746	7117	13627	5337	10324
13627	12744	9465	12557	8372	
18719	5742	19263	6232	7445	

sample standard deviation $s = \$4007$. With 95% confidence and $n - 1 = 69$ degrees of freedom, Table 8.2 can be used to obtain the appropriate value for $t_{.025}$. We want the t value in the row with 69 degrees of freedom, and the column corresponding to .025 in the upper tail. The value shown is $t_{.025} = 1.995$.

We use expression (8.2) to compute an interval estimate of the population mean credit card balance.

$$9312 \pm 1.995 \frac{4007}{\sqrt{70}}$$

$$9312 \pm 955$$

The point estimate of the population mean is \$9312, the margin of error is \$955, and the 95% confidence interval is $9312 - 955 = \$8357$ to $9312 + 955 = \$10{,}267$. Thus, we are 95% confident that the mean credit card balance for the population of all households is between \$8357 and \$10,267.

Using Excel

We will use the credit card balances in Table 8.3 to illustrate how Excel can be used to construct an interval estimate of the population mean for the σ unknown case. We start by summarizing the data using Excel's Descriptive Statistics tool described in Chapter 3. Refer to Figure 8.7 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet is in the foreground.

Enter Data: A label and the credit card balances are entered into cells A1:A71.

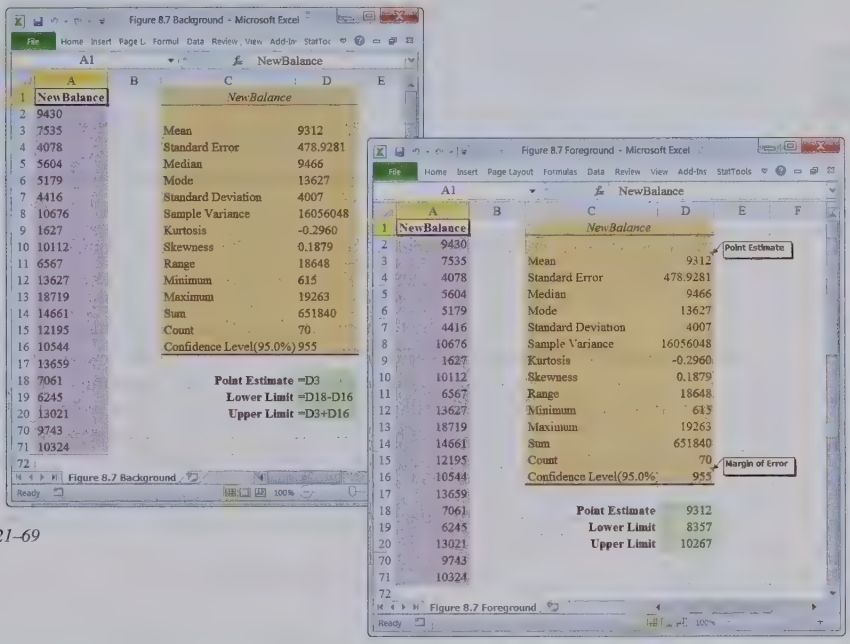
Apply Analysis Tools: The following steps describe how to use Excel's Descriptive Statistics tool for these data:

Step 1. Click the **Data** tab on the Ribbon
Step 2. In the **Analysis** group, click **Data Analysis**
Step 3. Choose **Descriptive Statistics** from the list of Analysis Tools
Step 4. When the Descriptive Statistics dialog box appears,
Enter A1:A71 in the **Input Range** box
Select **Grouped By Columns**
Select **Labels in First Row**
Select **Output Range**
Enter C1 in the **Output Range** box
Select **Summary Statistics**
Select **Confidence Level for Mean**
Enter 95 in the **Confidence Level for Mean** box
Click **OK**

The sample mean ($\bar{x}$) is in cell D3. The margin of error, labeled "Confidence Level(95%)," appears in cell D16. The value worksheet shows $\bar{x} = 9312$ and a margin of error equal to 955.

Enter Functions and Formulas: Cells D18:D20 provide the point estimate and the lower and upper limits for the confidence interval. Because the point estimate is just the sample mean, the formula =D3 is entered into cell D18. To compute the lower limit of the 95% confidence interval, $\bar{x}$ − (margin of error), we enter the formula =D18−D16 into cell D19. To compute the upper limit of the 95% confidence interval, $\bar{x}$ + (margin of error), we enter the formula =D18+D16 into cell D20. The value worksheet shows a lower limit of 8357 and an upper limit of 10,267. In other words, the 95% confidence interval for the population mean is from 8357 to 10,267.

FIGURE 8.7 EXCEL WORKSHEET: 95% CONFIDENCE INTERVAL FOR CREDIT CARD BALANCES



Note: Rows 21–69 are hidden.

Practical Advice

If the population follows a normal distribution, the confidence interval provided by expression (8.2) is exact and can be used for any sample size. If the population does not follow a normal distribution, the confidence interval provided by expression (8.2) will be approximate. In this case, the quality of the approximation depends on both the distribution of the population and the sample size.

Larger sample sizes are needed if the distribution of the population is highly skewed or includes outliers.

In most applications, a sample size of $n \geq 30$ is adequate when using expression (8.2) to develop an interval estimate of a population mean. However, if the population distribution is highly skewed or contains outliers, most statisticians would recommend increasing the sample size to 50 or more. If the population is not normally distributed but is roughly symmetric, sample sizes as small as 15 can be expected to provide good approximate confidence intervals. With smaller sample sizes, expression (8.2) should only be used if the analyst believes, or is willing to assume, that the population distribution is at least approximately normal.

Using a Small Sample

In the following example we develop an interval estimate for a population mean when the sample size is small. As we already noted, an understanding of the distribution of the population becomes a factor in deciding whether the interval estimation procedure provides acceptable results.

Scheer Industries is considering a new computer-assisted program to train maintenance employees to do machine repairs. In order to fully evaluate the program, the director of

TABLE 8.4 TRAINING TIME IN DAYS FOR A SAMPLE OF 20 SCHEER INDUSTRIES EMPLOYEES

WEB file Scheer

52	59	54	42
44	50	42	48
55	54	60	55
44	62	62	57
45	46	43	56

manufacturing requested an estimate of the population mean time required for maintenance employees to complete the computer-assisted training.

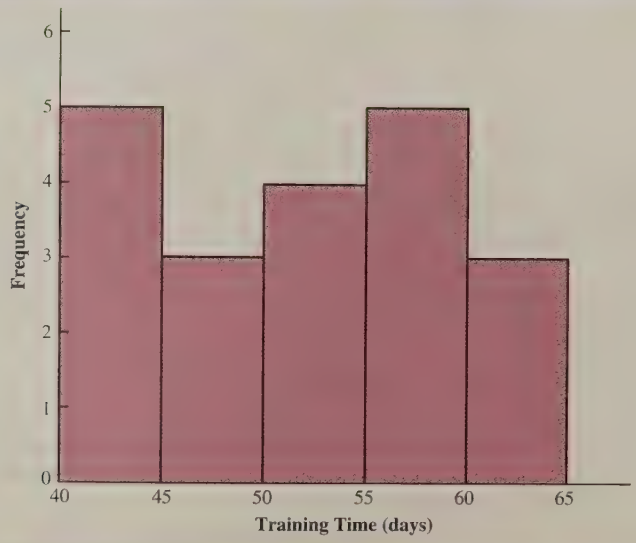
A sample of 20 employees is selected, with each employee in the sample completing the training program. Data on the training time in days for the 20 employees are shown in Table 8.4. A histogram of the sample data appears in Figure 8.8. What can we say about the distribution of the population based on this histogram? First, the sample data do not support the conclusion that the distribution of the population is normal, yet we do not see any evidence of skewness or outliers. Therefore, using the guidelines in the previous subsection, we conclude that an interval estimate based on the t distribution appears acceptable for the sample of 20 employees.

We continue by computing the sample mean and sample standard deviation as follows.

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{1030}{20} = 51.5 \text{ days}$$

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{889}{20-1}} = 6.84 \text{ days}$$

FIGURE 8.8 HISTOGRAM OF TRAINING TIMES FOR THE SCHEER INDUSTRIES SAMPLE



For a 95% confidence interval, we use Table 2 of Appendix B and $n - 1 = 19$ degrees of freedom to obtain $t_{.025} = 2.093$. Expression (8.2) provides the interval estimate of the population mean.

$$51.5 \pm 2.093\left(\frac{6.84}{\sqrt{20}}\right)$$

$$51.5 \pm 3.2$$

The point estimate of the population mean is 51.5 days. The margin of error is 3.2 days and the 95% confidence interval is $51.5 - 3.2 = 48.3$ days to $51.5 + 3.2 = 54.7$ days.

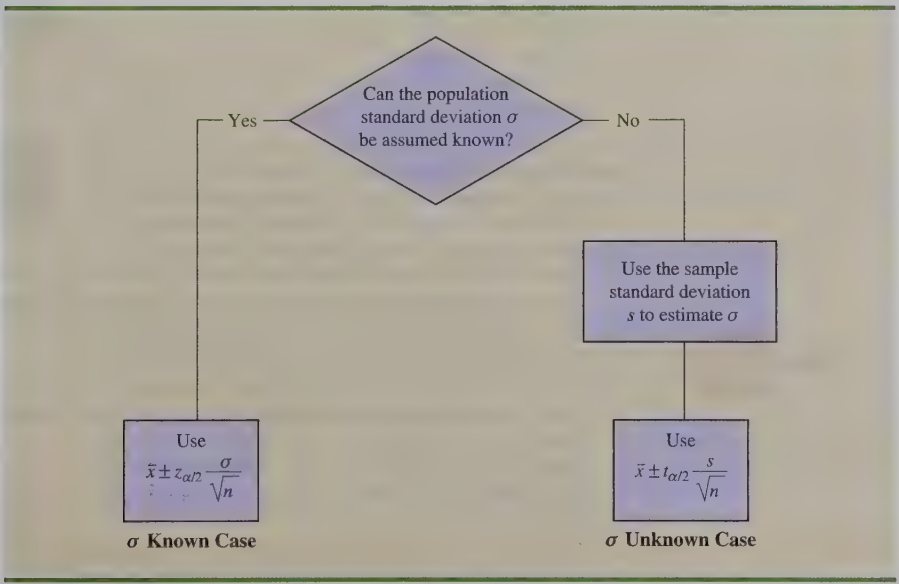
Using a histogram of the sample data to learn about the distribution of a population is not always conclusive, but in many cases it provides the only information available. The histogram, along with judgment on the part of the analyst, can often be used to decide whether expression (8.2) can be used to develop the interval estimate.

Summary of Interval Estimation Procedures

We provided two approaches to developing an interval estimate of a population mean. For the σ known case, σ and the standard normal distribution are used in expression (8.1) to compute the margin of error and to develop the interval estimate. For the σ unknown case, the sample standard deviation s and the t distribution are used in expression (8.2) to compute the margin of error and to develop the interval estimate.

A summary of the interval estimation procedures for the two cases is shown in Figure 8.9. In most applications, a sample size of $n \geq 30$ is adequate. If the population has a normal or approximately normal distribution, however, smaller sample sizes may be used.

FIGURE 8.9 SUMMARY OF INTERVAL ESTIMATION PROCEDURES FOR A POPULATION MEAN


Can the population standard deviation σ be assumed known?
Yes
No
Use the sample standard deviation s to estimate σ
Use $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$
σ Known Case
Use $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$
σ Unknown Case

For the σ unknown case a sample size of $n \geq 50$ is recommended if the population distribution is believed to be highly skewed or has outliers.

NOTES AND COMMENTS

1. When σ is known, the margin of error, $z_{\alpha/2}(\sigma/\sqrt{n})$, is fixed and is the same for all samples of size n. When σ is unknown, the margin of error, $t_{\alpha/2}(s/\sqrt{n})$, varies from sample to sample. This variation occurs because the sample standard deviation s varies depending upon the sample selected. A large value for s provides a larger margin of error, while a small value for s provides a smaller margin of error.
2. What happens to confidence interval estimates when the population is skewed? Consider a population that is skewed to the right with large data values stretching the distribution to the right. When such skewness exists, the sample mean $\bar{x}$ and the sample standard deviation s are positively correlated. Larger values of s tend to be associated with larger values of $\bar{x}$. Thus, when $\bar{x}$ is larger than the population mean, s tends to be larger than σ. This skewness causes the margin of error, $t_{\alpha/2}(s/\sqrt{n})$, to be larger than it would be with σ known. The confidence interval with the larger margin of error tends to include the population mean μ more often than it would if the true value of σ were used. But when $\bar{x}$ is smaller than the population mean, the correlation between $\bar{x}$ and s causes the margin of error to be small. In this case, the confidence interval with the smaller margin of error tends to miss the population mean more than it would if we knew σ and used it. For this reason, we recommend using larger sample sizes with highly skewed population distributions.

Exercises

Methods

11. For a t distribution with 16 degrees of freedom, find the area, or probability, in each region.
    a. To the right of 2.120
    b. To the left of 1.337
    c. To the left of -1.746
    d. To the right of 2.583
    e. Between -2.120 and 2.120
    f. Between -1.746 and 1.746
12. Find the t value(s) for each of the following cases.
    a. Upper tail area of .025 with 12 degrees of freedom
    b. Lower tail area of .05 with 50 degrees of freedom
    c. Upper tail area of .01 with 30 degrees of freedom
    d. Where 90% of the area falls between these two t values with 25 degrees of freedom
    e. Where 95% of the area falls between these two t values with 45 degrees of freedom

13. The following sample data are from a normal population: 10, 8, 12, 15, 13, 11, 6, 5.
    a. What is the point estimate of the population mean?
    b. What is the point estimate of the population standard deviation?
    c. With 95% confidence, what is the margin of error for the estimation of the population mean?
    d. What is the 95% confidence interval for the population mean?
14. A simple random sample with $n = 54$ provided a sample mean of 22.5 and a sample standard deviation of 4.4.
    a. Develop a 90% confidence interval for the population mean.
    b. Develop a 95% confidence interval for the population mean.

c. Develop a 99% confidence interval for the population mean.
d. What happens to the margin of error and the confidence interval as the confidence level is increased?

Applications

SELF test

15. Sales personnel for Skillings Distributors submit weekly reports listing the customer contacts made during the week. A sample of 65 weekly reports showed a sample mean of 19.5 customer contacts per week. The sample standard deviation was 5.2. Provide 90% and 95% confidence intervals for the population mean number of weekly customer contacts for the sales personnel.

16. The mean number of hours of flying time for pilots at Continental Airlines is 49 hours per month (*The Wall Street Journal,* February 25, 2003). Assume that this mean was based on actual flying times for a sample of 100 Continental pilots and that the sample standard deviation was 8.5 hours.
    a. At 95% confidence, what is the margin of error?
    b. What is the 95% confidence interval estimate of the population mean flying time for the pilots?
    c. The mean number of hours of flying time for pilots at United Airlines is 36 hours per month. Use your results from part (b) to discuss differences between the flying times for the pilots at the two airlines. *The Wall Street Journal* reported United Airlines as having the highest labor cost among all airlines. Does the information in this exercise provide insight as to why United Airlines might expect higher labor costs?

17. The International Air Transport Association surveys business travelers to develop quality ratings for transatlantic gateway airports. The maximum possible rating is 10. Suppose a simple random sample of 50 business travelers is selected and each traveler is asked to provide a rating for the Miami International Airport. The ratings obtained from the sample of 50 business travelers follow.

WEB file Miami

6	4	6	8	7	7	6	3	3	8	10	4	8
7	8	7	5	9	5	8	4	3	8	5	5	4
4	4	8	4	5	6	2	5	9	9	8	4	8
9	9	5	9	7	8	3	10	8	9	6		

Develop a 95% confidence interval estimate of the population mean rating for Miami.

WEB file JobSearch

18. Older people often have a hard time finding work. AARP reported on the number of weeks it takes a worker aged 55 plus to find a job. The data on number of weeks spent searching for a job contained in the file JobSearch are consistent with the AARP findings (*AARP Bulletin,* April 2008).
    a. Provide a point estimate of the population mean number of weeks it takes a worker aged 55 plus to find a job.
    b. At 95% confidence, what is the margin of error?
    c. What is the 95% confidence interval estimate of the mean?
    d. Discuss the degree of skewness found in the sample data. What suggestion would you make for a repeat of this study?

19. The average cost per night of a hotel room in New York City is $273 (*SmartMoney,* March 2009). Assume this estimate is based on a sample of 45 hotels and that the sample standard deviation is $65.
    a. With 95% confidence, what is the margin of error?
    b. What is the 95% confidence interval estimate of the population mean?
    c. Two years ago the average cost of a hotel room in New York City was $229. Discuss the change in cost over the two-year period.

WEB file
Program

20. Is your favorite TV program often interrupted by advertising? CNBC presented statistics on the average number of programming minutes in a half-hour sitcom (CNBC, February 23, 2006). The following data (in minutes) are representative of their findings.

21.06	22.24	20.62
21.66	21.23	23.86
23.82	20.30	21.52
21.52	21.91	23.14
20.02	22.20	21.20
22.37	22.19	22.34
23.36	23.44	

Assume the population is approximately normal. Provide a point estimate and a 95% confidence interval for the mean number of programming minutes during a half-hour television sitcom.

WEB file
Alcohol

21. Consumption of alcoholic beverages by young women of drinking age has been increasing in the United Kingdom, the United States, and Europe (*The Wall Street Journal,* February 15, 2006). Data (annual consumption in liters) consistent with the findings reported in *The Wall Street Journal* article are shown for a sample of 20 European young women.

266	82	199	174	97
170	222	115	130	169
164	102	113	171	0
93	0	93	110	130

Assuming the population is roughly symmetric, construct a 95% confidence interval for the mean annual consumption of alcoholic beverages by European young women.

22. Disney's *Hannah Montana: The Movie* opened on Easter weekend in April 2009. Over the three-day weekend, the movie became the number-one box office attraction (*The Wall Street Journal,* April 13, 2009). The ticket sales revenue in dollars for a sample of 25 theaters is as follows.

WEB file
TicketSales

20,200	10,150	13,000	11,320	9700
8350	7300	14,000	9940	11,200
10,750	6240	12,700	7430	13,500
13,900	4200	6750	6700	9330
13,185	9200	21,400	11,380	10,800

a. What is the 95% confidence interval estimate for the mean ticket sales revenue per theater? Interpret this result.
b. Using the movie ticket price of $7.16 per ticket, what is the estimate of the mean number of customers per theater?
c. The movie was shown in 3118 theaters. Estimate the total number of customers who saw *Hannah Montana: The Movie* and the total box office ticket sales for the three-day weekend.

8.3 Determining the Sample Size

If a desired margin of error is selected prior to sampling, the procedures in this section can be used to determine the sample size necessary to satisfy the margin of error requirement.

In providing practical advice in the two preceding sections, we commented on the role of the sample size in providing good approximate confidence intervals when the population is not normally distributed. In this section, we focus on another aspect of the sample size issue. We describe how to choose a sample size large enough to provide a desired margin of error. To understand how this process works, we return to the σ known case presented in Section 8.1. Using expression (8.1), the interval estimate is

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

The quantity $z_{\alpha/2}(\sigma/\sqrt{n})$ is the margin of error. Thus, we see that $z_{\alpha/2}$, the population standard deviation σ, and the sample size n combine to determine the margin of error. Once we select a confidence coefficient $1 - \alpha$, $z_{\alpha/2}$ can be determined. Then, if we have a value for σ, we can determine the sample size n needed to provide any desired margin of error. Development of the formula used to compute the required sample size n follows.

Let E = the desired margin of error:

$$E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

Solving for $\sqrt{n}$, we have

$$\sqrt{n} = \frac{z_{\alpha/2}\sigma}{E}$$

Squaring both sides of this equation, we obtain the following expression for the sample size.

Equation (8.3) can be used to provide a good sample size recommendation. However, judgment on the part of the analyst should be used to determine whether the final sample size should be adjusted upward.

SAMPLE SIZE FOR AN INTERVAL ESTIMATE OF A POPULATION MEAN

$$n = \frac{(z_{\alpha/2})^2\sigma^2}{E^2} \tag{8.3}$$

This sample size provides the desired margin of error at the chosen confidence level.

In equation (8.3), E is the margin of error that the user is willing to accept, and the value of $z_{\alpha/2}$ follows directly from the confidence level to be used in developing the interval estimate. Although user preference must be considered, 95% confidence is the most frequently chosen value ($z_{.025} = 1.96$).

Finally, use of equation (8.3) requires a value for the population standard deviation σ. However, even if σ is unknown, we can use equation (8.3) provided we have a preliminary or *planning value* for σ. In practice, one of the following procedures can be chosen.

A planning value for the population standard deviation σ must be specified before the sample size can be determined. Three methods of obtaining a planning value for σ are discussed here.

1. Use the estimate of the population standard deviation computed from data of previous studies as the planning value for σ.
2. Use a pilot study to select a preliminary sample. The sample standard deviation from the preliminary sample can be used as the planning value for σ.
3. Use judgment or a "best guess" for the value of σ. For example, we might begin by estimating the largest and smallest data values in the population. The difference between the largest and smallest values provides an estimate of the range for the data. Finally, the range divided by 4 is often suggested as a rough approximation of the standard deviation and thus an acceptable planning value for σ.

Let us demonstrate the use of equation (8.3) to determine the sample size by considering the following example. A previous study that investigated the cost of renting automobiles in the United States found a mean cost of approximately $55 per day for renting a midsize automobile. Suppose that the organization that conducted this study would like to conduct a new study in order to estimate the population mean daily rental cost for a midsize automobile in the United States. In designing the new study, the project director specifies that the population mean daily rental cost be estimated with a margin of error of $2 and a 95% level of confidence.

The project director specified a desired margin of error of $E = 2$, and the 95% level of confidence indicates $z_{.025} = 1.96$. Thus, we only need a planning value for the population standard deviation σ in order to compute the required sample size. At this point, an analyst

Equation (8.3) provides the minimum sample size needed to satisfy the desired margin of error requirement. If the computed sample size is not an integer, rounding up to the next integer value will provide a margin of error slightly smaller than required.

reviewed the sample data from the previous study and found that the sample standard deviation for the daily rental cost was \$9.65. Using 9.65 as the planning value for σ, we obtain

$$n = \frac{(z_{\alpha/2})^2\sigma^2}{E^2} = \frac{(1.96)^2(9.65)^2}{2^2} = 89.43$$

Thus, the sample size for the new study needs to be at least 89.43 midsize automobile rentals in order to satisfy the project director's \$2 margin-of-error requirement. In cases where the computed n is not an integer, we round up to the next integer value; hence, the recommended sample size is 90 midsize automobile rentals.

Exercises

Methods

23. How large a sample should be selected to provide a 95% confidence interval with a margin of error of 10? Assume that the population standard deviation is 40.

24. The range for a set of data is estimated to be 36.
    a. What is the planning value for the population standard deviation?
    b. At 95% confidence, how large a sample would provide a margin of error of 3?
    c. At 95% confidence, how large a sample would provide a margin of error of 2?

Applications

25. Refer to the Scheer Industries example in Section 8.2. Use 6.84 days as a planning value for the population standard deviation.
    a. Assuming 95% confidence, what sample size would be required to obtain a margin of error of 1.5 days?
    b. If the precision statement was made with 90% confidence, what sample size would be required to obtain a margin of error of 2 days?

26. The average cost of a gallon of unleaded gasoline in Greater Cincinnati was reported to be \$2.41 (*The Cincinnati Enquirer*, February 3, 2006). During periods of rapidly changing prices, the newspaper samples service stations and prepares reports on gasoline prices frequently. Assume the standard deviation is \$.15 for the price of a gallon of unleaded regular gasoline, and recommend the appropriate sample size for the newspaper to use if they wish to report a margin of error at 95% confidence.
    a. Suppose the desired margin of error is \$.07.
    b. Suppose the desired margin of error is \$.05.
    c. Suppose the desired margin of error is \$.03.

27. Annual starting salaries for college graduates with degrees in business administration are generally expected to be between \$30,000 and \$45,000. Assume that a 95% confidence interval estimate of the population mean annual starting salary is desired. What is the planning value for the population standard deviation? How large a sample should be taken if the desired margin of error is
    a. \$500?
    b. \$200?
    c. \$100?
    d. Would you recommend trying to obtain the \$100 margin of error? Explain.

28. An online survey by ShareBuilder, a retirement plan provider, and Harris Interactive reported that 60% of female business owners are not confident they are saving enough for

retirement (*SmallBiz*, Winter 2006). Suppose we would like to do a follow-up study to determine how much female business owners are saving each year toward retirement and want to use $100 as the desired margin of error for an interval estimate of the population mean. Use $1100 as a planning value for the standard deviation and recommend a sample size for each of the following situations.

a. A 90% confidence interval is desired for the mean amount saved.
b. A 95% confidence interval is desired for the mean amount saved.
c. A 99% confidence interval is desired for the mean amount saved.
d. When the desired margin of error is set, what happens to the sample size as the confidence level is increased? Would you recommend using a 99% confidence interval in this case? Discuss.

29. The travel-to-work time for residents of the 15 largest cities in the United States is reported in the *2003 Information Please Almanac*. Suppose that a preliminary simple random sample of residents of San Francisco is used to develop a planning value of 6.25 minutes for the population standard deviation.
    a. If we want to estimate the population mean travel-to-work time for San Francisco residents with a margin of error of 2 minutes, what sample size should be used? Assume 95% confidence.
    b. If we want to estimate the population mean travel-to-work time for San Francisco residents with a margin of error of 1 minute, what sample size should be used? Assume 95% confidence.

30. During the first quarter of 2003, the price/earnings (P/E) ratio for stocks listed on the New York Stock Exchange generally ranged from 5 to 60 (*The Wall Street Journal*, March 7, 2003). Assume that we want to estimate the population mean P/E ratio for all stocks listed on the exchange. How many stocks should be included in the sample if we want a margin of error of 3? Use 95% confidence.

8.4 Population Proportion

In the introduction to this chapter we said that the general form of an interval estimate of a population proportion p is

$$\bar{p} \pm \text{Margin of error}$$

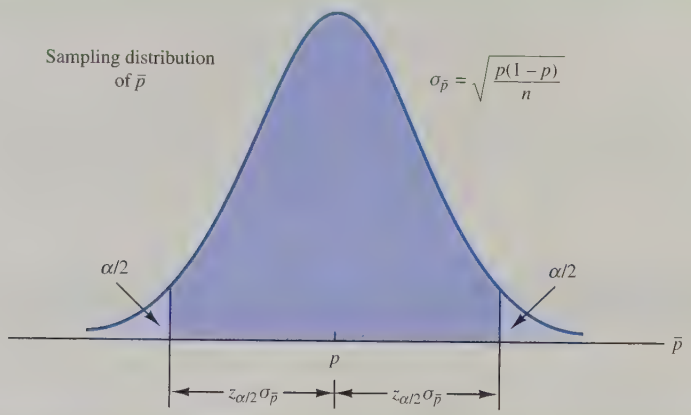
The sampling distribution of $\bar{p}$ plays a key role in computing the margin of error for this interval estimate.

In Chapter 7 we said that the sampling distribution of $\bar{p}$ can be approximated by a normal distribution whenever $np \geq 5$ and $n(1 - p) \geq 5$. Figure 8.10 shows the normal approximation of the sampling distribution of $\bar{p}$. The mean of the sampling distribution of $\bar{p}$ is the population proportion p, and the standard error of $\bar{p}$ is

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} \tag{8.4}$$

Because the sampling distribution of $\bar{p}$ is normally distributed, if we choose $z_{\alpha/2}\sigma_{\bar{p}}$ as the margin of error in an interval estimate of a population proportion, we know that $100(1 - \alpha)\%$ of the intervals generated will contain the true population proportion. But $\sigma_{\bar{p}}$ cannot be used directly in the computation of the margin of error because p will not be

FIGURE 8.10 NORMAL APPROXIMATION OF THE SAMPLING DISTRIBUTION OF $\bar{p}$



known; p is what we are trying to estimate. So $\bar{p}$ is substituted for p and the margin of error for an interval estimate of a population proportion is given by

$$\text{Margin of error} = z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \tag{8.5}$$

With this margin of error, the general expression for an interval estimate of a population proportion is as follows.

INTERVAL ESTIMATE OF A POPULATION PROPORTION

$$\bar{p} \pm z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \tag{8.6}$$

When developing confidence intervals for proportions, the quantity $z_{\alpha/2}\sqrt{\bar{p}(1-\bar{p})/n}$ provides the margin of error.

where $1 - \alpha$ is the confidence coefficient and $z_{\alpha/2}$ is the z value providing an area of $\alpha/2$ in the upper tail of the standard normal distribution.

WEB file
TeeTimes

The following example illustrates the computation of the margin of error and interval estimate for a population proportion. A national survey of 900 women golfers was conducted to learn how women golfers view their treatment at golf courses in the United States. The survey found that 396 of the women golfers were satisfied with the availability of tee times. Thus, the point estimate of the proportion of the population of women golfers who are satisfied with the availability of tee times is 396/900 = .44. Using expression (8.6) and a 95% confidence level,

$$\bar{p} \pm z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$.44 \pm 1.96\sqrt{\frac{.44(1-.44)}{900}}$$

$$.44 \pm .0324$$

Thus, the margin of error is .0324 and the 95% confidence interval estimate of the population proportion is .4076 to .4724. Using percentages, the survey results enable us to state with 95% confidence that between 40.76% and 47.24% of all women golfers are satisfied with the availability of tee times.

Using Excel

Excel can be used to construct an interval estimate of the population proportion of women golfers who are satisfied with the availability of tee times. The responses in the survey were recorded as a Yes or No for each woman surveyed. Refer to Figure 8.11 as we describe the tasks involved in constructing a 95% confidence interval. The formula worksheet is in the background; the value worksheet appears in the foreground.

Enter Data: The Yes/No data for the 900 women golfers are entered into cells A2:A901.

Enter Functions and Formulas: The descriptive statistics we need and the response of interest are provided in cells D3:D6. Because Excel's COUNT function only works with numerical data, we used the COUNTA function in cell D3 to compute the sample size. The response for which we want to develop an interval estimate, Yes or No, is entered into cell D4. Figure 8.11 shows that Yes has been entered into cell D4, indicating that we want to develop an interval estimate of the population proportion of women golfers who are satisfied with the availability of tee times. If we had wanted to develop an interval estimate of the population proportion of women golfers who are not satisfied with the availability of tee times, we would have entered No in cell D4. With Yes entered in cell D4, the COUNTIF function in cell D5 counts the number of Yes responses in the sample. The sample proportion is then computed in cell D6 by dividing the number of Yes responses in cell D5 by the sample size in cell D3.

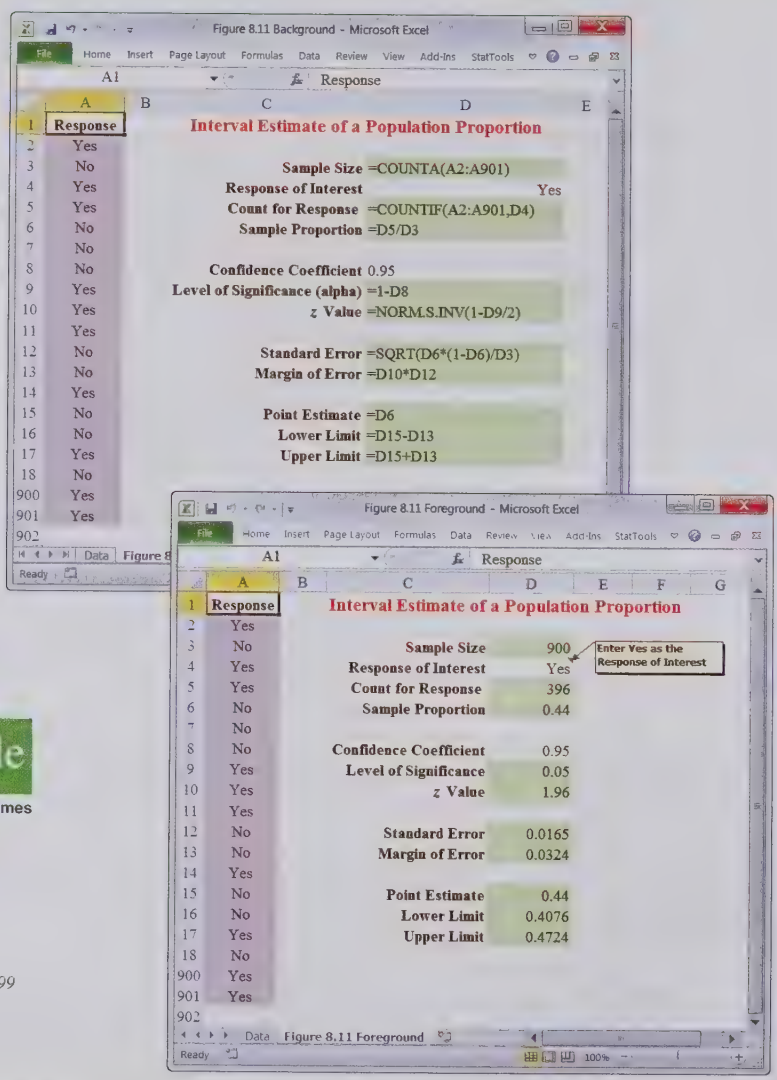
Cells D8:D10 are used to compute the appropriate z value. The confidence coefficient (0.95) is entered into cell D8 and the level of significance (α) is computed in cell D9 by entering the formula =1−D8. The z value corresponding to an upper tail area of $\alpha/2$ is computed by entering the formula =NORM.S.INV(1−D9/2) into cell D10. The value worksheet shows that $z_{.025} = 1.960$.

Cells D12:D13 provide the estimate of the standard error and the margin of error. In cell D12, we entered the formula =SQRT(D6*(1−D6)/D3) to compute the standard error using the sample proportion and the sample size as inputs. The formula =D10*D12 is entered into cell D13 to compute the margin of error.

Cells D15:D17 provide the point estimate and the lower and upper limits for a confidence interval. The point estimate in cell D15 is the sample proportion. The lower and upper limits in cells D16 and D17 are obtained by subtracting and adding the margin of error to the point estimate. We note that the 95% confidence interval for the proportion of women golfers who are satisfied with the availability of tee times is .4076 to .4724.

A template for other problems The worksheet in Figure 8.11 can be used as a template for developing confidence intervals about a population proportion p. To use this worksheet for another problem of this type, we must first enter the new problem data in column A. The response of interest would then be typed in cell D4 and the ranges for the formulas in cells D3 and D5 would be revised to correspond to the new data. After doing so, the point estimate and a 95% confidence interval will be displayed in cells D15:D17. If a confidence interval with a different confidence coefficient is desired, we simply change the value in cell D8.

FIGURE 8.11 EXCEL WORKSHEET: 95% CONFIDENCE INTERVAL FOR SURVEY OF WOMEN GOLFERS



WEB file
TeeTimes

Note: Rows 19 to 899 are hidden.

Determining the Sample Size

Let us consider the question of how large the sample size should be to obtain an estimate of a population proportion at a specified level of precision. The rationale for the sample size determination in developing interval estimates of p is similar to the rationale used in Section 8.3 to determine the sample size for estimating a population mean.

Previously in this section we said that the margin of error associated with an interval estimate of a population proportion is $z_{\alpha/2}\sqrt{\bar{p}(1-\bar{p})/n}$. The margin of error is based on the value of $z_{\alpha/2}$, the sample proportion $\bar{p}$, and the sample size n. Larger sample sizes provide a smaller margin of error and better precision.

Let E denote the desired margin of error.

$$E = z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Solving this equation for n provides a formula for the sample size that will provide a margin of error of size E.

$$n = \frac{(z_{\alpha/2})^2\bar{p}(1-\bar{p})}{E^2}$$

Note, however, that we cannot use this formula to compute the sample size that will provide the desired margin of error because $\bar{p}$ will not be known until after we select the sample. What we need, then, is a planning value for $\bar{p}$ that can be used to make the computation. Using p^* to denote the planning value for $\bar{p}$, the following formula can be used to compute the sample size that will provide a margin of error of size E.

SAMPLE SIZE FOR AN INTERVAL ESTIMATE OF A POPULATION PROPORTION

$$n = \frac{(z_{\alpha/2})^2p^*(1-p^*)}{E^2} \tag{8.7}$$

In practice, the planning value p^* can be chosen by one of the following procedures.

1. Use the sample proportion from a previous sample of the same or similar units.
2. Use a pilot study to select a preliminary sample. The sample proportion from this sample can be used as the planning value, p^*.
3. Use judgment or a "best guess" for the value of p^*.
4. If none of the preceding alternatives apply, use a planning value of $p^* = .50$.

Let us return to the survey of women golfers and assume that the company is interested in conducting a new survey to estimate the current proportion of the population of women golfers who are satisfied with the availability of tee times. How large should the sample be if the survey director wants to estimate the population proportion with a margin of error of .025 at 95% confidence? With $E = .025$ and $z_{\alpha/2} = 1.96$, we need a planning value p^* to answer the sample size question. Using the previous survey result of $\bar{p} = .44$ as the planning value p^*, equation (8.7) shows that

$$n = \frac{(z_{\alpha/2})^2p^*(1-p^*)}{E^2} = \frac{(1.96)^2(.44)(1-.44)}{(.025)^2} = 1514.5$$

Thus, the sample size must be at least 1514.5 women golfers to satisfy the margin of error requirement. Rounding up to the next integer value indicates that a sample of 1515 women golfers is recommended to satisfy the margin of error requirement.

The fourth alternative suggested for selecting a planning value p^* is to use $p^* = .50$. This value of p^* is frequently used when no other information is available. To understand why, note that the

TABLE 8.5 SOME POSSIBLE VALUES FOR $p^*(1 - p^*)$

p^*	$p^*(1 - p^*)$	
.10	(.10)(.90) = .09	
.30	(.30)(.70) = .21	
.40	(.40)(.60) = .24	
.50	(.50)(.50) = .25	← Largest value for $p^*(1 - p^*)$
.60	(.60)(.40) = .24	
.70	(.70)(.30) = .21	
.90	(.90)(.10) = .09	

numerator of equation (8.7) shows that the sample size is proportional to the quantity $p^*(1 - p^*)$. A larger value for the quantity $p^*(1 - p^*)$ will result in a larger sample size. Table 8.5 gives some possible values of $p^*(1 - p^*)$. Note that the largest value of $p^*(1 - p^*)$ occurs when $p^* = .50$. Thus, in case of any uncertainty about an appropriate planning value, we know that $p^* = .50$ will provide the largest sample size recommendation. In effect, we play it safe by recommending the largest necessary sample size. If the sample proportion turns out to be different from the .50 planning value, the margin of error will be smaller than anticipated. Thus, in using $p^* = .50$, we guarantee that the sample size will be sufficient to obtain the desired margin of error.

In the survey of women golfers example, a planning value of $p^* = .50$ would have provided the sample size

$$n = \frac{(z_{\alpha/2})^2 p^*(1 - p^*)}{E^2} = \frac{(1.96)^2(.50)(1 - .50)}{(.025)^2} = 1536.6$$

Thus, a slightly larger sample size of 1537 women golfers would be recommended.

NOTES AND COMMENTS

The desired margin of error for estimating a population proportion is almost always .10 or less. In national public opinion polls conducted by organizations such as Gallup and Harris, a .03 or .04 margin of error is common. With such margins of error, equation (8.7) will almost always provide a sample size that is large enough to satisfy the requirements of $np \geq 5$ and $n(1 - p) \geq 5$ for using a normal distribution as an approximation for the sampling distribution of $\bar{x}$.

Exercises

Methods

SELF test

31. A simple random sample of 400 individuals provides 100 Yes responses.
    a. What is the point estimate of the proportion of the population that would provide Yes responses?
    b. What is your estimate of the standard error of the proportion, $\sigma_{\bar{p}}$?
    c. Compute the 95% confidence interval for the population proportion.
32. A simple random sample of 800 elements generates a sample proportion $\bar{p} = .70$.
    a. Provide a 90% confidence interval for the population proportion.
    b. Provide a 95% confidence interval for the population proportion.

33. In a survey, the planning value for the population proportion is $p^* = .35$. How large a sample should be taken to provide a 95% confidence interval with a margin of error of .05?

34. At 95% confidence, how large a sample should be taken to obtain a margin of error of .03 for the estimation of a population proportion? Assume that past data are not available for developing a planning value for p^*.

Applications

SELF test

35. The Consumer Reports National Research Center conducted a telephone survey of 2000 adults to learn about the major economic concerns for the future (*Consumer Reports,* January 2009). The survey results showed that 1760 of the respondents think the future health of Social Security is a major economic concern.
    a. What is the point estimate of the population proportion of adults who think the future health of Social Security is a major economic concern.
    b. At 90% confidence, what is the margin of error?
    c. Develop a 90% confidence interval for the population proportion of adults who think the future health of Social Security is a major economic concern.
    d. Develop a 95% confidence interval for this population proportion.

36. According to statistics reported on CNBC, a surprising number of motor vehicles are not covered by insurance (CNBC, February 23, 2006). Sample results, consistent with the CNBC report, showed 46 of 200 vehicles were not covered by insurance.
    a. What is the point estimate of the proportion of vehicles not covered by insurance?
    b. Develop a 95% confidence interval for the population proportion.

37. Towers Perrin, a New York human resources consulting firm, conducted a survey of 1100 employees at medium-sized and large companies to determine how dissatisfied employees were with their jobs (*The Wall Street Journal,* January 29, 2003). Representative data are shown in the file JobSatisfaction. A response of Yes indicates the employee strongly disliked the current work experience.

WEB file
JobSatisfaction

    a. What is the point estimate of the proportion of the population of employees who strongly dislike their current work experience?
    b. At 95% confidence, what is the margin of error?
    c. What is the 95% confidence interval for the proportion of the population of employees who strongly dislike their current work experience?
    d. Towers Perrin estimates that it costs employers one-third of an hourly employee's annual salary to find a successor and as much as 1.5 times the annual salary to find a successor for a highly compensated employee. What message did this survey send to employers?

38. According to Thomson Financial, through January 25, 2006, the majority of companies reporting profits had beaten estimates (*BusinessWeek,* February 6, 2006). A sample of 162 companies showed that 104 beat estimates, 29 matched estimates, and 29 fell short.
    a. What is the point estimate of the proportion that fell short of estimates?
    b. Determine the margin of error and provide a 95% confidence interval for the proportion that beat estimates.
    c. How large a sample is needed if the desired margin of error is .05?

SELF test

39. The percentage of people not covered by health care insurance in 2003 was 15.6% (*Statistical Abstract of the United States,* 2006). A congressional committee has been charged with conducting a sample survey to obtain more current information.
    a. What sample size would you recommend if the committee's goal is to estimate the current proportion of individuals without health care insurance with a margin of error of .03? Use a 95% confidence level.
    b. Repeat part (a) using a 99% confidence level.

40. For many years businesses have struggled with the rising cost of health care. But recently, the increases have slowed due to less inflation in health care prices and employees paying for a larger portion of health care benefits. A recent Mercer survey showed that 52% of U.S. employers were likely to require higher employee contributions for health care coverage in 2009 (*BusinessWeek,* February 16, 2009). Suppose the survey was based on a sample of 800 companies. Compute the margin of error and a 95% confidence interval for the proportion of companies likely to require higher employee contributions for health care coverage in 2009.

41. America's young people are heavy Internet users; 87% of Americans ages 12 to 17 are Internet users (*The Cincinnati Enquirer,* February 7, 2006). MySpace was voted the most popular website by 9% in a sample survey of Internet users in this age group. Suppose 1400 youths participated in the survey. What is the margin of error, and what is the interval estimate of the population proportion for which MySpace is the most popular website? Use a 95% confidence level.

42. A poll for the presidential campaign sampled 491 potential voters in June. A primary purpose of the poll was to obtain an estimate of the proportion of potential voters who favored each candidate. Assume a planning value of $p^* = .50$ and a 95% confidence level.
    a. For $p^* = .50$, what was the planned margin of error for the June poll?
    b. Closer to the November election, better precision and smaller margins of error are desired. Assume the following margins of error are requested for surveys to be conducted during the presidential campaign. Compute the recommended sample size for each survey.

Survey	Margin of Error
September	.04
October	.03
Early November	.02
Pre-Election Day	.01

43. A Phoenix Wealth Management/Harris Interactive survey of 1500 individuals with net worth of $1 million or more provided a variety of statistics on wealthy people (*BusinessWeek,* September 22, 2003). The previous three-year period had been bad for the stock market, which motivated some of the questions asked.
    a. The survey reported that 53% of the respondents lost 25% or more of their portfolio value over the past three years. Develop a 95% confidence interval for the proportion of wealthy people who lost 25% or more of their portfolio value over the past three years.
    b. The survey reported that 31% of the respondents feel they have to save more for retirement to make up for what they lost. Develop a 95% confidence interval for the population proportion.
    c. Five percent of the respondents gave $25,000 or more to charity over the previous year. Develop a 95% confidence interval for the proportion who gave $25,000 or more to charity.
    d. Compare the margin of error for the interval estimates in parts (a), (b), and (c). How is the margin of error related to $\bar{p}$? When the same sample is being used to estimate a variety of proportions, which of the proportions should be used to choose the planning value p^*? Why do you think $p^* = .50$ is often used in these cases?

Summary

In this chapter we presented methods for developing interval estimates of a population mean and a population proportion. A point estimator may or may not provide a good estimate of a population parameter. The use of an interval estimate provides a measure of the precision

of an estimate. Both the interval estimate of the population mean and the population proportion are of the form: point estimate ± margin of error.

We presented interval estimates for a population mean for two cases. In the σ known case, historical data or other information is used to develop an estimate of σ prior to taking a sample. Analysis of new sample data then proceeds based on the assumption that σ is known. In the σ unknown case, the sample data are used to estimate both the population mean and the population standard deviation. The final choice of which interval estimation procedure to use depends upon the analyst's understanding of which method provides the best estimate of σ.

In the σ known case, the interval estimation procedure is based on the assumed value of σ and the use of the standard normal distribution. In the σ unknown case, the interval estimation procedure uses the sample standard deviation s and the t distribution. In both cases the quality of the interval estimates obtained depends on the distribution of the population and the sample size. If the population is normally distributed the interval estimates will be exact in both cases, even for small sample sizes. If the population is not normally distributed, the interval estimates obtained will be approximate. Larger sample sizes will provide better approximations, but the more highly skewed the population is, the larger the sample size needs to be to obtain a good approximation. Practical advice about the sample size necessary to obtain good approximations was included in Sections 8.1 and 8.2. In most cases a sample of size 30 or more will provide good approximate confidence intervals.

The general form of the interval estimate for a population proportion is $\bar{p}$ ± margin of error. In practice the sample sizes used for interval estimates of a population proportion are generally large. Thus, the interval estimation procedure is based on the standard normal distribution.

Often a desired margin of error is specified prior to developing a sampling plan. We showed how to choose a sample size large enough to provide the desired precision.

Glossary

Interval estimate An estimate of a population parameter that provides an interval believed to contain the value of the parameter. For the interval estimates in this chapter, it has the form: point estimate ± margin of error.

Margin of error The ± value added to and subtracted from a point estimate in order to develop an interval estimate of a population parameter.

σ known The case when historical data or other information provides a good value for the population standard deviation prior to taking a sample. The interval estimation procedure uses this known value of σ in computing the margin of error.

Confidence level The confidence associated with an interval estimate. For example, if an interval estimation procedure provides intervals such that 95% of the intervals formed using the procedure will include the population parameter, the interval estimate is said to be constructed at the 95% confidence level.

Confidence coefficient The confidence level expressed as a decimal value. For example, .95 is the confidence coefficient for a 95% confidence level.

Confidence interval Another name for an interval estimate.

Level of significance The probability that the interval estimation procedure will generate an interval that does not contain μ.

σ unknown The more common case when no good basis exists for estimating the population standard deviation prior to taking the sample. The interval estimation procedure uses the sample standard deviation s in computing the margin of error.

t distribution A family of probability distributions that can be used to develop an interval estimate of a population mean whenever the population standard deviation σ is unknown and is estimated by the sample standard deviation s.

Degrees of freedom A parameter of the t distribution. When the t distribution is used in the computation of an interval estimate of a population mean, the appropriate t distribution has $n - 1$ degrees of freedom, where n is the size of the simple random sample.

Key Formulas

Interval Estimate of a Population Mean: σ Known

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \tag{8.1}$$

Interval Estimate of a Population Mean: σ Unknown

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} \tag{8.2}$$

Sample Size for an Interval Estimate of a Population Mean

$$n = \frac{(z_{\alpha/2})^2 \sigma^2}{E^2} \tag{8.3}$$

Interval Estimate of a Population Proportion

$$\bar{p} \pm z_{\alpha/2} \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} \tag{8.6}$$

Sample Size for an Interval Estimate of a Population Proportion

$$n = \frac{(z_{\alpha/2})^2 p^*(1 - p^*)}{E^2} \tag{8.7}$$

Supplementary Exercises

44. A sample survey of 54 discount brokers showed that the mean price charged for a trade of 100 shares at $50 per share was $33.77 (*AAII Journal*, February 2006). The survey is conducted annually. With the historical data available, assume a known population standard deviation of $15.
    a. Using the sample data, what is the margin of error associated with a 95% confidence interval?
    b. Develop a 95% confidence interval for the mean price charged by discount brokers for a trade of 100 shares at $50 per share.

45. A survey conducted by the American Automobile Association showed that a family of four spends an average of $215.60 per day while on vacation. Suppose a sample of 64 families

of four vacationing at Niagara Falls resulted in a sample mean of $252.45 per day and a sample standard deviation of $74.50.

a. Develop a 95% confidence interval estimate of the mean amount spent per day by a family of four visiting Niagara Falls.
b. Based on the confidence interval from part (a), does it appear that the population mean amount spent per day by families visiting Niagara Falls differs from the mean reported by the American Automobile Association? Explain.

46. The 92 million Americans of age 50 and over control 50 percent of all discretionary income (*AARP Bulletin*, March 2008). AARP estimated that the average annual expenditure on restaurants and carryout food was $1873 for individuals in this age group. Suppose this estimate is based on a sample of 80 persons and that the sample standard deviation is $550.
    a. At 95% confidence, what is the margin of error?
    b. What is the 95% confidence interval for the population mean amount spent on restaurants and carryout food?
    c. What is your estimate of the total amount spent by Americans of age 50 and over on restaurants and carryout food?
    d. If the amount spent on restaurants and carryout food is skewed to the right, would you expect the median amount spent to be greater or less than $1873?

47. Many stock market observers say that when the P/E ratio for stocks gets over 20 the market is overvalued. The P/E ratio is the stock price divided by the most recent 12 months of earnings. Suppose you are interested in seeing whether the current market is overvalued and would also like to know what proportion of companies pay dividends. A random sample of 30 companies listed on the New York Stock Exchange (NYSE) is provided (*Barron's*, January 19, 2004).

WEB file
NYSEStocks

Company	Dividend	P/E Ratio	Company	Dividend	P/E Ratio
Albertsons	Yes	14	NY Times A	Yes	25
BRE Prop	Yes	18	Omnicare	Yes	25
CityNtl	Yes	16	PallCp	Yes	23
DelMonte	No	21	PubSvcEnt	Yes	11
EnrgzHldg	No	20	SensientTch	Yes	11
Ford Motor	Yes	22	SmtProp	Yes	12
Gildan A	No	12	TJX Cos	Yes	21
HudsnUtdBcp	Yes	13	Thomson	Yes	30
IBM	Yes	22	USB Hldg	Yes	12
JeffPilot	Yes	16	US Restr	Yes	26
KingswayFin	No	6	Varian Med	No	41
Libbey	Yes	13	Visx	No	72
MasoniteIntl	No	15	Waste Mgt	No	23
Motorola	Yes	68	Wiley A	Yes	21
Ntl City	Yes	10	Yum Brands	No	18

    a. What is a point estimate of the P/E ratio for the population of stocks listed on the New York Stock Exchange? Develop a 95% confidence interval.
    b. Based on your answer to part (a), do you believe that the market is overvalued?
    c. What is a point estimate of the proportion of companies on the NYSE that pay dividends? Is the sample size large enough to justify using the normal distribution to construct a confidence interval for this proportion? Why or why not?

WEB file
Flights

48. US Airways conducted a number of studies that indicated a substantial savings could be obtained by encouraging Dividend Miles frequent flyer customers to redeem miles and

schedule award flights online (*US Airways Attaché,* February 2003). One study collected data on the amount of time required to redeem miles and schedule an award flight over the telephone. A sample showing the time in minutes required for each of 150 award flights scheduled by telephone is contained in the data set Flights. Use Excel to help answer the following questions.

a. What is the sample mean number of minutes required to schedule an award flight by telephone?
b. What is the 95% confidence interval for the population mean time to schedule an award flight by telephone?
c. Assume a telephone ticket agent works 7.5 hours per day. How many award flights can one ticket agent be expected to handle a day?
d. Discuss why this information supported US Airways' plans to use an online system to reduce costs.

49. A recent article reported that there are approximately 11 minutes of actual playing time in a typical National Football League (NFL) game (*The Wall Street Journal*, January 15, 2010). The article included information about the amount of time devoted to replays, the amount of time devoted to commercials, and the amount of time the players spend standing around between plays. Data consistent with the findings published in *The Wall Street Journal* are in the file named Standing. These data provide the amount of time players spend standing around between plays for a sample of 60 NFL games.
    a. Use the Standing data set to develop a point estimate of the number of minutes during an NFL game that players are standing around between plays. Compare this to the actual playing time reported in the article. Are you surprised?
    b. What is the sample standard deviation?
    c. Develop a 95% confidence interval for the number of minutes players spend standing around between plays.

50. Mileage tests are conducted for a particular model of automobile. If a 98% confidence interval with a margin of error of 1 mile per gallon is desired, how many automobiles should be used in the test? Assume that preliminary mileage tests indicate the standard deviation is 2.6 miles per gallon.

51. In developing patient appointment schedules, a medical center wants to estimate the mean time that a staff member spends with each patient. How large a sample should be taken if the desired margin of error is two minutes at a 95% level of confidence? How large a sample should be taken for a 99% level of confidence? Use a planning value for the population standard deviation of eight minutes.

52. Annual salary plus bonus data for chief executive officers are presented in the *BusinessWeek* Annual Pay Survey. A preliminary sample showed that the standard deviation is $675 with data provided in thousands of dollars. How many chief executive officers should be in a sample if we want to estimate the population mean annual salary plus bonus with a margin of error of $100,000? (*Note:* The desired margin of error would be $E = 100$ if the data are in thousands of dollars.) Use 95% confidence.

53. The National Center for Education Statistics reported that 47% of college students work to pay for tuition and living expenses. Assume that a sample of 450 college students was used in the study.
    a. Provide a 95% confidence interval for the population proportion of college students who work to pay for tuition and living expenses.
    b. Provide a 99% confidence interval for the population proportion of college students who work to pay for tuition and living expenses.
    c. What happens to the margin of error as the confidence is increased from 95% to 99%?

54. A *USA Today*/CNN/Gallup survey of 369 working parents found 200 who said they spend too little time with their children because of work commitments.

a. What is the point estimate of the proportion of the population of working parents who feel they spend too little time with their children because of work commitments?
b. At 95% confidence, what is the margin of error?
c. What is the 95% confidence interval estimate of the population proportion of working parents who feel they spend too little time with their children because of work commitments?

55. Which would be hardest for you to give up: your computer or your television? In a recent survey of 1677 U.S. Internet users, 74% of the young tech elite (average age of 22) say their computer would be very hard to give up (*PC Magazine,* February 3, 2004). Only 48% say their television would be very hard to give up.
a. Develop a 95% confidence interval for the proportion of the young tech elite that would find it very hard to give up their computer.
b. Develop a 99% confidence interval for the proportion of the young tech elite that would find it very hard to give up their television.
c. In which case, part (a) or part (b), is the margin of error larger? Explain why.

56. Cincinnati/Northern Kentucky International Airport had the second highest on-time arrival rate for 2005 among the nation's busiest airports (*The Cincinnati Enquirer,* February 3, 2006). Assume the findings were based on 455 on-time arrivals out of a sample of 550 flights.
a. Develop a point estimate of the on-time arrival rate (proportion of flights arriving on time) for the airport.
b. Construct a 95% confidence interval for the on-time arrival rate of the population of all flights at the airport during 2005.

57. The *2003 Statistical Abstract of the United States* reported the percentage of people 18 years of age and older who smoke. Suppose that a study designed to collect new data on smokers and nonsmokers uses a preliminary estimate of the proportion who smoke of .30.
a. How large a sample should be taken to estimate the proportion of smokers in the population with a margin of error of .02? Use 95% confidence.
b. Assume that the study uses your sample size recommendation in part (a) and finds 520 smokers. What is the point estimate of the proportion of smokers in the population?
c. What is the 95% confidence interval for the proportion of smokers in the population?

58. A well-known bank credit card firm wishes to estimate the proportion of credit card holders who carry a nonzero balance at the end of the month and incur an interest charge. Assume that the desired margin of error is .03 at 98% confidence.
a. How large a sample should be selected if it is anticipated that roughly 70% of the firm's card holders carry a nonzero balance at the end of the month?
b. How large a sample should be selected if no planning value for the proportion could be specified?

59. Workers in several industries were surveyed to determine the proportion of workers who feel their industry is understaffed. In the government sector, 37% of the respondents said they were understaffed, in the health care sector 33% said they were understaffed, and in the education sector 28% said they were understaffed (*USA Today,* January 11, 2010). Suppose that 200 workers were surveyed in each industry.
a. Construct a 95% confidence interval for the proportion of workers in each of these industries who feel their industry is understaffed.
b. Assuming the sample size will be the same in each industry, how large would the sample need to be to ensure that the margin of error is .05 or less for each of the three confidence intervals?

60. Although airline schedules and cost are important factors for business travelers when choosing an airline carrier, a *USA Today* survey found that business travelers list an airline's frequent flyer program as the most important factor. From a sample of $n = 1993$

business travelers who responded to the survey, 618 listed a frequent flyer program as the most important factor.

a. What is the point estimate of the proportion of the population of business travelers who believe a frequent flyer program is the most important factor when choosing an airline carrier?
b. Develop a 95% confidence interval estimate of the population proportion.
c. How large a sample would be required to report the margin of error of .01 at 95% confidence? Would you recommend that *USA Today* attempt to provide this degree of precision? Why or why not?

Case Problem 1 *Young Professional* Magazine

Young Professional magazine was developed for a target audience of recent college graduates who are in their first 10 years in a business/professional career. In its two years of publication, the magazine has been fairly successful. Now the publisher is interested in expanding the magazine's advertising base. Potential advertisers continually ask about the demographics and interests of subscribers to *Young Professional*. To collect this information, the magazine commissioned a survey to develop a profile of its subscribers. The survey results will be used to help the magazine choose articles of interest and provide advertisers with a profile of subscribers. As a new employee of the magazine, you have been asked to help analyze the survey results.

WEB file
Professional

Some of the survey questions follow:

1. What is your age?
2. Are you: Male_____ Female_____
3. Do you plan to make any real estate purchases in the next two years? Yes_____ No_____
4. What is the approximate total value of financial investments, exclusive of your home, owned by you or members of your household?
5. How many stock/bond/mutual fund transactions have you made in the past year?
6. Do you have broadband access to the Internet at home? Yes_____ No_____
7. Please indicate your total household income last year.
8. Do you have children? Yes_____ No_____

The file entitled Professional contains the responses to these questions. Table 8.6 shows the portion of the file pertaining to the first five survey respondents.

Managerial Report

Prepare a managerial report summarizing the results of the survey. In addition to statistical summaries, discuss how the magazine might use these results to attract advertisers. You

TABLE 8.6 PARTIAL SURVEY RESULTS FOR *YOUNG PROFESSIONAL* MAGAZINE

Age	Gender	Real Estate Purchases	Value of Investments($)	Number of Transactions	Broadband Access	Household Income($)	Children
38	Female	No	12200	4	Yes	75200	Yes
30	Male	No	12400	4	Yes	70300	Yes
41	Female	No	26800	5	Yes	48200	No
28	Female	Yes	19600	6	No	95300	No
31	Female	Yes	15100	5	No	73300	Yes
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

might also comment on how the survey results could be used by the magazine's editors to identify topics that would be of interest to readers. Your report should address the following issues, but do not limit your analysis to just these areas.

1. Develop appropriate descriptive statistics to summarize the data.
2. Develop 95% confidence intervals for the mean age and household income of subscribers.
3. Develop 95% confidence intervals for the proportion of subscribers who have broadband access at home and the proportion of subscribers who have children.
4. Would *Young Professional* be a good advertising outlet for online brokers? Justify your conclusion with statistical data.
5. Would this magazine be a good place to advertise for companies selling educational software and computer games for young children?
6. Comment on the types of articles you believe would be of interest to readers of *Young Professional.*

Case Problem 2 Gulf Real Estate Properties

Gulf Real Estate Properties, Inc., is a real estate firm located in southwest Florida. The company, which advertises itself as "expert in the real estate market," monitors condominium sales by collecting data on location, list price, sale price, and number of days it takes to sell each unit. Each condominium is classified as *Gulf View* if it is located directly on the Gulf of Mexico or *No Gulf View* if it is located on the bay or a golf course, near but not on the Gulf. Sample data from the Multiple Listing Service in Naples, Florida, provided recent sales data for 40 Gulf View condominiums and 18 No Gulf View condominiums.* Prices are in thousands of dollars. The data are shown in Table 8.7.

Managerial Report

1. Use appropriate descriptive statistics to summarize each of the three variables for the 40 Gulf View condominiums.
2. Use appropriate descriptive statistics to summarize each of the three variables for the 18 No Gulf View condominiums.
3. Compare your summary results. Discuss any specific statistical results that would help a real estate agent understand the condominium market.
4. Develop a 95% confidence interval estimate of the population mean sales price and population mean number of days to sell for Gulf View condominiums. Interpret your results.
5. Develop a 95% confidence interval estimate of the population mean sales price and population mean number of days to sell for No Gulf View condominiums. Interpret your results.
6. Assume the branch manager requested estimates of the mean selling price of Gulf View condominiums with a margin of error of $40,000 and the mean selling price of No Gulf View condominiums with a margin of error of $15,000. Using 95% confidence, how large should the sample sizes be?
7. Gulf Real Estate Properties just signed contracts for two new listings: a Gulf View condominium with a list price of $589,000 and a No Gulf View condominium with a list price of $285,000. What is your estimate of the final selling price and number of days required to sell each of these units?

*Data based on condominium sales reported in the Naples MLS (Coldwell Banker, June 2000).

TABLE 8.7 SALES DATA FOR GULF REAL ESTATE PROPERTIES

Gulf View Condominiums			No Gulf View Condominiums		
List Price	Sale Price	Days to Sell	List Price	Sale Price	Days to Sell
495.0	475.0	130	217.0	217.0	182
379.0	350.0	71	148.0	135.5	338
529.0	519.0	85	186.5	179.0	122
552.5	534.5	95	239.0	230.0	150
334.9	334.9	119	279.0	267.5	169
550.0	505.0	92	215.0	214.0	58
169.9	165.0	197	279.0	259.0	110
210.0	210.0	56	179.9	176.5	130
975.0	945.0	73	149.9	144.9	149
314.0	314.0	126	235.0	230.0	114
315.0	305.0	88	199.8	192.0	120
885.0	800.0	282	210.0	195.0	61
975.0	975.0	100	226.0	212.0	146
469.0	445.0	56	149.9	146.5	137
329.0	305.0	49	160.0	160.0	281
365.0	330.0	48	322.0	292.5	63
332.0	312.0	88	187.5	179.0	48
520.0	495.0	161	247.0	227.0	52
425.0	405.0	149			
675.0	669.0	142			
409.0	400.0	28			
649.0	649.0	29			
319.0	305.0	140			
425.0	410.0	85			
359.0	340.0	107			
469.0	449.0	72			
895.0	875.0	129			
439.0	430.0	160			
435.0	400.0	206			
235.0	227.0	91			
638.0	618.0	100			
629.0	600.0	97			
329.0	309.0	114			
595.0	555.0	45			
339.0	315.0	150			
215.0	200.0	48			
395.0	375.0	135			
449.0	425.0	53			
499.0	465.0	86			
439.0	428.5	158			

WEB file GulfProp

Case Problem 3 Metropolitan Research, Inc.

Metropolitan Research, Inc., a consumer research organization, conducts surveys designed to evaluate a wide variety of products and services available to consumers. In one particular study, Metropolitan looked at consumer satisfaction with the performance of automobiles produced by a major Detroit manufacturer. A questionnaire sent to owners of one of the manufacturer's full-sized cars revealed several complaints about early transmission problems. To

learn more about the transmission failures, Metropolitan used a sample of actual transmission repairs provided by a transmission repair firm in the Detroit area. The following data show the actual number of miles driven for 50 vehicles at the time of transmission failure.

WEB file
Auto

85,092	32,609	59,465	77,437	32,534	64,090	32,464	59,902
39,323	89,641	94,219	116,803	92,857	63,436	65,605	85,861
64,342	61,978	67,998	59,817	101,769	95,774	121,352	69,568
74,276	66,998	40,001	72,069	25,066	77,098	69,922	35,662
74,425	67,202	118,444	53,500	79,294	64,544	86,813	116,269
37,831	89,341	73,341	85,288	138,114	53,402	85,586	82,256
77,539	88,798						

Managerial Report

1. Use appropriate descriptive statistics to summarize the transmission failure data.
2. Develop a 95% confidence interval for the mean number of miles driven until transmission failure for the population of automobiles with transmission failure. Provide a managerial interpretation of the interval estimate.
3. Discuss the implication of your statistical findings in terms of the belief that some owners of the automobiles experienced early transmission failures.
4. How many repair records should be sampled if the research firm wants the population mean number of miles driven until transmission failure to be estimated with a margin of error of 5000 miles? Use 95% confidence.
5. What other information would you like to gather to evaluate the transmission failure problem more fully?

Appendix Interval Estimation with StatTools

In this appendix we show how StatTools can be used to develop an interval estimate of a population mean for the σ unknown case and determine the sample size needed to provide a desired margin of error.

Interval Estimation of Population Mean: σ Unknown Case

In this case the population standard deviation σ will be estimated by the sample standard deviation s. We use the credit card balance data in Table 8.3 to illustrate. Begin by using the Data Set Manager to create a StatTools data set for these data using the procedure described in the appendix to Chapter 1. The following steps can be used to compute a 95% confidence interval estimate of the population mean.

WEB file
NewBalance

Step 1. Click the **StatTools** tab on the Ribbon
Step 2. In the **Analyses** group, click **Statistical Inference**
Step 3. Choose the **Confidence Interval** option
Step 4. Choose **Mean/Std. Deviation**
Step 5. When the StatTools - Confidence Interval for Mean/Std. Deviation dialog box appears,
For **Analysis Type** choose **One-Sample Analysis**
In the **Variables** section, select **NewBalance**
In the **Confidence Intervals to Calculate** section,
Select the **For the Mean** option
Select 95% for the **Confidence Level**
Click **OK**

Some descriptive statistics and the confidence interval will appear.

Determining the Sample Size

In Section 8.3 we showed how to determine the sample size needed to provide a desired margin of error. The example used involved a study designed to estimate the population mean daily rental cost for a midsize automobile in the United States. The project director specified that the population mean daily rental cost be estimated with a margin of error of $2 and a 95% level of confidence. Sample data from a previous study provided a sample standard deviation of $9.65; this value was used as the planning value for the population standard deviation. The following steps can be used to compute the recommended sample size required to provide a 95% confidence interval estimate of the population mean with a margin of error of $2.

Step 1. Click the **StatTools** tab on the Ribbon
Step 2. In the **Analyses** group, click **Statistical Inference**
Step 3. Choose the **Sample Size Selection** option
Step 4. When the StatTools - Sample Size Selection dialog box appears,
In the **Parameter to Estimate** section, select **Mean**
In the **Confidence Interval Specification** section,
Select **95%** for the **Confidence Level**
Enter **2** in the **Half-Length of Interval** box
Enter **9.65** in the **Estimated Std Dev** box
Click **OK**

The half-length of interval is the margin of error.

The output showing a recommended sample size of 90 will appear.

CHAPTER 9

Hypothesis Tests

CONTENTS

STATISTICS IN PRACTICE: JOHN MORRELL & COMPANY

9.1 DEVELOPING NULL AND ALTERNATIVE HYPOTHESES
- The Alternative Hypothesis as a Research Hypothesis
- The Null Hypothesis as an Assumption to Be Challenged
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STATISTICS in PRACTICE

JOHN MORRELL & COMPANY*

CINCINNATI, OHIO

John Morrell & Company, which began in England in 1827, is considered the oldest continuously operating meat manufacturer in the United States. It is a wholly owned and independently managed subsidiary of Smithfield Foods, Smithfield, Virginia. John Morrell & Company offers an extensive product line of processed meats and fresh pork to consumers under 13 regional brands, including John Morrell, E-Z-Cut, Tobin's First Prize, Dinner Bell, Hunter, Kretschmar, Rath, Rodeo, Shenson, Farmers Hickory Brand, Iowa Quality, and Peyton's. Each regional brand enjoys high brand recognition and loyalty among consumers.

Market research at Morrell provides management with up-to-date information on the company's various products and how the products compare with competing brands of similar products. A recent study compared a Beef Pot Roast made by Morrell to similar beef products from two major competitors. In the three-product comparison test, a sample of consumers was used to indicate how the products rated in terms of taste, appearance, aroma, and overall preference.

One research question concerned whether the Beef Pot Roast made by Morrell was the preferred choice of more than 50% of the consumer population. Letting p indicate the population proportion preferring Morrell's product, the hypothesis test for the research question is as follows:

$$H_0: p \leq .50$$
$$H_a: p > .50$$

The null hypothesis H_0 indicates the preference for Morrell's product is less than or equal to 50%. If the sample data support rejecting H_0 in favor of the alternative hypothesis H_a, Morrell will draw the research conclusion that in a three-product comparison, its Beef Pot Roast is preferred by more than 50% of the consumer population.

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Hypothesis testing helps John Morrell & Company analyze market research about its products.

In an independent taste test study using a sample of 224 consumers in Cincinnati, Milwaukee, and Los Angeles, 150 consumers selected the Beef Pot Roast made by Morrell as the preferred product. Using statistical hypothesis testing procedures, the null hypothesis H_0 was rejected. The study provided statistical evidence supporting H_a and the conclusion that the Morrell product is preferred by more than 50% of the consumer population.

The point estimate of the population proportion was $\bar{p} = 150/224 = .67$. Thus, the sample data provided support for a food magazine advertisement showing that in a three-product taste comparison, Beef Pot Roast made by Morrell was "preferred 2 to 1 over the competition."

In this chapter we will discuss how to formulate hypotheses and how to conduct tests like the one used by Morrell. Through the analysis of sample data, we will be able to determine whether a hypothesis should or should not be rejected.

*The authors are indebted to Marty Butler, Vice President of Marketing, John Morrell, for providing this Statistics in Practice.

In Chapters 7 and 8 we showed how a sample could be used to develop point and interval estimates of population parameters. In this chapter we continue the discussion of statistical inference by showing how hypothesis testing can be used to determine whether a statement about the value of a population parameter should or should not be rejected.

In hypothesis testing we begin by making a tentative assumption about a population parameter. This tentative assumption is called the **null hypothesis** and is denoted by H_0.

We then define another hypothesis, called the **alternative hypothesis**, which is the opposite of what is stated in the null hypothesis. The alternative hypothesis is denoted by H_a. The hypothesis testing procedure uses data from a sample to test the two competing statements indicated by H_0 and H_a.

This chapter shows how hypothesis tests can be conducted about a population mean and a population proportion. We begin by providing examples that illustrate approaches to developing null and alternative hypotheses.

9.1 Developing Null and Alternative Hypotheses

It is not always obvious how the null and alternative hypotheses should be formulated. Care must be taken to structure the hypotheses appropriately so that the hypothesis testing conclusion provides the information the researcher or decision maker wants. The context of the situation is very important in determining how the hypotheses should be stated. All hypothesis testing applications involve collecting a sample and using the sample results to provide evidence for drawing a conclusion. Good questions to consider when formulating the null and alternative hypotheses are, What is the purpose of collecting the sample? What conclusions are we hoping to make?

Learning to formulate hypotheses correctly will take some practice. Expect some initial confusion over the proper choice of the null and alternative hypotheses. The examples in this section are intended to provide guidelines.

In the chapter introduction, we stated that the null hypothesis H_0 is a tentative assumption about a population parameter such as a population mean or a population proportion. The alternative hypothesis H_a is a statement that is the opposite of what is stated in the null hypothesis. In some situations it is easier to identify the alternative hypothesis first and then develop the null hypothesis. In other situations it is easier to identify the null hypothesis first and then develop the alternative hypothesis. We will illustrate these situations in the following examples.

The Alternative Hypothesis as a Research Hypothesis

Many applications of hypothesis testing involve an attempt to gather evidence in support of a research hypothesis. In these situations, it is often best to begin with the alternative hypothesis and make it the conclusion that the researcher hopes to support. Consider a particular automobile that currently attains a fuel efficiency of 24 miles per gallon in city driving. A product research group has developed a new fuel injection system designed to increase the miles-per-gallon rating. The group will run controlled tests with the new fuel injection system looking for statistical support for the conclusion that the new fuel injection system provides more miles per gallon than the current system.

Several new fuel injection units will be manufactured, installed in test automobiles, and subjected to research-controlled driving conditions. The sample mean miles per gallon for these automobiles will be computed and used in a hypothesis test to determine if it can be concluded that the new system provides more than 24 miles per gallon. In terms of the population mean miles per gallon μ, the research hypothesis $\mu > 24$ becomes the alternative hypothesis. Since the current system provides an average or mean of 24 miles per gallon, we will make the tentative assumption that the new system is not any better than the current system and choose $\mu \leq 24$ as the null hypothesis. The null and alternative hypotheses are:

$$H_0: \mu \leq 24$$
$$H_a: \mu > 24$$

If the sample results lead to the conclusion to reject H_0, the inference can be made that $H_a: \mu > 24$ is true. The researchers have the statistical support to state that the new

The conclusion that the research hypothesis is true is made if the sample data provide sufficient evidence to show that the null hypothesis can be rejected.

fuel injection system increases the mean number of miles per gallon. The production of automobiles with the new fuel injection system should be considered. However, if the sample results lead to the conclusion that H_0 cannot be rejected, the researchers cannot conclude that the new fuel injection system is better than the current system. Production of automobiles with the new fuel injection system on the basis of better gas mileage cannot be justified. Perhaps more research and further testing can be conducted.

Successful companies stay competitive by developing new products, new methods, new systems, and the like that are better than what is currently available. Before adopting something new, it is desirable to conduct research to determine if there is statistical support for the conclusion that the new approach is indeed better. In such cases, the research hypothesis is stated as the alternative hypothesis. For example, a new teaching method is developed that is believed to be better than the current method. The alternative hypothesis is that the new method is better. The null hypothesis is that the new method is no better than the old method. A new sales force bonus plan is developed in an attempt to increase sales. The alternative hypothesis is that the new bonus plan increases sales. The null hypothesis is that the new bonus plan does not increase sales. A new drug is developed with the goal of lowering blood pressure more than an existing drug. The alternative hypothesis is that the new drug lowers blood pressure more than the existing drug. The null hypothesis is that the new drug does not provide lower blood pressure than the existing drug. In each case, rejection of the null hypothesis H_0 provides statistical support for the research hypothesis. We will see many examples of hypothesis tests in research situations such as these throughout this chapter and in the remainder of the text.

The Null Hypothesis as an Assumption to Be Challenged

Of course, not all hypothesis tests involve research hypotheses. In the following discussion we consider applications of hypothesis testing where we begin with a belief or an assumption that a statement about the value of a population parameter is true. We will then use a hypothesis test to challenge the assumption and determine if there is statistical evidence to conclude that the assumption is incorrect. In these situations, it is helpful to develop the null hypothesis first. The null hypothesis H_0 expresses the belief or assumption about the value of the population parameter. The alternative hypothesis H_a is that the belief or assumption is incorrect.

As an example, consider the situation of a manufacturer of soft drink products. The label on a soft drink bottle states that it contains 67.6 fluid ounces. We consider the label correct provided the population mean filling weight for the bottles is *at least* 67.6 fluid ounces. Without any reason to believe otherwise, we would give the manufacturer the benefit of the doubt and assume that the statement provided on the label is correct. Thus, in a hypothesis test about the population mean fluid weight per bottle, we would begin with the assumption that the label is correct and state the null hypothesis as $\mu \geq 67.6$. The challenge to this assumption would imply that the label is incorrect and the bottles are being underfilled. This challenge would be stated as the alternative hypothesis $\mu < 67.6$. Thus, the null and alternative hypotheses are:

$$H_0: \mu \geq 67.6$$
$$H_a: \mu < 67.6$$

A manufacturer's product information is usually assumed to be true and stated as the null hypothesis. The conclusion that the information is incorrect can be made if the null hypothesis is rejected.

A government agency with the responsibility for validating manufacturing labels could select a sample of soft drink bottles, compute the sample mean filling weight, and use the sample results to test the preceding hypotheses. If the sample results lead to the conclusion to reject H_0, the inference that H_a: $\mu < 67.6$ is true can be made. With this statistical support, the agency is justified in concluding that the label is incorrect and underfilling of the

bottles is occurring. Appropriate action to force the manufacturer to comply with labeling standards would be considered. However, if the sample results indicate H_0 cannot be rejected, the assumption that the manufacturer's labeling is correct cannot be rejected. With this conclusion, no action would be taken.

Let us now consider a variation of the soft drink bottle-filling example by viewing the same situation from the manufacturer's point of view. The bottle-filling operation has been designed to fill soft drink bottles with 67.6 fluid ounces as stated on the label. The company does not want to underfill the containers because that could result in an underfilling complaint from customers or, perhaps, a government agency. However, the company does not want to overfill containers either because putting more soft drink than necessary into the containers would be an unnecessary cost. The company's goal would be to adjust the bottle-filling operation so that the population mean filling weight per bottle is 67.6 fluid ounces as specified on the label.

Although this is the company's goal, from time to time any production process can get out of adjustment. If this occurs in our example, underfilling or overfilling of the soft drink bottles will occur. In either case, the company would like to know about it in order to correct the situation by readjusting the bottle-filling operation to the designed 67.6 fluid ounces. In this hypothesis testing application, we would begin with the assumption that the production process is operating correctly and state the null hypothesis as $\mu = 67.6$ fluid ounces. The alternative hypothesis that challenges this assumption is that $\mu \neq 67.6$, which indicates either overfilling or underfilling is occurring. The null and alternative hypotheses for the manufacturer's hypothesis test are:

$$H_0: \mu = 67.6$$
$$H_a: \mu \neq 67.6$$

Suppose that the soft drink manufacturer uses a quality control procedure to periodically select a sample of bottles from the filling operation and computes the sample mean filling weight per bottle. If the sample results lead to the conclusion to reject H_0, the inference is made that $H_a: \mu \neq 67.6$ is true. We conclude that the bottles are not being filled properly and the production process should be adjusted to restore the population mean to 67.6 fluid ounces per bottle. However, if the sample results indicate H_0 cannot be rejected, the assumption that the manufacturer's bottle-filling operation is functioning properly cannot be rejected. In this case, no further action would be taken and the production operation would continue to run.

The two preceding forms of the soft drink manufacturing hypothesis test show that the null and alternative hypotheses may vary depending upon the point of view of the researcher or decision maker. To formulate hypotheses correctly it is important to understand the context of the situation and structure the hypotheses to provide the information the researcher or decision maker wants.

Summary of Forms for Null and Alternative Hypotheses

The hypothesis tests in this chapter involve two population parameters: the population mean and the population proportion. Depending on the situation, hypothesis tests about a population parameter may take one of three forms: two use inequalities in the null hypothesis; the third uses an equality in the null hypothesis. For hypothesis tests involving a population mean, we let μ_0 denote the hypothesized value and we must choose one of the following three forms for the hypothesis test.

The three possible forms of hypotheses H_0 and H_a are shown here. Note that the equality always *appears in the null hypothesis H_0.*

$$H_0: \mu \geq \mu_0 \qquad H_0: \mu \leq \mu_0 \qquad H_0: \mu = \mu_0$$
$$H_a: \mu < \mu_0 \qquad H_a: \mu > \mu_0 \qquad H_a: \mu \neq \mu_0$$

For reasons that will be clear later, the first two forms are called one-tailed tests. The third form is called a two-tailed test.

In many situations, the choice of H_0 and H_a is not obvious and judgment is necessary to select the proper form. However, as the preceding forms show, the equality part of the expression (either $\geq$, $\leq$, or $=$) *always* appears in the null hypothesis. In selecting the proper form of H_0 and H_a, keep in mind that the alternative hypothesis is often what the test is attempting to establish. Hence, asking whether the user is looking for evidence to support $\mu < \mu_0$, $\mu > \mu_0$, or $\mu \neq \mu_0$ will help determine H_a. The following exercises are designed to provide practice in choosing the proper form for a hypothesis test involving a population mean.

Exercises

1. The manager of the Danvers-Hilton Resort Hotel stated that the mean guest bill for a weekend is \$600 or less. A member of the hotel's accounting staff noticed that the total charges for guest bills have been increasing in recent months. The accountant will use a sample of future weekend guest bills to test the manager's claim.
   a. Which form of the hypotheses should be used to test the manager's claim? Explain.

$$H_0: \mu \geq 600 \qquad H_0: \mu \leq 600 \qquad H_0: \mu = 600$$
$$H_a: \mu < 600 \qquad H_a: \mu > 600 \qquad H_a: \mu \neq 600$$

   b. What conclusion is appropriate when H_0 cannot be rejected?
   c. What conclusion is appropriate when H_0 can be rejected?

SELF test

2. The manager of an automobile dealership is considering a new bonus plan designed to increase sales volume. Currently, the mean sales volume is 14 automobiles per month. The manager wants to conduct a research study to see whether the new bonus plan increases sales volume. To collect data on the plan, a sample of sales personnel will be allowed to sell under the new bonus plan for a one-month period.
   a. Develop the null and alternative hypotheses most appropriate for this situation.
   b. Comment on the conclusion when H_0 cannot be rejected.
   c. Comment on the conclusion when H_0 can be rejected.

3. A production line operation is designed to fill cartons with laundry detergent to a mean weight of 32 ounces. A sample of cartons is periodically selected and weighed to determine whether underfilling or overfilling is occurring. If the sample data lead to a conclusion of underfilling or overfilling, the production line will be shut down and adjusted to obtain proper filling.
   a. Formulate the null and alternative hypotheses that will help in deciding whether to shut down and adjust the production line.
   b. Comment on the conclusion and the decision when H_0 cannot be rejected.
   c. Comment on the conclusion and the decision when H_0 can be rejected.

4. Because of high production-changeover time and costs, a director of manufacturing must convince management that a proposed manufacturing method reduces costs before the new method can be implemented. The current production method operates with a mean cost of \$220 per hour. A research study will measure the cost of the new method over a sample production period.
   a. Develop the null and alternative hypotheses most appropriate for this study.
   b. Comment on the conclusion when H_0 cannot be rejected.
   c. Comment on the conclusion when H_0 can be rejected.

9.2 Type I and Type II Errors

The null and alternative hypotheses are competing statements about the population. Either the null hypothesis H_0 is true or the alternative hypothesis H_a is true, but not both. Ideally the hypothesis testing procedure should lead to the acceptance of H_0 when H_0 is true and the rejection of H_0 when H_a is true. Unfortunately, the correct conclusions are not always possible. Because hypothesis tests are based on sample information, we must allow for the possibility of errors. Table 9.1 illustrates the two kinds of errors that can be made in hypothesis testing.

The first row of Table 9.1 shows what can happen if the conclusion is to accept H_0. If H_0 is true, this conclusion is correct. However, if H_a is true, we make a **Type II error**; that is, we accept H_0 when it is false. The second row of Table 9.1 shows what can happen if the conclusion is to reject H_0. If H_0 is true, we make a **Type I error**; that is, we reject H_0 when it is true. However, if H_a is true, rejecting H_0 is correct.

Recall the hypothesis testing illustration discussed in Section 9.1, in which an automobile product research group developed a new fuel injection system designed to increase the miles-per-gallon rating of a particular automobile. With the current model obtaining an average of 24 miles per gallon, the hypothesis test was formulated as follows.

$$H_0: \mu \leq 24$$
$$H_a: \mu > 24$$

The alternative hypothesis, $H_a: \mu > 24$, indicates that the researchers are looking for sample evidence to support the conclusion that the population mean miles per gallon with the new fuel injection system is greater than 24.

In this application, the Type I error of rejecting H_0 when it is true corresponds to the researchers claiming that the new system improves the miles-per-gallon rating ($\mu > 24$) when in fact the new system is not any better than the current system. In contrast, the Type II error of accepting H_0 when it is false corresponds to the researchers concluding that the new system is not any better than the current system ($\mu \leq 24$) when in fact the new system improves miles-per-gallon performance.

For the miles-per-gallon rating hypothesis test, the null hypothesis is $H_0: \mu \leq 24$. Suppose the null hypothesis is true as an equality; that is, $\mu = 24$. The probability of making a Type I error when the null hypothesis is true as an equality is called the **level of significance**. Thus, for the miles-per-gallon rating hypothesis test, the level of significance is the probability of rejecting $H_0: \mu \leq 24$ when $\mu = 24$. Because of the importance of this concept, we now restate the definition of level of significance.

TABLE 9.1 ERRORS AND CORRECT CONCLUSIONS IN HYPOTHESIS TESTING

		Population Condition	
		H_0 True	H_a True
Conclusion	Accept H_0	Correct Conclusion	Type II Error
	Reject H_0	Type I Error	Correct Conclusion

LEVEL OF SIGNIFICANCE

The level of significance is the probability of making a Type I error when the null hypothesis is true as an equality.

The Greek symbol α (alpha) is used to denote the level of significance, and common choices for α are .05 and .01.

In practice, the person responsible for the hypothesis test specifies the level of significance. By selecting α, that person is controlling the probability of making a Type I error. If the cost of making a Type I error is high, small values of α are preferred. If the cost of making a Type I error is not too high, larger values of α are typically used. Applications of hypothesis testing that only control for the Type I error are called *significance tests*. Many applications of hypothesis testing are of this type.

Although most applications of hypothesis testing control for the probability of making a Type I error, they do not always control for the probability of making a Type II error. Hence, if we decide to accept H_0, we cannot determine how confident we can be with that decision. Because of the uncertainty associated with making a Type II error when conducting significance tests, statisticians usually recommend that we use the statement "do not reject H_0" instead of "accept H_0." Using the statement "do not reject H_0" carries the recommendation to withhold both judgment and action. In effect, by not directly accepting H_0, the statistician avoids the risk of making a Type II error. Whenever the probability of making a Type II error has not been determined and controlled, we will not make the statement "accept H_0." In such cases, only two conclusions are possible: *do not reject* H_0 or *reject* H_0.

If the sample data are consistent with the null hypothesis H_0, we will follow the practice of concluding "do not reject H_0." This conclusion is preferred over "accept H_0," because the conclusion to accept H_0 puts us at risk of making a Type II error.

Although controlling for a Type II error in hypothesis testing is not common, it can be done. More advanced texts describe procedures for determining and controlling the probability of making a Type II error.[1] If proper controls have been established for this error, action based on the "accept H_0" conclusion can be appropriate.

NOTES AND COMMENTS

Walter Williams, syndicated columnist and professor of economics at George Mason University, points out that the possibility of making a Type I or a Type II error is always present in decision making (*The Cincinnati Enquirer*, August 14, 2005). He notes that the Food and Drug Administration runs the risk of making these errors in its drug approval process. The FDA must either approve a new drug or not approve it. Thus, the FDA runs the risk of approving a new drug that is not safe and effective, or failing to approve a new drug that is safe and effective. Regardless of the decision made, the possibility of making a costly error cannot be eliminated.

Exercises

SELF test

5. Nielsen reported that young men in the United States watch 56.2 minutes of prime-time TV daily (*The Wall Street Journal Europe*, November 18, 2003). A researcher believes that young men in Germany spend more time watching prime-time TV. A sample of German young men will be selected by the researcher and the time they spend watching TV in one

[1] See, for example, D. R. Anderson, D. J. Sweeney, and T. A. Williams, *Statistics for Business and Economics*, 11th edition (Cincinnati: South-Western, 2011).

day will be recorded. The sample results will be used to test the following null and alternative hypotheses.

$$H_0: \mu \leq 56.2$$
$$H_a: \mu > 56.2$$

a. What is the Type I error in this situation? What are the consequences of making this error?
b. What is the Type II error in this situation? What are the consequences of making this error?

6. The label on a 3-quart container of orange juice states that the orange juice contains an average of 1 gram of fat or less. Answer the following questions for a hypothesis test that could be used to test the claim on the label.
   a. Develop the appropriate null and alternative hypotheses.
   b. What is the Type I error in this situation? What are the consequences of making this error?
   c. What is the Type II error in this situation? What are the consequences of making this error?
7. Carpetland salespersons average $8000 per week in sales. Steve Contois, the firm's vice president, proposes a compensation plan with new selling incentives. Steve hopes that the results of a trial selling period will enable him to conclude that the compensation plan increases the average sales per salesperson.
   a. Develop the appropriate null and alternative hypotheses.
   b. What is the Type I error in this situation? What are the consequences of making this error?
   c. What is the Type II error in this situation? What are the consequences of making this error?
8. Suppose a new production method will be implemented if a hypothesis test supports the conclusion that the new method reduces the mean operating cost per hour.
   a. State the appropriate null and alternative hypotheses if the mean cost for the current production method is $220 per hour.
   b. What is the Type I error in this situation? What are the consequences of making this error?
   c. What is the Type II error in this situation? What are the consequences of making this error?

9.3 Population Mean: σ Known

In Chapter 8 we said that the σ known case corresponds to applications in which historical data and/or other information are available that enable us to obtain a good estimate of the population standard deviation prior to sampling. In such cases the population standard deviation can, for all practical purposes, be considered known. In this section we show how to conduct a hypothesis test about a population mean for the σ known case.

The methods presented in this section are exact if the sample is selected from a population that is normally distributed. In cases where it is not reasonable to assume the population is normally distributed, these methods are still applicable if the sample size is large enough. We provide some practical advice concerning the population distribution and the sample size at the end of this section.

One-Tailed Test

One-tailed tests about a population mean take one of the following two forms.

Lower Tail Test	Upper Tail Test
$H_0: \mu \geq \mu_0$	$H_0: \mu \leq \mu_0$
$H_a: \mu < \mu_0$	$H_a: \mu > \mu_0$

Let us consider an example involving a lower tail test.

The Federal Trade Commission (FTC) periodically conducts statistical studies designed to test the claims that manufacturers make about their products. For example, the label on a large can of Hilltop Coffee states that the can contains 3 pounds of coffee. The FTC knows that Hilltop's production process cannot place exactly 3 pounds of coffee in each can, even if the mean filling weight for the population of all cans filled is 3 pounds per can. However, as long as the population mean filling weight is at least 3 pounds per can, the rights of consumers will be protected. Thus, the FTC interprets the label information on a large can of coffee as a claim by Hilltop that the population mean filling weight is at least 3 pounds per can. We will show how the FTC can check Hilltop's claim by conducting a lower tail hypothesis test.

The first step is to develop the null and alternative hypotheses for the test. If the population mean filling weight is at least 3 pounds per can, Hilltop's claim is correct. This establishes the null hypothesis for the test. However, if the population mean weight is less than 3 pounds per can, Hilltop's claim is incorrect. This establishes the alternative hypothesis. With μ denoting the population mean filling weight, the null and alternative hypotheses are as follows:

$$H_0: \mu \geq 3$$
$$H_a: \mu < 3$$

Note that the hypothesized value of the population mean is $\mu_0 = 3$.

If the sample data indicate that H_0 cannot be rejected, the statistical evidence does not support the conclusion that a label violation has occurred. Hence, no action should be taken against Hilltop. However, if the sample data indicate that H_0 can be rejected, we will conclude that the alternative hypothesis, H_a: $\mu < 3$, is true. In this case a conclusion of underfilling and a charge of a label violation against Hilltop would be justified.

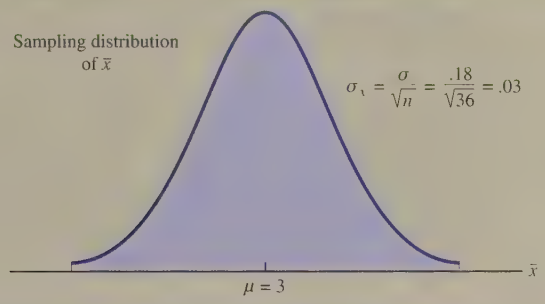
Suppose a sample of 36 cans of coffee is selected and the sample mean $\bar{x}$ is computed as an estimate of the population mean μ. If the value of the sample mean $\bar{x}$ is less than 3 pounds, the sample results will cast doubt on the null hypothesis. What we want to know is how much less than 3 pounds must $\bar{x}$ be before we would be willing to declare the difference significant and risk making a Type I error by falsely accusing Hilltop of a label violation. A key factor in addressing this issue is the value the decision maker selects for the level of significance.

As noted in the preceding section, the level of significance, denoted by α, is the probability of making a Type I error by rejecting H_0 when the null hypothesis is true as an equality. The decision maker must specify the level of significance. If the cost of making a Type I error is high, a small value should be chosen for the level of significance. If the cost is not high, a larger value is more appropriate. In the Hilltop Coffee study, the director of the FTC's testing program made the following statement: "If the company is meeting its weight specifications at $\mu = 3$, I do not want to take action against them. But, I am willing to risk a 1% chance of making such an error." From the director's statement, we set the level of significance for the hypothesis test at $\alpha = .01$. Thus, we must design the hypothesis test so that the probability of making a Type I error when $\mu = 3$ is .01.

For the Hilltop Coffee study, by developing the null and alternative hypotheses and specifying the level of significance for the test, we carry out the first two steps required in conducting every hypothesis test. We are now ready to perform the third step of hypothesis testing: collect the sample data and compute the value of what is called a test statistic.

Test statistic For the Hilltop Coffee study, previous FTC tests show that the population standard deviation can be assumed known with a value of $\sigma = .18$. In addition, these tests also show that the population of filling weights can be assumed to have a normal

FIGURE 9.1 SAMPLING DISTRIBUTION OF $\bar{x}$ FOR THE HILLTOP COFFEE STUDY WHEN THE NULL HYPOTHESIS IS TRUE AS AN EQUALITY ($\mu = 3$)



distribution. From the study of sampling distributions in Chapter 7 we know that if the population from which we are sampling is normally distributed, the sampling distribution of $\bar{x}$ will also be normally distributed. Thus, for the Hilltop Coffee study, the sampling distribution of $\bar{x}$ is normally distributed. With a known value of $\sigma = .18$ and a sample size of $n = 36$, Figure 9.1 shows the sampling distribution of $\bar{x}$ when the null hypothesis is true as an equality, that is, when $\mu = \mu_0 = 3$.[2] Note that the standard error of $\bar{x}$ is given by $\sigma_{\bar{x}} = \sigma/\sqrt{n} = .18/\sqrt{36} = .03$.

The standard error of $\bar{x}$ is the standard deviation of the sampling distribution of $\bar{x}$.

Because the sampling distribution of $\bar{x}$ is normally distributed, the sampling distribution of

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{\bar{x} - 3}{.03}$$

is a standard normal distribution. A value of $z = -1$ means that the value of $\bar{x}$ is one standard error below the hypothesized value of the mean, a value of $z = -2$ means that the value of $\bar{x}$ is two standard errors below the hypothesized value of the mean, and so on. We can use the standard normal probability table to find the lower tail probability corresponding to any z value. For instance, the lower tail area at $z = -3.00$ is .0013. Hence, the probability of obtaining a value of z that is three or more standard errors below the mean is .0013. As a result, the probability of obtaining a value of $\bar{x}$ that is 3 or more standard errors below the hypothesized population mean $\mu_0 = 3$ is also .0013. Such a result is unlikely if the null hypothesis is true.

For hypothesis tests about a population mean in the σ known case, we use the standard normal random variable z as a **test statistic** to determine whether $\bar{x}$ deviates from the hypothesized value of μ enough to justify rejecting the null hypothesis. With $\sigma_{\bar{x}} = \sigma/\sqrt{n}$, the test statistic is as follows.

TEST STATISTIC FOR HYPOTHESIS TESTS ABOUT A POPULATION MEAN: σ KNOWN

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \tag{9.1}$$

[2] In constructing sampling distributions for hypothesis tests, it is assumed that H_0 is satisfied as an equality.

The key question for a lower tail test is, How small must the test statistic z be before we choose to reject the null hypothesis? Two approaches can be used to answer this question: the p-value approach and the critical value approach.

p-value approach The p-value approach uses the value of the test statistic z to compute a probability called a ***p*-value**.

A small p-value indicates the value of the test statistic is unusual given the assumption that H_0 is true.

p-VALUE

A p-value is a probability that provides a measure of the evidence against the null hypothesis provided by the sample. Smaller p-values indicate more evidence against H_0.

The p-value is used to determine whether the null hypothesis should be rejected.

Let us see how the p-value is computed and used. The value of the test statistic is used to compute the p-value. The method used depends on whether the test is a lower tail, an upper tail, or a two-tailed test. For a lower tail test, the p-value is the probability of obtaining a value for the test statistic as small as or smaller than that provided by the sample. Thus, to compute the p-value for the lower tail test in the σ known case, we must find, using the standard normal distribution, the probability that z is less than or equal to the value of the test statistic. After computing the p-value, we must then decide whether it is small enough to reject the null hypothesis; as we will show, this decision involves comparing the p-value to the level of significance.

Coffee

Let us now compute the p-value for the Hilltop Coffee lower tail test. Suppose the sample of 36 Hilltop coffee cans provides a sample mean of $\bar{x} = 2.92$ pounds. Is $\bar{x} = 2.92$ small enough to cause us to reject H_0? Because this is a lower tail test, the p-value is the area under the standard normal curve for values of $z \leq$ the value of the test statistic. Using $\bar{x} = 2.92$, $\sigma = .18$, and $n = 36$, we compute the value of the test statistic z.

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{2.92 - 3}{.18/\sqrt{36}} = -2.67$$

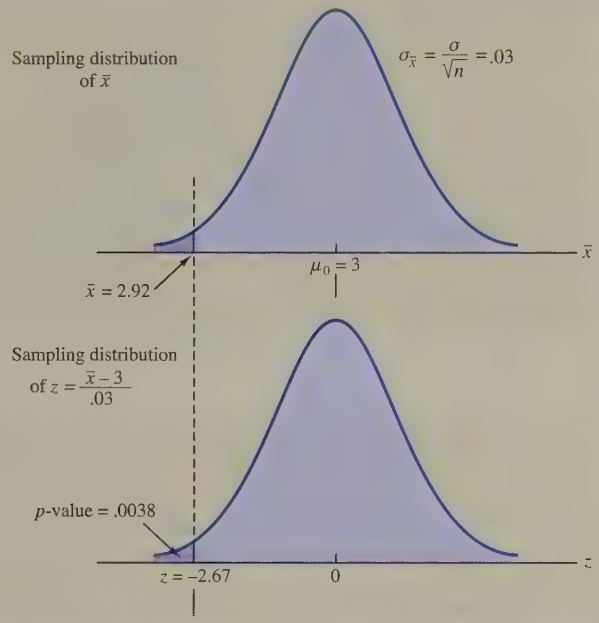
Thus, the p-value is the probability that z is less than or equal to -2.67 (the lower tail area corresponding to the value of the test statistic).

Using the standard normal probability table, we find that the lower tail area at $z = -2.67$ is .0038. Figure 9.2 shows that $\bar{x} = 2.92$ corresponds to $z = -2.67$ and a p-value = .0038. This p-value indicates a small probability of obtaining a sample mean of $\bar{x} = 2.92$ (and a test statistic of -2.67) or smaller when sampling from a population with $\mu = 3$. This p-value does not provide much support for the null hypothesis, but is it small enough to cause us to reject H_0? The answer depends upon the level of significance for the test.

As noted previously, the director of the FTC's testing program selected a value of .01 for the level of significance. The selection of $\alpha = .01$ means that the director is willing to tolerate a probability of .01 of rejecting the null hypothesis when it is true as an equality ($\mu_0 = 3$). The sample of 36 coffee cans in the Hilltop Coffee study resulted in a p-value = .0038, which means that the probability of obtaining a value of $\bar{x} = 2.92$ or less when the null hypothesis is true as an equality is .0038. Because .0038 is less than or equal to $\alpha = .01$, we reject H_0. Therefore, we find sufficient statistical evidence to reject the null hypothesis at the .01 level of significance.

We can now state the general rule for determining whether the null hypothesis can be rejected when using the p-value approach. For a level of significance α, the rejection rule

FIGURE 9.2 p-VALUE FOR THE HILLTOP COFFEE STUDY WHEN $\bar{x} = 2.92$ AND $z = -2.67$



using the p-value approach is as follows:

REJECTION RULE USING p-VALUE

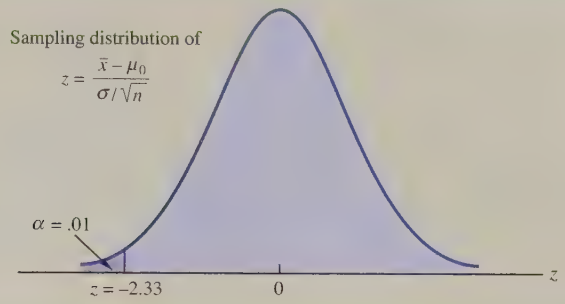
Reject H_0 if p-value $\leq \alpha$

In the Hilltop Coffee test, the p-value of .0038 resulted in the rejection of the null hypothesis. Although the basis for making the rejection decision involves a comparison of the p-value to the level of significance specified by the FTC director, the observed p-value of .0038 means that we would reject H_0 for any value of $\alpha \geq .0038$. For this reason, the p-value is also called the *observed level of significance.*

Different decision makers may express different opinions concerning the cost of making a Type I error and may choose a different level of significance. By providing the p-value as part of the hypothesis testing results, another decision maker can compare the reported p-value to his or her own level of significance and possibly make a different decision with respect to rejecting H_0.

Critical value approach The critical value approach requires that we first determine a value for the test statistic called the **critical value**. For a lower tail test, the critical value serves as a benchmark for determining whether the value of the test statistic is small enough to reject the null hypothesis. It is the value of the test statistic that corresponds to an area of

FIGURE 9.3 CRITICAL VALUE = −2.33 FOR THE HILLTOP COFFEE HYPOTHESIS TEST



α (the level of significance) in the lower tail of the sampling distribution of the test statistic. In other words, the critical value is the largest value of the test statistic that will result in the rejection of the null hypothesis. Let us return to the Hilltop Coffee example and see how this approach works.

In the σ known case, the sampling distribution for the test statistic z is a standard normal distribution. Therefore, the critical value is the value of the test statistic that corresponds to an area of $\alpha = .01$ in the lower tail of a standard normal distribution. Using the standard normal probability table, we find that $z = -2.33$ provides an area of .01 in the lower tail (see Figure 9.3). Thus, if the sample results in a value of the test statistic that is less than or equal to −2.33, the corresponding p-value will be less than or equal to .01; in this case, we should reject the null hypothesis. Hence, for the Hilltop Coffee study the critical value rejection rule for a level of significance of .01 is

$$\text{Reject } H_0 \text{ if } z \leq -2.33$$

In the Hilltop Coffee example, $\bar{x} = 2.92$ and the test statistic is $z = -2.67$. Because $z = -2.67 < -2.33$, we can reject H_0 and conclude that Hilltop Coffee is underfilling cans.

We can generalize the rejection rule for the critical value approach to handle any level of significance. The rejection rule for a lower tail test follows.

REJECTION RULE FOR A LOWER TAIL TEST: CRITICAL VALUE APPROACH

$$\text{Reject } H_0 \text{ if } z \leq -z_\alpha$$

where $-z_\alpha$ is the critical value; that is, the z value that provides an area of α in the lower tail of the standard normal distribution.

Summary The p-value approach to hypothesis testing and the critical value approach will always lead to the same rejection decision; that is, whenever the p-value is less than or equal to α, the value of the test statistic will be less than or equal to the critical value. The advantage of the p-value approach is that the p-value tells us *how* significant the results are (the observed level of significance). If we use the critical value approach, we only know that the results are significant at the stated level of significance.

At the beginning of this section, we said that one-tailed tests about a population mean take one of the following two forms:

Lower Tail Test	Upper Tail Test
$H_0: \mu \geq \mu_0$	$H_0: \mu \leq \mu_0$
$H_a: \mu < \mu_0$	$H_a: \mu > \mu_0$

We used the Hilltop Coffee study to illustrate how to conduct a lower tail test. We can use the same general approach to conduct an upper tail test. The test statistic z is still computed using equation (9.1). But, for an upper tail test, the p-value is the probability of obtaining a value for the test statistic as large as or larger than that provided by the sample. Thus, to compute the p-value for the upper tail test in the σ known case, we must find, using the standard normal distribution, the probability that z is greater than or equal to the value of the test statistic. Using the critical value approach causes us to reject the null hypothesis if the value of the test statistic is greater than or equal to the critical value z_α; in other words, we reject H_0 if $z \geq z_\alpha$.

Let us summarize the steps involved in computing p-values for one-tailed hypothesis tests.

COMPUTATION OF p-VALUES FOR ONE-TAILED TESTS

1. Compute the value of the test statistic using equation (9.1).
2. **Lower tail test:** Using the standard normal distribution, compute the probability that z is less than or equal to the value of the test statistic (area in the lower tail).
3. **Upper tail test:** Using the standard normal distribution, compute the probability that z is greater than or equal to the value of the test statistic (area in the upper tail).

Two-Tailed Test

In hypothesis testing, the general form for a **two-tailed test** about a population mean is as follows:

$$H_0: \mu = \mu_0$$
$$H_a: \mu \neq \mu_0$$

In this subsection we show how to conduct a two-tailed test about a population mean for the σ known case. As an illustration, we consider the hypothesis testing situation facing MaxFlight, Inc.

The U.S. Golf Association (USGA) establishes rules that manufacturers of golf equipment must meet if their products are to be acceptable for use in USGA events. MaxFlight uses a high-technology manufacturing process to produce golf balls with a mean driving distance of 295 yards. Sometimes, however, the process gets out of adjustment and produces golf balls with a mean driving distance different from 295 yards. When the mean distance falls below 295 yards, the company worries about losing sales because the golf balls do not provide as much distance as advertised. When the mean distance passes 295 yards, MaxFlight's golf balls may be rejected by the USGA for exceeding the overall distance standard concerning carry and roll.

MaxFlight's quality control program involves taking periodic samples of 50 golf balls to monitor the manufacturing process. For each sample, a hypothesis test is conducted to determine whether the process has fallen out of adjustment. Let us develop the null and alternative hypotheses. We begin by assuming that the process is functioning correctly; that is, the golf balls being produced have a mean distance of 295 yards. This assumption

establishes the null hypothesis. The alternative hypothesis is that the mean distance is not equal to 295 yards. With a hypothesized value of $\mu_0 = 295$, the null and alternative hypotheses for the MaxFlight hypothesis test are as follows:

$$H_0: \mu = 295$$
$$H_a: \mu \neq 295$$

If the sample mean $\bar{x}$ is significantly less than 295 yards or significantly greater than 295 yards, we will reject H_0. In this case, corrective action will be taken to adjust the manufacturing process. On the other hand, if $\bar{x}$ does not deviate from the hypothesized mean $\mu_0 = 295$ by a significant amount, H_0 will not be rejected and no action will be taken to adjust the manufacturing process.

The quality control team selected $\alpha = .05$ as the level of significance for the test. Data from previous tests conducted when the process was known to be in adjustment show that the population standard deviation can be assumed known with a value of $\sigma = 12$. Thus, with a sample size of $n = 50$, the standard error of $\bar{x}$ is

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{50}} = 1.7$$

Because the sample size is large, the central limit theorem (see Chapter 7) allows us to conclude that the sampling distribution of $\bar{x}$ can be approximated by a normal distribution. Figure 9.4 shows the sampling distribution of $\bar{x}$ for the MaxFlight hypothesis test with a hypothesized population mean of $\mu_0 = 295$.

WEB file
GolfTest

Suppose that a sample of 50 golf balls is selected and that the sample mean is $\bar{x} = 297.6$ yards. This sample mean provides support for the conclusion that the population mean is larger than 295 yards. Is this value of $\bar{x}$ enough larger than 295 to cause us to reject H_0 at the .05 level of significance? In the previous section we described two approaches that can be used to answer this question: the *p*-value approach and the critical value approach.

p-value approach Recall that the *p*-value is a probability used to determine whether the null hypothesis should be rejected. For a two-tailed test, values of the test statistic in *either* tail provide evidence against the null hypothesis. For a two-tailed test, the *p*-value is the probability of obtaining a value for the test statistic *as unlikely as or more unlikely than* that provided by the sample. Let us see how the *p*-value is computed for the MaxFlight hypothesis test.

FIGURE 9.4 SAMPLING DISTRIBUTION OF $\bar{x}$ FOR THE MAXFLIGHT HYPOTHESIS TEST

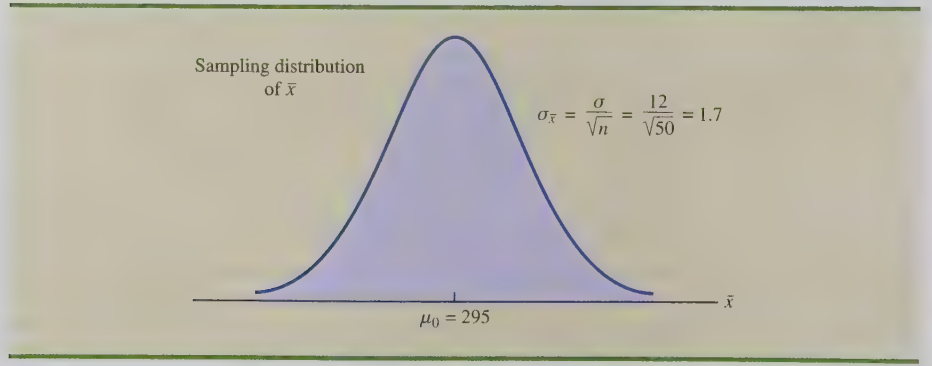
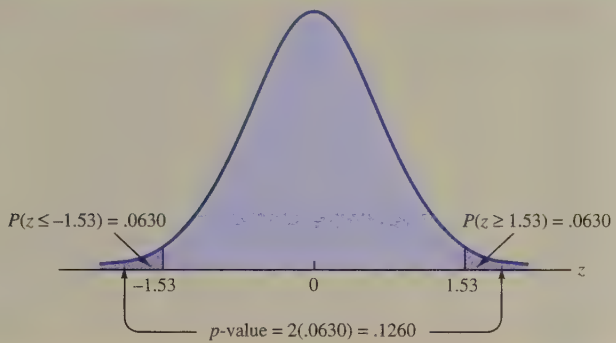


FIGURE 9.5 p-VALUE FOR THE MAXFLIGHT HYPOTHESIS TEST



First we compute the value of the test statistic. For the σ known case, the test statistic z is a standard normal random variable. Using equation (9.1) with $\bar{x} = 297.6$, the value of the test statistic is

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{297.6 - 295}{12/\sqrt{50}} = 1.53$$

Now to compute the p-value we must find the probability of obtaining a value for the test statistic *at least as unlikely as* $z = 1.53$. Clearly values of $z \geq 1.53$ are *at least as unlikely.* But, because this is a two-tailed test, values of $z \leq -1.53$ are also *at least as unlikely as* the value of the test statistic provided by the sample. In Figure 9.5, we see that the two-tailed p-value in this case is given by $P(z \leq -1.53) + P(z \geq 1.53)$. Because the normal curve is symmetric, we can compute this probability by finding $P(z \geq 1.53)$ and doubling it. The table for the standard normal distribution shows that $P(z < 1.53)$ is .9370. Thus, $P(z \geq 1.53)$ is $1.0000 - .9370 = .0630$. Doubling this, we find that the p-value for the MaxFlight two-tailed hypothesis test is p-value $= 2(.0630) = .1260$.

Next we compare the p-value to the level of significance to see whether the null hypothesis should be rejected. With a level of significance of $\alpha = .05$, we do not reject H_0 because the p-value $= .1260 > .05$. Because the null hypothesis is not rejected, no action will be taken to adjust the MaxFlight manufacturing process.

Let us summarize the steps involved in computing p-values for two-tailed hypothesis tests.

COMPUTATION OF p-VALUES FOR TWO-TAILED TESTS

1. Compute the value of the test statistic using equation (9.1).
2. If the value of the test statistic is in the upper tail, compute the probability that z is greater than or equal to the value of the test statistic (the upper tail area). If the value of the test statistic is in the lower tail, compute the probability that z is less than or equal to the value of the test statistic (the lower tail area).
3. Double the probability (or tail area) from step 2 to obtain the p-value.

Critical value approach Before leaving this section, let us see how the test statistic z can be compared to a critical value to make the hypothesis testing decision for a two-tailed test.

FIGURE 9.6 CRITICAL VALUES FOR THE MAXFLIGHT HYPOTHESIS TEST

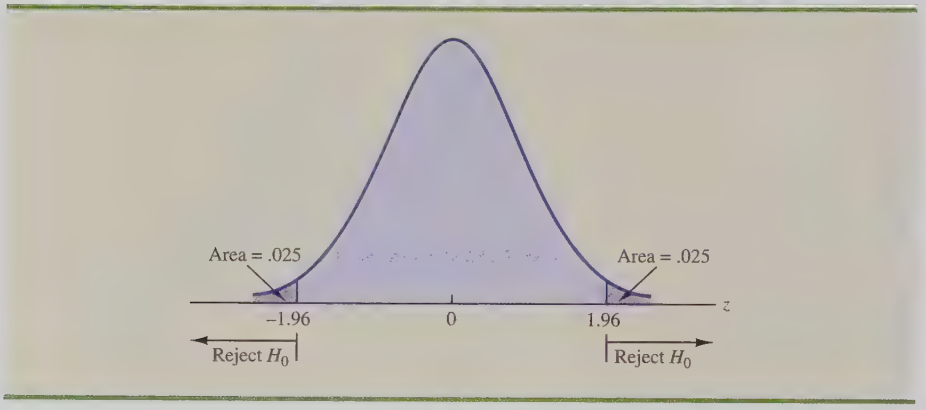


Figure 9.6 shows that the critical values for the test will occur in both the lower and upper tails of the standard normal distribution. With a level of significance of $\alpha = .05$, the area in each tail corresponding to the critical values is $\alpha/2 = .05/2 = .025$. Using the standard normal probability table, we find the critical values for the test statistic are $-z_{.025} = -1.96$ and $z_{.025} = 1.96$. Thus, using the critical value approach, the two-tailed rejection rule is

$$\text{Reject } H_0 \text{ if } z \leq -1.96 \text{ or if } z \geq 1.96$$

Because the value of the test statistic for the MaxFlight study is $z = 1.53$, the statistical evidence will not permit us to reject the null hypothesis at the .05 level of significance.

Using Excel

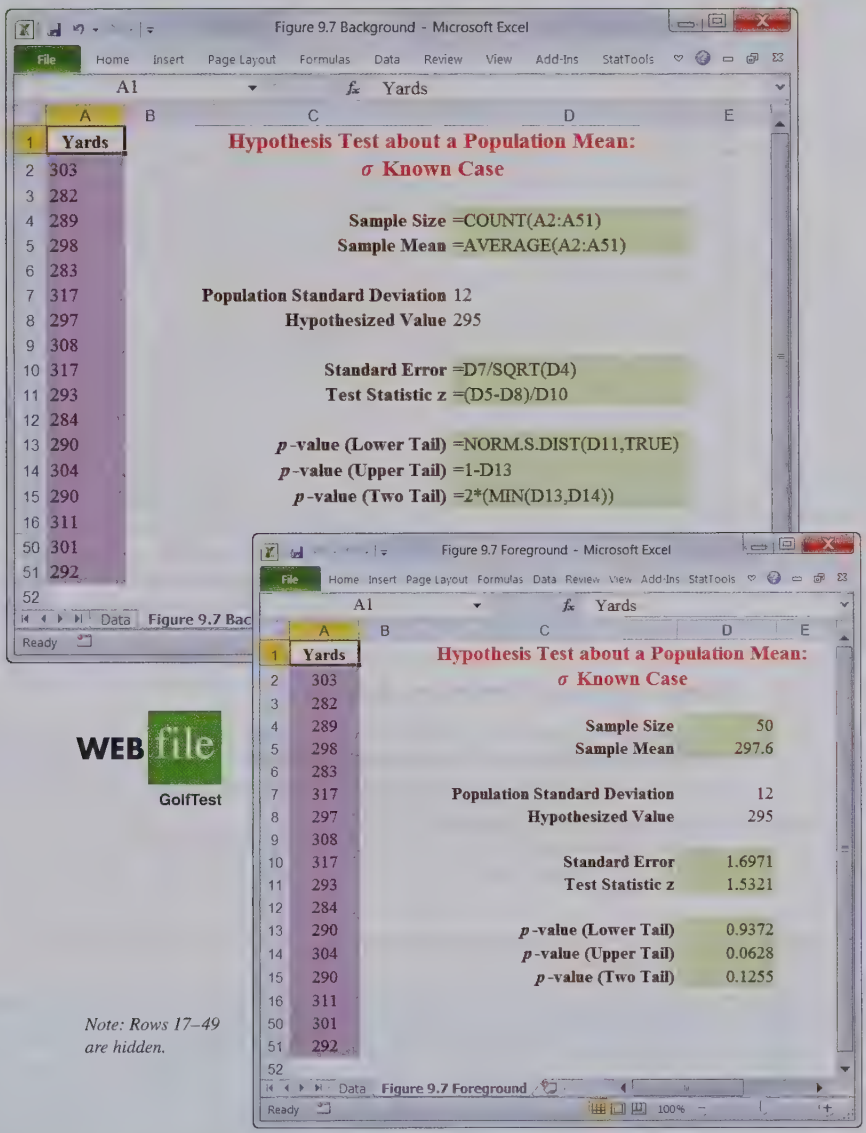
Excel can be used to conduct one-tailed and two-tailed hypothesis tests about a population mean for the σ known case using the p-value approach. Recall that the method used to compute a p-value depends upon whether the test is lower tail, upper tail, or two-tailed. Therefore, in the Excel procedure we describe we will use the sample results to compute three p-values: p-value (Lower Tail), p-value (Upper Tail), and p-value (Two Tail). The user can then choose α and draw a conclusion using whichever p-value is appropriate for the type of hypothesis test being conducted. We will illustrate using the MaxFlight two-tailed hypothesis test. Refer to Figure 9.7 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet is in the foreground.

Enter Data: A label and the distance data for the sample of 50 golf balls are entered into cells A1:A51.

Enter Functions and Formulas: The descriptive statistics needed are provided in cells D4 and D5. Excel's COUNT and AVERAGE functions compute the sample size and the sample mean, respectively. The value of the known population standard deviation (12) is entered into cell D7, and the hypothesized value of the population mean (295) is entered into cell D8.

The standard error is obtained in cell D10 by entering the formula =D7/SQRT(D4). The formula =(D5-D8)/D10 entered into cell D11 computes the test statistic z(1.5321). To compute the p-value for a lower tail test, we enter the formula =NORM.S.DIST(D11,TRUE) into cell D13. The p-value for an upper tail test is then computed in cell D14 as 1 minus the p-value for the lower tail test. Finally, the p-value for a two-tailed test is computed in cell D15

FIGURE 9.7 EXCEL WORKSHEET: HYPOTHESIS TEST FOR THE σ KNOWN CASE



as two times the minimum of the two one-tailed p-values. The value worksheet shows that p-value (Lower Tail) = 0.9372, p-value (Upper Tail) = 0.0628, and p-value (Two Tail) = 0.1255.

The development of the worksheet is now complete. For the two-tailed MaxFlight problem we cannot reject H_0: $\mu = 295$ using $\alpha = .05$ because the p-value (Two Tail) = 0.1255 is greater than α. Thus, the quality control manager has no reason to doubt that

the manufacturing process is producing golf balls with a population mean distance of 295 yards.

A template for other problems The worksheet in Figure 9.7 can be used as a template for conducting any one-tailed and two-tailed hypothesis tests for the σ known case. Just enter the appropriate data in column A, adjust the ranges for the formulas in cells D4 and D5, enter the population standard deviation in cell D7, and enter the hypothesized value in cell D8. The standard error, the test statistic, and the three p-values will then appear. Depending on the form of the hypothesis test (lower tail, upper tail, or two-tailed), we can then choose the appropriate p-value to make the rejection decision.

We can further simplify the use of Figure 9.7 as a template for other problems by eliminating the need to enter new data ranges in cells D4 and D5. To do so we rewrite the cell formulas as follows:

Cell D4: =COUNT(A:A)
Cell D5: =AVERAGE(A:A)

The GolfTest data set includes a worksheet entitled Template that uses the A:A method for entering the data ranges.

With the A:A method of specifying data ranges, Excel's COUNT function will count the number of numerical values in column A and Excel's AVERAGE function will compute the average of the numerical values in column A. Thus, to solve a new problem it is only necessary to enter the new data in column A, enter the value of the known population standard deviation in cell D7, and enter the hypothesized value of the population mean in cell D8.

The worksheet can also be used as a template for text exercises in which n, $\bar{x}$, and σ are given. Just ignore the data in column A and enter the values for n, $\bar{x}$, and σ into cells D4, D5, and D7, respectively. Then enter the appropriate hypothesized value for the population mean into cell D8. The p-values corresponding to lower tail, upper tail, and two-tailed hypothesis tests will then appear in cells D13:D15.

Summary and Practical Advice

We presented examples of a lower tail test and a two-tailed test about a population mean. Based upon these examples, we can now summarize the hypothesis testing procedures about a population mean for the σ known case as shown in Table 9.2. Note that μ_0 is the hypothesized value of the population mean.

The hypothesis testing steps followed in the two examples presented in this section are common to every hypothesis test.

TABLE 9.2 SUMMARY OF HYPOTHESIS TESTS ABOUT A POPULATION MEAN: σ KNOWN CASE

	Lower Tail Test	Upper Tail Test	Two-Tailed Test
Hypotheses	$H_0: \mu \geq \mu_0$ $H_a: \mu < \mu_0$	$H_0: \mu \leq \mu_0$ $H_a: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_a: \mu \neq \mu_0$
Test Statistic	$z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$
Rejection Rule: p-Value Approach	Reject H_0 if p-value $\leq \alpha$	Reject H_0 if p-value $\leq \alpha$	Reject H_0 if p-value $\leq \alpha$
Rejection Rule: Critical Value Approach	Reject H_0 if $z \leq -z_\alpha$	Reject H_0 if $z \geq z_\alpha$	Reject H_0 if $z \leq -z_{\alpha/2}$ or if $z \geq z_{\alpha/2}$

STEPS OF HYPOTHESIS TESTING

Step 1. Develop the null and alternative hypotheses.
Step 2. Specify the level of significance.
Step 3. Collect the sample data and compute the value of the test statistic.

p-Value Approach

Step 4. Use the value of the test statistic to compute the *p*-value.
Step 5. Reject H_0 if the *p*-value $\leq \alpha$.

Critical Value Approach

Step 4. Use the level of significance to determine the critical value and the rejection rule.
Step 5. Use the value of the test statistic and the rejection rule to determine whether to reject H_0.

Practical advice about the sample size for hypothesis tests is similar to the advice we provided about the sample size for interval estimation in Chapter 8. In most applications, a sample size of $n \geq 30$ is adequate when using the hypothesis testing procedure described in this section. In cases where the sample size is less than 30, the distribution of the population from which we are sampling becomes an important consideration. If the population is normally distributed, the hypothesis testing procedure that we described is exact and can be used for any sample size. If the population is not normally distributed but is at least roughly symmetric, sample sizes as small as 15 can be expected to provide acceptable results.

Relationship Between Interval Estimation and Hypothesis Testing

In Chapter 8 we showed how to develop a confidence interval estimate of a population mean. For the σ known case, the $(1 - \alpha)\%$ confidence interval estimate of a population mean is given by

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

In this chapter we showed that a two-tailed hypothesis test about a population mean takes the following form:

$$H_0: \mu = \mu_0$$
$$H_a: \mu \neq \mu_0$$

where μ_0 is the hypothesized value for the population mean.

Suppose that we follow the procedure described in Chapter 8 for constructing a $100(1 - \alpha)\%$ confidence interval for the population mean. We know that $100(1 - \alpha)\%$ of the confidence intervals generated will contain the population mean and $100\alpha\%$ of the confidence intervals generated will not contain the population mean. Thus, if we reject H_0 whenever the confidence interval does not contain μ_0, we will be rejecting the null hypothesis when it is true ($\mu = \mu_0$) with probability α. Recall that the level of significance is the

probability of rejecting the null hypothesis when it is true. So constructing a $100(1 - \alpha)\%$ confidence interval and rejecting H_0 whenever the interval does not contain μ_0 is equivalent to conducting a two-tailed hypothesis test with α as the level of significance. The procedure for using a confidence interval to conduct a two-tailed hypothesis test can now be summarized.

A CONFIDENCE INTERVAL APPROACH TO TESTING A HYPOTHESIS OF THE FORM

$$H_0: \mu = \mu_0$$
$$H_a: \mu \neq \mu_0$$

1. Select a simple random sample from the population and use the value of the sample mean $\bar{x}$ to develop the confidence interval for the population mean μ.

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

For a two-tailed hypothesis test, the null hypothesis can be rejected if the confidence interval does not include μ_0.

2. If the confidence interval contains the hypothesized value μ_0, do not reject H_0. Otherwise, reject[3] H_0.

Let us illustrate by conducting the MaxFlight hypothesis test using the confidence interval approach. The MaxFlight hypothesis test takes the following form:

$$H_0: \mu = 295$$
$$H_a: \mu \neq 295$$

To test these hypotheses with a level of significance of $\alpha = .05$, we sampled 50 golf balls and found a sample mean distance of $\bar{x} = 297.6$ yards. Recall that the population standard deviation is $\sigma = 12$. Using these results with $z_{.025} = 1.96$, we find that the 95% confidence interval estimate of the population mean is

$$\bar{x} \pm z_{.025} \frac{\sigma}{\sqrt{n}}$$
$$297.6 \pm 1.96 \frac{12}{\sqrt{50}}$$
$$297.6 \pm 3.3$$

or

$$294.3 \text{ to } 300.9$$

This finding enables the quality control manager to conclude with 95% confidence that the mean distance for the population of golf balls is between 294.3 and 300.9 yards. Because the hypothesized value for the population mean, $\mu_0 = 295$, is in this interval, the hypothesis testing conclusion is that the null hypothesis, $H_0: \mu = 295$, cannot be rejected.

[3]To be consistent with the rule for rejecting H_0 when the p-value $\leq \alpha$, we would also reject H_0 using the confidence interval approach if μ_0 happens to be equal to one of the endpoints of the $100(1 - \alpha)\%$ confidence interval.

Note that this discussion and example pertain to two-tailed hypothesis tests about a population mean. However, the same confidence interval and two-tailed hypothesis testing relationship exists for other population parameters. The relationship can also be extended to one-tailed tests about population parameters. Doing so, however, requires the development of one-sided confidence intervals, which are rarely used in practice.

NOTES AND COMMENTS

We have shown how to use *p*-values. The smaller the *p*-value the greater the evidence against H_0 and the more the evidence in favor of H_a. Here are some guidelines statisticians suggest for interpreting small *p*-values.

- Less than .01—Overwhelming evidence to conclude that H_a is true
- Between .01 and .05—Strong evidence to conclude that H_a is true
- Between .05 and .10—Weak evidence to conclude that H_a is true
- Greater than .10—Insufficient evidence to conclude that H_a is true

Exercises

Note to Student: Some of the exercises that follow ask you to use the *p*-value approach and others ask you to use the critical value approach. Both methods will provide the same hypothesis testing conclusion. We provide exercises with both methods to give you practice using both. In later sections and in following chapters, we will generally emphasize the *p*-value approach as the preferred method, but you may select either based on personal preference.

Methods

9. Consider the following hypothesis test:

$$H_0: \mu \geq 20$$
$$H_a: \mu < 20$$

A sample of 50 provided a sample mean of 19.4. The population standard deviation is 2.

a. Compute the value of the test statistic.
b. What is the *p*-value?
c. Using $\alpha = .05$, what is your conclusion?
d. What is the rejection rule using the critical value? What is your conclusion?

10. Consider the following hypothesis test:

$$H_0: \mu \leq 25$$
$$H_a: \mu > 25$$

A sample of 40 provided a sample mean of 26.4. The population standard deviation is 6.

a. Compute the value of the test statistic.
b. What is the *p*-value?
c. At $\alpha = .01$, what is your conclusion?
d. What is the rejection rule using the critical value? What is your conclusion?

11. Consider the following hypothesis test:

$$H_0: \mu = 15$$
$$H_a: \mu \neq 15$$

A sample of 50 provided a sample mean of 14.15. The population standard deviation is 3.
   a. Compute the value of the test statistic.
   b. What is the *p*-value?
   c. At $\alpha = .05$, what is your conclusion?
   d. What is the rejection rule using the critical value? What is your conclusion?

12. Consider the following hypothesis test:

$$H_0: \mu \geq 80$$
$$H_a: \mu < 80$$

A sample of 100 is used and the population standard deviation is 12. Compute the *p*-value and state your conclusion for each of the following sample results. Use $\alpha = .01$.
   a. $\bar{x} = 78.5$
   b. $\bar{x} = 77$
   c. $\bar{x} = 75.5$
   d. $\bar{x} = 81$

13. Consider the following hypothesis test:

$$H_0: \mu \leq 50$$
$$H_a: \mu > 50$$

A sample of 60 is used and the population standard deviation is 8. Use the critical value approach to state your conclusion for each of the following sample results. Use $\alpha = .05$.
   a. $\bar{x} = 52.5$
   b. $\bar{x} = 51$
   c. $\bar{x} = 51.8$

14. Consider the following hypothesis test:

$$H_0: \mu = 22$$
$$H_a: \mu \neq 22$$

A sample of 75 is used and the population standard deviation is 10. Compute the *p*-value and state your conclusion for each of the following sample results. Use $\alpha = .01$.
   a. $\bar{x} = 23$
   b. $\bar{x} = 25.1$
   c. $\bar{x} = 20$

Applications

15. Individuals filing federal income tax returns prior to March 31 received an average refund of $1056. Consider the population of "last-minute" filers who mail their tax return during the last five days of the income tax period (typically April 10 to April 15).
   a. A researcher suggests that a reason individuals wait until the last five days is that on average these individuals receive lower refunds than do early filers. Develop appropriate hypotheses such that rejection of H_0 will support the researcher's contention.

b. For a sample of 400 individuals who filed a tax return between April 10 and 15, the sample mean refund was \$910. Based on prior experience, a population standard deviation of $\sigma = \$1600$ may be assumed. What is the p-value?
c. At $\alpha = .05$, what is your conclusion?
d. Repeat the preceding hypothesis test using the critical value approach.

16. In a study entitled How Undergraduate Students Use Credit Cards, it was reported that undergraduate students have a mean credit card balance of \$3173 (*Sallie Mae*, April 2009). This figure was an all-time high and had increased 44% over the previous five years. Assume that a current study is being conducted to determine if it can be concluded that the mean credit card balance for undergraduate students has continued to increase compared to the April 2009 report. Based on previous studies, use a population standard deviation $\sigma = \$1000$.
a. State the null and alternative hypotheses.
b. What is the p-value for a sample of 180 undergraduate students with a sample mean credit card balance of \$3325?
c. Using a .05 level of significance, what is your conclusion?

17. Wall Street securities firms paid out record year-end bonuses of \$125,500 per employee for 2005 (*Fortune,* February 6, 2006). Suppose we would like to take a sample of employees at the Jones & Ryan securities firm to see whether the mean year-end bonus is different from the reported mean of \$125,500 for the population.
a. State the null and alternative hypotheses you would use to test whether the year-end bonuses paid by Jones & Ryan were different from the population mean.
b. Suppose a sample of 40 Jones & Ryan employees showed a sample mean year-end bonus of \$118,000. Assume a population standard deviation of $\sigma = \$30{,}000$ and compute the p-value.
c. With $\alpha = .05$ as the level of significance, what is your conclusion?
d. Repeat the preceding hypothesis test using the critical value approach.

18. The average annual total return for U.S. Diversified Equity mutual funds from 1999 to 2003 was 4.1% (*BusinessWeek,* January 26, 2004). A researcher would like to conduct a hypothesis test to see whether the returns for mid-cap growth funds over the same period are significantly different from the average for U.S. Diversified Equity funds.
a. Formulate the hypotheses that can be used to determine whether the mean annual return for mid-cap growth funds differ from the mean for U.S. Diversified Equity funds.
b. A sample of 40 mid-cap growth funds provides a mean return of $\bar{x} = 3.4\%$. Assume the population standard deviation for mid-cap growth funds is known from previous studies to be $\sigma = 2\%$. Use the sample results to compute the test statistic and p-value for the hypothesis test.
c. At $\alpha = .05$, what is your conclusion?

19. The Internal Revenue Service (IRS) provides a toll-free help line for taxpayers to call in and get answers to questions as they prepare their tax returns. In recent years, the IRS has been inundated with taxpayer calls and has redesigned its phone service as well as posting answers to frequently asked questions on its website (*The Cincinnati Enquirer,* January 7, 2010). According to a report by a taxpayer advocate, callers using the new system can expect to wait on hold for an unreasonably long time of 12 minutes before being able to talk to an IRS employee. Suppose you select a sample of 50 callers after the new phone service has been implemented; the sample results show a mean waiting time of 10 minutes before an IRS employee comes on line. Based upon data from past years, you decide it is reasonable to assume that the standard deviation of waiting times is 8 minutes. Using your sample results, can you conclude that the actual mean waiting time turned out to be significantly less than the 12-minute claim made by the taxpayer advocate? Use $\alpha = .05$.

20. For the United States, the mean monthly Internet bill is $32.79 per household (CNBC, January 18, 2006). A sample of 50 households in a southern state showed a sample mean of $30.63. Use a population standard deviation of $\sigma = \$5.60$.
    a. Formulate hypotheses for a test to determine whether the sample data support the conclusion that the mean monthly Internet bill in the southern state is less than the national mean of $32.79.
    b. What is the value of the test statistic?
    c. What is the p-value?
    d. At $\alpha = .01$, what is your conclusion?

21. Fowle Marketing Research, Inc., bases charges to a client on the assumption that telephone surveys can be completed in a mean time of 15 minutes or less. If a longer mean survey time is necessary, a premium rate is charged. A sample of 35 surveys provided the survey times shown in the file named Fowle. Based upon past studies, the population standard deviation is assumed known with $\sigma = 4$ minutes. Is the premium rate justified?
    a. Formulate the null and alternative hypotheses for this application.
    b. Compute the value of the test statistic.
    c. What is the p-value?
    d. At $\alpha = .01$, what is your conclusion?

22. CCN and ActMedia provided a television channel targeted to individuals waiting in supermarket checkout lines. The channel showed news, short features, and advertisements. The length of the program was based on the assumption that the population mean time a shopper stands in a supermarket checkout line is 8 minutes. A sample of actual waiting times will be used to test this assumption and determine whether actual mean waiting time differs from this standard.
    a. Formulate the hypotheses for this application.
    b. A sample of 120 shoppers showed a sample mean waiting time of 8.4 minutes. Assume a population standard deviation of $\sigma = 3.2$ minutes. What is the p-value?
    c. At $\alpha = .05$, what is your conclusion?
    d. Compute a 95% confidence interval for the population mean. Does it support your conclusion?

9.4 Population Mean: σ Unknown

In this section we describe how to conduct hypothesis tests about a population mean for the σ unknown case. Because the σ unknown case corresponds to situations in which an estimate of the population standard deviation cannot be developed prior to sampling, the sample must be used to develop an estimate of both μ and σ. Thus, to conduct a hypothesis test about a population mean for the σ unknown case, the sample mean $\bar{x}$ is used as an estimate of μ and the sample standard deviation s is used as an estimate of σ.

The steps of the hypothesis testing procedure for the σ unknown case are the same as those for the σ known case described in Section 9.3. But, with σ unknown, the computation of the test statistic and p-value is a bit different. Recall that for the σ known case, the sampling distribution of the test statistic has a standard normal distribution. For the σ unknown case, however, the sampling distribution of the test statistic follows the t distribution; it has slightly more variability because the sample is used to develop estimates of both μ and σ.

In Section 8.2 we showed that an interval estimate of a population mean for the σ unknown case is based on a probability distribution known as the t distribution. Hypothesis tests about a population mean for the σ unknown case are also based on the

t distribution. For the σ unknown case, the test statistic has a t distribution with $n - 1$ degrees of freedom.

TEST STATISTIC FOR HYPOTHESIS TESTS ABOUT A POPULATION MEAN: σ UNKNOWN

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \tag{9.2}$$

In Chapter 8 we said that the t distribution is based on an assumption that the population from which we are sampling has a normal distribution. However, research shows that this assumption can be relaxed considerably when the sample size is large enough. We provide some practical advice concerning the population distribution and sample size at the end of the section.

One-Tailed Test

Let us consider an example of a one-tailed test about a population mean for the σ unknown case. A business travel magazine wants to classify transatlantic gateway airports according to the mean rating for the population of business travelers. A rating scale with a low score of 0 and a high score of 10 will be used, and airports with a population mean rating greater than 7 will be designated as superior service airports. The magazine staff surveyed a sample of 60 business travelers at each airport to obtain the ratings data. The sample for London's Heathrow Airport provided a sample mean rating of $\bar{x} = 7.25$ and a sample standard deviation of $s = 1.052$. Do the data indicate that Heathrow should be designated as a superior service airport?

WEB file
AirRating

We want to develop a hypothesis test for which the decision to reject H_0 will lead to the conclusion that the population mean rating for the Heathrow Airport is *greater* than 7. Thus, an upper tail test with H_a: $\mu > 7$ is required. The null and alternative hypotheses for this upper tail test are as follows:

$$H_0: \mu \leq 7$$
$$H_a: \mu > 7$$

We will use $\alpha = .05$ as the level of significance for the test.

Using equation (9.2) with $\bar{x} = 7.25, \mu_0 = 7, s = 1.052$, and $n = 60$, the value of the test statistic is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{7.25 - 7}{1.052/\sqrt{60}} = 1.84$$

The sampling distribution of t has $n - 1 = 60 - 1 = 59$ degrees of freedom. Because the test is an upper tail test, the p-value is $P(t \geq 1.84)$, that is, the upper tail area corresponding to the value of the test statistic.

The t distribution table provided in most textbooks will not contain sufficient detail to determine the exact p-value, such as the p-value corresponding to $t = 1.84$. For instance, using Table 2 in Appendix B, the t distribution with 59 degrees of freedom provides the following information.

Area in Upper Tail	.20	.10	.05	.025	.01	.005
t Value (59 df)	.848	1.296	1.671	2.001	2.391	2.662

$t = 1.84$

We see that $t = 1.84$ is between 1.671 and 2.001. Although the table does not provide the exact p-value, the values in the "Area in Upper Tail" row show that the p-value must be less than .05 and greater than .025. With a level of significance of $\alpha = .05$, this placement is all we need to know to make the decision to reject the null hypothesis and conclude that Heathrow should be classified as a superior service airport.

It is cumbersome to use a t table to compute p-values, and only approximate values are obtained. We describe how to compute exact p-values using Excel's T.DIST function in the Using Excel subsection which follows. The exact upper tail p-value for the Heathrow Airport hypothesis test is .0354. With $.0354 < .05$, we reject the null hypothesis and conclude that Heathrow should be classified as a superior service airport.

Two-Tailed Test

To illustrate how to conduct a two-tailed test about a population mean for the σ unknown case, let us consider the hypothesis testing situation facing Holiday Toys. The company manufactures and distributes its products through more than 1000 retail outlets. In planning production levels for the coming winter season, Holiday must decide how many units of each product to produce prior to knowing the actual demand at the retail level. For this year's most important new toy, Holiday's marketing director is expecting demand to average 40 units per retail outlet. Prior to making the final production decision based upon this estimate, Holiday decided to survey a sample of 25 retailers in order to develop more information about the demand for the new product. Each retailer was provided with information about the features of the new toy along with the cost and the suggested selling price. Then each retailer was asked to specify an anticipated order quantity.

With μ denoting the population mean order quantity per retail outlet, the sample data will be used to conduct the following two-tailed hypothesis test:

$$H_0: \mu = 40$$
$$H_a: \mu \neq 40$$

If H_0 cannot be rejected, Holiday will continue its production planning based on the marketing director's estimate that the population mean order quantity per retail outlet will be $\mu = 40$ units. However, if H_0 is rejected, Holiday will immediately reevaluate its production plan for the product. A two-tailed hypothesis test is used because Holiday wants to reevaluate the production plan if the population mean quantity per retail outlet is less than anticipated or greater than anticipated. Because no historical data are available (it's a new product), the population mean μ and the population standard deviation must both be estimated using $\bar{x}$ and s from the sample data.

Orders

The sample of 25 retailers provided a mean of $\bar{x} = 37.4$ and a standard deviation of $s = 11.79$ units. Before going ahead with the use of the t distribution, the analyst constructed a histogram of the sample data in order to check on the form of the population distribution. The histogram of the sample data showed no evidence of skewness or any extreme outliers, so the analyst concluded that the use of the t distribution with $n - 1 = 24$ degrees of freedom was appropriate. Using equation (9.2) with $\bar{x} = 37.4$, $\mu_0 = 40$, $s = 11.79$, and $n = 25$, the value of the test statistic is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{37.4 - 40}{11.79/\sqrt{25}} = -1.10$$

Because we have a two-tailed test, the p-value is two times the area under the curve of the t distribution for $t \leq -1.10$. Using Table 2 in Appendix B, the t distribution table for 24 degrees of freedom provides the following information.

Area in Upper Tail	.20	.10	.05	.025	.01	.005
t-Value (24 *df*)	.857	1.318	1.711	2.064	2.492	2.797

$t = 1.10$

The t distribution table only contains positive t values (corresponding to areas in the upper tail). Because the t distribution is symmetric, however, the upper tail area for $t = 1.10$ is the same as the lower tail area for $t = -1.10$. We see that $t = 1.10$ is between 0.857 and 1.318. From the "Area in Upper Tail" row, we see that the area in the tail to the right of $t = 1.10$ is between .20 and .10. When we double these amounts, we see that the p-value must be between .40 and .20. With a level of significance of $\alpha = .05$, we now know that the p-value is greater than α. Therefore, H_0 cannot be rejected. Sufficient evidence is not available to conclude that Holiday should change its production plan for the coming season.

In the Using Excel subsection which follows, we show how to compute the exact p-value for this hypothesis test using Excel. The p-value obtained is .2811. With a level of significance of $\alpha = .05$, we cannot reject H_0 because $.2811 > .05$.

The test statistic can also be compared to the critical value to make the two-tailed hypothesis testing decision. With $\alpha = .05$ and the t distribution with 24 degrees of freedom, $-t_{.025} = -2.064$ and $t_{.025} = 2.064$ are the critical values for the two-tailed test. The rejection rule using the test statistic is

$$\text{Reject } H_0 \text{ if } t \leq -2.064 \text{ or if } t \geq 2.064$$

Based on the test statistic $t = -1.10$, H_0 cannot be rejected. This result indicates that Holiday should continue its production planning for the coming season based on the expectation that $\mu = 40$.

Using Excel

Excel can be used to conduct one-tailed and two-tailed hypothesis tests about a population mean for the σ unknown case. The approach is similar to the procedure used in the σ known case. The sample data and the test statistic (t) are used to compute three p-values: p-value (Lower Tail), p-value (Upper Tail), and p-value (Two Tail). The user can then choose α and draw a conclusion using whichever p-value is appropriate for the type of hypothesis test being conducted.

Let's start by showing how to use Excel's T.DIST function to compute a lower tail p-value. The T.DIST function has three inputs; its general form is as follows:

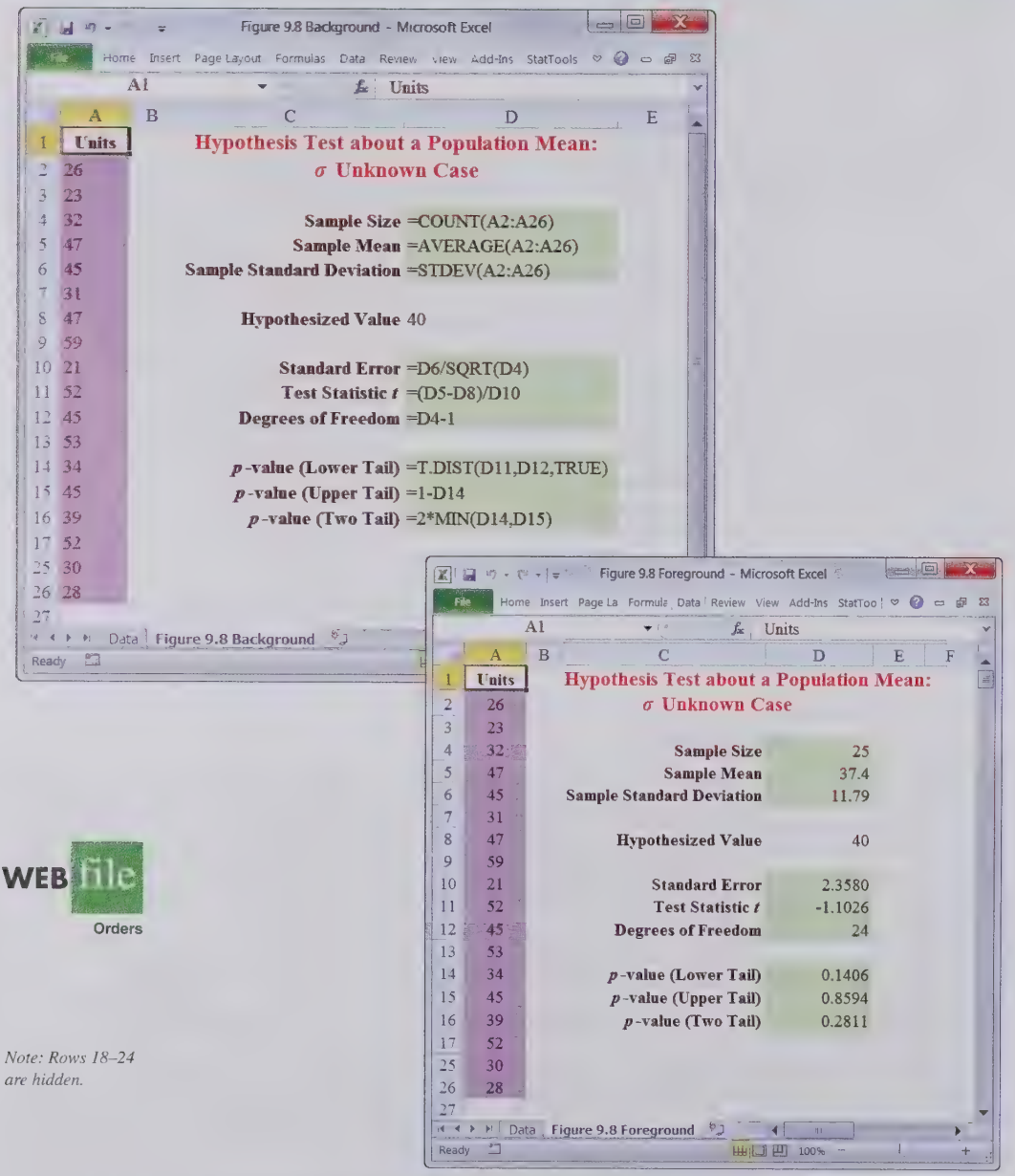
T.DIST(test statistic, degrees of freedom, cumulative)

For the first input, we enter the value of the test statistic, for the second input we enter the number of degrees of freedom. For the third input, we enter TRUE if we want a cumulative probability and FALSE if we want the height of the curve. When we want to compute a lower tail p-value, we enter TRUE.

Once the lower tail p-value has been computed, it is easy to compute the upper tail and the two-tailed p-values. The upper tail p-value is just 1 minus the lower tail p-value. And the two-tailed p-value is given by two times the smaller of the lower and upper tail p-values.

Let us now construct an Excel worksheet to conduct the two-tailed hypothesis test for the Holiday Toys study. Refer to Figure 9.8 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet is in the foreground.

FIGURE 9.8 EXCEL WORKSHEET: HYPOTHESIS TEST FOR THE σ UNKNOWN CASE



WEB file
Orders

Note: Rows 18–24 are hidden.

Enter Data: A label and the order quantity data for the sample of 25 retailers are entered into cells A1:A26.

Enter Functions and Formulas: The descriptive statistics needed are provided in cells D4:D6. Excel's COUNT, AVERAGE, and STDEV functions compute the sample size, the sample mean, and the sample standard deviation, respectively. The hypothesized value of the population mean (40) is entered into cell D8.

Using the sample standard deviation as an estimate of the population standard deviation, an estimate of the standard error is obtained in cell D10 by dividing the sample standard deviation in cell D6 by the square root of the sample size in cell D4. The formula =(D5-D8)/D10 entered into cell D11 computes the test statistic t(-1.1026). The degrees of freedom are computed in cell D12 as the sample size in cell D4 minus 1.

To compute the p-value for a lower tail test, we enter the following formula into cell D14:

=T.DIST(D11,D12,TRUE)

The p-value for an upper tail test is then computed in cell D15 as 1 minus the p-value for the lower tail test. Finally, the p-value for a two-tailed test is computed in cell D16 as two times the minimum of the two one-tailed p-values. The value worksheet shows that the three p-values are p-value (Lower Tail) = 0.1406, p-value (Upper Tail) = 0.8594, and p-value (Two Tail) = 0.2811.

The development of the worksheet is now complete. For the two-tailed Holiday Toys problem we cannot reject H_0: $\mu = 40$ using $\alpha = .05$ because the p-value (Two Tail) = 0.2811 is greater than α. This result indicates that Holiday should continue its production planning for the coming season based on the expectation that $\mu = 40$. The worksheet in Figure 9.8 can also be used for any one-tailed hypothesis test involving the t distribution. If a lower tail test is required, compare the p-value (Lower Tail) with α to make the rejection decision. If an upper tail test is required, compare the p-value (Upper Tail) with α to make the rejection decision.

A template for other problems The worksheet in Figure 9.8 can be used as a template for any hypothesis tests about a population mean for the σ unknown case. Just enter the appropriate data in column A, adjust the ranges for the formulas in cells D4:D6, and enter the hypothesized value in cell D8. The standard error, the test statistic, and the three p-values will then appear. Depending on the form of the hypothesis test (lower tail, upper tail, or two-tailed), we can then choose the appropriate p-value to make the rejection decision.

We can further simplify the use of Figure 9.8 as a template for other problems by eliminating the need to enter new data ranges in cells D4:D6. To do so we rewrite the cell formulas as follows:

Cell D4: =COUNT(A:A)
Cell D5: =AVERAGE(A:A)
Cell D6: =STDEV(A:A)

The Orders data set includes a worksheet entitled Template that uses the A:A method for entering the data ranges.

With the A:A method of specifying data ranges, Excel's COUNT function will count the number of numeric values in column A, Excel's AVERAGE function will compute the average of the numeric values in column A, and Excel's STDEV function will compute the standard deviation of the numeric values in Column A. Thus, to solve a new problem it is only necessary to enter the new data in column A and enter the hypothesized value of the population mean in cell D8.

TABLE 9.3 SUMMARY OF HYPOTHESIS TESTS ABOUT A POPULATION MEAN: σ UNKNOWN CASE

	Lower Tail Test	Upper Tail Test	Two-Tailed Test
Hypotheses	$H_0: \mu \geq \mu_0$ $H_a: \mu < \mu_0$	$H_0: \mu \leq \mu_0$ $H_a: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_a: \mu \neq \mu_0$
Test Statistic	$t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$
Rejection Rule: *p*-Value Approach	Reject H_0 if *p*-value $\leq \alpha$	Reject H_0 if *p*-value $\leq \alpha$	Reject H_0 if *p*-value $\leq \alpha$
Rejection Rule: Critical Value Approach	Reject H_0 if $t \leq -t_\alpha$	Reject H_0 if $t \geq t_\alpha$	Reject H_0 if $t \leq -t_{\alpha/2}$ or if $t \geq t_{\alpha/2}$

Summary and Practical Advice

Table 9.3 provides a summary of the hypothesis testing procedures about a population mean for the σ unknown case. The key difference between these procedures and the ones for the σ known case is that s is used, instead of σ, in the computation of the test statistic. For this reason, the test statistic follows the t distribution.

The applicability of the hypothesis testing procedures of this section is dependent on the distribution of the population being sampled from and the sample size. When the population is normally distributed, the hypothesis tests described in this section provide exact results for any sample size. When the population is not normally distributed, the procedures are approximations. Nonetheless, we find that sample sizes of 30 or greater will provide good results in most cases. If the population is approximately normal, small sample sizes (e.g., $n < 15$) can provide acceptable results. If the population is highly skewed or contains outliers, sample sizes approaching 50 are recommended.

Exercises

Methods

23. Consider the following hypothesis test:

$$H_0: \mu \leq 12$$
$$H_a: \mu > 12$$

A sample of 25 provided a sample mean $\bar{x} = 14$ and a sample standard deviation $s = 4.32$.

a. Compute the value of the test statistic.
b. Use the t distribution table (Table 2 in Appendix B) to compute a range for the *p*-value.
c. At $\alpha = .05$, what is your conclusion?
d. What is the rejection rule using the critical value? What is your conclusion?

24. Consider the following hypothesis test:

SELF test

$$H_0: \mu = 18$$
$$H_a: \mu \neq 18$$

A sample of 48 provided a sample mean $\bar{x} = 17$ and a sample standard deviation $s = 4.5$.

a. Compute the value of the test statistic.
b. Use the t distribution table (Table 2 in Appendix B) to compute a range for the p-value.
c. At $\alpha = .05$, what is your conclusion?
d. What is the rejection rule using the critical value? What is your conclusion?

25. Consider the following hypothesis test:

$$H_0: \mu \geq 45$$
$$H_a: \mu < 45$$

A sample of 36 is used. Identify the p-value and state your conclusion for each of the following sample results. Use $\alpha = .01$.

a. $\bar{x} = 44$ and $s = 5.2$
b. $\bar{x} = 43$ and $s = 4.6$
c. $\bar{x} = 46$ and $s = 5.0$

26. Consider the following hypothesis test:

$$H_0: \mu = 100$$
$$H_a: \mu \neq 100$$

A sample of 65 is used. Identify the p-value and state your conclusion for each of the following sample results. Use $\alpha = .05$.

a. $\bar{x} = 103$ and $s = 11.5$
b. $\bar{x} = 96.5$ and $s = 11.0$
c. $\bar{x} = 102$ and $s = 10.5$

Applications

SELF test

27. The Employment and Training Administration reported that the U.S. mean unemployment insurance benefit was $238 per week (*The World Almanac*, 2003). A researcher in the state of Virginia anticipated that sample data would show evidence that the mean weekly unemployment insurance benefit in Virginia was below the national average.

a. Develop appropriate hypotheses such that rejection of H_0 will support the researcher's contention.
b. For a sample of 100 individuals, the sample mean weekly unemployment insurance benefit was $231 with a sample standard deviation of $80. What is the p-value?
c. At $\alpha = .05$, what is your conclusion?
d. Repeat the preceding hypothesis test using the critical value approach.

28. A shareholders' group, in lodging a protest, claimed that the mean tenure for a chief executive officer (CEO) was at least nine years. A survey of companies reported in *The Wall Street Journal* found a sample mean tenure of $\bar{x} = 7.27$ years for CEOs with a standard deviation of $s = 6.38$ years (*The Wall Street Journal*, January 2, 2007).

a. Formulate hypotheses that can be used to challenge the validity of the claim made by the shareholders' group.
b. Assume 85 companies were included in the sample. What is the p-value for your hypothesis test?
c. At $\alpha = .01$, what is your conclusion?

WEB file
Diamonds

29. The cost of a one-carat VS2 clarity, H color diamond from Diamond Source USA is $5600 (Diamond Source website, March 2003). A midwestern jeweler makes calls to contacts in the diamond district of New York City to see whether the mean price of diamonds there differs from $5600.

a. Formulate hypotheses that can be used to determine whether the mean price in New York City differs from $5600.
b. A sample of 25 New York City contacts provided the prices shown in the file named Diamonds. What is the p-value?
c. At $\alpha = .05$, can the null hypothesis be rejected? What is your conclusion?
d. Repeat the preceding hypothesis test using the critical value approach.

30. Time Warner Inc.'s CNN has been the longtime ratings leader of cable television news. Nielsen Media Research indicated that the mean CNN viewing audience was 600,000 viewers per day during 2002 (*The Wall Street Journal,* March 10, 2003). Assume that for a sample of 40 days during the first half of 2003, the daily audience was 612,000 viewers with a sample standard deviation of 65,000 viewers.
a. What are the hypotheses if CNN management would like information on any change in the CNN viewing audience?
b. What is the p-value?
c. Select your own level of significance. What is your conclusion?
d. What recommendation would you make to CNN management in this application?

31. The Coca-Cola Company reported that the mean per capita annual sales of its beverages in the United States was 423 eight-ounce servings (Coca-Cola Company website, February 3, 2009). Suppose you are curious whether the consumption of Coca-Cola beverages is higher in Atlanta, Georgia, the location of Coca-Cola's corporate headquarters. A sample of 36 individuals from the Atlanta area showed a sample mean annual consumption of 460.4 eight-ounce servings with a standard deviation of $s = 101.9$ ounces. Using $\alpha = .05$, do the sample results support the conclusion that mean annual consumption of Coca-Cola beverage products is higher in Atlanta?

UsedCars

32. According to the National Automobile Dealers Association, the mean price for used cars is $10,192. A manager of a Kansas City used car dealership reviewed a sample of 50 recent used car sales at the dealership in an attempt to determine whether the population mean price for used cars at this particular dealership differed from the national mean. The prices for the sample of 50 cars are shown in the file named UsedCars.
a. Formulate the hypotheses that can be used to determine whether a difference exists in the mean price for used cars at the dealership.
b. What is the p-value?
c. At $\alpha = .05$, what is your conclusion?

33. Annual per capita consumption of milk is 21.6 gallons (*Statistical Abstract of the United States: 2006*). Being from the Midwest, you believe milk consumption is higher there and wish to support your opinion. A sample of 16 individuals from the midwestern town of Webster City showed a sample mean annual consumption of 24.1 gallons with a standard deviation of $s = 4.8$.
a. Develop a hypothesis test that can be used to determine whether the mean annual consumption in Webster City is higher than the national mean.
b. What is a point estimate of the difference between mean annual consumption in Webster City and the national mean?
c. At $\alpha = .05$, test for a significant difference. What is your conclusion?

34. Joan's Nursery specializes in custom-designed landscaping for residential areas. The estimated labor cost associated with a particular landscaping proposal is based on the number of plantings of trees, shrubs, and so on to be used for the project. For cost-estimating purposes, managers use two hours of labor time for the planting of a medium-sized tree. Actual times from a sample of 10 plantings during the past month follow (times in hours).

1.7	1.5	2.6	2.2	2.4	2.3	2.6	3.0	1.4	2.3

With a .05 level of significance, test to see whether the mean tree-planting time differs from two hours.

a. State the null and alternative hypotheses.
b. Compute the sample mean.
c. Compute the sample standard deviation.
d. What is the p-value?
e. What is your conclusion?

9.5 Population Proportion

In this section we show how to conduct a hypothesis test about a population proportion p. Using p_0 to denote the hypothesized value for the population proportion, the three forms for a hypothesis test about a population proportion are as follows.

$$\begin{array}{lll} H_0: p \geq p_0 & H_0: p \leq p_0 & H_0: p = p_0 \\ H_a: p < p_0 & H_a: p > p_0 & H_a: p \neq p_0 \end{array}$$

The first form is called a lower tail test, the second form is called an upper tail test, and the third form is called a two-tailed test.

Hypothesis tests about a population proportion are based on the difference between the sample proportion $\bar{p}$ and the hypothesized population proportion p_0. The methods used to conduct the hypothesis test are similar to those used for hypothesis tests about a population mean. The only difference is that we use the sample proportion and its standard error to compute the test statistic. The p-value approach or the critical value approach is then used to determine whether the null hypothesis should be rejected.

Let us consider an example involving a situation faced by Pine Creek golf course. Over the past year, 20% of the players at Pine Creek were women. In an effort to increase the proportion of women players, Pine Creek implemented a special promotion designed to attract women golfers. One month after the promotion was implemented, the course manager requested a statistical study to determine whether the proportion of women players at Pine Creek had increased. Because the objective of the study is to determine whether the proportion of women golfers increased, an upper tail test with $H_a: p > .20$ is appropriate. The null and alternative hypotheses for the Pine Creek hypothesis test are as follows:

$$\begin{aligned} H_0: p &\leq .20 \\ H_a: p &> .20 \end{aligned}$$

If H_0 can be rejected, the test results will give statistical support for the conclusion that the proportion of women golfers increased and the promotion was beneficial. The course manager specified that a level of significance of $\alpha = .05$ be used in carrying out this hypothesis test.

The next step of the hypothesis testing procedure is to select a sample and compute the value of an appropriate test statistic. To show how this step is done for the Pine Creek upper tail test, we begin with a general discussion of how to compute the value of the test statistic for any form of a hypothesis test about a population proportion. The sampling distribution of $\bar{p}$, the point estimator of the population parameter p, is the basis for developing the test statistic.

When the null hypothesis is true as an equality, the expected value of $\bar{p}$ equals the hypothesized value p_0; that is, $E(\bar{p}) = p_0$. The standard error of $\bar{p}$ is given by

$$\sigma_{\bar{p}} = \sqrt{\frac{p_0(1 - p_0)}{n}}$$

In Chapter 7 we said that if $np \geq 5$ and $n(1 - p) \geq 5$, the sampling distribution of $\bar{p}$ can be approximated by a normal distribution.[4] Under these conditions, which usually apply in practice, the quantity

$$z = \frac{\bar{p} - p_0}{\sigma_{\bar{p}}} \tag{9.3}$$

has a standard normal probability distribution. With $\sigma_{\bar{p}} = \sqrt{p_0(1 - p_0)/n}$, the standard normal random variable z is the test statistic used to conduct hypothesis tests about a population proportion.

TEST STATISTIC FOR HYPOTHESIS TESTS ABOUT A POPULATION PROPORTION

$$z = \frac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1 - p_0)}{n}}} \tag{9.4}$$

WEB file
WomenGolf

We can now compute the test statistic for the Pine Creek hypothesis test. Suppose a random sample of 400 players was selected, and that 100 of the players were women. The proportion of women golfers in the sample is

$$\bar{p} = \frac{100}{400} = .25$$

Using equation (9.4), the value of the test statistic is

$$z = \frac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1 - p_0)}{n}}} = \frac{.25 - .20}{\sqrt{\dfrac{.20(1 - .20)}{400}}} = \frac{.05}{.02} = 2.50$$

Because the Pine Creek hypothesis test is an upper tail test, the p-value is the probability that z is greater than or equal to $z = 2.50$; that is, it is the upper tail area corresponding to $z \geq 2.50$. Using the standard normal probability table, we find that the lower tail area for $z = 2.50$ is .9938. Thus, the p-value for the Pine Creek test is $1.0000 - .9938 = .0062$. Figure 9.9 shows this p-value calculation.

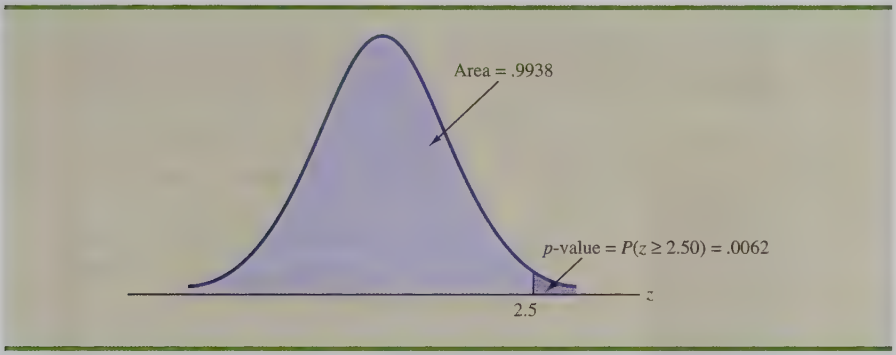
Recall that the course manager specified a level of significance of $\alpha = .05$. A p-value $= .0062 < .05$ gives sufficient statistical evidence to reject H_0 at the .05 level of significance. Thus, the test provides statistical support for the conclusion that the special promotion increased the proportion of women players at the Pine Creek golf course.

The decision whether to reject the null hypothesis can also be made using the critical value approach. The critical value corresponding to an area of .05 in the upper tail of a normal probability distribution is $z_{.05} = 1.645$. Thus, the rejection rule using the critical value approach is to reject H_0 if $z \geq 1.645$. Because $z = 2.50 > 1.645$, H_0 is rejected.

Again, we see that the p-value approach and the critical value approach lead to the same hypothesis testing conclusion, but the p-value approach provides more information. With a p-value $= .0062$, the null hypothesis would be rejected for any level of significance greater than or equal to .0062.

[4] In most applications involving hypothesis tests of a population proportion, sample sizes are large enough to use the normal approximation. The exact sampling distribution of $\bar{p}$ is discrete, with the probability for each value of $\bar{p}$ given by the binomial distribution. So hypothesis testing is a bit more complicated for small samples when the normal approximation cannot be used.

FIGURE 9.9 CALCULATION OF THE p-VALUE FOR THE PINE CREEK HYPOTHESIS TEST



Using Excel

Excel can be used to conduct one-tailed and two-tailed hypothesis tests about a population proportion using the p-value approach. The procedure is similar to the approach used with Excel in conducting hypothesis tests about a population mean. The primary difference is that the test statistic is based on the sampling distribution of $\bar{x}$ for hypothesis tests about a population mean and on the sampling distribution of $\bar{p}$ for hypothesis tests about a population proportion. Thus, although different formulas are used to compute the test statistic needed to make the hypothesis testing decision, the computations of the critical value and the p-value for the tests are identical.

We will illustrate the procedure by showing how Excel can be used to conduct the upper tail hypothesis test for the Pine Creek golf course study. Refer to Figure 9.10 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet is in the foreground.

Enter Data: A label and the gender of each golfer in the study are entered into cells A1:A401.

Enter Functions and Formulas: The descriptive statistics needed are provided in cells D3, D5, and D6. Because the data are not numeric, Excel's COUNTA function, not the COUNT function, is used in cell D3 to determine the sample size. We entered Female in cell D4 to identify the response for which we wish to compute a proportion. The COUNTIF function is then used in cell D5 to determine the number of responses of the type identified in cell D4. The sample proportion is then computed in cell D6 by dividing the response count by the sample size.

The hypothesized value of the population proportion (.20) is entered into cell D8. The standard error is obtained in cell D10 by entering the formula =SQRT(D8*(1-D8)/D3). The formula =(D6-D8)/D10 entered into cell D11 computes the test statistic z(2.50). To compute the p-value for a lower tail test, we enter the formula =NORM.S.DIST(D11,TRUE) into cell D13. The p-value for an upper tail test is then computed in cell D14 as 1 minus the p-value for the lower tail test. Finally, the p-value for a two-tailed test is computed in cell D15 as two times the minimum of the two one-tailed p-values. The value worksheet shows that the three p-values are as follows: p-value (Lower Tail) = 0.9938; p-value (Upper Tail) = 0.0062; and p-value (Two Tail) = 0.0124.

The development of the worksheet is now complete. For the Pine Creek upper tail hypothesis test, we reject the null hypothesis that the population proportion is .20 or less because the p-value (Upper Tail) = 0.0062 is less than $\alpha = .05$. Indeed, with this p-value we would reject the null hypothesis for any level of significance of .0062 or greater.

FIGURE 9.10 EXCEL WORKSHEET: HYPOTHESIS TEST FOR PINE CREEK GOLF COURSE

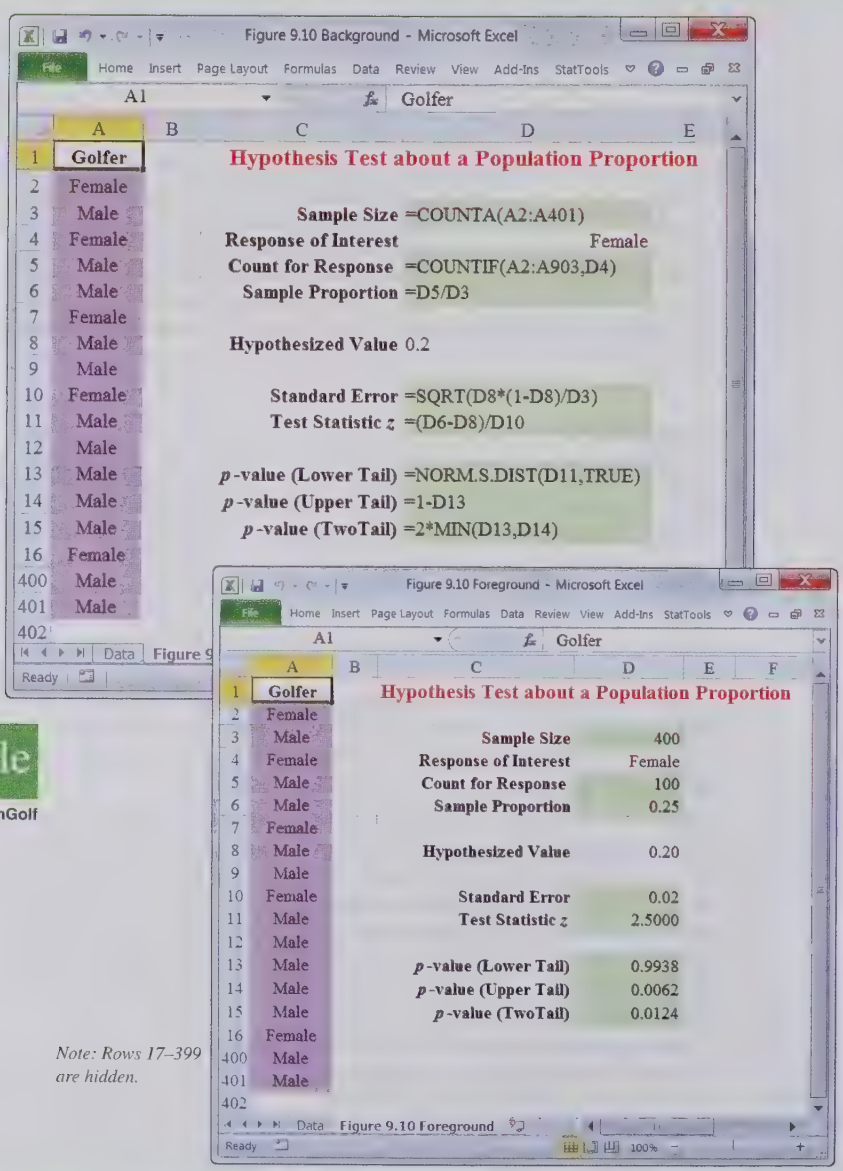


Figure 9.10 Background - Microsoft Excel
Hypothesis Test about a Population Proportion
Sample Size =COUNTA(A2:A401)
Response of Interest Female
Count for Response =COUNTIF(A2:A903,D4)
Sample Proportion =D5/D3
Hypothesized Value 0.2
Standard Error =SQRT(D8*(1-D8)/D3)
Test Statistic z =(D6-D8)/D10
p-value (Lower Tail) =NORM.S.DIST(D11,TRUE)
p-value (Upper Tail) =1-D13
p-value (TwoTail) =2*MIN(D13,D14)
Figure 9.10 Foreground - Microsoft Excel
Hypothesis Test about a Population Proportion
Sample Size 400
Response of Interest Female
Count for Response 100
Sample Proportion 0.25
Hypothesized Value 0.20
Standard Error 0.02
Test Statistic z 2.5000
p-value (Lower Tail) 0.9938
p-value (Upper Tail) 0.0062
p-value (TwoTail) 0.0124
Note: Rows 17–399 are hidden.

WEB file
WomenGolf

A template for other problems The worksheet in Figure 9.10 can be used as a template for hypothesis tests about a population proportion whenever $np \geq 5$ and $n(1 - p) \geq 5$. Just enter the appropriate data in column A, adjust the ranges for the formulas in cells D3 and D5, enter the appropriate response in cell D4, and enter the hypothesized value in cell D8. The standard error, the test statistic, and the three p-values will then appear. Depending on

TABLE 9.4 SUMMARY OF HYPOTHESIS TESTS ABOUT A POPULATION PROPORTION

	Lower Tail Test	Upper Tail Test	Two-Tailed Test
Hypotheses	$H_0: p \geq p_0$ $H_a: p < p_0$	$H_0: p \leq p_0$ $H_a: p > p_0$	$H_0: p = p_0$ $H_a: p \neq p_0$
Test Statistic	$z = \dfrac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$	$z = \dfrac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$	$z = \dfrac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$
Rejection Rule: *p*-Value Approach	Reject H_0 if p-value $\leq \alpha$	Reject H_0 if p-value $\leq \alpha$	Reject H_0 if p-value $\leq \alpha$
Rejection Rule: Critical Value Approach	Reject H_0 if $z \leq -z_\alpha$	Reject H_0 if $z \geq z_\alpha$	Reject H_0 if $z \leq -z_{\alpha/2}$ or if $z \geq z_{\alpha/2}$

the form of the hypothesis test (lower tail, upper tail, or two-tailed), we can then choose the appropriate p-value to make the rejection decision.

Summary

The procedure used to conduct a hypothesis test about a population proportion is similar to the procedure used to conduct a hypothesis test about a population mean. Although we only illustrated how to conduct a hypothesis test about a population proportion for an upper tail test, similar procedures can be used for lower tail and two-tailed tests. Table 9.4 provides a summary of the hypothesis tests about a population proportion. We assume that $np \geq 5$ and $n(1 - p) \geq 5$; thus the normal probability distribution can be used to approximate the sampling distribution of $\bar{p}$.

Exercises

Methods

35. Consider the following hypothesis test:

$$H_0: p = .20$$
$$H_a: p \neq .20$$

A sample of 400 provided a sample proportion $\bar{p} = .175$.

a. Compute the value of the test statistic.
b. What is the p-value?
c. At $\alpha = .05$, what is your conclusion?
d. What is the rejection rule using the critical value? What is your conclusion?

SELF test

36. Consider the following hypothesis test:

$$H_0: p \geq .75$$
$$H_a: p < .75$$

A sample of 300 items was selected. Compute the p-value and state your conclusion for each of the following sample results. Use $\alpha = .05$.

a. $\bar{p} = .68$
b. $\bar{p} = .72$
c. $\bar{p} = .70$
d. $\bar{p} = .77$

Applications

37. A study found that, in 2005, 12.5% of U.S. workers belonged to unions (*The Wall Street Journal,* January 21, 2006). Suppose a sample of 400 U.S. workers is collected in 2006 to determine whether union efforts to organize have increased union membership.
    a. Formulate the hypotheses that can be used to determine whether union membership increased in 2006.
    b. If the sample results show that 52 of the workers belonged to unions, what is the p-value for your hypothesis test?
    c. At $\alpha = .05$, what is your conclusion?

38. A study by *Consumer Reports* showed that 64% of supermarket shoppers believe supermarket brands to be as good as national name brands. To investigate whether this result applies to its own product, the manufacturer of a national name-brand ketchup asked a sample of shoppers whether they believed that supermarket ketchup was as good as the national brand ketchup.
    a. Formulate the hypotheses that could be used to determine whether the percentage of supermarket shoppers who believe that the supermarket ketchup was as good as the national brand ketchup differed from 64%.
    b. If a sample of 100 shoppers showed 52 stating that the supermarket brand was as good as the national brand, what is the p-value?
    c. At $\alpha = .05$, what is your conclusion?
    d. Should the national brand ketchup manufacturer be pleased with this conclusion? Explain.

AgeGroup

39. According to the Pew Internet & American Life Project, 75% of American adults use the Internet (Pew Internet website, April 19, 2008). The Pew project authors also reported on the percentage of Americans who use the Internet by age group. The data in the file AgeGroup are consistent with their findings. These data were obtained from a sample of 100 Internet users in the 30–49 age group and 200 Internet users in the 50–64 age group. A Yes indicates the survey repondent had used the Internet; a No indicates the survey repondent had not.
    a. Formulate hypotheses that could be used to determine whether the percentage of Internet users in the two age groups differs from the overall average of 75%.
    b. Estimate the proportion of Internet users in the 30–49 age group. Does this proportion differ significantly from the overall proportion of .75? Use $\alpha = .05$.
    c. Estimate the proportion of Internet users in the 50–64 age group. Does this proportion differ significantly from the overall proportion of .75? Use $\alpha = .05$.
    d. Would you expect the proportion of users in the 18–29 age group to be larger or smaller than the proportion for the 30–49 age group? Support your conclusion with the results obtained in parts (b) and (c).

40. In 2008, 46% of business owners gave a holiday gift to their employees. A 2009 survey of business owners indicated that 35% plan to provide a holiday gift to their employees (Radio WEZV, Myrtle Beach, South Carolina, November 11, 2009). Suppose the survey results are based on a sample of 60 business owners.
    a. How many business owners in the survey plan to provide a holiday gift to their employees?
    b. Suppose the business owners in the sample do as they plan. Compute the p-value for a hypothesis test that can be used to determine if the proportion of business owners providing holiday gifts has decreased from the 2008 level.

c. Using a .05 level of significance, would you conclude that the proportion of business owners providing gifts has decreased? What is the smallest level of significance for which you could draw such a conclusion?

41. Speaking to a group of analysts in January 2006, a brokerage firm executive claimed that at least 70% of investors are currently confident of meeting their investment objectives. A UBS Investor Optimism Survey, conducted over the period January 2 to January 15, found that 67% of investors were confident of meeting their investment objectives (CNBC, January 20, 2006).
    a. Formulate the hypotheses that can be used to test the validity of the brokerage firm executive's claim.
    b. Assume the UBS Investor Optimism Survey collected information from 300 investors. What is the p-value for the hypothesis test?
    c. At $\alpha = .05$, should the executive's claim be rejected?

42. According to the University of Nevada Center for Logistics Management, 6% of all merchandise sold in the United States gets returned (*BusinessWeek*, January 15, 2007). A Houston department store sampled 80 items sold in January and found that 12 of the items were returned.
    a. Construct a point estimate of the proportion of items returned for the population of sales transactions at the Houston store.
    b. Construct a 95% confidence interval for the porportion of returns at the Houston store.
    c. Is the proportion of returns at the Houston store significantly different from the returns for the nation as a whole? Provide statistical support for your answer.

Eagle

43. Eagle Outfitters is a chain of stores specializing in outdoor apparel and camping gear. It is considering a promotion that involves mailing discount coupons to all its credit card customers. This promotion will be considered a success if more than 10% of those receiving the coupons use them. Before going national with the promotion, coupons were sent to a sample of 100 credit card customers.
    a. Develop hypotheses that can be used to test whether the population proportion of those who will use the coupons is sufficient to go national.
    b. The file Eagle contains the sample data. Develop a point estimate of the population proportion.
    c. Use $\alpha = .05$ to conduct your hypothesis test. Should Eagle go national with the promotion?

Drowsy

44. In a cover story, *BusinessWeek* published information about sleep habits of Americans (*BusinessWeek,* January 26, 2004). The article noted that sleep deprivation causes a number of problems, including highway deaths. Fifty-one percent of adult drivers admit to driving while drowsy. A researcher hypothesized that this issue was an even bigger problem for night shift workers.
    a. Formulate the hypotheses that can be used to help determine whether more than 51% of the population of night shift workers admit to driving while drowsy.
    b. A sample of 400 night shift workers identified those who admitted to driving while drowsy. See the Drowsy file. What is the sample proportion? What is the p-value?
    c. At $\alpha = .01$, what is your conclusion?

45. Many investors and financial analysts believe the Dow Jones Industrial Average (DJIA) provides a good barometer of the overall stock market. On January 31, 2006, 9 of the 30 stocks making up the DJIA increased in price (*The Wall Street Journal,* February 1, 2006). On the basis of this fact, a financial analyst claims we can assume that 30% of the stocks traded on the New York Stock Exchange (NYSE) went up the same day.
    a. Formulate null and alternative hypotheses to test the analyst's claim.
    b. A sample of 50 stocks traded on the NYSE that day showed that 24 went up. What is your point estimate of the population proportion of stocks that went up?
    c. Conduct your hypothesis test using $\alpha = .01$ as the level of significance. What is your conclusion?

Summary

Hypothesis testing is a statistical procedure that uses sample data to determine whether a statement about the value of a population parameter should or should not be rejected. The hypotheses are two competing statements about a population parameter. One statement is called the null hypothesis (H_0), and the other statement is called the alternative hypothesis (H_a). In Section 9.1 we provided guidelines for developing hypotheses for situations frequently encountered in practice.

Whenever historical data or other information provides a basis for assuming that the population standard deviation is known, the hypothesis testing procedure for the population mean is based on the standard normal distribution. Whenever σ is unknown, the sample standard deviation s is used to estimate σ and the hypothesis testing procedure is based on the t distribution. In both cases, the quality of results depends on both the form of the population distribution and the sample size. If the population has a normal distribution, both hypothesis testing procedures are applicable, even with small sample sizes. If the population is not normally distributed, larger sample sizes are needed. General guidelines about the sample size were provided in Sections 9.3 and 9.4. In the case of hypothesis tests about a population proportion, the hypothesis testing procedure uses a test statistic based on the standard normal distribution.

In all cases, the value of the test statistic can be used to compute a p-value for the test. A p-value is a probability used to determine whether the null hypothesis should be rejected. If the p-value is less than or equal to the level of significance α, the null hypothesis can be rejected.

Hypothesis testing conclusions can also be made by comparing the value of the test statistic to a critical value. For lower tail tests, the null hypothesis is rejected if the value of the test statistic is less than or equal to the critical value. For upper tail tests, the null hypothesis is rejected if the value of the test statistic is greater than or equal to the critical value. Two-tailed tests consist of two critical values: one in the lower tail of the sampling distribution and one in the upper tail. In this case, the null hypothesis is rejected if the value of the test statistic is less than or equal to the critical value in the lower tail or greater than or equal to the critical value in the upper tail.

Glossary

Null hypothesis The hypothesis tentatively assumed true in the hypothesis testing procedure.
Alternative hypothesis The hypothesis concluded to be true if the null hypothesis is rejected.
Type I error The error of rejecting H_0 when it is true.
Type II error The error of accepting H_0 when it is false.
Level of significance The probability of making a Type I error when the null hypothesis is true as an equality.
One-tailed test A hypothesis test in which rejection of the null hypothesis occurs for values of the test statistic in one tail of its sampling distribution.
Test statistic A statistic whose value helps determine whether a null hypothesis should be rejected.
***p*-value** A probability that provides a measure of the evidence against the null hypothesis provided by the sample. Smaller p-values indicate more evidence against H_0. For a lower tail test, the p-value is the probability of obtaining a value for the test statistic as small as or smaller than that provided by the sample. For an upper tail test, the p-value is the probability of obtaining a value for the test statistic as large as or larger than that provided by the sample. For a two-tailed test, the p-value is the probability of obtaining a value for the test statistic at least as unlikely as or more unlikely than that provided by the sample.

Critical value A value that is compared with the test statistic to determine whether H_0 should be rejected.

Two-tailed test A hypothesis test in which rejection of the null hypothesis occurs for values of the test statistic in either tail of its sampling distribution.

Key Formulas

Test Statistic for Hypothesis Tests About a Population Mean: σ Known

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \tag{9.1}$$

Test Statistic for Hypothesis Tests About a Population Mean: σ Unknown

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \tag{9.2}$$

Test Statistic for Hypothesis Tests About a Population Proportion

$$z = \frac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1 - p_0)}{n}}} \tag{9.4}$$

Supplementary Exercises

46. A production line operates with a mean filling weight of 16 ounces per container. Overfilling or underfilling presents a serious problem and when detected requires the operator to shut down the production line to readjust the filling mechanism. From past data, a population standard deviation $\sigma = .8$ ounces is assumed. A quality control inspector selects a sample of 30 items every hour and at that time makes the decision of whether to shut down the line for readjustment. The level of significance is $\alpha = .05$.
    a. State the hypothesis test for this quality control application.
    b. If a sample mean of $\bar{x} = 16.32$ ounces were found, what is the p-value? What action would you recommend?
    c. If a sample mean of $\bar{x} = 15.82$ ounces were found, what is the p-value? What action would you recommend?
    d. Use the critical value approach. What is the rejection rule for the preceding hypothesis testing procedure? Repeat parts (b) and (c). Do you reach the same conclusion?

47. At Western University the historical mean of scholarship examination scores for freshman applications is 900. A historical population standard deviation $\sigma = 180$ is assumed known. Each year, the assistant dean uses a sample of applications to determine whether the mean examination score for the new freshman applications has changed.
    a. State the hypotheses.
    b. What is the 95% confidence interval estimate of the population mean examination score if a sample of 200 applications provided a sample mean of $\bar{x} = 935$?
    c. Use the confidence interval to conduct a hypothesis test. Using $\alpha = .05$, what is your conclusion?
    d. What is the p-value?

48. *Playbill* is a magazine distributed around the country to people attending musicals and other theatrical productions. The mean annual household income for the population of *Playbill* readers is $119,155 (*Playbill,* January 2006). Assume the standard deviation is

σ = $20,700. A San Francisco civic group has asserted that the mean for theater goers in the Bay Area is higher. A sample of 60 theater attendees in the Bay Area showed a sample mean household income of $126,100.

a. Develop hypotheses that can be used to determine whether the sample data support the conclusion that theater attendees in the Bay Area have a higher mean household income than that for all *Playbill* readers.
b. What is the *p*-value based on the sample of 60 theater attendees in the Bay Area?
c. Use $\alpha = .01$ as the level of significance. What is your conclusion?

49. On Friday, Wall Street traders were anxiously awaiting the federal government's release of numbers on the January increase in nonfarm payrolls. The early consensus estimate among economists was for a growth of 250,000 new jobs (CNBC, February 3, 2006). However, a sample of 20 economists taken Thursday afternoon provided a sample mean of 266,000 with a sample standard deviation of 24,000. Financial analysts often call such a sample mean, based on late-breaking news, the *whisper number*. Treat the "consensus estimate" as the population mean. Conduct a hypothesis test to determine whether the whisper number justifies a conclusion of a statistically significant increase in the consensus estimate of economists. Use $\alpha = .01$ as the level of significance.

WEB file FirstBirth

50. Data released by the National Center for Health Statistics showed that the mean age at which women had their first child was 25.0 in 2006 (*The Wall Street Journal*, February 4, 2009). The reporter, Sue Shellenbarger, noted that this was the first decrease in the average age at which women had their first child in several years. A recent sample of 42 women provided the data in the website file named FirstBirth concerning the age at which these women had their first child. Do the data indicate a change from 2006 in the mean age at which women had their first child? Use $\alpha = .05$.

WEB file WeeklyPay

51. A recent issue of the *AARP Bulletin* reported that the average weekly pay for a woman with a high school diploma was $520 (*AARP Bulletin*, January–February, 2010). Suppose you would like to determine if the average weekly pay for all working women is significantly greater than that for women with a high school diploma. Data providing the weekly pay for a sample of 50 working women are available in the file named WeeklyPay. These data are consistent with the findings reported in the article mentioned above.

a. State the hypotheses that should be used to test whether the mean weekly pay for all women is significantly greater than the mean weekly pay for women with a high school diploma.
b. Use the data in the file named WeeklyPay to compute the sample mean, the test statistic, and the *p*-value.
c. Use $\alpha = .05$. What is your conclusion?
d. Repeat the hypothesis test using the critical value approach.

52. The chamber of commerce of a Florida Gulf Coast community advertises that area residential property is available at a mean cost of $125,000 or less per lot. Suppose a sample of 32 properties provided a sample mean of $130,000 per lot and a sample standard deviation of $12,500. Use a .05 level of significance to test the validity of the advertising claim.

WEB file Gasoline

53. The U.S. Energy Administration reported that the mean price for a gallon of regular gasoline in the United States was $2.357 (U.S. Energy Administration, January 30, 2006). Data for a sample of regular gasoline prices at 50 service stations in the Lower Atlantic states are contained in the data file named Gasoline. Conduct a hypothesis test to determine whether the mean price for a gallon of gasoline in the Lower Atlantic states is different from the national mean. Use $\alpha = .05$ for the level of significance, and state your conclusion.

54. On December 25, 2009, an airline passenger was subdued while attempting to blow up a Northwest Airlines flight headed for Detroit, Michigan. The passenger had smuggled explosives hidden in his underwear past a metal detector at an airport screening facility. As a result, the Transportation Security Administration (TSA) proposed installing full-body

scanners to replace the metal detectors at the nation's largest airports. This proposal resulted in strong objections from privacy advocates, who considered the scanners an invasion of privacy. On January 5–6, 2010, *USA Today* conducted a poll of 542 adults to learn what proportion of airline travelers approved of using full-body scanners (*USA Today,* January 11, 2010). The poll results showed that 455 of the respondents felt that full-body scanners would improve airline security and 423 indicated that they approved of using the devices.

a. Conduct a hypothesis test to determine if the results of the poll justify concluding that over 80% of airline travelers feel that the use of full-body scanners will improve airline security. Use $\alpha = .05$.

b. Suppose the TSA will go forward with the installation and mandatory use of full-body scanners if over 75% of airline travelers approve of using the devices. You have been told to conduct a statistical analysis using the poll results to determine if the TSA should go forward with mandatory use of the full-body scanners. Because this is viewed as a very sensitive decision, use $\alpha = .01$. What is your recommendation?

55. A recent article concerning bullish and bearish sentiment about the stock market reported that 41% of investors responding to an American Institute of Individual Investors (AAII) poll were bullish on the market and 26% were bearish (*USA Today,* January 11, 2010). The article also reported that the long-term average measure of bullishness is .39 or 39%. Suppose the AAII poll used a sample size of 450. Using .39 (the long-term average) as the population proportion of investors that are bullish, conduct a hypothesis test to determine if the current proportion of investors that are bullish is significantly greater than the long-term average proportion.

a. State the appropriate hypotheses for your significance test.

b. Use the sample results to compute the test statistic and the p-value.

c. Using $\alpha = .10$, what is your conclusion?

56. Virtual call centers are staffed by individuals working out of their homes. Most home agents earn $10 to $15 per hour without benefits versus $7 to $9 per hour with benefits at a traditional call center (*BusinessWeek,* January 23, 2006). Regional Airways is considering employing home agents, but only if a level of customer satisfaction greater than 80% can be maintained. A test was conducted with home service agents. In a sample of 300 customers, 252 reported that they were satisfied with service.

a. Develop hypotheses for a test to determine whether the sample data support the conclusion that customer service with home agents meets the Regional Airways criterion.

b. What is your point estimate of the percentage of satisfied customers?

c. What is the p-value provided by the sample data?

d. What is your hypothesis testing conclusion? Use $\alpha = .05$ as the level of significance.

57. During the 2004 election year, new polling results were reported daily. In an IBD/TIPP poll of 910 adults, 503 respondents reported that they were optimistic about the national outlook, and President Bush's leadership index jumped 4.7 points to 55.3 (*Investor's Business Daily,* January 14, 2004).

a. What is the sample proportion of respondents who are optimistic about the national outlook?

b. A campaign manager wants to claim that this poll indicates that the majority of adults are optimistic about the national outlook. Construct a hypothesis test so that rejection of the null hypothesis will permit the conclusion that the proportion optimistic is greater than 50%.

c. Use the polling data to compute the p-value for the hypothesis test in part (b). Explain to the manager what this p-value means about the level of significance of the results.

58. A radio station in Myrtle Beach announced that at least 90% of the hotels and motels would be full for the Memorial Day weekend. The station advised listeners to make reservations in advance if they planned to be in the resort over the weekend. On Saturday night a sample of 58 hotels and motels showed 49 with a no-vacancy sign and 9 with vacancies. What is

your reaction to the radio station's claim after seeing the sample evidence? Use $\alpha = .05$ in making the statistical test. What is the p-value?

59. According to the federal government, 24% of workers covered by their company's health care plan were not required to contribute to the premium (*Statistical Abstract of the United States: 2006*). A recent study found that 81 out of 400 workers sampled were not required to contribute to their company's health care plan.
    a. Develop hypotheses that can be used to test whether the percent of workers not required to contribute to their company's health care plan has declined.
    b. What is a point estimate of the proportion receiving free company-sponsored health care insurance?
    c. Has a statistically significant decline occurred in the proportion of workers receiving free company-sponsored health care insurance? Use $\alpha = .05$.

Case Problem 1 Quality Associates, Inc.

Quality Associates, Inc., a consulting firm, advises its clients about sampling and statistical procedures that can be used to control their manufacturing processes. In one particular application, a client gave Quality Associates a sample of 800 observations taken during a time in which that client's process was operating satisfactorily. The sample standard deviation for these data was .21; hence, with so much data, the population standard deviation was assumed to be .21. Quality Associates then suggested that random samples of size 30 be taken periodically to monitor the process on an ongoing basis. By analyzing the new samples, the client could quickly learn whether the process was operating satisfactorily. When the process was not operating satisfactorily, corrective action could be taken to eliminate the problem. The design specification indicated the mean for the process should be 12. The hypothesis test suggested by Quality Associates follows.

$$H_0: \mu = 12$$
$$H_a: \mu \neq 12$$

Corrective action will be taken any time H_0 is rejected.

The samples below were collected at hourly intervals during the first day of operation of the new statistical process control procedure. These data are available in the data set Quality.

Managerial Report

1. Conduct a hypothesis test for each sample at the .01 level of significance and determine what action, if any, should be taken. Provide the test statistic and p-value for each test.

WEB file Quality

Sample 1	Sample 2	Sample 3	Sample 4
11.55	11.62	11.91	12.02
11.62	11.69	11.36	12.02
11.52	11.59	11.75	12.05
11.75	11.82	11.95	12.18
11.90	11.97	12.14	12.11

11.64	11.71	11.72	12.07
11.80	11.87	11.61	12.05
12.03	12.10	11.85	11.64
11.94	12.01	12.16	12.39
11.92	11.99	11.91	11.65
12.13	12.20	12.12	12.11
12.09	12.16	11.61	11.90
11.93	12.00	12.21	12.22
12.21	12.28	11.56	11.88
12.32	12.39	11.95	12.03
11.93	12.00	12.01	12.35
11.85	11.92	12.06	12.09
11.76	11.83	11.76	11.77
12.16	12.23	11.82	12.20
11.77	11.84	12.12	11.79
12.00	12.07	11.60	12.30
12.04	12.11	11.95	12.27
11.98	12.05	11.96	12.29
12.30	12.37	12.22	12.47
12.18	12.25	11.75	12.03
11.97	12.04	11.96	12.17
12.17	12.24	11.95	11.94
11.85	11.92	11.89	11.97
12.30	12.37	11.88	12.23
12.15	12.22	11.93	12.25

2. Compute the standard deviation for each of the four samples. Does the assumption of .21 for the population standard deviation appear reasonable?
3. Compute limits for the sample mean $\bar{x}$ around $\mu = 12$ such that, as long as a new sample mean is within those limits, the process will be considered to be operating satisfactorily. If $\bar{x}$ exceeds the upper limit or if $\bar{x}$ is below the lower limit, corrective action will be taken. These limits are referred to as upper and lower control limits for quality control purposes.
4. Discuss the implications of changing the level of significance to a larger value. What mistake or error could increase if the level of significance is increased?

Case Problem 2 Ethical Behavior of Business Students at Bayview University

During the global recession of 2008 and 2009, there were many accusations of unethical behavior by Wall Street executives, financial managers, and other corporate officers. At that time, an article appeared that suggested that part of the reason for such unethical business behavior may stem from the fact that cheating has become more prevalent among business students (*Chronicle of Higher Education*, February 10, 2009). The article reported that 56% of business students admitted to cheating at some time during their academic career as compared to 47% of nonbusiness students.

Cheating has been a concern of the dean of the College of Business at Bayview University for several years. Some faculty members in the college believe that cheating is more widespread at Bayview than at other universities, while other faculty members think that cheating is not a major problem in the college. To resolve some of these issues, the dean commissioned a study to assess the current ethical behavior of business students at Bayview. As part of this study, an anonymous exit survey was administered to a sample of 90 business students from this year's graduating class. Responses to the following questions were used to obtain data regarding three types of cheating.

During your time at Bayview, did you ever present work copied off the Internet as your own?

Yes _____ No _____

During your time at Bayview, did you ever copy answers off another student's exam?

Yes _____ No _____

During your time at Bayview, did you ever collaborate with other students on projects that were supposed to be completed individually?

Yes _____ No _____

Any student who answered Yes to one or more of these questions was considered to have been involved in some type of cheating. A portion of the data collected follows. The complete data set is in the file named Bayview.

Bayview

Student	Copied from Internet	Copied on Exam	Collaborated on Individual Project	Gender
1	No	No	No	Female
2	No	No	No	Male
3	Yes	No	Yes	Male
4	Yes	Yes	No	Male
5	No	No	Yes	Male
6	Yes	No	No	Female
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
88	No	No	No	Male
89	No	Yes	Yes	Male
90	No	No	No	Female

Managerial Report

Prepare a report for the dean of the college that summarizes your assessment of the nature of cheating by business students at Bayview University. Be sure to include the following items in your report.

1. Use descriptive statistics to summarize the data and comment on your findings.
2. Develop 95% confidence intervals for the proportion of all students, the proportion of male students, and the proportion of female students who were involved in some type of cheating.

3. Conduct a hypothesis test to determine if the proportion of business students at Bayview University who were involved in some type of cheating is less than that of business students at other institutions as reported by the *Chronicle of Higher Education.*
4. Conduct a hypothesis test to determine if the proportion of business students at Bayview University who were involved in some form of cheating is less than that of nonbusiness students at other institutions as reported by the *Chronicle of Higher Education.*
5. What advice would you give to the dean based upon your analysis of the data?

Appendix Hypothesis Testing with StatTools

In this appendix we show how StatTools can be used to conduct hypothesis tests about a population mean for the σ unknown case

Population Mean: σ Unknown Case

In this case the population standard deviation σ will be estimated by the sample standard deviation s. We use the example discussed in Section 9.4 involving ratings that 60 business travelers gave for Heathrow Airport.

Begin by using the Data Set Manager to create a StatTools data set for these data using the procedure described in the appendix in Chapter 1. The following steps can be used to test the hypothesis H_0: $\mu \leq 7$ against H_a: $\mu > 7$.

Step 1. Click the **StatTools** tab on the Ribbon
Step 2. In the **Analyses** group, click **Statistical Inference**
Step 3. Choose the **Hypothesis Test** option
Step 4. Choose **Mean/Std. Deviation . . .**
Step 5. When the StatTools - Hypothesis Test for Mean/Std. Deviation dialog box appears,
For **Analysis Type,** choose **One-Sample Analysis**
In the **Variables** section, select **Rating**
In the **Hypothesis Tests to Perform** section,
Select the **Mean** option
Enter 7 in the **Null Hypothesis Value** box
Select **Greater Than Null Value (One-Tailed Test)** in the **Alternative Hypothesis Type** box
If selected, remove the check in the **Standard Deviation** box
Click **OK**

The results from the hypothesis test will appear. They include the p-value and the value of the test statistic.

CHAPTER 10

Comparisons Involving Means, Experimental Design, and Analysis of Variance

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STATISTICS in PRACTICE

U.S. FOOD AND DRUG ADMINISTRATION

WASHINGTON, D.C.

It is the responsibility of the U.S. Food and Drug Administration (FDA), through its Center for Drug Evaluation and Research (CDER), to ensure that drugs are safe and effective. But CDER does not do the actual testing of new drugs itself. It is the responsibility of the company seeking to market a new drug to test it and submit evidence that it is safe and effective. CDER statisticians and scientists then review the evidence submitted.

Companies seeking approval of a new drug conduct extensive statistical studies to support their application. The testing process in the pharmaceutical industry usually consists of three stages: (1) preclinical testing, (2) testing for long-term usage and safety, and (3) clinical efficacy testing. At each successive stage, the chance that a drug will pass the rigorous tests decreases; however, the cost of further testing increases dramatically. Industry surveys indicate that on average the research and development for one new drug costs $250 million and takes 12 years. Hence, it is important to eliminate unsuccessful new drugs in the early stages of the testing process, as well as to identify promising ones for further testing.

Statistics plays a major role in pharmaceutical research, where government regulations are stringent and rigorously enforced. In preclinical testing, a two- or three-population statistical study typically is used to determine whether a new drug should continue to be studied in the long-term usage and safety program. The populations may consist of the new drug, a control, and a standard drug. The preclinical testing process begins when a new drug is sent to the pharmacology group for evaluation of efficacy—the capacity of the drug to produce the desired effects. As part of the process, a statistician is asked to design an experiment that can be used to test the new drug. The design must specify the sample size and the statistical methods of analysis. In a two-population study, one sample is used to obtain data on the efficacy of the new drug (population 1) and a second sample is used to obtain data on the efficacy of a standard drug (population 2). Depending on the intended use, the new and standard drugs are tested in such disciplines as neurology, cardiology, and immunology. In most studies, the statistical method involves hypothesis testing for the difference between the means of the new drug population and the standard drug population. If a new drug lacks efficacy or produces undesirable effects in comparison with the standard drug, the new drug is rejected and withdrawn from further testing. Only new drugs that show promising comparisons with the standard drugs are forwarded to the long-term usage and safety testing program.

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Statistical methods are used to test and develop new drugs.

Further data collection and multipopulation studies are conducted in the long-term usage and safety testing program and in the clinical testing programs. The FDA requires that statistical methods be defined prior to such testing to avoid data-related biases. In addition, to avoid human biases, some of the clinical trials are double or triple blind. That is, neither the subject nor the investigator knows what drug is administered to whom. If the new drug meets all requirements in relation to the standard drug, a new drug application (NDA) is filed with the FDA. The application is rigorously scrutinized by statisticians and scientists at the agency.

In this chapter you will learn how to construct interval estimates and make hypothesis tests about means with two or more populations. Techniques will be presented for analyzing independent random samples as well as matched samples.

In Chapters 8 and 9 we showed how to develop interval estimates and conduct hypothesis tests for situations involving a single population mean and a single population proportion. In Sections 10.1–10.3 we continue our discussion of statistical inference by showing how interval estimates and hypothesis tests can be developed for situations involving two populations, when the difference between the two population means is of prime importance. For example, we may want to develop an interval estimate of the difference between the mean starting salary for a population of men and the mean starting salary for a population of women or conduct a hypothesis test to determine whether any difference is present between the two population means.

In Section 10.4 we introduce the basic principles of an experimental study and show how they are used in a completely randomized design. We also provide a conceptual overview of the statistical procedure called analysis of variance (ANOVA). In Section 10.5 we show how ANOVA can be used to test for the equality of k population means using data obtained from a completely randomized design as well as data obtained from an observational study. So, in this sense, ANOVA extends the statistical material in Sections 10.1–10.3 from two population means to three or more population means.

We begin our discussion of statistical inference about two populations by showing how to develop interval estimates and conduct hypothesis tests about the difference between the means of two populations when the standard deviations of the two populations are assumed known.

10.1 Inferences About the Difference Between Two Population Means: σ_1 and σ_2 Known

Letting μ_1 denote the mean of population 1 and μ_2 denote the mean of population 2, we will focus on inferences about the difference between the means: $\mu_1 - \mu_2$. To make an inference about this difference, we select a simple random sample of n_1 elements from population 1 and a second simple random sample of n_2 elements from population 2. The two samples, taken separately and independently, are referred to as **independent simple random samples**. In this section, we assume that information is available such that the two population standard deviations, σ_1 and σ_2, can be assumed known prior to collecting the samples. We refer to this situation as the σ_1 and σ_2 known case. In the following example we show how to compute a margin of error and develop an interval estimate of the difference between the two population means when σ_1 and σ_2 are known.

Interval Estimation of $\mu_1 - \mu_2$

HomeStyle sells furniture at two stores: one is located in the inner city and the other is located in a suburban shopping center. The regional manager noticed that products that sell well in one store do not always sell well in the other. The manager believes this situation may be attributable to differences in customer demographics at the two locations. Customers may differ in age, education, income, and so on. Suppose the manager asks us to investigate the difference between the mean ages of the customers who shop at the two stores.

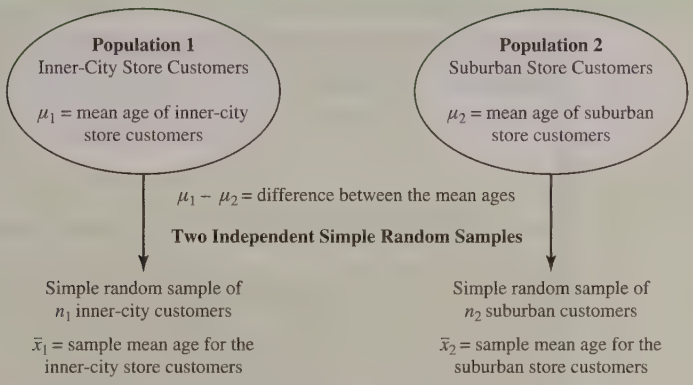
Let us define population 1 as all customers who shop at the inner-city store and population 2 as all customers who shop at the suburban store.

μ_1 = mean of population 1 (i.e., the mean age of all customers who shop at the inner-city store)

μ_2 = mean of population 2 (i.e., the mean age of all customers who shop at the suburban store)

The difference between the two population means is $\mu_1 - \mu_2$.

FIGURE 10.1 ESTIMATING THE DIFFERENCE BETWEEN TWO POPULATION MEANS


Population 1
Inner-City Store Customers
μ_1 = mean age of inner-city store customers
Population 2
Suburban Store Customers
μ_2 = mean age of suburban store customers
$\mu_1 - \mu_2$ = difference between the mean ages
Two Independent Simple Random Samples
Simple random sample of n_1 inner-city customers
$\bar{x}_1$ = sample mean age for the inner-city store customers
Simple random sample of n_2 suburban customers
$\bar{x}_2$ = sample mean age for the suburban store customers
$\bar{x}_1 - \bar{x}_2$ = point estimator of $\mu_1 - \mu_2$

To estimate $\mu_1 - \mu_2$, we will select a simple random sample of n_1 customers from population 1 and a simple random sample of n_2 customers from population 2. We then compute the two sample means.

$\bar{x}_1$ = sample mean age for the simple random sample of n_1 inner-city customers
$\bar{x}_2$ = sample mean age for the simple random sample of n_2 suburban customers

The point estimator of the difference between the two population means is the difference between the two sample means.

POINT ESTIMATOR OF THE DIFFERENCE BETWEEN TWO POPULATION MEANS

$$\bar{x}_1 - \bar{x}_2 \tag{10.1}$$

Figure 10.1 provides an overview of the process used to estimate the difference between two population means based on two independent simple random samples.

The standard error of $\bar{x}_1 - \bar{x}_2$ is the standard deviation of the sampling distribution of $\bar{x}_1 - \bar{x}_2$.

As with other point estimators, the point estimator $\bar{x}_1 - \bar{x}_2$ has a standard error that describes the variation in the sampling distribution of the estimator. With two independent simple random samples, the standard error of $\bar{x}_1 - \bar{x}_2$ is as follows:

STANDARD ERROR OF $\bar{x}_1 - \bar{x}_2$

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \tag{10.2}$$

If both populations have a normal distribution, or if the sample sizes are large enough that the central limit theorem enables us to conclude that the sampling distributions of $\bar{x}_1$ and $\bar{x}_2$ can be approximated by a normal distribution, the sampling distribution of $\bar{x}_1 - \bar{x}_2$ will have a normal distribution with mean given by $\mu_1 - \mu_2$.

As we showed in Chapter 8, an interval estimate is given by a point estimate ± a margin of error. When estimating the difference between two population means, the interval estimate takes the following form:

$$\bar{x}_1 - \bar{x}_2 \pm \text{Margin of error}$$

With the sampling distribution of $\bar{x}_1 - \bar{x}_2$ having a normal distribution, we can write the margin of error as follows:

The margin of error is given by multiplying the standard error by $z_{\alpha/2}$.

$$\text{Margin of error} = z_{\alpha/2}\sigma_{\bar{x}_1-\bar{x}_2} = z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \qquad \textbf{(10.3)}$$

Thus, the interval estimate of the difference between two population means is as follows:

INTERVAL ESTIMATE OF THE DIFFERENCE BETWEEN TWO POPULATION MEANS: σ_1 AND σ_2 KNOWN

$$\bar{x}_1 - \bar{x}_2 \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \qquad \textbf{(10.4)}$$

where $1 - \alpha$ is the confidence coefficient.

Let us return to the HomeStyle example. Based on data from previous customer demographic studies, the two population standard deviations are assumed known with $\sigma_1 = 9$ years and $\sigma_2 = 10$ years. The data collected from the two independent simple random samples of HomeStyle customers provided the following results.

WEB file
HomeStyle

	Inner-City Store	Suburban Store
Sample Size	$n_1 = 36$	$n_2 = 49$
Sample Mean	$\bar{x}_1 = 40$ years	$\bar{x}_2 = 35$ years

Using expression (10.1), we find that the point estimate of the difference between the mean ages of the two populations is $\bar{x}_1 - \bar{x}_2 = 40 - 35 = 5$ years. Thus, we estimate that the customers at the inner-city store have a mean age 5 years greater than the mean age of the suburban store customers. We can now use expression (10.4) to compute the margin of error and provide the interval estimate of $\mu_1 - \mu_2$. Using 95% confidence and $z_{\alpha/2} = z_{.025} = 1.96$, we have

$$\bar{x}_1 - \bar{x}_2 \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$40 - 35 \pm 1.96\sqrt{\frac{9^2}{36} + \frac{10^2}{49}}$$

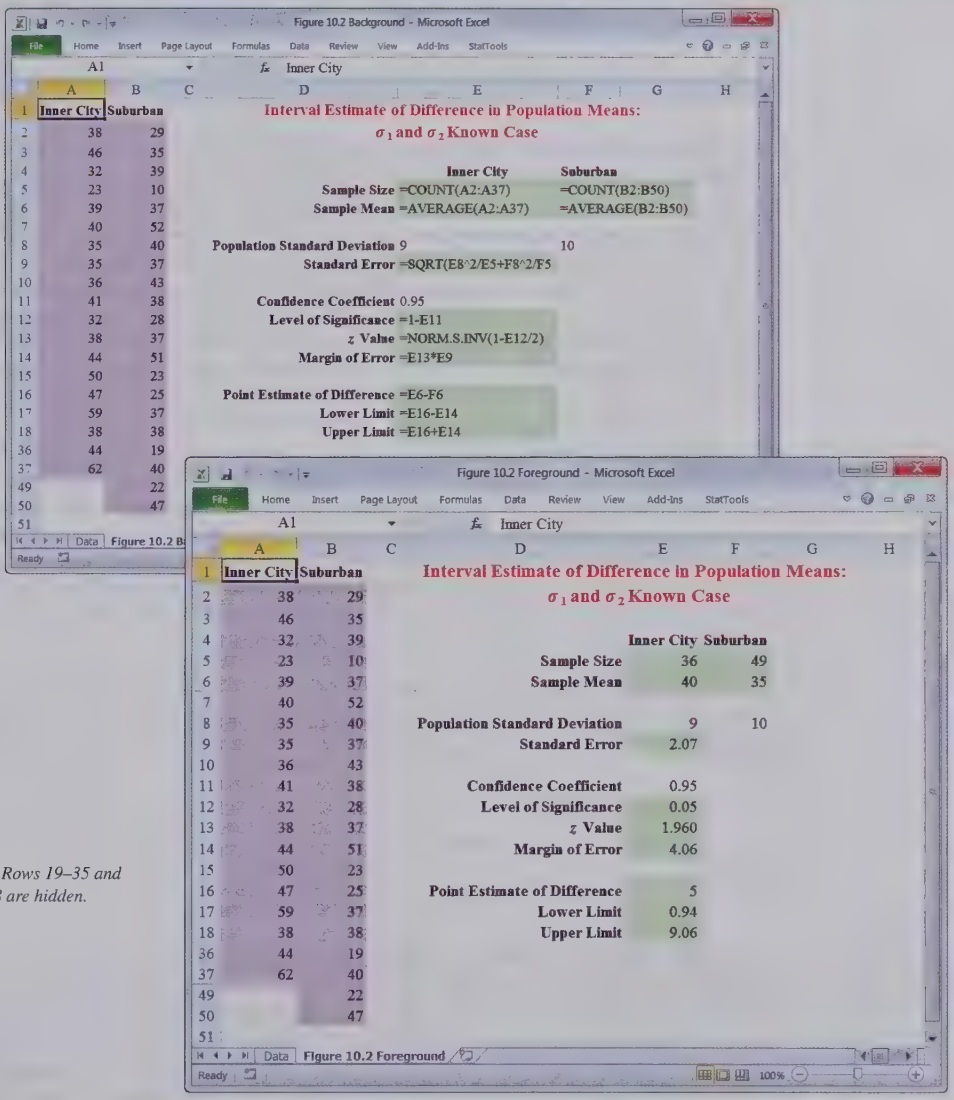
$$5 \pm 4.06$$

Thus, the margin of error is 4.06 years and the 95% confidence interval estimate of the difference between the two population means is $5 - 4.06 = .94$ years to $5 + 4.06 = 9.06$ years.

Using Excel to Construct a Confidence Interval

Excel's data analysis tools do not provide a procedure for developing interval estimates involving two population means. However, we can develop an Excel worksheet that can be used as a template to construct interval estimates. We will illustrate by constructing an

FIGURE 10.2 EXCEL WORKSHEET: CONSTRUCTING A 95% CONFIDENCE INTERVAL FOR HOMESTYLE FURNITURE STORES



Note: Rows 19–35 and 38–48 are hidden.

interval estimate of the difference between the population means in the HomeStyle Furniture Stores study. Refer to Figure 10.2 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet is in the foreground.

Enter Data: Column A contains the age data for the simple random sample of 36 inner-city customers, and column B contains the age data for the simple random sample of 49 suburban customers.

Enter Functions and Formulas: The descriptive statistics needed are provided in cells E5:F6. The known population standard deviations are entered into cells E8 and F8. Using the two population standard deviations and the sample sizes, the standard error of the point estimator, $\bar{x}_1 - \bar{x}_2$, is computed using equation (10.2) by entering the following formula into cell E9:

=SQRT(E8^2/E5+F8^2/F5)

Cells E11:E14 are used to compute the appropriate z value and the margin of error. The confidence coefficient is entered into cell E11 (.95) and the corresponding level of significance ($\alpha = 1 -$ confidence coefficient) is computed in cell E12. In cell E13, we used the NORM.S.INV function to compute the z value needed for the interval estimate. The margin of error is computed in cell E14 by multiplying the z value by the standard error.

In cell E16 the difference in the sample means is used to compute the point estimate of the difference in the two population means. The lower limit of the confidence interval is computed in cell E17 (.94) and the upper limit is computed in cell E18 (9.06); thus, the 95% confidence interval estimate of the difference in the two population means is .94 to 9.06.

A template for other problems This worksheet can be used as a template for developing interval estimates of the difference in population means when the population standard deviations are assumed known. For another problem of this type, we must first enter the new problem data in columns A and B. The data ranges in cells E5:F6 must be modified in order to compute the sample means and sample sizes for the new data. Also, the assumed known population standard deviations must be entered into cells E8 and F8. After doing so, the point estimate and a 95% confidence interval will be displayed in cells E16:E18. If a confidence interval with a different confidence coefficient is desired, we simply change the value in cell E11.

We can further simplify the use of Figure 10.2 as a template for other problems by eliminating the need to enter new data ranges in cells E5:F6. We rewrite the cell formulas as follows:

Cell E5: =COUNT(A:A)
Cell F5: =COUNT(B:B)
Cell E6: =AVERAGE(A:A)
Cell F6: =AVERAGE(B:B)

The HomeStyle webfile includes a worksheet entitled Template that uses the A:A and B:B methods for entering the data ranges.

Using the A:A method of specifying data ranges in cells E5 and E6, Excel's COUNT function will count the number of numeric values in column A and Excel's AVERAGE function will compute the average of the numeric values in column A. Similarly, using the B:B method of specifying data ranges in cells F5 and F6, Excel's COUNT function will count the number of numeric values in column B and Excel's AVERAGE function will compute the average of the numeric values in column B. Thus, to solve a new problem it is only necessary to enter the new data into columns A and B and enter the known population standard deviations in cells E8 and F8.

This worksheet can also be used as a template for text exercises in which the sample sizes, sample means, and population standard deviations are given. In this type of situation, no change in the data is necessary. We simply replace the values in cells E5:F6 and E8:F8 with the given values of the sample sizes, sample means, and population standard deviations. If something other than a 95% confidence interval is desired, the confidence coefficient in cell E11 must also be changed.

Hypothesis Tests About $\mu_1 - \mu_2$

Let us consider hypothesis tests about the difference between two population means. Using D_0 to denote the hypothesized difference between μ_1 and μ_2, the three forms for a hypothesis test are as follows:

$$H_0: \mu_1 - \mu_2 \geq D_0 \qquad H_0: \mu_1 - \mu_2 \leq D_0 \qquad H_0: \mu_1 - \mu_2 = D_0$$
$$H_a: \mu_1 - \mu_2 < D_0 \qquad H_a: \mu_1 - \mu_2 > D_0 \qquad H_a: \mu_1 - \mu_2 \neq D_0$$

In most applications, $D_0 = 0$. Using the two-tailed test as an example, when $D_0 = 0$ the null hypothesis is $H_0: \mu_1 - \mu_2 = 0$. In this case, the null hypothesis is that μ_1 and μ_2 are equal. Rejection of H_0 leads to the conclusion that $H_a: \mu_1 - \mu_2 \neq 0$ is true; that is, μ_1 and μ_2 are not equal.

The steps for conducting hypothesis tests presented in Chapter 9 are applicable here. We must choose a level of significance, compute the value of the test statistic, and find the *p*-value to determine whether the null hypothesis should be rejected. With two independent simple random samples, we showed that the point estimator $\bar{x}_1 - \bar{x}_2$ has a standard error $\sigma_{\bar{x}_1 - \bar{x}_2}$ given by expression (10.2) and the sampling distribution of $\bar{x}_1 - \bar{x}_2$ can be described by a normal distribution. In this case, the test statistic for the difference between two population means when σ_1 and σ_2 are known is as follows.

TEST STATISTIC FOR HYPOTHESIS TESTS ABOUT $\mu_1 - \mu_2$: σ_1 AND σ_2 KNOWN

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \qquad \textbf{(10.5)}$$

Let us demonstrate the use of this test statistic in the following hypothesis testing example.

As part of a study to evaluate differences in education quality between two training centers, a standardized examination is given to individuals who are trained at the centers. The difference between the mean examination scores is used to assess quality differences between the centers. The population means for the two centers are as follows.

μ_1 = the mean examination score for the population of individuals trained at center A

μ_2 = the mean examination score for the population of individuals trained at center B

We begin with the tentative assumption that no difference exists between the training quality provided at the two centers. Hence, in terms of the mean examination scores, the null hypothesis is that $\mu_1 - \mu_2 = 0$. If sample evidence leads to the rejection of this hypothesis, we will conclude that the mean examination scores differ for the two populations. This conclusion indicates a quality differential between the two centers and suggests that a follow-up study investigating the reason for the differential may be warranted. The null and alternative hypotheses for this two-tailed test are written as follows.

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_a: \mu_1 - \mu_2 \neq 0$$

WEB file
ExamScores

The standardized examination given previously in a variety of settings always resulted in an examination score standard deviation near 10 points. Thus, we will use this information to assume that the population standard deviations are known with $\sigma_1 = 10$ and $\sigma_2 = 10$. An $\alpha = .05$ level of significance is specified for the study.

Independent simple random samples of $n_1 = 30$ individuals from training center A and $n_2 = 40$ individuals from training center B are taken. The respective sample means are $\bar{x}_1 = 82$ and $\bar{x}_2 = 78$. Do these data suggest a significant difference between the population means at the two training centers? To help answer this question, we compute the test statistic using equation (10.5).

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(82 - 78) - 0}{\sqrt{\frac{10^2}{30} + \frac{10^2}{40}}} = 1.66$$

Next let us compute the p-value for this two-tailed test. Because the test statistic z is in the upper tail, we first compute the upper tail area at $z = 1.66$. Using the standard normal distribution table, the area to the left of $z = 1.66$ is .9515. Thus, the area in the upper tail of the distribution is $1.0000 - .9515 = .0485$. Because this test is a two-tailed test, we must double the tail area: p-value $= 2(.0485) = .0970$. Following the usual rule to reject H_0 if p-value $\leq \alpha$, we see that the p-value of .0970 does not allow us to reject H_0 at the .05 level of significance. The sample results do not provide sufficient evidence to conclude that the training centers differ in quality.

In this chapter we will use the p-value approach to hypothesis testing as described in Chapter 9. However, if you prefer, the test statistic and the critical value rejection rule may be used. With $\alpha = .05$ and $z_{\alpha/2} = z_{.025} = 1.96$, the rejection rule employing the critical value approach would be reject H_0 if $z \leq -1.96$ or if $z \geq 1.96$. With $z = 1.66$, we reach the same do not reject H_0 conclusion.

In the preceding example, we demonstrated a two-tailed hypothesis test about the difference between two population means. Lower tail and upper tail tests can also be considered. These tests use the same test statistic as given in equation (10.5). The procedure for computing the p-value and the rejection rules for these one-tailed tests are the same as those presented in Chapter 9.

Using Excel to Conduct a Hypothesis Test

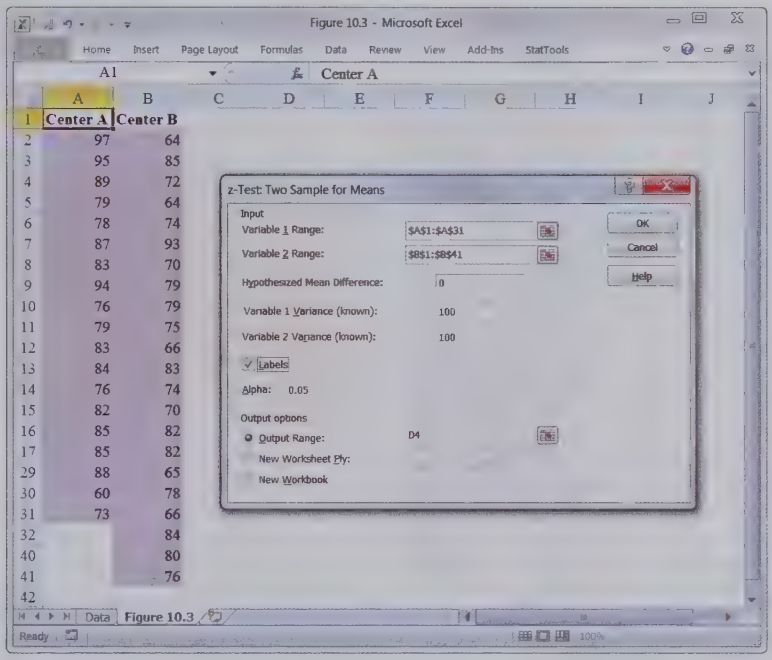
The Excel tool used to conduct the hypothesis test to determine whether there is a significant difference in population means when σ_1 and σ_2 are assumed known is called *z-Test: Two Sample for Means*. We illustrate using the sample data for exam scores at center A and at center B. With an assumed known standard deviation of 10 points at each center, the known variance of exam scores for each of the two populations is equal to $10^2 = 100$. Refer to the Excel worksheets shown in Figure 10.3 and Figure 10.4 as we describe the tasks involved.

Enter Data: Column A in Figure 10.3 contains the examination score data for the simple random sample of 30 individuals trained at center A, and column B contains the examination score data for the simple random sample of 40 individuals trained at center B.

Apply Tools: The following steps will provide the information needed to conduct the hypothesis test to see whether there is a significant difference in test scores at the two centers.

Step 1. Click the **Data** tab on the Ribbon
Step 2. In the **Analysis** group, click **Data Analysis**
Step 3. Choose **z-Test: Two Sample for Means** from the list of Analysis Tools
Step 4. When the z-Test: Two Sample for Means dialog box appears (Figure 10.3),
Enter A1:A31 in the **Variable 1 Range** box
Enter B1:B41 in the **Variable 2 Range** box
Enter 0 in the **Hypothesized Mean Difference** box
Enter 100 in the **Variable 1 Variance (known)** box

FIGURE 10.3 DIALOG BOX FOR EXCEL'S z-TEST: TWO SAMPLE FOR MEANS TOOL



Note: Rows 18–28 and 33–39 are hidden.

Enter 100 in the **Variable 2 Variance (known)** box
Select **Labels**
Enter .05 in the **Alpha** box
Select **Output Range** and enter D4 in the box
Click **OK**

The value of the test statistic shown here (1.6562) and the p-value (.0977) differ slightly from those shown previously, because we rounded the test statistic to two places (1.66) in the text.

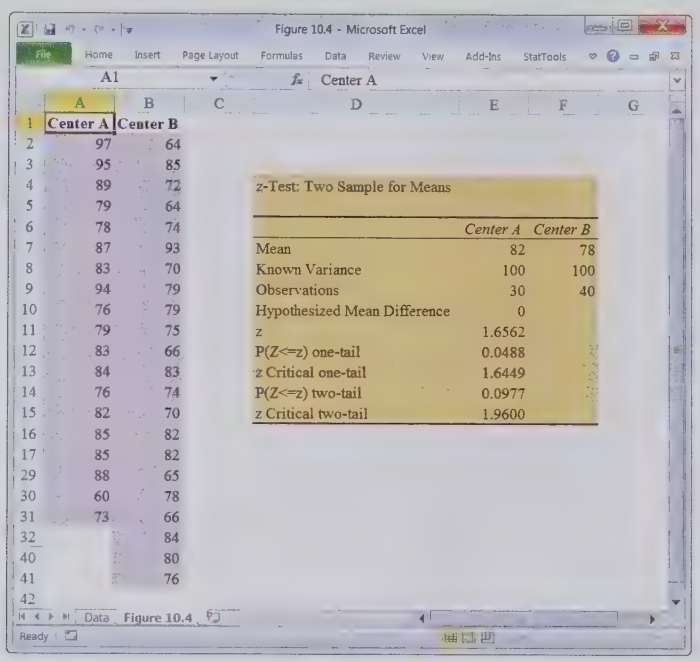
The results are shown in Figure 10.4. Descriptive statistics for the two samples are shown in cells E7:F9. The value of the test statistic, 1.6562, is shown in cell E11. The *p*-value for the test, labeled P(Z<=z) two-tail, is shown in cell E14. Because the *p*-value, .0977, is greater than the level of significance, $\alpha = .05$, we cannot conclude that the means for the two populations are different.

The z-Test: Two Sample for Means tool can also be used to conduct one-tailed hypothesis tests. The only change required to make the hypothesis testing decision is that we need to use the *p*-value for a one-tailed test, labeled P(Z<=z) one-tail (see cell E12).

Practical Advice

In most applications of the interval estimation and hypothesis testing procedures presented in this section, random samples with $n_1 \geq 30$ and $n_2 \geq 30$ are adequate. In cases where either or both sample sizes are less than 30, the distributions of the populations become important considerations. In general, with smaller sample sizes, it is more important for the analyst to be satisfied that it is reasonable to assume that the distributions of the two populations are at least approximately normal.

FIGURE 10.4 EXCEL RESULTS FOR THE HYPOTHESIS TEST ABOUT EQUALITY OF EXAM SCORES AT TWO TRAINING CENTERS



Exercises

Methods

SELF test

1. The following results are for two independent random samples taken from two populations.

Sample 1	Sample 2
$n_1 = 50$	$n_2 = 35$
$\bar{x}_1 = 13.6$	$\bar{x}_2 = 11.6$
$\sigma_1 = 2.2$	$\sigma_2 = 3.0$

   a. What is the point estimate of the difference between the two population means?
   b. Provide a 90% confidence interval for the difference between the two population means.
   c. Provide a 95% confidence interval for the difference between the two population means.

SELF test

2. Consider the following hypothesis test.

$$H_0: \mu_1 - \mu_2 \leq 0$$
$$H_a: \mu_1 - \mu_2 > 0$$

The following results are for two independent samples taken from the two populations.

Sample 1	Sample 2
$n_1 = 40$	$n_2 = 50$
$\bar{x}_1 = 25.2$	$\bar{x}_2 = 22.8$
$\sigma_1 = 5.2$	$\sigma_2 = 6.0$

a. What is the value of the test statistic?
b. What is the p-value?
c. With $\alpha = .05$, what is your hypothesis testing conclusion?

3. Consider the following hypothesis test.

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_a: \mu_1 - \mu_2 \neq 0$$

The following results are for two independent samples taken from the two populations.

Sample 1	Sample 2
$n_1 = 80$	$n_2 = 70$
$\bar{x}_1 = 104$	$\bar{x}_2 = 106$
$\sigma_1 = 8.4$	$\sigma_2 = 7.6$

a. What is the value of the test statistic?
b. What is the p-value?
c. With $\alpha = .05$, what is your hypothesis testing conclusion?

Applications

SELF test

4. *Condé Nast Traveler* conducts an annual survey in which readers rate their favorite cruise ship. All ships are rated on a 100-point scale, with higher values indicating better service. A sample of 37 ships that carry fewer than 500 passengers resulted in an average rating of 85.36, and a sample of 44 ships that carry 500 or more passengers provided an average rating of 81.40 (*Condé Nast Traveler,* February 2008). Assume that the population standard deviation is 4.55 for ships that carry fewer than 500 passengers and 3.97 for ships that carry 500 or more passengers.
   a. What is the point estimate of the difference between the population mean rating for ships that carry fewer than 500 passengers and the population mean rating for ships that carry 500 or more passengers?
   b. At 95% confidence, what is the margin of error?
   c. What is a 95% confidence interval estimate of the difference between the population mean ratings for the two sizes of ships?

5. The average expenditure on Valentine's Day was expected to be $100.89 (*USA Today,* February 13, 2006). Do male and female consumers differ in the amounts they spend? The average expenditure in a sample survey of 40 male consumers was $135.67, and the average expenditure in a sample survey of 30 female consumers was $68.64. Based on past surveys, the standard deviation for male consumers is assumed to be $35, and the standard deviation for female consumers is assumed to be $20.
   a. What is the point estimate of the difference between the population mean expenditure for males and the population mean expenditure for females?
   b. At 99% confidence, what is the margin of error?
   c. Develop a 99% confidence interval for the difference between the two population means.

WEB file
Hotel

6. Suppose that you are responsible for making arrangements for a business convention. Because of budget cuts due to the recent recession, you have been charged with choosing a city for the convention that has the least expensive hotel rooms. You have narrowed your choices to Atlanta and Houston. The file named Hotel contains samples of prices for rooms in Atlanta and Houston that are consistent with the results reported by Smith Travel Research (*SmartMoney,* March 2009). Because considerable historical data on the prices of rooms in both cities are available, the population standard deviations for the prices can be assumed to be $20 in Atlanta and $25 in Houston. Based on the sample data, can you conclude that the mean price of a hotel room in Atlanta is lower than one in Houston?

7. During the 2003 season, Major League Baseball took steps to speed up the play of baseball games in order to maintain fan interest (*CNN Headline News,* September 30, 2003). The following results come from a sample of 60 games played during the summer of 2002 and a sample of 50 games played during the summer of 2003. The sample mean shows the mean duration of the games included in each sample.

2002 Season	2003 Season
$n_1 = 60$	$n_2 = 50$
$\bar{x}_1 = 2$ hours, 52 minutes	$\bar{x}_2 = 2$ hours, 46 minutes

   a. A research hypothesis was that the steps taken during the 2003 season would reduce the population mean duration of baseball games. Formulate the null and alternative hypotheses.
   b. What is the point estimate of the reduction in the mean duration of games during the 2003 season?
   c. Historical data indicate a population standard deviation of 12 minutes is a reasonable assumption for both years. Conduct the hypothesis test and report the *p*-value. At a .05 level of significance, what is your conclusion?
   d. Provide a 95% confidence interval estimate of the reduction in the mean duration of games during the 2003 season.
   e. What was the percentage reduction in the mean time of baseball games during the 2003 season? Should management be pleased with the results of the statistical analysis? Discuss. Should the length of baseball games continue to be an issue in future years? Explain.

8. Will improving customer service result in higher stock prices for the companies providing the better service? "When a company's satisfaction score has improved over the prior year's results and is above the national average (currently 75.7), studies show its shares have a good chance of outperforming the broad stock market in the long run" (*BusinessWeek,* March 2, 2009). The following satisfaction scores of three companies for the 4th quarters of 2007 and 2008 were obtained from the American Customer Satisfaction Index. Assume that the scores are based on a poll of 60 customers from each company. Because the polling has been done for several years, the standard deviation can be assumed to equal 6 points in each case.

Company	2007 Score	2008 Score
Rite Aid	73	76
Expedia	75	77
JC Penney	77	78

   a. For Rite Aid, is the increase in the satisfaction score from 2007 to 2008 statistically significant? Use $\alpha = .05$. What can you conclude?
   b. Can you conclude that the 2008 score for Rite Aid is above the national average of 75.7? Use $\alpha = .05$.
   c. For Expedia, is the increase from 2007 to 2008 statistically significant? Use $\alpha = .05$.
   d. When conducting a hypothesis test with the values given for the standard deviation, sample size, and α, how large must the increase from 2007 to 2008 be for it to be statistically significant?
   e. Use the result of part (d) to state whether the increase for JC Penney from 2007 to 2008 is statistically significant.

10.2 Inferences About the Difference Between Two Population Means: σ_1 and σ_2 Unknown

In this section we extend the discussion of inferences about the difference between two population means to the case when the two population standard deviations, σ_1 and σ_2, are unknown. In this case, we will use the sample standard deviations, s_1 and s_2, to estimate the unknown population standard deviations. When we use the sample standard deviations, the interval estimation and hypothesis testing procedures will be based on the t distribution rather than the standard normal distribution.

Interval Estimation of $\mu_1 - \mu_2$

In the following example we show how to compute a margin of error and develop an interval estimate of the difference between two population means when σ_1 and σ_2 are unknown. Clearwater National Bank is conducting a study designed to identify differences between checking account practices by customers at two of its branch banks. A simple random sample of 28 checking accounts is selected from the Cherry Grove Branch and an independent simple random sample of 22 checking accounts is selected from the Beechmont Branch. The current checking account balance is recorded for each of the checking accounts. A summary of the account balances follows:

WEB file
CheckAcct

	Cherry Grove	Beechmont
Sample Size	$n_1 = 28$	$n_2 = 22$
Sample Mean	$\bar{x}_1 = \$1025$	$\bar{x}_2 = \$910$
Sample Standard Deviation	$s_1 = \$150$	$s_2 = \$125$

Clearwater National Bank would like to estimate the difference between the mean checking account balance maintained by the population of Cherry Grove customers and the population of Beechmont customers. Let us develop the margin of error and an interval estimate of the difference between these two population means.

In Section 10.1, we provided the following interval estimate for the case when the population standard deviations, σ_1 and σ_2, are known.

$$\bar{x}_1 - \bar{x}_2 \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

When σ_1 and σ_2 are estimated by s_1 and s_2, the t distribution is used to make inferences about the difference between two population means.

With σ_1 and σ_2 unknown, we will use the sample standard deviations s_1 and s_2 to estimate σ_1 and σ_2 and replace $z_{\alpha/2}$ with $t_{\alpha/2}$. As a result, the interval estimate of the difference between two population means is given by the following expression:

INTERVAL ESTIMATE OF THE DIFFERENCE BETWEEN TWO POPULATION MEANS: σ_1 AND σ_2 UNKNOWN

$$\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \qquad (10.6)$$

where $1 - \alpha$ is the confidence coefficient.

In this expression, the use of the t distribution is an approximation, but it provides excellent results and is relatively easy to use. The only difficulty that we encounter in using

expression (10.6) is determining the appropriate degrees of freedom for $t_{\alpha/2}$. The formula used is as follows:

DEGREES OF FREEDOM: t DISTRIBUTION WITH TWO INDEPENDENT RANDOM SAMPLES

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2} \qquad (10.7)$$

The Excel worksheet in Figure 10.5 makes this computation easy.

Let us return to the Clearwater National Bank example and show how to use expression (10.6) to provide a 95% confidence interval estimate of the difference between the population mean checking account balances at the two branch banks. The sample data show $n_1 = 28$, $\bar{x}_1 = \$1025$, and $s_1 = \$150$ for the Cherry Grove Branch, and $n_2 = 22$, $\bar{x}_2 = \$910$, and $s_2 = \$125$ for the Beechmont Branch. The calculation for degrees of freedom for $t_{\alpha/2}$ is as follows:

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2} = \frac{\left(\frac{150^2}{28} + \frac{125^2}{22}\right)^2}{\frac{1}{28 - 1}\left(\frac{150^2}{28}\right)^2 + \frac{1}{22 - 1}\left(\frac{125^2}{22}\right)^2} = 47.8$$

We round the noninteger degrees of freedom *down* to 47 to provide a larger t value and a more conservative (wider) interval estimate. Using the t distribution table with 47 degrees of freedom, we find $t_{.025} = 2.012$. Using expression (10.6), we develop the 95% confidence interval estimate of the difference between the two population means as follows.

$$\bar{x}_1 - \bar{x}_2 \pm t_{.025}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$1025 - 910 \pm 2.012\sqrt{\frac{150^2}{28} + \frac{125^2}{22}}$$

$$115 \pm 78$$

The point estimate of the difference between the population mean checking account balances at the two branches is \$115 and the margin of error is \$78. Thus the 95% confidence interval estimate of the difference between the two population means is $115 - 78 = \$37$ to $115 + 78 = \$193$.

This suggestion should help if you are using equation (10.7) to calculate the degrees of freedom by hand.

The computation of the degrees of freedom in equation (10.7) is cumbersome if you are doing the calculation by hand. However, note that the expressions s_1^2/n_1 and s_2^2/n_2 appear in both expression (10.6) and equation (10.7). These values only need to be computed once in order to evaluate both (10.6) and (10.7).

Using Excel to Construct a Confidence Interval

Excel's data analysis tools do not provide a procedure for developing interval estimates involving two population means. However, we can develop an Excel worksheet that can be used as a template to construct interval estimates. We will illustrate by constructing an interval estimate of the difference between the population means in the Clearwater National Bank study. Refer to Figure 10.5 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet is in the foreground.

FIGURE 10.5 EXCEL WORKSHEET: CONSTRUCTING A 95% CONFIDENCE INTERVAL FOR CLEARWATER NATIONAL BANK

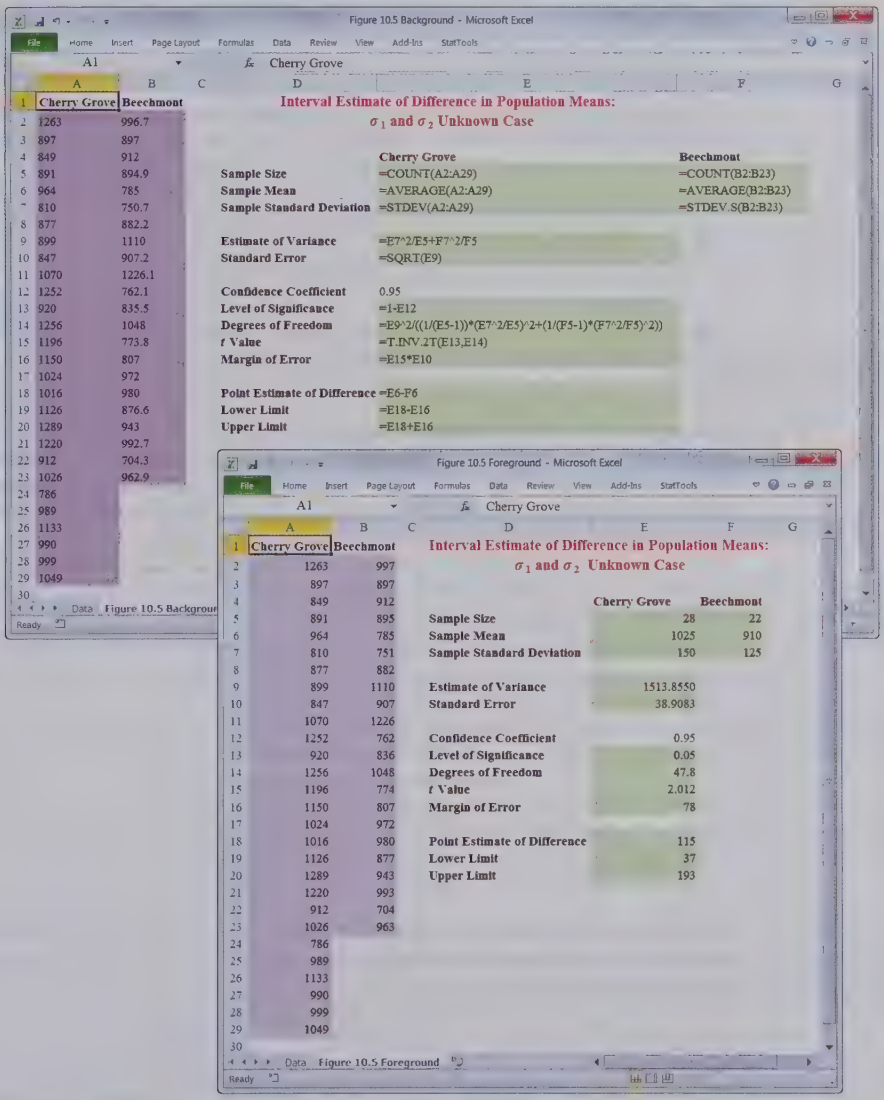


Figure 10.5 Background - Microsoft Excel
Cherry Grove
Interval Estimate of Difference in Population Means:
σ1 and σ2 Unknown Case
Cherry Grove
Beechmont
Sample Size =COUNT(A2:A29) =COUNT(B2:B23)
Sample Mean =AVERAGE(A2:A29) =AVERAGE(B2:B23)
Sample Standard Deviation =STDEV(A2:A29) =STDEV.S(B2:B23)
Estimate of Variance =E7^2/E5+F7^2/F5
Standard Error =SQRT(E9)
Confidence Coefficient 0.95
Level of Significance =1-E12
Degrees of Freedom =E9^2/((1/(E5-1))*(E7^2/E5)^2+(1/(F5-1)*(F7^2/F5)^2))
t Value =T.INV.2T(E13,E14)
Margin of Error =E15*E10
Point Estimate of Difference =E6-F6
Lower Limit =E18-E16
Upper Limit =E18+E16
Figure 10.5 Foreground - Microsoft Excel
Interval Estimate of Difference in Population Means:
σ1 and σ2 Unknown Case
Sample Size 28 22
Sample Mean 1025 910
Sample Standard Deviation 150 125
Estimate of Variance 1513.8550
Standard Error 38.9083
Confidence Coefficient 0.95
Level of Significance 0.05
Degrees of Freedom 47.8
t Value 2.012
Margin of Error 78
Point Estimate of Difference 115
Lower Limit 37
Upper Limit 193

Enter Data: Column A contains the account balances for the simple random sample of 28 customers at the Cherry Grove Branch, and column B contains the account balances for the simple random sample of 22 customers at the Beechmont Branch.

Enter Functions and Formulas: The descriptive statistics needed are provided in cells E5:F7. Using the two sample standard deviations and the sample sizes, an estimate of the

variance of the point estimator $\bar{x}_1 - \bar{x}_2$ is computed by entering the following formula into cell E9:

=E7^2/E5+F7^2/F5

An estimate of the standard error is then computed in cell E10 by taking the square root of the variance.

Cells E12:E16 are used to compute the appropriate t value and the margin of error. The confidence coefficient is entered into cell E12 (.95) and the corresponding level of significance is computed in cell E13 ($\alpha = .05$). In cell E14, we used formula (10.7) to compute the degrees of freedom (47.8). In cell E15, we used the T.INV.2T function to compute the t value needed for the interval estimate. The margin of error is computed in cell E16 by multiplying the t value by the standard error.

In cell E18 the difference in the sample means is used to compute the point estimate of the difference in the two population means (115). The lower limit of the confidence interval is computed in cell E19 (37) and the upper limit is computed in cell E20 (193); thus, the 95% confidence interval estimate of the difference in the two population means is 37 to 193.

A template for other problems This worksheet can be used as a template for developing interval estimates of the difference in population means when the population standard deviations are unknown. For another problem of this type, we must first enter the new problem data in columns A and B. The data ranges in cells E5:F7 must be modified in order to compute the sample means, sample sizes, and sample standard deviations for the new data. After doing so the point estimate and a 95% confidence interval will be displayed in cells E18:E20. If a confidence interval with a different confidence coefficient is desired, we simply change the value in cell E12.

We can further simplify the use of Figure 10.5 as a template for other problems by eliminating the need to enter new data ranges in cells E5:F7. We rewrite the cell formulas as follows:

Cell E5: =COUNT(A:A)
Cell F5: =COUNT(B:B)
Cell E6: =AVERAGE(A:A)
Cell F6: =AVERAGE(B:B)
Cell E7: =STDEV.S(A:A)
Cell F7: =STDEV.S(B:B)

The CheckAcct data set includes a worksheet entitled Template that uses the A:A and B:B methods for entering the data ranges.

Using the A:A method of specifying data ranges in cells E5:E7, Excel's COUNT function will count the number of numeric values in column A, Excel's AVERAGE function will compute the average of the numeric values in column A, and Excel's STDEV function will compute the standard deviation of the numeric values in column A. Similarly, using the B:B method of specifying data ranges in cells F5:F7, Excel's COUNT function will count the number of numeric values in column B, Excel's AVERAGE function will compute the average of the numeric values in column B, and Excel's STDEV function will compute the standard deviation of the numeric values in column B. Thus, to solve a new problem it is only necessary to enter the new data into columns A and B.

This worksheet can also be used as a template for text exercises in which the sample sizes, sample means, and sample standard deviations are given. In this type of situation, no change in the data is necessary. We simply replace the values in cells E5:F7 with the given values of the sample sizes, sample means, and sample standard deviations. If something other than a 95% confidence interval is desired, the confidence coefficient in cell E12 must also be changed.

Hypothesis Tests About $\mu_1 - \mu_2$

Let us now consider hypothesis tests about the difference between the means of two populations when the population standard deviations σ_1 and σ_2 are unknown. Letting D_0 denote the hypothesized difference between μ_1 and μ_2, Section 10.1 showed that the test statistic used for the case where σ_1 and σ_2 are known is as follows.

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

The test statistic, z, follows the standard normal distribution.

When σ_1 and σ_2 are unknown, we use s_1 as an estimator of σ_1 and s_2 as an estimator of σ_2. Substituting these sample standard deviations for σ_1 and σ_2 provides the following test statistic when σ_1 and σ_2 are unknown.

TEST STATISTIC FOR HYPOTHESIS TESTS ABOUT $\mu_1 - \mu_2$: σ_1 AND σ_2 UNKNOWN

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \qquad (10.8)$$

The degrees of freedom for t are given by equation (10.7).

Let us demonstrate the use of this test statistic in the following hypothesis testing example.

Consider a new computer software package developed to help systems analysts reduce the time required to design, develop, and implement an information system. To evaluate the benefits of the new software package, a random sample of 24 systems analysts is selected. Each analyst is given specifications for a hypothetical information system. Then 12 of the analysts are instructed to produce the information system by using current technology. The other 12 analysts are trained in the use of the new software package and then instructed to use it to produce the information system.

This study involves two populations: a population of systems analysts using the current technology and a population of systems analysts using the new software package. In terms of the time required to complete the information system design project, the population means are as follow.

μ_1 = the mean project completion time for systems analysts using the current technology

μ_2 = the mean project completion time for systems analysts using the new software package

The researcher in charge of the new software evaluation project hopes to show that the new software package will provide a shorter mean project completion time. Thus, the researcher is looking for evidence to conclude that μ_2 is less than μ_1; in this case, the difference between the two population means, $\mu_1 - \mu_2$, will be greater than zero. The research hypothesis $\mu_1 - \mu_2 > 0$ is stated as the alternative hypothesis. Thus, the hypothesis test becomes

$$H_0: \mu_1 - \mu_2 \leq 0$$
$$H_a: \mu_1 - \mu_2 > 0$$

We will use $\alpha = .05$ as the level of significance.

TABLE 10.1 COMPLETION TIME DATA AND SUMMARY STATISTICS FOR THE SOFTWARE TESTING STUDY

WEB file
SoftwareTest

	Current Technology	New Software
	300	274
	280	220
	344	308
	385	336
	372	198
	360	300
	288	315
	321	258
	376	318
	290	310
	301	332
	283	263
Summary Statistics		
Sample size	$n_1 = 12$	$n_2 = 12$
Sample mean	$\bar{x}_1 = 325$ hours	$\bar{x}_2 = 286$ hours
Sample standard deviation	$s_1 = 40$	$s_2 = 44$

Suppose that the 24 analysts complete the study with the results shown in Table 10.1. Using the test statistic in equation (10.8), we have

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(325 - 286) - 0}{\sqrt{\frac{40^2}{12} + \frac{44^2}{12}}} = 2.27$$

Computing the degrees of freedom using equation (10.7), we have

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2} = \frac{\left(\frac{40^2}{12} + \frac{44^2}{12}\right)^2}{\frac{1}{12 - 1}\left(\frac{40^2}{12}\right)^2 + \frac{1}{12 - 1}\left(\frac{44^2}{12}\right)^2} = 21.8$$

Rounding down, we will use a t distribution with 21 degrees of freedom. This row of the t distribution table is as follows:

Area in Upper Tail	.20	.10	.05	.025	.01	.005
t Value (21 df)	.859	1.323	1.721	2.080	2.518	2.831

$t = 2.27$

Using the t distribution table, we can only determine a range for the p-value. Use of Excel (see Figure 10.6) shows the p-value = .0166.

With an upper tail test, the p-value is the area in the upper tail to the right of $t = 2.27$. From the preceding results, we see that the p-value is between .025 and .01. Thus, the p-value is less than $\alpha = .05$ and H_0 is rejected. The sample results enable the researcher to conclude that $\mu_1 - \mu_2 > 0$, or $\mu_2 < \mu_1$. Thus, the research study supports the conclusion that the new software package provides a smaller population mean completion time.

Using Excel to Conduct a Hypothesis Test

The Excel tool used to conduct a hypothesis test to determine whether there is a significant difference in population means when the population standard deviations are unknown is called *t-Test: Two-Sample Assuming Unequal Variances*. We illustrate using the sample data for the software evaluation study. Twelve systems analysts developed an information system using current technology, and 12 systems analysts developed an information system using a new software package. A one-tailed hypothesis test is to be conducted to see whether the mean completion time is shorter using the new software package. Refer to the Excel worksheets shown in Figure 10.6 and Figure 10.7 as we describe the tasks involved.

Enter Data: Column A in Figure 10.6 contains the completion time data for the simple random sample of 12 individuals using the current technology, and column B contains the completion time data for the simple random sample of 12 individuals using the new software.

Apply Tools: The following steps will provide the information needed to conduct the hypothesis test to see whether there is a significant difference in favor of the new software.

Step 1. Click the **Data** tab on the Ribbon
Step 2. In the **Analysis** group, click **Data Analysis**
Step 3. Choose **t-Test: Two-Sample Assuming Unequal Variances** from the list of Analysis Tools

FIGURE 10.6 DIALOG BOX FOR EXCEL'S t-TEST: TWO-SAMPLE ASSUMING UNEQUAL VARIANCES TOOL

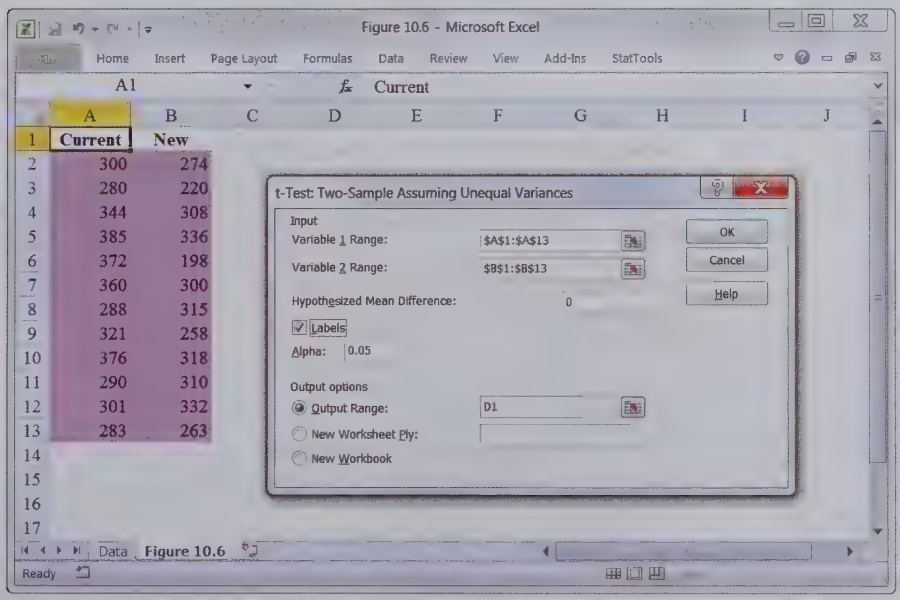
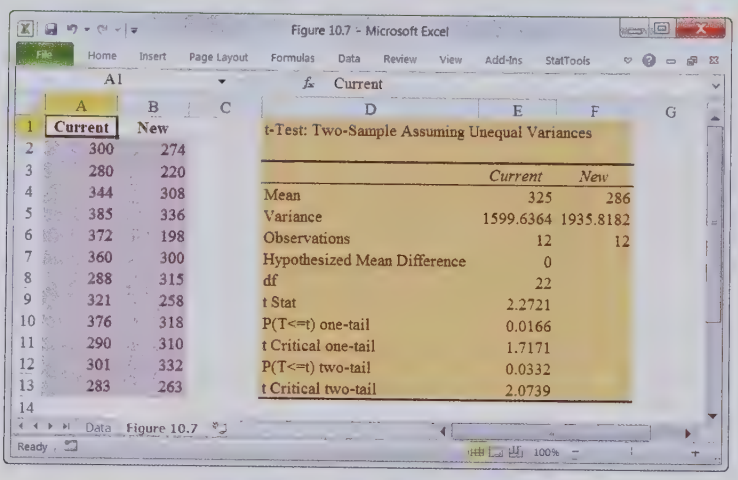


FIGURE 10.7 EXCEL RESULTS FOR THE HYPOTHESIS TEST ABOUT EQUALITY OF MEAN PROJECT COMPLETION TIMES



Step 4. When the t-Test: Two-Sample Assuming Unequal Variances dialog box appears (Figure 10.7),

Enter A1:A13 in the **Variable 1 Range** box
Enter B1:B13 in the **Variable 2 Range** box
Enter 0 in the **Hypothesized Mean Difference** box
Select **Labels**
Enter .05 in the **Alpha** box
Select **Output Range** and enter D1 in the box
Click **OK**

The results are shown in Figure 10.7. Descriptive statistics for the two samples are shown in cells E4:F6. The value of the test statistic, 2.2721, is shown in cell E9. The p-value for the test, labeled P(T<=t) one-tail, is shown in cell E10. Because the p-value, .0166, is less than the level of significance $\alpha = .05$, we can conclude that the mean completion time for the population using the new software package is smaller.

The t-Test: Two-Sample Assuming Unequal Variances tool can also be used to conduct two-tailed hypothesis tests. The only change required to make the hypothesis testing decision is that we need to use the p-value for a two-tailed test, labeled P(T<=t) two-tail (see cell E12).

Practical Advice

The interval estimation and hypothesis testing procedures presented in this section are robust and can be used with relatively small sample sizes. In most applications, sample sizes that are equal or nearly equal and result in a total sample size of $n_1 + n_2 \geq 20$ will provide very good results even if the populations are not normal. However, larger sample sizes are recommended if the distributions of the populations are highly skewed or contain outliers. Smaller sample sizes should only be used if the analyst is satisfied that the distributions of the populations are at least approximately normal.

NOTES AND COMMENTS

Another approach used to make inferences about the difference between two population means when σ_1 and σ_2 are unknown is based on the assumption that the two population standard deviations are *equal* ($\sigma_1 = \sigma_2 = \sigma$). Under this assumption, the two sample standard deviations are combined to provide the following *pooled sample variance:*

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

The t test statistic becomes

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

and has $n_1 + n_2 - 2$ degrees of freedom. At this point, the computation of the p-value and the interpretation of the sample results are identical to the procedures discussed earlier in this section.

A difficulty with this procedure is that the assumption that the two population standard deviations are equal is usually difficult to verify. Unequal population standard deviations are frequently encountered. Using the pooled procedure may not provide satisfactory results, especially if the sample sizes n_1 and n_2 are quite different.

The t procedure that we presented in this section does not require the assumption of equal population standard deviations and can be applied whether the population standard deviations are equal or not. It is a more general procedure and is recommended for most applications.

Exercises

Methods

SELF test

9. The following results are for independent random samples taken from two populations.

Sample 1	Sample 2
$n_1 = 20$	$n_2 = 30$
$\bar{x}_1 = 22.5$	$\bar{x}_2 = 20.1$
$s_1 = 2.5$	$s_2 = 4.8$

a. What is the point estimate of the difference between the two population means?
b. What is the degrees of freedom for the t distribution?
c. At 95% confidence, what is the margin of error?
d. What is the 95% confidence interval for the difference between the two population means?

SELF test

10. Consider the following hypothesis test.

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_a: \mu_1 - \mu_2 \neq 0$$

The following results are from independent samples taken from two populations.

Sample 1	Sample 2
$n_1 = 35$	$n_2 = 40$
$\bar{x}_1 = 13.6$	$\bar{x}_2 = 10.1$
$s_1 = 5.2$	$s_2 = 8.5$

a. What is the value of the test statistic?
b. What is the value of the degrees of freedom for the t distribution?
c. What is the p-value?
d. At $\alpha = .05$, what is your conclusion?

11. Consider the following data for two independent random samples taken from two normal populations.

Sample 1	10	7	13	7	9	8
Sample 2	8	7	8	4	6	9

a. Compute the two sample means.
b. Compute the two sample standard deviations.
c. What is the point estimate of the difference between the two population means?
d. What is the 90% confidence interval estimate of the difference between the two population means?

Applications

SELF test

12. The U.S. Department of Transportation provides the number of miles that residents of the 75 largest metropolitan areas travel per day in a car. Suppose that for a simple random sample of 50 Buffalo residents the mean is 22.5 miles a day and the standard deviation is 8.4 miles a day, and for an independent simple random sample of 40 Boston residents the mean is 18.6 miles a day and the standard deviation is 7.4 miles a day.
a. What is the point estimate of the difference between the mean number of miles that Buffalo residents travel per day and the mean number of miles that Boston residents travel per day?
b. What is the 95% confidence interval for the difference between the two population means?

13. FedEx and United Parcel Service (UPS) are the world's two leading cargo carriers by volume and revenue (*The Wall Street Journal*, January 27, 2004). According to the Airports Council International, the Memphis International Airport (FedEx) and the Louisville International Airport (UPS) are 2 of the 10 largest cargo airports in the world. The following random samples show the tons of cargo per day handled by these airports. Data are in thousands of tons.

WEB file
Cargo

Memphis					
9.1	15.1	8.8	10.0	7.5	10.5
8.3	9.1	6.0	5.8	12.1	9.3
Louisville					
4.7	5.0	4.2	3.3	5.5	
2.2	4.1	2.6	3.4	7.0	

a. Compute the sample mean and sample standard deviation for each airport.
b. What is the point estimate of the difference between the two population means? Interpret this value in terms of the higher-volume airport and a comparison of the volume difference between the two airports.
c. Develop a 95% confidence interval of the difference between the daily population means for the two airports.

14. Are nursing salaries in Tampa, Florida, lower than those in Dallas, Texas? Salary data provided by *http://www.salary.com* shows staff nurses in Tampa earn less than staff nurses in Dallas (*The Tampa Tribune*, January 15, 2007). Suppose that in a follow-up study of 40 staff nurses in Tampa and 50 staff nurses in Dallas you obtain the following results.

Tampa Bay	Dallas
$n_1 = 40$	$n_2 = 50$
$\bar{x}_1 = \$56{,}100$	$\bar{x}_2 = \$59{,}400$
$s_1 = \$6000$	$s_2 = \$7000$

a. Formulate hypotheses so that, if the null hypothesis is rejected, we can conclude that salaries for staff nurses in Tampa Bay are significantly lower than for those in Dallas. Use $\alpha = .05$.
b. What is the value of the test statistic?
c. What is the p-value?
d. What is your conclusion?

15. Commercial real estate prices and rental rates suffered substantial declines in the past year (*Newsweek,* July 27, 2009). These declines were particularly severe in Asia; annual lease rates in Tokyo, Hong Kong, and Singapore declined by 40% or more. Even with such large declines, annual lease rates in Asia were still higher than in many cities in Europe and the United States. Annual lease rates for a sample of 30 commercial properties in Hong Kong showed an average of $1114 per square meter with a standard deviation of $230. Annual lease rates for a sample of 40 commercial properties in Paris showed an average lease rate of $989 per square meter with a standard deviation of $195.
a. On the basis of the sample results, can we conclude that the mean annual lease rate is higher in Hong Kong than in Paris? Develop appropriate null and alternative hypotheses.
b. Use $\alpha = .01$. What is your conclusion? Are rental rates higher in Hong Kong than in Paris?

16. *Consumer Reports* rated 15 midpriced family sedans and 15 affordable family sedans. The following data show the road-test scores (1 to 100 scale with 100 the highest rating) for the 30 cars (*Consumer Reports,* February 2008).

WEB file
RoadTest

Midpriced Family Sedans	Affordable Family Sedans
89	85
88	79
87	78
86	77
84	76
77	75
77	74
75	69
70	69
69	69
64	64
58	62
56	59
48	49
46	44

a. Formulate hypotheses so that if the null hypothesis is rejected, we can conclude that the mean road-test score for midpriced family sedans is significantly greater than the mean road-test score for affordable family sedans. Use $\alpha = .05$.
b. What is the point estimate of the difference between the mean road-test score for midpriced family sedans and the mean road-test score for affordable family sedans?
c. Compute the p-value for the hypotheses formulated in part (a).
d. Using the p-value computed in part (c) and $\alpha = .05$, what is your conclusion.

17. Periodically, Merrill Lynch customers are asked to evaluate Merrill Lynch financial consultants and services (2010 Merrill Lynch Client Satisfaction Survey). Higher ratings on the client satisfaction survey indicate better service with 7 the maximum service rating. Independent samples of service ratings for two financial consultants are summarized here. Consultant A has 10 years of experience, whereas consultant B has 1 year of experience.

Use $\alpha = .05$ and test to see whether the consultant with more experience has the higher population mean service rating.

Consultant A	Consultant B
$n_1 = 16$	$n_2 = 10$
$\bar{x}_1 = 6.82$	$\bar{x}_2 = 6.25$
$s_1 = .64$	$s_2 = .75$

a. State the null and alternative hypotheses.
b. Compute the value of the test statistic.
c. What is the p-value?
d. What is your conclusion?

18. The Tire Rack, America's leading online distributor of tires and wheels, conducts extensive testing to provide customers with products that are right for their vehicle, driving style, and conditions in which they drive. In addition, they have maintained an independent consumer survey to help drivers help each other by sharing their long-term tire experiences. The data shown below show survey ratings (1 to 10 scale with 10 the highest rating) for two all-season passenger tires, the Dunlop SP60 and the Michelin MX4 (Tire Rack website, April 18, 2008).

WEB file TireRack

Dunlop

7.7	8.1	9.1	8.6	7.4	8.3	9.7	9.5	9.0	8.2
9.8	8.0	10.0	8.5	9.3	7.7	8.9	9.6	7.9	10.0
8.3	8.1	5.1	7.3	8.0	8.5	9.4	7.6	8.4	4.2
7.6	5.9								

Michelin

8.0	7.2	7.1	4.4	8.5	6.6	7.4	9.5	6.6	8.0
7.3	6.4	8.9	8.6	5.9	7.2	6.6	2.3	7.7	5.5
7.6	7.3	8.6	8.1	8.9	9.0	8.0	8.6	6.6	6.9
8.1	3.9	5.2	6.6	6.0	6.4	8.3	6.6	3.4	8.4

a. Formulate the hypotheses that can be used to determine whether the sample data support the hypothesis that the mean rating for Dunlop SP60 tires is greater than the mean rating for Michelin MX4 tires.
b. What is the point estimate of the difference between the means for the two populations?
c. Compute the p-value for the hypothesis test.
d. At $\alpha = .05$, what is your conclusion?

10.3 Inferences About the Difference Between Two Population Means: Matched Samples

Suppose employees at a manufacturing company can use two different methods to perform a production task. To maximize production output, the company wants to identify the method with the shorter population mean completion time. Let μ_1 denote the population mean completion time for production method 1 and μ_2 denote the population mean completion time for production method 2. With no preliminary indication of the preferred production method, we begin by tentatively assuming that the two production methods have the same population mean completion time. Thus, the null hypothesis is H_0: $\mu_1 - \mu_2 = 0$. If this hypothesis is rejected, we can conclude that the population mean completion times

differ. In this case, the method providing the shorter mean completion time would be recommended. The null and alternative hypotheses are written as follows:

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_a: \mu_1 - \mu_2 \neq 0$$

In choosing the sampling procedure that will be used to collect production time data and test the hypotheses, we consider two alternative designs. One is based on **independent simple random samples** and the other is based on **matched samples**.

1. *Independent sample design:* A simple random sample of workers is selected and each worker in the sample uses method 1. A second independent simple random sample of workers is selected and each worker in this sample uses method 2. The test of the difference between population means is based on the procedures in Section 10.2.
2. *Matched sample design:* One simple random sample of workers is selected. Each worker first uses one method and then uses the other method. The order of the two methods is assigned randomly to the workers, with some workers performing method 1 first and others performing method 2 first. Each worker provides a pair of data values, one value for method 1 and another value for method 2.

In the matched sample design the two production methods are tested under similar conditions (i.e., with the same workers); hence this design often leads to a smaller sampling error than the independent sample design. The primary reason is that in a matched sample design, variation between workers is eliminated because the same workers are used for both production methods.

Let us demonstrate the analysis of a matched sample design by assuming it is the method used to test the difference between population means for the two production methods. A random sample of six workers is used. The data on completion times for the six workers are given in Table 10.2. Note that each worker provides a pair of data values, one for each production method. Also note that the last column contains the difference in completion times d_i for each worker in the sample.

The key to the analysis of the matched sample design is to realize that we consider only the column of differences. Therefore, we have six data values (.6, −.2, .5, .3, .0, and .6) that will be used to analyze the difference between population means of the two production methods.

Let μ_d = the mean of the *difference* values for the population of workers. With this notation, the null and alternative hypotheses are rewritten as follows:

$$H_0: \mu_d = 0$$
$$H_a: \mu_d \neq 0$$

If H_0 is rejected, we can conclude that the population mean completion times differ.

TABLE 10.2 TASK COMPLETION TIMES FOR A MATCHED SAMPLE DESIGN

Worker	Completion Time for Method 1 (minutes)	Completion Time for Method 2 (minutes)	Difference in Completion Times (d_i)
1	6.0	5.4	.6
2	5.0	5.2	−.2
3	7.0	6.5	.5
4	6.2	5.9	.3
5	6.0	6.0	.0
6	6.4	5.8	.6

WEB file
Matched

Other than the use of the d notation, the formulas for the sample mean and sample standard deviation are the same ones used previously in the text.

The d notation is a reminder that the matched sample provides *difference* data. The sample mean and sample standard deviation for the six difference values in Table 10.2 follow.

$$\bar{d} = \frac{\Sigma d_i}{n} = \frac{1.8}{6} = .30$$

$$s_d = \sqrt{\frac{\Sigma(d_i - \bar{d})^2}{n-1}} = \sqrt{\frac{.56}{5}} = .3347$$

With the small sample of $n = 6$ workers, we need to make the assumption that the population of differences has a normal distribution. This assumption is necessary so that we may use the t distribution for hypothesis testing and interval estimation procedures. Based on this assumption, the following test statistic has a t distribution with $n - 1$ degrees of freedom.

It is not necessary to make the assumption that the population has a normal distribution if the sample size is large. Sample size guidelines for using the t distribution were presented in Chapters 8 and 9.

TEST STATISTIC FOR HYPOTHESIS TESTS INVOLVING MATCHED SAMPLES

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} \qquad (10.9)$$

Once the difference data are computed, the t distribution procedure for matched samples is the same as the one-population estimation and hypothesis testing procedures described in Chapters 8 and 9.

Let us use equation (10.9) to test the hypotheses H_0: $\mu_d = 0$ and H_a: $\mu_d \neq 0$, using $\alpha = .05$. Substituting the sample results $\bar{d} = .30$, $s_d = .3347$, and $n = 6$ into equation (10.9), we compute the value of the test statistic.

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{.30 - 0}{.3347/\sqrt{6}} = 2.20$$

Now let us compute the p-value for this two-tailed test. Because $t = 2.20 > 0$, the test statistic is in the upper tail of the t distribution. With $t = 2.20$, the area in the upper tail to the right of the test statistic can be found by using the t distribution table with degrees of freedom $= n - 1 = 6 - 1 = 5$. Information from the 5 degrees of freedom row of the t distribution table is as follows:

Area in Upper Tail	.20	.10	.05	.025	.01	.005
t Value (5 *df*)	0.920	1.476	2.015	2.571	3.365	4.032

$t = 2.20$

Thus, we see that the area in the upper tail is between .05 and .025. Because this test is a two-tailed test, we double these values to conclude that the p-value is between .10 and .05. This p-value is greater than $\alpha = .05$. Thus, the null hypothesis H_0: $\mu_d = 0$ is not rejected. Using Excel and the data in Table 10.2, we find the exact p-value = .0795.

We can also obtain an interval estimate of the difference between the two population means by using the single population methodology of Chapter 8. At 95% confidence, the calculation follows.

$$\bar{d} \pm t_{.025}\frac{s_d}{\sqrt{n}}$$

$$.3 \pm 2.571\left(\frac{.3347}{\sqrt{6}}\right)$$

$$.3 \pm .35$$

Thus, the margin of error is .35 and the 95% confidence interval for the difference between the population means of the two production methods is −.05 minute to .65 minute.

Using Excel to Conduct a Hypothesis Test

Excel's t-Test: Paired Two Sample for Means tool can be used to conduct a hypothesis test about the difference between the population means when a matched sample design is used. We illustrate by conducting the hypothesis test involving the two production methods. Refer to the Excel worksheets shown in Figure 10.8 and Figure 10.9 as we describe the tasks involved.

Enter Data: Column A in Figure 10.8 is used to identify each of the six workers who participated in the study. Column B contains the completion time data for each worker using method 1, and column C contains the completion time data for each worker using method 2.

Apply Tools: The following steps describe how to use Excel's t-Test: Paired Two Sample for Means tool to conduct the hypothesis test about the difference between the means of the two production methods

Step 1. Click the **Data** tab on the Ribbon
Step 2. In the **Analysis** group, click **Data Analysis**
Step 3. Choose **t-Test: Paired Two Sample for Means** from the list of Analysis Tools
Step 4. When the t-Test: Paired Two Sample for Means dialog box appears (Figure 10.8),
Enter B1:B7 in the **Variable 1 Range** box
Enter C1:C7 in the **Variable 2 Range** box

FIGURE 10.8 DIALOG BOX FOR EXCEL'S t-TEST: PAIRED TWO SAMPLE FOR MEANS TOOL

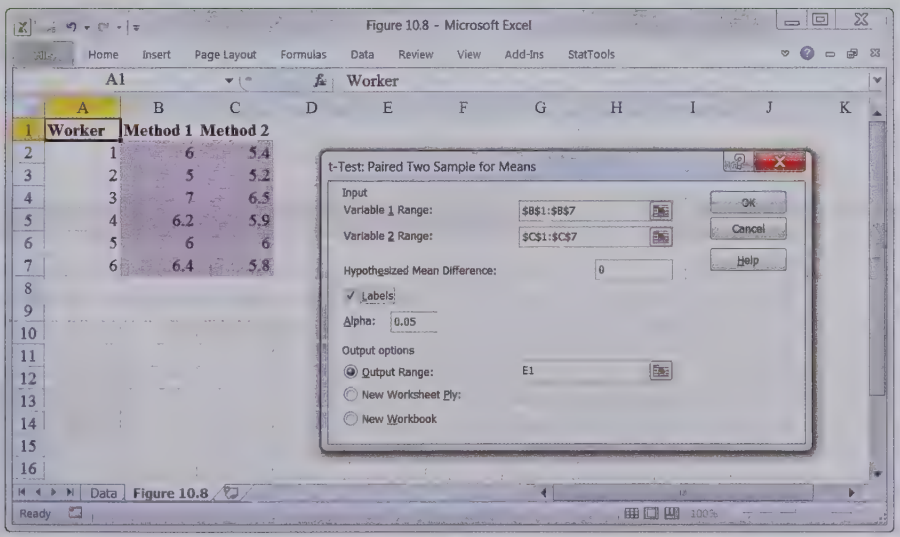
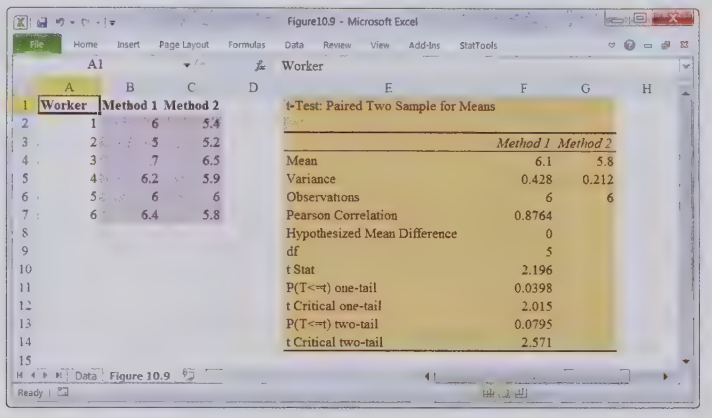


FIGURE 10.9 EXCEL RESULTS FOR THE HYPOTHESIS TEST IN THE MATCHED SAMPLES STUDY



Enter 0 in the **Hypothesized Mean Difference** box
Select **Labels**
Enter .05 in the **Alpha** box
Select **Output Range**
Enter E1 in the **Output Range** box (to identify the upper left corner of the section of the worksheet where the output will appear)
Click **OK**

The results are shown in cells E1:G14 of the worksheet shown in Figure 10.9. The *p*-value for the test, labeled P(T<= t) two-tail, is shown in cell F13. Because the *p*-value, .0795, is greater than the level of significance $\alpha = .05$, we cannot reject the null hypothesis that the mean completion times are equal.

The same procedure can also be used to conduct one-tailed hypothesis tests. The only change required to make the hypothesis testing decision is that we need to use the *p*-value for a one-tailed test, labeled P(T<= t) one-tail (see cell F11).

NOTES AND COMMENTS

1. In the example presented in this section, workers performed the production task with first one method and then the other method. This example illustrates a matched sample design in which each sampled element (worker) provides a pair of data values. It is also possible to use different but "similar" elements to provide the pair of data values. For example, a worker at one location could be matched with a similar worker at another location (similarity based on age, education, gender, experience, etc.). The pairs of workers would provide the difference data that could be used in the matched sample analysis.
2. A matched sample procedure for inferences about two population means generally provides better precision than the independent sample approach; therefore, it is the recommended design. However, in some applications the matching cannot be achieved, or perhaps the time and cost associated with matching are excessive. In such cases, the independent sample design should be used.

Exercises

Methods

SELF test

19. Consider the following hypothesis test.

$$H_0: \mu_d \leq 0$$
$$H_a: \mu_d > 0$$

The following data are from matched samples taken from two populations.

	Population	
Element	1	2
1	21	20
2	28	26
3	18	18
4	20	20
5	26	24

a. Compute the difference value for each element.
b. Compute $\bar{d}$.
c. Compute the standard deviation s_d.
d. Conduct a hypothesis test using $\alpha = .05$. What is your conclusion?

20. The following data are from matched samples taken from two populations.

	Population	
Element	1	2
1	11	8
2	7	8
3	9	6
4	12	7
5	13	10
6	15	15
7	15	14

a. Compute the difference value for each element.
b. Compute $\bar{d}$.
c. Compute the standard deviation s_d.
d. What is the point estimate of the difference between the two population means?
e. Provide a 95% confidence interval for the difference between the two population means.

Applications

SELF test

21. A market research firm used a sample of individuals to rate the purchase potential of a particular product before and after the individuals saw a new television commercial about the product. The purchase potential ratings were based on a 0 to 10 scale, with higher values indicating a higher purchase potential. The null hypothesis stated that the mean rating "after" would be less than or equal to the mean rating "before." Rejection of this hypothesis would show that the commercial improved the mean purchase potential rating. Use $\alpha = .05$ and the following data to test the hypothesis and comment on the value of the commercial.

	Purchase Rating			Purchase Rating	
Individual	After	Before	Individual	After	Before
1	6	5	5	3	5
2	6	4	6	9	8
3	7	7	7	7	5
4	4	3	8	6	6

22. Per-share earnings data comparing the current quarter's earnings with the previous quarter are in the file named Earnings2005 (*The Wall Street Journal,* January 27, 2006). Provide a 95% confidence interval estimate of the difference between the population mean for the current quarter versus the previous quarter. Have earnings increased?

23. Bank of America's Consumer Spending Survey collected data on annual credit card charges in seven different categories of expenditures: transportation, groceries, dining out, household expenses, home furnishings, apparel, and entertainment (*US Airways Attaché,* December 2003). Using data from a sample of 42 credit card accounts, assume that each account was used to identify the annual credit card charges for groceries (population 1) and the annual credit card charges for dining out (population 2). Using the difference data, the sample mean difference was $\bar{d} = \$850$, and the sample standard deviation was $s_d = \$1123$.
    a. Formulate the null and alternative hypotheses to test for no difference between the population mean credit card charges for groceries and the population mean credit card charges for dining out.
    b. Use a .05 level of significance. Can you conclude that the population means differ? What is the *p*-value?
    c. Which category, groceries or dining out, has a higher population mean annual credit card charge? What is the point estimate of the difference between the population means? What is the 95% confidence interval estimate of the difference between the population means?

24. Airline travelers often choose which airport to fly from based on flight cost. Cost data (in dollars) for a sample of flights to eight cities from Dayton, Ohio, and Louisville, Kentucky, were collected to help determine which of the two airports was more costly to fly from (*The Cincinnati Enquirer,* February 19, 2006). A researcher argued that it is significantly more costly to fly out of Dayton than Louisville. Use the sample data to see whether they support the researcher's argument. Use $\alpha = .05$ as the level of significance.

Destination	Dayton	Louisville
Chicago-O'Hare	$319	$142
Grand Rapids, Michigan	192	213
Portland, Oregon	503	317
Atlanta	256	387
Seattle	339	317
South Bend, Indiana	379	167
Miami	268	273
Dallas–Ft. Worth	288	274

25. In recent years, a growing array of entertainment options competes for consumer time. By 2004, cable television and radio surpassed broadcast television, recorded music, and the daily newspaper to become the two entertainment media with the greatest usage (*The Wall Street Journal,* January 26, 2004). Researchers used a sample of 15 individuals and collected data on the hours per week spent watching cable television and hours per week spent listening to the radio.

WEB file
TVRadio

Individual	Television	Radio	Individual	Television	Radio
1	22	25	9	21	21
2	8	10	10	23	23
3	25	29	11	14	15
4	22	19	12	14	18
5	12	13	13	14	17
6	26	28	14	16	15
7	22	23	15	24	23
8	19	21			

a. Use a .05 level of significance and test for a difference between the population mean usage for cable television and radio. What is the p-value?
b. What is the sample mean number of hours per week spent watching cable television? What is the sample mean number of hours per week spent listening to radio? Which medium has the greater usage?

26. Scores in the first and fourth (final) rounds for a sample of 20 golfers who competed in PGA tournaments are shown in the following table (*Golfweek*, February 14, 2009, and February 28, 2009). Suppose you would like to determine if the mean score for the first round of a PGA Tour event is significantly different than the mean score for the fourth and final round. Does the pressure of playing in the final round cause scores to go up? Or does the increased player concentration cause scores to come down?

WEB file
GolfScores

Player	First Round	Final Round	Player	First Round	Final Round
Michael Letzig	70	72	Aron Price	72	72
Scott Verplank	71	72	Charles Howell	72	70
D. A. Points	70	75	Jason Dufner	70	73
Jerry Kelly	72	71	Mike Weir	70	77
Soren Hansen	70	69	Carl Pettersson	68	70
D. J. Trahan	67	67	Bo Van Pelt	68	65
Bubba Watson	71	67	Ernie Els	71	70
Reteif Goosen	68	75	Cameron Beckman	70	68
Jeff Klauk	67	73	Nick Watney	69	68
Kenny Perry	70	69	Tommy Armour III	67	71

a. Use $\alpha = .10$ to test for a statistically significantly difference between the population means for first- and fourth-round scores. What is the p-value? What is your conclusion?
b. What is the point estimate of the difference between the two population means? For which round is the population mean score lower?
c. What is the margin of error for a 90% confidence interval estimate for the difference between the population means? Could this confidence interval have been used to test the hypothesis in part (a)? Explain.

27. A manufacturer produces both a deluxe and a standard model of an automatic sander designed for home use. Selling prices obtained from a sample of retail outlets follow.

	Model Price ($)			Model Price ($)	
Retail Outlet	Deluxe	Standard	Retail Outlet	Deluxe	Standard
1	39	27	5	40	30
2	39	28	6	39	34
3	45	35	7	35	29
4	38	30			

a. The manufacturer's suggested retail prices for the two models show a $10 price differential. Use a .05 level of significance and test that the mean difference between the prices of the two models is $10.
b. What is the 95% confidence interval for the difference between the mean prices of the two models?

10.4 An Introduction to Experimental Design and Analysis of Variance

In Chapter 1 we stated that statistical studies can be classified as either experimental or observational. In an experimental statistical study, an experiment is conducted to generate the data. An experiment begins with identifying a variable of interest. Then one or more other variables, thought to be related, are identified and controlled, and data are collected about how those variables influence the variable of interest.

In an observational study, data are usually obtained through sample surveys and not a controlled experiment. Good design principles are still employed, but the rigorous controls associated with an experimental statistical study are often not possible. For instance, in a study of the relationship between smoking and lung cancer the researcher cannot assign a smoking habit to subjects. The researcher is restricted to simply observing the effects of smoking on people who already smoke and the effects of not smoking on people who already do not smoke.

Sir Ronald Alymer Fisher (1890–1962) invented the branch of statistics known as experimental design. In addition to being accomplished in statistics, he was a noted scientist in the field of genetics.

In this section we introduce the basic principles of an experimental study and show how they are used in a completely randomized design. We also provide a conceptual overview of the statistical procedure called analysis of variance (ANOVA). In the following section we show how ANOVA can be used to test for the equality of k population means using data obtained from a completely randomized design as well as data obtained from an observational study. So, in this sense, ANOVA extends the statistical material in the preceding sections from two population means to three or more population means. In later chapters, we will see that ANOVA plays a key role in analyzing the results of regression studies involving both experimental and observational data.

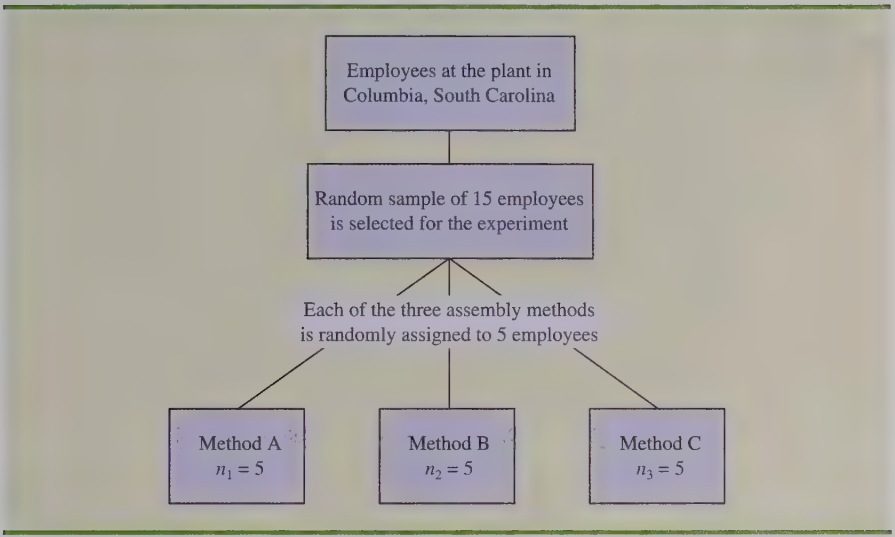
Cause-and-effect relationships can be difficult to establish in observational studies; such relationships are easier to establish in experimental studies.

As an example of an experimental statistical study, let us consider the problem facing Chemitech, Inc. Chemitech developed a new filtration system for municipal water supplies. The components for the new filtration system will be purchased from several suppliers, and Chemitech will assemble the components at its plant in Columbia, South Carolina. The industrial engineering group is responsible for determining the best assembly method for the new filtration system. After considering a variety of possible approaches, the group narrows the alternatives to three: method A, method B, and method C. These methods differ in the sequence of steps used to assemble the system. Managers at Chemitech want to determine which assembly method can produce the greatest number of filtration systems per week.

In the Chemitech experiment, assembly method is the independent variable or **factor**. Because three assembly methods correspond to this factor, we say that three treatments are associated with this experiment; each **treatment** corresponds to one of the three assembly methods. The Chemitech problem is an example of a **single-factor experiment**; it involves one categorical factor (method of assembly). More complex experiments may consist of multiple factors; some factors may be categorical and others may be quantitative.

The three assembly methods or treatments define the three populations of interest for the Chemitech experiment. One population is all Chemitech employees who use assembly method A, another is those who use method B, and the third is those who use method C. Note that for each population the dependent or **response variable** is the number of filtration systems produced per week, and the primary statistical objective of the experiment is to determine whether the mean number of units produced per week is the same for all three populations (methods).

FIGURE 10.10 COMPLETELY RANDOMIZED DESIGN FOR EVALUATING THE CHEMITECH ASSEMBLY METHOD EXPERIMENT


Employees at the plant in Columbia, South Carolina
Random sample of 15 employees is selected for the experiment
Each of the three assembly methods is randomly assigned to 5 employees
Method A $n_1 = 5$
Method B $n_2 = 5$
Method C $n_3 = 5$

Randomization is the process of assigning the treatments to the experimental units at random. Prior to the work of Sir R. A. Fisher, treatments were assigned on a systematic or subjective basis.

Suppose a random sample of three employees is selected from all assembly workers at the Chemitech production facility. In experimental design terminology, the three randomly selected workers are the **experimental units**. The experimental design that we will use for the Chemitech problem is called a **completely randomized design**. This type of design requires that each of the three assembly methods or treatments be assigned randomly to one of the experimental units or workers. For example, method A might be randomly assigned to the second worker, method B to the first worker, and method C to the third worker. The concept of *randomization,* as illustrated in this example, is an important principle of all experimental designs.

To obtain additional data for each assembly method, we must repeat or replicate the basic experimental process. Suppose, for example, that instead of selecting just 3 workers at random we selected 15 workers and then randomly assigned each of the three treatments to 5 of the workers. Because each method of assembly is assigned to 5 workers, we say that five replicates have been obtained. The process of *replication* is another important principle of experimental design. Figure 10.10 shows the completely randomized design for the Chemitech experiment.

Data Collection

Once we are satisfied with the experimental design, we proceed by collecting and analyzing the data. In the Chemitech case, the employees would be instructed in how to perform the assembly method assigned to them and then would begin producing the new filtration systems using that method. After this assignment and training, the number of units produced by each employee during one week is as shown in Table 10.3. The sample means, sample variances, and sample standard deviations for each assembly method are also provided. Thus, the sample mean number of units produced using method A is 62; the sample mean using method B is 66; and the sample mean using method C is 52. From these data, method B appears to result in higher production rates than either of the other methods.

TABLE 10.3 NUMBER OF UNITS PRODUCED BY 15 WORKERS

WEB file
Chemitech

	Method		
	A	B	C
	58	58	48
	64	69	57
	55	71	59
	66	64	47
	67	68	49
Sample mean	62	66	52
Sample variance	27.5	26.5	31.0
Sample standard deviation	5.244	5.148	5.568

The real issue is whether the three sample means observed are different enough for us to conclude that the means of the populations corresponding to the three methods of assembly are different. To write this question in statistical terms, we introduce the following notation:

μ_1 = mean number of units produced per week using method A
μ_2 = mean number of units produced per week using method B
μ_3 = mean number of units produced per week using method C

Although we will never know the actual values of μ_1, μ_2, and μ_3, we want to use the sample means to test the following hypotheses.

If H_0 is rejected, we cannot conclude that all population means are different. Rejecting H_0 means that at least two population means have different values.

H_0: $\mu_1 = \mu_2 = \mu_3$
H_a: Not all population means are equal

As we will demonstrate shortly, ANOVA is the statistical procedure used to determine whether the observed differences in the three sample means are large enough to reject H_0.

Assumptions for Analysis of Variance

Three assumptions are required to use analysis of variance.

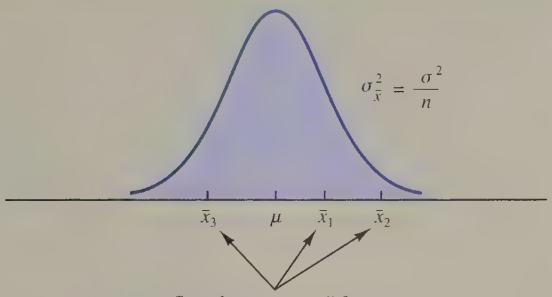
If the sample sizes are equal, analysis of variance is not sensitive to departures from the assumption of normally distributed populations.

1. **For each population, the response variable is normally distributed.** Implication: In the Chemitech experiment the number of units produced per week (response variable) must be normally distributed for each assembly method.
2. **The variance of the response variable, denoted σ^2, is the same for all the populations.** Implication: In the Chemitech experiment, the variance of the number of units produced per week must be the same for each assembly method.
3. **The observations must be independent.** Implication: In the Chemitech experiment, the number of units produced per week for each employee must be independent of the number of units produced per week for any other employee.

Analysis of Variance: A Conceptual Overview

If the means for the three populations are equal, we would expect the three sample means to be close together. In fact, the closer the three sample means are to one another, the more evidence we have for the conclusion that the population means are equal. Alternatively, the more the sample means differ, the more evidence we have for the conclusion that the population means are not equal. In other words, if the variability among the sample means is "small," it supports H_0; if the variability among the sample means is "large," it supports H_a.

FIGURE 10.11 SAMPLING DISTRIBUTION OF $\bar{x}$ GIVEN H_0 IS TRUE



If the null hypothesis, H_0: $\mu_1 = \mu_2 = \mu_3$, is true, we can use the variability among the sample means to develop an estimate of σ^2. First, note that if the assumptions for analysis of variance are satisfied, each sample will have come from the same normal distribution with mean μ and variance σ^2. Recall from Chapter 7 that the sampling distribution of the sample mean $\bar{x}$ for a simple random sample of size n from a normal population will be normally distributed with mean μ and variance σ^2/n. Figure 10.11 illustrates such a sampling distribution.

Thus, if the null hypothesis is true, we can think of each of the three sample means, $\bar{x}_1 = 62$, $\bar{x}_2 = 66$, and $\bar{x}_3 = 52$ from Table 10.3, as values drawn at random from the sampling distribution shown in Figure 10.11. In this case, the mean and variance of the three $\bar{x}$ values can be used to estimate the mean and variance of the sampling distribution. When the sample sizes are equal, as in the Chemitech experiment, the best estimate of the mean of the sampling distribution of $\bar{x}$ is the mean or average of the sample means. Thus, in the Chemitech experiment, an estimate of the mean of the sampling distribution of $\bar{x}$ is $(62 + 66 + 52)/3 = 60$. We refer to this estimate as the *overall sample mean.* An estimate of the variance of the sampling distribution of $\bar{x}$, $\sigma_{\bar{x}}^2$, is provided by the variance of the three sample means.

$$s_{\bar{x}}^2 = \frac{(62 - 60)^2 + (66 - 60)^2 + (52 - 60)^2}{3 - 1} = \frac{104}{2} = 52$$

Because $\sigma_{\bar{x}}^2 = \sigma^2/n$, solving for σ^2 gives

$$\sigma^2 = n\sigma_{\bar{x}}^2$$

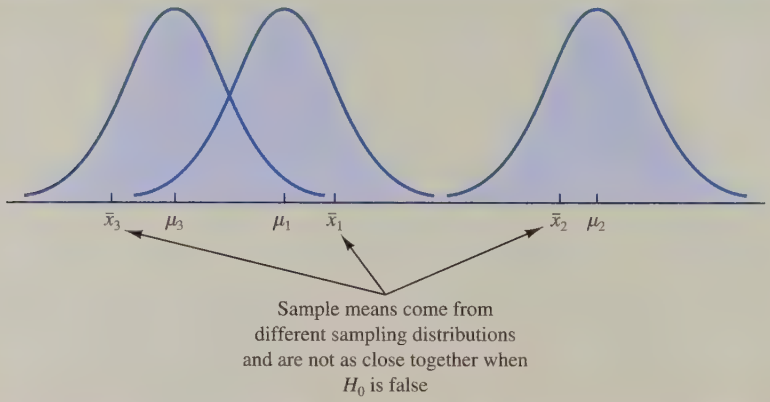
Hence,

$$\text{Estimate of } \sigma^2 = n(\text{Estimate of } \sigma_{\bar{x}}^2) = ns_{\bar{x}}^2 = 5(52) = 260$$

The result, $ns_{\bar{x}}^2 = 260$, is referred to as the *between-treatments* estimate of σ^2.

The between-treatments estimate of σ^2 is based on the assumption that the null hypothesis is true. In this case, each sample comes from the same population, and there is only one sampling distribution of $\bar{x}$. To illustrate what happens when H_0 is false, suppose the

FIGURE 10.12 SAMPLING DISTRIBUTIONS OF $\bar{x}$ GIVEN H_0 IS FALSE



population means all differ. Note that because the three samples are from normal populations with different means, they will result in three different sampling distributions. Figure 10.12 shows that in this case, the sample means are not as close together as they were when H_0 was true. Thus, $s_{\bar{x}}^2$ will be larger, causing the between-treatments estimate of σ^2 to be larger. In general, when the population means are not equal, the between-treatments estimate will overestimate the population variance σ^2.

The variation within each of the samples also has an effect on the conclusion we reach in analysis of variance. When a simple random sample is selected from each population, each of the sample variances provides an unbiased estimate of σ^2. Hence, we can combine or pool the individual estimates of σ^2 into one overall estimate. The estimate of σ^2 obtained in this way is called the *pooled* or *within-treatments* estimate of σ^2. Because each sample variance provides an estimate of σ^2 based only on the variation within each sample, the within-treatments estimate of σ^2 is not affected by whether the population means are equal. When the sample sizes are equal, the within-treatments estimate of σ^2 can be obtained by computing the average of the individual sample variances. For the Chemitech experiment we obtain

$$\text{Within-treatments estimate of } \sigma^2 = \frac{27.5 + 26.5 + 31.0}{3} = \frac{85}{3} = 28.33$$

In the Chemitech experiment, the between-treatments estimate of σ^2 (260) is much larger than the within-treatments estimate of σ^2 (28.33). In fact, the ratio of these two estimates is $260/28.33 = 9.18$. Recall, however, that the between-treatments approach provides a good estimate of σ^2 only if the null hypothesis is true; if the null hypothesis is false, the between-treatments approach overestimates σ^2. The within-treatments approach provides a good estimate of σ^2 in either case. Thus, if the null hypothesis is true, the two estimates will be similar and their ratio will be close to 1. If the null hypothesis is false, the between-treatments estimate will be larger than the within-treatments estimate, and their ratio will be large. In the next section we will show how large this ratio must be to reject H_0.

In summary, the logic behind ANOVA is based on the development of two independent estimates of the common population variance σ^2. One estimate of σ^2 is based on the variability among the sample means themselves, and the other estimate of σ^2 is based on the variability of the data within each sample. By comparing these two estimates of σ^2, we will be able to determine whether the population means are equal.

NOTES AND COMMENTS

1. Randomization in experimental design is the analog of probability sampling in an observational study.
2. In many medical experiments, potential bias is eliminated by using a double-blind experimental design. With this design, neither the physician applying the treatment nor the subject knows which treatment is being applied. Many other types of experiments could benefit from this type of design.
3. In this section we provided a conceptual overview of how analysis of variance can be used to test for the equality of k population means for a completely randomized experimental design. We will see that the same procedure can also be used to test for the equality of k population means for an observational or nonexperimental study.
4. In Sections 10.1 and 10.2 we presented statistical methods for testing the hypothesis that the means of two populations are equal. ANOVA can also be used to test the hypothesis that the means of two populations are equal. In practice, however, analysis of variance is usually not used except when dealing with three or more population means.

10.5 Analysis of Variance and the Completely Randomized Design

In this section we show how analysis of variance can be used to test for the equality of k population means for a completely randomized design. The general form of the hypotheses tested is

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k$$
$$H_a: \text{Not all population means are equal}$$

where

$$\mu_j = \text{mean of the } j\text{th population}$$

We assume that a simple random sample of size n_j has been selected from each of the k populations or treatments. For the resulting sample data, let

$$x_{ij} = \text{value of observation } i \text{ for treatment } j$$
$$n_j = \text{number of observations for treatment } j$$
$$\bar{x}_j = \text{sample mean for treatment } j$$
$$s_j^2 = \text{sample variance for treatment } j$$
$$s_j = \text{sample standard deviation for treatment } j$$

The formulas for the sample mean and sample variance for treatment j are as follows:

$$\bar{x}_j = \frac{\sum_{i=1}^{n_j} x_{ij}}{n_j} \tag{10.10}$$

$$s_j^2 = \frac{\sum_{i-1}^{n_j} (x_{ij} - \bar{x}_j)^2}{n_j - 1} \tag{10.11}$$

The overall sample mean, denoted $\bar{\bar{x}}$, is the sum of all the observations divided by the total number of observations. That is,

$$\bar{\bar{x}} = \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} x_{ij}}{n_T} \tag{10.12}$$

where

$$n_T = n_1 + n_2 + \cdots + n_k \tag{10.13}$$

If the size of each sample is n, $n_T = kn$; in this case equation (10.12) reduces to

$$\bar{\bar{x}} = \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} x_{ij}}{kn} = \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} x_{ij}/n}{k} = \frac{\sum_{j=1}^{k} \bar{x}_j}{k} \tag{10.14}$$

In other words, whenever the sample sizes are the same, the overall sample mean is just the average of the k sample means.

Because each sample in the Chemitech experiment consists of $n = 5$ observations, the overall sample mean can be computed by using equation (10.14). For the data in Table 10.3 we obtained the following result:

$$\bar{\bar{x}} = \frac{62 + 66 + 52}{3} = 60$$

If the null hypothesis is true ($\mu_1 = \mu_2 = \mu_3 = \mu$), the overall sample mean of 60 is the best estimate of the population mean μ.

Between-Treatments Estimate of Population Variance

In the preceding section, we introduced the concept of a between-treatments estimate of σ^2 and showed how to compute it when the sample sizes were equal. This estimate of σ^2 is called the *mean square due to treatments* and is denoted MSTR. The general formula for computing MSTR is

$$\text{MSTR} = \frac{\sum_{j=1}^{k} n_j(\bar{x}_j - \bar{\bar{x}})^2}{k - 1} \tag{10.15}$$

The numerator in equation (10.15) is called the *sum of squares due to treatments* and is denoted SSTR. The denominator, $k - 1$, represents the degrees of freedom associated with SSTR. Hence, the mean square due to treatments can be computed using the following formula.

MEAN SQUARE DUE TO TREATMENTS

$$\text{MSTR} = \frac{\text{SSTR}}{k - 1} \tag{10.16}$$

where

$$\text{SSTR} = \sum_{j=1}^{k} n_j(\bar{x}_j - \bar{\bar{x}})^2 \tag{10.17}$$

If H_0 is true, MSTR provides an unbiased estimate of σ^2. However, if the means of the k populations are not equal, MSTR is not an unbiased estimate of σ^2; in fact, in that case, MSTR should overestimate σ^2.

For the Chemitech data in Table 10.3, we obtain the following results:

$$\text{SSTR} = \sum_{j=1}^{k} n_j(\bar{x}_j - \bar{\bar{x}})^2 = 5(62 - 60)^2 + 5(66 - 60)^2 + 5(52 - 60)^2 = 520$$

$$\text{MSTR} = \frac{\text{SSTR}}{k - 1} = \frac{520}{2} = 260$$

Within-Treatments Estimate of Population Variance

Earlier, we introduced the concept of a within-treatments estimate of σ^2 and showed how to compute it when the sample sizes were equal. This estimate of σ^2 is called the *mean square due to error* and is denoted MSE. The general formula for computing MSE is

$$\text{MSE} = \frac{\sum_{j=1}^{k}(n_j - 1)s_j^2}{n_T - k} \tag{10.18}$$

The numerator in equation (10.18) is called the *sum of squares due to error* and is denoted SSE. The denominator of MSE is referred to as the degrees of freedom associated with SSE. Hence, the formula for MSE can also be stated as follows.

MEAN SQUARE DUE TO ERROR

$$\text{MSE} = \frac{\text{SSE}}{n_T - k} \tag{10.19}$$

where

$$\text{SSE} = \sum_{j=1}^{k}(n_j - 1)s_j^2 \tag{10.20}$$

Note that MSE is based on the variation within each of the treatments; it is not influenced by whether the null hypothesis is true. Thus, MSE always provides an unbiased estimate of σ^2.

For the Chemitech data in Table 10.3 we obtain the following results:

$$\text{SSE} = \sum_{j=1}^{k}(n_j - 1)s_j^2 = (5 - 1)27.5 + (5 - 1)26.5 + (5 - 1)31 = 340$$

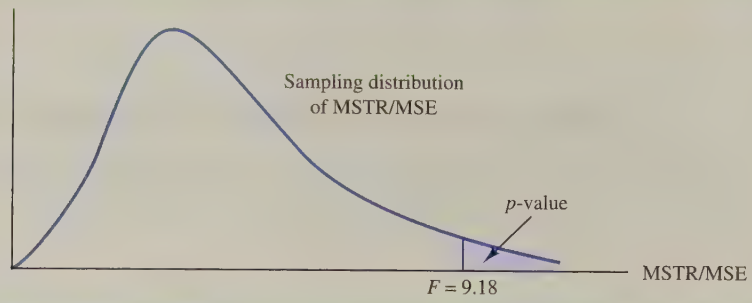
$$\text{MSE} = \frac{\text{SSE}}{n_T - k} = \frac{340}{15 - 3} = \frac{340}{12} = 28.33$$

Comparing the Variance Estimates: The F Test

If the null hypothesis is true, MSTR and MSE provide two independent, unbiased estimates of σ^2. If the ANOVA assumptions are also valid, the sampling distribution of MSTR/MSE is an ***F* distribution** with numerator degrees of freedom equal to $k - 1$ and denominator degrees of freedom equal to $n_T - k$. The general shape of the F distribution is shown in Figure 10.13. If the null hypothesis is true, the value of MSTR/MSE should appear to be from this F distribution.

However, if the null hypothesis is false, the value of MSTR/MSE will be inflated because MSTR overestimates σ^2. Hence, we will reject H_0 if the resulting value of MSTR/MSE appears to be too large to have been selected from an F distribution with $k - 1$ numerator degrees of freedom and $n_T - k$ denominator degrees of freedom. Because the decision to

FIGURE 10.13 COMPUTATION OF p-VALUE USING THE SAMPLING DISTRIBUTION OF MSTR/MSE



reject H_0 is based on the value of MSTR/MSE, the test statistic used to test for the equality of k population means is as follows.

TEST STATISTIC FOR THE EQUALITY OF k POPULATION MEANS

$$F = \frac{\text{MSTR}}{\text{MSE}} \tag{10.21}$$

The test statistic follows an F distribution with $k - 1$ degrees of freedom in the numerator and $n_T - k$ degrees of freedom in the denominator.

Let us return to the Chemitech experiment and use a level of significance $\alpha = .05$ to conduct the hypothesis test. The value of the test statistic is

$$F = \frac{\text{MSTR}}{\text{MSE}} = \frac{260}{28.33} = 9.18$$

The numerator degrees of freedom is $k - 1 = 3 - 1 = 2$ and the denominator degrees of freedom is $n_T - k = 15 - 3 = 12$. Because we will only reject the null hypothesis for large values of the test statistic, the p-value is the upper tail area of the F distribution to the right of the test statistic $F = 9.18$. Figure 10.13 shows the sampling distribution of $F = \text{MSTR/MSE}$, the value of the test statistic, and the upper tail area that is the p-value for the hypothesis test.

From Table 4 of Appendix B we find the following areas in the upper tail of an F distribution with 2 numerator degrees of freedom and 12 denominator degrees of freedom.

Area in Upper Tail	**.10**	**.05**	**.025**	**.01**
***F* Value ($df_1 = 2, df_2 = 12$)**	2.81	3.89	5.10	6.93

$F = 9.18$

Using Excel, p-value = F.DIST.RT(9.18,2,12) = .004.

Because $F = 9.18$ is greater than 6.93, the area in the upper tail at $F = 9.18$ is less than .01. Thus, the p-value is less than .01. Excel can be used to show that the exact p-value is .004. With p-value $\leq \alpha = .05$, H_0 is rejected. The test provides sufficient evidence to conclude that the means of the three populations are not equal. In other words, analysis of variance supports the conclusion that the population mean number of units produced per week for the three assembly methods are not equal.

As with other hypothesis testing procedures, the critical value approach may also be used. With $\alpha = .05$, the critical F value occurs with an area of .05 in the upper tail of an F distribution with 2 and 12 degrees of freedom. From the F distribution table, we find $F_{.05} = 3.89$. Hence, the appropriate upper tail rejection rule for the Chemitech experiment is

$$\text{Reject } H_0 \text{ if } F \geq 3.89$$

With $F = 9.18$, we reject H_0 and conclude that the means of the three populations are not equal. A summary of the overall procedure for testing for the equality of k population means follows.

TEST FOR THE EQUALITY OF k POPULATION MEANS

H_0: $\mu_1 = \mu_2 = \cdots = \mu_k$

H_a: Not all population means are equal

TEST STATISTIC

$$F = \frac{\text{MSTR}}{\text{MSE}}$$

REJECTION RULE

p-value approach:	Reject H_0 if p-value $\leq \alpha$
Critical value approach:	Reject H_0 if $F \geq F_\alpha$

where the value of F_α is based on an F distribution with $k - 1$ numerator degrees of freedom and $n_T - k$ denominator degrees of freedom.

ANOVA Table

The results of the preceding calculations can be displayed conveniently in a table referred to as the analysis of variance or **ANOVA table**. The general form of the ANOVA table for a completely randomized design is shown in Table 10.4; Table 10.5 is the corresponding ANOVA table for the Chemitech experiment. The sum of squares associated with the source of variation referred to as "Total" is called the total sum of squares (SST). Note that the results for the Chemitech experiment suggest that SST = SSTR + SSE, and that the degrees of freedom associated with this total sum of squares is the sum of the degrees of freedom associated with the sum of squares due to treatments and the sum of squares due to error.

We point out that SST divided by its degrees of freedom $n_T - 1$ is nothing more than the overall sample variance that would be obtained if we treated the entire set of 15 observations

TABLE 10.4 ANOVA TABLE FOR A COMPLETELY RANDOMIZED DESIGN

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Treatments	SSTR	$k - 1$	$\text{MSTR} = \dfrac{\text{SSTR}}{k - 1}$	$\dfrac{\text{MSTR}}{\text{MSE}}$	
Error	SSE	$n_T - k$	$\text{MSE} = \dfrac{\text{SSE}}{n_T - k}$		
Total	SST	$n_T - 1$			

TABLE 10.5 ANOVA TABLE FOR THE CHEMITECH EXPERIMENT

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Treatments	520	2	260.00	9.18	.004
Error	340	12	28.33		
Total	860	14			

as one data set. With the entire data set as one sample, the formula for computing the total sum of squares, SST, is

$$\text{SST} = \sum_{j=1}^{k} \sum_{i=1}^{n_j} (x_{ij} - \bar{\bar{x}})^2 \qquad \textbf{(10.22)}$$

It can be shown that the results we observed for the analysis of variance table for the Chemitech experiment also apply to other problems. That is,

$$\text{SST} = \text{SSTR} + \text{SSE} \qquad \textbf{(10.23)}$$

Analysis of variance can be thought of as a statistical procedure for partitioning the total sum of squares into separate components.

In other words, SST can be partitioned into two sums of squares: the sum of squares due to treatments and the sum of squares due to error. Note also that the degrees of freedom corresponding to SST, $n_T - 1$, can be partitioned into the degrees of freedom corresponding to SSTR, $k - 1$, and the degrees of freedom corresponding to SSE, $n_T - k$. The analysis of variance can be viewed as the process of **partitioning** the total sum of squares and the degrees of freedom into their corresponding sources: treatments and error. Dividing the sum of squares by the appropriate degrees of freedom provides the variance estimates, the F value, and the p-value used to test the hypothesis of equal population means.

Using Excel

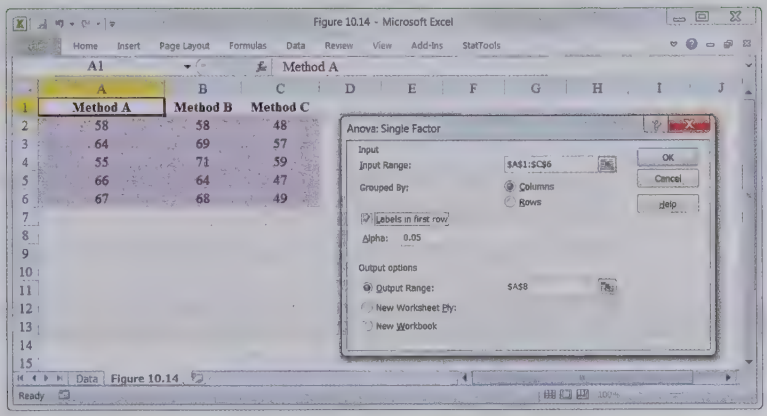
Excel's Anova: Single Factor tool can be used to conduct a hypothesis test about the difference between the population means for the Chemitech experiment. Refer to the Excel worksheets shown in Figure 10.14 and Figure 10.15 as we describe the tasks involved.

Enter Data: Columns A, B, and C in Figure 10.14 contain labels and the number of filtration systems produced per week for each method of assembly.

Apply Tools: The following steps describe how to use Excel's Anova: Single Factor tool to test the hypothesis that the mean number of units produced per week is the same for all three methods of assembly.

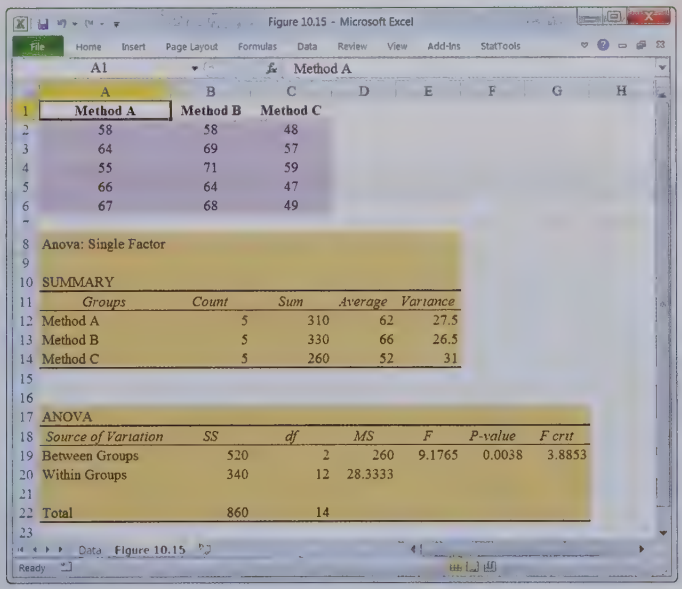
Step 1. Click the **Data** tab on the Ribbon
Step 2. In the **Analysis** group, click **Data Analysis**
Step 3. Choose **Anova: Single Factor** from the list of Analysis Tools
Step 4. When the Anova: Single Factor dialog box appears (Figure 10.14),
Enter A1:C6 in the **Input Range** box
Select **Grouped By: Columns**
Select **Labels in First Row**
Enter .05 in the **Alpha** box
Select **Output Range**
Enter A8 in the **Output Range** box (to identify the upper left corner of the section of the worksheet where the output will appear)
Click **OK**

FIGURE 10.14 DIALOG BOX FOR EXCEL'S ANOVA: SINGLE FACTOR TOOL



The output, titled *Anova: Single Factor,* appears in cells A8:G22 of the worksheet in Figure 10.15. Cells A10:E14 provide a summary of the statistics for each method. Note that the sample mean and sample variance for each method of assembly are the same as shown in Table 10.3. The ANOVA table, shown in cells A17:G22, is basically the same as the ANOVA table shown in Table 10.5. Excel identifies the treatments source of variation using

FIGURE 10.15 EXCEL RESULTS FOR THE HYPOTHESIS TEST ABOUT EQUALITY OF THREE POPULATION MEANS



the label *Between Groups* and the error source of variation using the label *Within Groups*. In addition, the Excel output provides the *p*-value associated with the test as well as the critical *F* value.

We can use the *p*-value shown in cell F19, 0.0038, to make the hypothesis testing decision. Thus, at the $\alpha = .05$ level of significance, we reject H_0 because the *p*-value = $0.0038 < \alpha = .05$. Hence, using the *p*-value approach we still conclude that the mean number of units produced per week are not the same for the three assembly methods.

Testing for the Equality of *k* Population Means: An Observational Study

We have shown how analysis of variance can be used to test for the equality of *k* population means for a completely randomized experimental design. It is important to understand that ANOVA can also be used to test for the equality of three or more population means using data obtained from an observational study. As an example, let us consider the situation at National Computer Products, Inc. (NCP).

NCP manufactures printers and fax machines at plants located in Atlanta, Dallas, and Seattle. To measure how much employees at these plants know about quality management, a random sample of 6 employees was selected from each plant and the employees selected were given a quality awareness examination. The examination scores for these 18 employees are shown in Table 10.6. The sample means, sample variances, and sample standard deviations for each group are also provided. Managers want to use these data to test the hypothesis that the mean examination score is the same for all three plants.

We define population 1 as all employees at the Atlanta plant, population 2 as all employees at the Dallas plant, and population 3 as all employees at the Seattle plant. Let

μ_1 = mean examination score for population 1
μ_2 = mean examination score for population 2
μ_3 = mean examination score for population 3

Although we will never know the actual values of μ_1, μ_2, and μ_3, we want to use the sample results to test the following hypotheses:

H_0: $\mu_1 = \mu_2 = \mu_3$
H_a: Not all population means are equal

TABLE 10.6 EXAMINATION SCORES FOR 18 EMPLOYEES

	Plant 1 Atlanta	Plant 2 Dallas	Plant 3 Seattle
	85	71	59
	75	75	64
	82	73	62
	76	74	69
	71	69	75
	85	82	67
Sample mean	79	74	66
Sample variance	34	20	32
Sample standard deviation	5.83	4.47	5.66

WEB file NCP

Exercise 35 will ask you to analyze the NCP data using the analysis of variance procedure.

Note that the hypothesis test for the NCP observational study is exactly the same as the hypothesis test for the Chemitech experiment. Indeed, the same analysis of variance methodology we used to analyze the Chemitech experiment can also be used to analyze the data from the NCP observational study.

Even though the same ANOVA methodology is used for the analysis, it is worth noting how the NCP observational statistical study differs from the Chemitech experimental statistical study. The individuals who conducted the NCP study had no control over how the plants were assigned to individual employees. That is, the plants were already in operation and a particular employee worked at one of the three plants. All that NCP could do was to select a random sample of 6 employees from each plant and administer the quality awareness examination. To be classified as an experimental study, NCP would have had to be able to randomly select 18 employees and then assign the plants to each employee in a random fashion.

NOTES AND COMMENTS

1. The overall sample mean can also be computed as a weighted average of the k sample means.

$$\bar{\bar{x}} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + \cdots + n_k\bar{x}_k}{n_T}$$

In problems where the sample means are provided, this formula is simpler than equation (10.12) for computing the overall mean.

2. If each sample consists of n observations, equation (10.15) can be written as

$$\text{MSTR} = \frac{n\sum_{j=1}^{k}(\bar{x}_j - \bar{\bar{x}})^2}{k-1} = n\left[\frac{\sum_{j=1}^{k}(\bar{x}_j - \bar{\bar{x}})^2}{k-1}\right] = ns_{\bar{x}}^2$$

Note that this result is the same as presented in Section 10.4 when we introduced the concept of the between-treatments estimate of σ^2. Equation (10.15) is simply a generalization of this result to the unequal sample-size case.

3. If each sample has n observations, $n_T = kn$; thus, $n_T - k = k(n-1)$, and equation (10.18) can be rewritten as

$$\text{MSE} = \frac{\sum_{j=1}^{k}(n-1)s_j^2}{k(n-1)} = \frac{(n-1)\sum_{j=1}^{k}s_j^2}{k(n-1)} = \frac{\sum_{j=1}^{k}s_j^2}{k}$$

In other words, if the sample sizes are the same, MSE is just the average of the k sample variances. Note that it is the same result we used in Section 10.4 when we introduced the concept of the within-treatments estimate of σ^2.

Exercises

Methods

SELF test

28. The following data are from a completely randomized design.

	Treatment		
	A	B	C
	162	142	126
	142	156	122
	165	124	138
	145	142	140
	148	136	150
	174	152	128
Sample mean	156	142	134
Sample variance	164.4	131.2	110.4

a. Compute the sum of squares between treatments.
b. Compute the mean square between treatments.
c. Compute the sum of squares due to error.
d. Compute the mean square due to error.
e. Set up the ANOVA table for this problem.
f. At the $\alpha = .05$ level of significance, test whether the means for the three treatments are equal.

29. In a completely randomized design, seven experimental units were used for each of the five levels of the factor. Complete the following ANOVA table.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	*F*	*p*-value
Treatments	300				
Error					
Total	460				

30. Refer to exercise 29.
a. What hypotheses are implied in this problem?
b. At the $\alpha = .05$ level of significance, can we reject the null hypothesis in part (a)? Explain.

31. In an experiment designed to test the output levels of three different treatments, the following results were obtained: SST = 400, SSTR = 150, $n_T = 19$. Set up the ANOVA table and test for any significant difference between the mean output levels of the three treatments. Use $\alpha = .05$.

32. In a completely randomized design, 12 experimental units were used for the first treatment, 15 for the second treatment, and 20 for the third treatment. Complete the following analysis of variance. At a .05 level of significance, is there a significant difference between the treatments?

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	*F*	*p*-value
Treatments	1200				
Error					
Total	1800				

33. Develop the analysis of variance computations for the following completely randomized design. At $\alpha = .05$, is there a significant difference between the treatment means?

WEB file
Exer33

	Treatment		
	A	B	C
	136	107	92
	120	114	82
	113	125	85
	107	104	101
	131	107	89
	114	109	117
	129	97	110
	102	114	120
		104	98
		89	106
$\bar{x}_j$	119	107	100
s_j^2	146.86	96.44	173.78

Applications

34. Three different methods for assembling a product were proposed by an industrial engineer. To investigate the number of units assembled correctly with each method, 30 employees were randomly selected and randomly assigned to the three proposed methods in such a way that each method was used by 10 workers. The number of units assembled correctly was recorded, and the analysis of variance procedure was applied to the resulting data set. The following results were obtained: SST = 10,800; SSTR = 4560.
    a. Set up the ANOVA table for this problem.
    b. Use $\alpha = .05$ to test for any significant difference in the means for the three assembly methods.

35. Refer to the NCP data in Table 10.6. Set up the ANOVA table and test for any significant difference in the mean examination score for the three plants. Use $\alpha = .05$.

36. To study the effect of temperature on yield, in pounds, for a chemical process, five batches were produced at each of three temperature levels. The results follow. Construct an analysis of variance table. Use a .05 level of significance to test whether the temperature level has an effect on the mean yield of the process.

Temperature		
50°C	60°C	70°C
34	30	23
24	31	28
36	34	28
39	23	30
32	27	31

37. Auditors must make judgments about various aspects of an audit on the basis of their own direct experience, indirect experience, or a combination of the two. In a study, auditors were asked to make judgments about the frequency of errors to be found in an audit. The judgments by the auditors were then compared to the actual results. Suppose the following data were obtained from a similar study; lower scores indicate better judgments.

WEB file
AudJudg

Direct	Indirect	Combination
17.0	16.6	25.2
18.5	22.2	24.0
15.8	20.5	21.5
18.2	18.3	26.8
20.2	24.2	27.5
16.0	19.8	25.8
13.3	21.2	24.2

Use $\alpha = .05$ to test to see whether the basis for the judgment affects the quality of the judgment. What is your conclusion?

38. Four different paints are advertised as having the same drying time. To check the manufacturer's claims, five samples were tested for each of the paints. The time in minutes until the paint was dry enough for a second coat to be applied was recorded. The following data were obtained.

WEB file
Paint

Paint 1	Paint 2	Paint 3	Paint 4
128	144	133	150
137	133	143	142
135	142	137	135
124	146	136	140
141	130	131	153

At the $\alpha = .05$ level of significance, test to see whether the mean drying time is the same for each type of paint.

39. The *Consumer Reports* Restaurant Customer Satisfaction Survey is based upon 148,599 visits to full-service restaurant chains (Consumer Reports website). One of the variables in the study is meal price, the average amount paid per person for dinner and drinks, minus the tip. Suppose a reporter for the *Sun Coast Times* thought that it would be of interest to her readers to conduct a similar study for restaurants located on the Grand Strand section in Myrtle Beach, South Carolina. The reporter selected a sample of eight seafood restaurants, eight Italian restaurants, and eight steakhouses. The following data show the meal prices ($) obtained for the 24 restaurants sampled. Use $\alpha = .05$ to test whether there is a significant difference among the mean meal price for the three types of restaurants.

WEB file
GrandStrand

Italian	Seafood	Steakhouse
$12	$16	$24
13	18	19
15	17	23
17	26	25
18	23	21
20	15	22
17	19	27
24	18	31

Summary

In this chapter we discussed procedures for developing interval estimates and conducting hypothesis tests involving two populations. First, we showed how to make inferences about the difference between two population means when independent simple random samples are selected. We first considered the case where the population standard deviations σ_1 and σ_2 could be assumed known. The standard normal distribution z was used to develop the interval estimate and served as the test statistic for hypothesis tests. We then considered the case where the population standard deviations were unknown and estimated by the sample standard deviations s_1 and s_2. In this case, the t distribution was used to develop the interval estimate and the t value served as the test statistic for hypothesis tests.

Inferences about the difference between two population means were then discussed for the matched sample design. In the matched sample design each element provides a pair of data values, one from each population. The difference between the paired data values is then used in the statistical analysis. The matched sample design is generally preferred to the independent sample design because the matched-sample procedure often improves the precision of the estimate.

In the final two sections we provided an introduction to experimental design and analysis of variance (ANOVA). Experimental studies differ from observational studies in the sense that an experiment is conducted to generate the data. The completely randomized design was described and the analysis of variance was used to test for a treatment effect. The

same analysis of variance procedure can be used to test for the difference among k population means in an observational study.

Glossary

Independent simple random samples Samples selected from two populations in such a way that the elements making up one sample are chosen independently of the elements making up the other sample.

Matched samples One simple random sample of elements is selected and two data values are obtained for each element. For example, to compare two production methods, one simple random sample of n workers is selected. Each worker first uses one method and then the other method. The order of the two methods is assigned randomly.

Factor Another word for the independent variable of interest.

Treatments Different levels of a factor.

Single-factor experiment An experiment involving only one factor with k populations or treatments.

Response variable Another word for the dependent variable of interest.

Experimental units The objects of interest in the experiment.

Completely randomized design An experimental design in which the treatments are randomly assigned to the experimental units.

***F* distribution** A probability distribution based on the ratio of two independent estimates of the variance of a normal population. The F distribution is used in hypothesis tests about the equality of k population means.

ANOVA table A table used to summarize the analysis of variance computations and results. It contains columns showing the source of variation, the sum of squares, the degrees of freedom, the mean square, the F value(s), and the p-value(s).

Key Formulas

Point Estimator of the Difference Between Two Population Means

$$\bar{x}_1 - \bar{x}_2 \tag{10.1}$$

Standard Error of $\bar{x}_1 - \bar{x}_2$

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \tag{10.2}$$

Interval Estimate of the Difference Between Two Population Means: σ_1 and σ_2 Known

$$\bar{x}_1 - \bar{x}_2 \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \tag{10.4}$$

Test Statistic for Hypothesis Tests About $\mu_1 - \mu_2$: σ_1 and σ_2 Known

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \tag{10.5}$$

Interval Estimate of the Difference Between Two Population Means: σ_1 and σ_2 Unknown

$$\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \tag{10.6}$$

Degrees of Freedom: t Distribution with Two Independent Random Samples

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2} \tag{10.7}$$

Test Statistic for Hypothesis Tests About $\mu_1 - \mu_2$: σ_1 and σ_2 Unknown

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \tag{10.8}$$

Test Statistic for Hypothesis Tests Involving Matched Samples

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} \tag{10.9}$$

Sample Mean for Treatment j

$$\bar{x}_j = \frac{\sum_{i=1}^{n_j} x_{ij}}{n_j} \tag{10.10}$$

Sample Variance for Treatment j

$$s_j^2 = \frac{\sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2}{n_j - 1} \tag{10.11}$$

Overall Sample Mean

$$\bar{\bar{x}} = \frac{\sum_{j=1}^{k} \sum_{i=1}^{n_j} x_{ij}}{n_T} \tag{10.12}$$

$$n_T = n_1 + n_2 + \cdots + n_k \tag{10.13}$$

Mean Square Due to Treatments

$$\text{MSTR} = \frac{\text{SSTR}}{k - 1} \tag{10.16}$$

Sum of Squares Due to Treatments

$$\text{SSTR} = \sum_{j=1}^{k} n_j (\bar{x}_j - \bar{\bar{x}})^2 \tag{10.17}$$

Mean Square Due to Error

$$\text{MSE} = \frac{\text{SSE}}{n_T - k} \tag{10.19}$$

Sum of Squares Due to Error

$$\text{SSE} = \sum_{j=1}^{k} (n_j - 1) s_j^2 \tag{10.20}$$

Test Statistic for the Equality of k Population Means

$$F = \frac{\text{MSTR}}{\text{MSE}} \tag{10.21}$$

Total Sum of Squares

$$\text{SST} = \sum_{j=1}^{k} \sum_{i=1}^{n_j} (x_{ij} - \bar{\bar{x}})^2 \tag{10.22}$$

Partitioning of Sum of Squares

$$\text{SST} = \text{SSTR} + \text{SSE} \tag{10.23}$$

Supplementary Exercises

40. How much is the cost of a hospital stay increasing? The mean cost of one day in a semiprivate room was reported to be $4848 in 2005 and $5260 in 2006 (*The Wall Street Journal*, January 2, 2007). Assume the estimate for 2005 is a sample mean based on a sample size of 80 and the estimate for 2006 is a sample mean based on a sample size of 60.
    a. Develop a point estimate of the increase in the cost of a semiprivate hospital room from 2005 to 2006.
    b. Historical data indicate that a population standard deviation of $800 is a reasonable assumption for both years. Compute the margin of error for your estimate in part (a). Use 95% confidence.
    c. Develop a 95% confidence interval estimate of the increase in cost for a semiprivate room.

WEB file
HomePrices

41. Home values tend to increase over time under normal conditions, but the recession of 2008 and 2009 has reportedly caused the resale price of existing homes to fall nationwide (*BusinessWeek*, March 9, 2009). You would like to see if the data support this conclusion. The file HomePrices contains data on 30 existing home sales in 2006 and 40 existing home sales in 2009.
    a. Provide a point estimate of the difference between the population mean prices for the two years.
    b. Develop a 99% confidence interval estimate of the difference between the resale prices of houses in 2006 and 2009.
    c. Would you feel justified in concluding that resale prices of existing homes have declined from 2006 to 2009? Why or why not?

42. Mutual funds are classified as *load* or *no-load* funds. Load funds require an investor to pay an initial fee based on a percentage of the amount invested in the fund. The no-load funds do not require this initial fee. Some financial advisors argue that the load mutual funds may be worth the extra fee because these funds provide a higher mean rate of return than the no-load mutual funds. A sample of 30 load mutual funds and a sample of 30 no-load mutual funds were selected. Data were collected on the annual return for the funds over a five-year period. The data are contained in the data set Mutual. The data for the first five load and first five no-load mutual funds are as follows.

WEB file
Mutual

Mutual Funds—Load	Return	Mutual Funds—No Load	Return
American National Growth	15.51	Amana Income Fund	13.24
Arch Small Cap Equity	14.57	Berger One Hundred	12.13
Bartlett Cap Basic	17.73	Columbia International Stock	12.17
Calvert World International	10.31	Dodge & Cox Balanced	16.06
Colonial Fund A	16.23	Evergreen Fund	17.61

a. Formulate H_0 and H_a such that rejection of H_0 leads to the conclusion that the load mutual funds have a higher mean annual return over the five-year period.
b. Use the 60 mutual funds in the data set Mutual to conduct the hypothesis test. What is the p-value? At $\alpha = .05$, what is your conclusion?

43. The National Association of Home Builders provided data on the cost of the most popular home remodeling projects. Sample data on cost in thousands of dollars for two types of remodeling projects are as follows.

Kitchen	Master Bedroom	Kitchen	Master Bedroom
25.2	18.0	23.0	17.8
17.4	22.9	19.7	24.6
22.8	26.4	16.9	21.0
21.9	24.8	21.8	
19.7	26.9	23.6	

a. Develop a point estimate of the difference between the population mean remodeling costs for the two types of projects.
b. Develop a 90% confidence interval for the difference between the two population means.

44. In early 2009, the economy was experiencing a recession. But how was the recession affecting the stock market? Shown are data from a sample of 15 companies. Shown for each company is the price per share of stock on January 1 and April 30 (*The Wall Street Journal*, May 1, 2009).

WEB file
PriceChange

Company	January 1 ($)	April 30 ($)
Applied Materials	10.13	12.21
Bank of New York	28.33	25.48
Chevron	73.97	66.10
Cisco Systems	16.30	19.32
Coca-Cola	45.27	43.05
Comcast	16.88	15.46
Ford Motors	2.29	5.98
General Electric	16.20	12.65
Johnson & Johnson	59.83	52.36
JP Morgan Chase	31.53	33.00
Microsoft	19.44	20.26
Oracle	17.73	19.34
Pfizer	17.71	13.36
Philip Morris	43.51	36.18
Procter & Gamble	61.82	49.44

a. What is the change in the mean price per share of stock over the four-month period?
b. Provide a 90% confident interval estimate of the change in the mean price per share of stock. Interpret the results.
c. What was the percentage change in the mean price per share of stock over the four-month period?
d. If this same percentage change were to occur for the next four months and again for the four months after that, what would be the mean price per share of stock at the end of the year 2009?

45. In a completely randomized experimental design, three brands of paper towels were tested for their ability to absorb water. Equal-size towels were used, with four sections of towels tested per brand. The absorbency rating data follow. At a .05 level of significance, does there appear to be a difference in the ability of the brands to absorb water?

Brand		
x	*y*	*z*
91	99	83
100	96	88
88	94	89
89	99	76

46. A study reported in the *Journal of Small Business Management* concluded that self-employed individuals do not experience higher job satisfaction than individuals who are not self-employed. In this study, job satisfaction is measured using 18 items, each of which is rated using a Likert-type scale with 1–5 response options ranging from strong agreement to strong disagreement. A higher score on this scale indicates a higher degree of job satisfaction. The sum of the ratings for the 18 items, ranging from 18 to 90, is used as the measure of job satisfaction. Suppose that this approach was used to measure the job satisfaction for lawyers, physical therapists, cabinetmakers, and systems analysts. The results obtained for a sample of 10 individuals from each profession follow.

WEB file SatisJob

Lawyer	Physical Therapist	Cabinetmaker	Systems Analyst
44	55	54	44
42	78	65	73
74	80	79	71
42	86	69	60
53	60	79	64
50	59	64	66
45	62	59	41
48	52	78	55
64	55	84	76
38	50	60	62

At the $\alpha = .05$ level of significance, test for any difference in the job satisfaction among the four professions.

47. *Money* magazine reports percentage returns and expense ratios for stock and bond funds. The following data are the expense ratios for 10 midcap stock funds, 10 small-cap stock funds, 10 hybrid stock funds, and 10 specialty stock funds (*Money,* March 2003).

WEB file Funds

Midcap	Small-Cap	Hybrid	Specialty
1.2	2.0	2.0	1.6
1.1	1.2	2.7	2.7
1.0	1.7	1.8	2.6
1.2	1.8	1.5	2.5
1.3	1.5	2.5	1.9
1.8	2.3	1.0	1.5
1.4	1.9	0.9	1.6
1.4	1.3	1.9	2.7
1.0	1.2	1.4	2.2
1.4	1.3	0.3	0.7

Use $\alpha = .05$ to test for any significant difference in the mean expense ratio among the four types of stock funds.

48. The U.S. Census Bureau computes quarterly vacancy and homeownership rates by state and metropolitan statistical area. Each metropolitan statistical area (MSA) has at least one urbanized area of 50,000 or more inhabitants. The following data are the rental vacancy rates (%) for MSAs in four geographic regions of the United States for the first quarter of 2008 (U.S. Census Bureau website, January 2009).

Midwest	Northeast	South	West
16.2	2.7	16.6	7.9
10.1	11.5	8.5	6.6
8.6	6.6	12.1	6.9
12.3	7.9	9.8	5.6
10.0	5.3	9.3	4.3
16.9	10.7	9.1	15.2
16.9	8.6	5.6	5.7
5.4	5.5	9.4	4.0
18.1	12.7	11.6	12.3
11.9	8.3	15.6	3.6
11.0	6.7	18.3	11.0
9.6	14.2	13.4	12.1
7.6	1.7	6.5	8.7
12.9	3.6	11.4	5.0
12.2	11.5	13.1	4.7
13.6	16.3	4.4	3.3
		8.2	3.4
		24.0	5.5
		12.2	
		22.6	
		12.0	
		14.5	
		12.6	
		9.5	
		10.1	

Use $\alpha = .05$ to test whether the mean vacancy rate is the same for each geographic region.

49. Three different assembly methods have been proposed for a new product. A completely randomized experimental design was chosen to determine which assembly method results in the greatest number of parts produced per hour, and 30 workers were randomly selected and assigned to use one of the proposed methods. The number of units produced by each worker follows.

	Method	
A	B	C
97	93	99
73	100	94
93	93	87
100	55	66
73	77	59
91	91	75
100	85	84
86	73	72
92	90	88
95	83	86

Use these data and test to see whether the mean number of parts produced is the same with each method. Use $\alpha = .05$.

50. In a study conducted to investigate browsing activity by shoppers, each shopper was initially classified as a nonbrowser, light browser, or heavy browser. For each shopper, the study obtained a measure to determine how comfortable the shopper was in a store. Higher scores indicated greater comfort. Suppose the following data were collected.

WEB file
Browsing

Nonbrowser	Light Browser	Heavy Browser
4	5	5
5	6	7
6	5	5
3	4	7
3	7	4
4	4	6
5	6	5
4	5	7

Use $\alpha = .05$ to test for differences among comfort levels for the three types of browsers.

Case Problem 1 Par, Inc.

Par, Inc., is a major manufacturer of golf equipment. Management believes that Par's market share could be increased with the introduction of a cut-resistant, longer-lasting golf ball. Therefore, the research group at Par has been investigating a new golf ball coating designed to resist cuts and provide a more durable ball. The tests with the coating have been promising.

One of the researchers voiced concern about the effect of the new coating on driving distances. Par would like the new cut-resistant ball to offer driving distances comparable to those of the current-model golf ball. To compare the driving distances for the two balls, 40 balls of both the new and current models were subjected to distance tests. The testing was performed with a mechanical hitting machine so that any difference between the mean distances for the two models could be attributed to a difference in the two models. The results of the tests, with distances measured to the nearest yard, follow. These data are available on the website that accompanies this text in the file named Golf.

WEB file
Golf

Model		Model		Model		Model	
Current	New	Current	New	Current	New	Current	New
264	277	270	272	263	274	281	283
261	269	287	259	264	266	274	250
267	263	289	264	284	262	273	253
272	266	280	280	263	271	263	260
258	262	272	274	260	260	275	270
283	251	275	281	283	281	267	263
258	262	265	276	255	250	279	261
266	289	260	269	272	263	274	255
259	286	278	268	266	278	276	263
270	264	275	262	268	264	262	279

Managerial Report

1. Formulate and present the rationale for a hypothesis test that Par could use to compare the driving distances of the current and new golf balls.
2. Analyze the data to provide the hypothesis testing conclusion. What is the p-value for your test? What is your recommendation for Par, Inc.?
3. Provide descriptive statistical summaries of the data for each model.
4. What is the 95% confidence interval for the population mean of each model, and what is the 95% confidence interval for the difference between the means of the two populations?
5. Do you see a need for larger sample sizes and more testing with the golf balls? Discuss.

Case Problem 2 Wentworth Medical Center

As part of a long-term study of individuals 65 years of age or older, sociologists and physicians at the Wentworth Medical Center in upstate New York investigated the relationship between geographic location and depression. A sample of 60 individuals, all in reasonably good health, was selected; 20 individuals were residents of Florida, 20 were residents of New York, and 20 were residents of North Carolina. Each of the individuals sampled was given a standardized test to measure depression. The data collected follow; higher test scores indicate higher levels of depression. These data are available on the website that accompanies this text in the file named Medical1.

A second part of the study considered the relationship between geographic location and depression for individuals 65 years of age or older who had a chronic health condition such as arthritis, hypertension, and/or heart ailment. A sample of 60 individuals with such conditions was identified. Again, 20 were residents of Florida, 20 were residents of New York, and 20 were residents of North Carolina. The levels of depression recorded for this study follow. These data are available on the website that accompanies this text in the file named Medical2.

Medical1

Medical2

Data from Medical1			Data from Medical2		
Florida	New York	North Carolina	Florida	New York	North Carolina
3	8	10	13	14	10
7	11	7	12	9	12
7	9	3	17	15	15
3	7	5	17	12	18
8	8	11	20	16	12
8	7	8	21	24	14
8	8	4	16	18	17
5	4	3	14	14	8
5	13	7	13	15	14
2	10	8	17	17	16
6	6	8	12	20	18
2	8	7	9	11	17
6	12	3	12	23	19
6	8	9	15	19	15
9	6	8	16	17	13
7	8	12	15	14	14
5	5	6	13	9	11
4	7	3	10	14	12
7	7	8	11	13	13
3	8	11	17	11	11

Managerial Report

1. Use descriptive statistics to summarize the data from the two studies. What are your preliminary observations about the depression scores?
2. Use analysis of variance on both data sets. State the hypotheses being tested in each case. What are your conclusions?
3. Use inferences about individual treatment means where appropriate. What are your conclusions?

Appendix Comparisons Involving Means Using StatTools

In this appendix we show how StatTools can be used to develop interval estimates and conduct hypothesis tests about the difference between two population means for the σ_1 and σ_2 unknown case. We also show how StatTools can be used to test for the equality of k population means for a completely randomized design.

Interval Estimation of μ_1 and μ_2

We will use the data for the checking account balances example presented in Section 10.2. Begin by using the Data Set Manager to create a StatTools data set for these data using the procedure described in the appendix in Chapter 1. The following steps can be used to compute a 95% confidence interval estimate of the difference between the two population means.

Step 1. Click the **StatTools** tab on the Ribbon
Step 2. In the **Analyses** group, click **Statistical Inference**
Step 3. Select the **Confidence Interval** option
Step 4. Choose **Mean/Std. Deviation**
Step 5. When the StatTools - Confidence Interval for Mean/Std. Deviation dialog box appears,

For **Analysis Type,** choose **Two-Sample Analysis**
In the **Variables** section,
  Select **Cherry Grove**
  Select **Beechmont**
In the **Confidence Intervals to Calculate** section,
  Select the **For the Difference of Means** option
  Select 95% for the **Confidence Level**
Click **OK**

Because the sample size for Cherry Grove ($n_1 = 28$) differs from the sample size for Beechmont ($n_2 = 22$), StatTools will inform you of this difference after you click OK in step 5. A dialog box will appear saying "The variable Beechmont contains missing data, which the analysis will ignore." Click OK. A Choose Variable Ordering dialog box then appears, indicating that the analysis will compare the difference between the Cherry Grove data set and the Beechmont data set. Click OK and the StatTools interval estimation output will appear.

Hypothesis Tests About μ_1 and μ_2

We will use the software evaluation example and the completion time data presented in Table 10.1. Begin by using the Data Set Manager to create a StatTools data set for these data using the procedure described in the appendix in Chapter 1. The following steps can be used to test the hypothesis H_0: $\mu_1 - \mu_2 \leq 0$ against H_a: $\mu_1 - \mu_2 > 0$.

Step 1. Click the **StatTools** tab on the Ribbon
Step 2. In the **Analyses** group, click **Statistical Inference**
Step 3. Choose the **Hypothesis Test** option
Step 4. Choose **Mean/Std. Deviation**
Step 5. When the StatTools - Hypothesis Test for Mean/Std. Deviation dialog box appears,
For **Analysis Type,** choose **Two-Sample Analysis**
In the **Variables** section,
Select **Current**
Select **New**
In the **Hypothesis Tests to Perform** section,
Select **Difference of Means**
Enter 0 in the **Null Hypothesis Value** box
Select **Greater Than Null Value (One-Tailed Test)** in the **Alternative Hypothesis** box
Click **OK**
When the Choose Variable Ordering dialog box appears, click **OK**

The results of the hypothesis test will then appear.

Inferences About the Difference Between Two Population Means: Matched Samples

StatTools can be used to develop interval estimates and conduct hypothesis tests for the difference between population means for the matched samples case. We will use the matched-sample completion times in Table 10.2 to illustrate.

Begin by using the Data Set Manager to create a StatTools data set for these data using the procedure described in the appendix in Chapter 1. The following steps can be used to compute a 95% confidence interval estimate of the difference between the population mean completion times.

Step 1. Click the **StatTools** tab on the Ribbon
Step 2. In the **Analyses** group, click **Statistical Inference**
Step 3. Choose the **Confidence Interval** option
Step 4. When the StatTools - Confidence Interval dialog box appears,
For **Analysis Type,** choose **Paired-Sample Analysis**
In the **Variables** section,
Select **Method 1**
Select **Method 2**
In the **Confidence Intervals to Calculate** section,
Select the **For the Difference of Means** option
Select 95% for the **Confidence Level**
If selected, remove the check in the **For the Standard Deviation** box
Click **OK**
When the Choose Variable Ordering dialog box appears, click **OK**

The confidence interval will appear.

Conducting hypothesis tests for the matched samples case is very similar to conducting hypothesis tests for the difference in two means shown above. After selecting the Hypothesis Test option in step 3, select the Paired-Sample Analysis option in step 4.

Analysis of a Completely Randomized Design

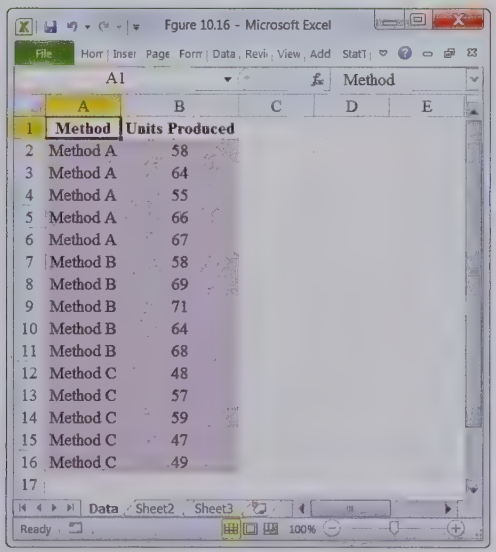
StatTools can be used to test for the equality of k population means for a completely randomized design. We use the Chemitech data in Table 10.3 to illustrate. Begin by using the Data Set Manager to create a StatTools data set for these data using the procedure described in the appendix in Chapter 1. The following steps can be used to test for the equality of the three population means.

WEB file
Chemitech

Step 1. Click the **StatTools** tab on the Ribbon
Step 2. In the **Analyses** group, click **Statistical Inference**
Step 3. Choose the **One-Way ANOVA** option
Step 4. When the One-Way ANOVA dialog box appears,
In the **Variables** section,
Click the **Format button** and select **Unstacked**
Select **Method A**
Select **Method B**
Select **Method C**
Select 95% in the **Confidence Level** box
Click **OK**

Note that in step 4 we selected the Unstacked option after clicking the Format button. The Unstacked option means that the data for the three treatments appear in separate columns of the worksheet. In a stacked format, only two columns would be used. For example, the data could have been organized as shown in Figure 10.16. Data are frequently recorded in a stacked format. For stacked data, simply select the Stacked option after clicking the Format button.

FIGURE 10.16 CHEMITECH DATA IN STACKED FORMAT



CHAPTER 11

Comparisons Involving Proportions and a Test of Independence

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- Interval Estimation of $p_1 - p_2$
- Using Excel to Construct a Confidence Interval
- Hypothesis Tests About $p_1 - p_2$
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11.3 TEST OF INDEPENDENCE
- Using Excel to Conduct a Test of Independence

STATISTICS in PRACTICE

UNITED WAY*

ROCHESTER, NEW YORK

United Way of Greater Rochester is a nonprofit organization dedicated to improving the quality of life for all people in the seven counties it serves by meeting the community's most important human care needs.

The annual United Way/Red Cross fund-raising campaign, conducted each spring, funds hundreds of programs offered by more than 200 service providers. These providers meet a wide variety of human needs—physical, mental, and social—and serve people of all ages, backgrounds, and economic means.

Because of enormous volunteer involvement, United Way of Greater Rochester is able to hold its operating costs at just eight cents of every dollar raised.

The United Way of Greater Rochester decided to conduct a survey to learn more about community perceptions of charities. Focus-group interviews were held with professional, service, and general worker groups to get preliminary information on perceptions. The information obtained was then used to help develop the questionnaire for the survey. The questionnaire was pretested, modified, and distributed to 440 individuals; 323 completed questionnaires were obtained.

A variety of descriptive statistics, including frequency distributions and crosstabulations, were provided from the data collected. An important part of the analysis involved the use of contingency tables and chi-square tests of independence. One use of such statistical tests was to determine whether perceptions of administrative expenses were independent of occupation.

The hypotheses for the test of independence were:

H_0: Perception of United Way administrative expenses is independent of the occupation of the respondent.

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Statistical surveys help United Way adjust its program to better meet the needs of the people it serves.

H_a: Perception of United Way administrative expenses is not independent of the occupation of the respondent.

Two questions in the survey provided the data for the statistical test. One question obtained data on perceptions of the percentage of funds going to administrative expenses (up to 10%, 11–20%, and 21% or more). The other question asked for the occupation of the respondent.

The chi-square test at a .05 level of significance led to rejection of the null hypothesis of independence and to the conclusion that perceptions of United Way's administrative expenses did vary by occupation. Actual administrative expenses were less than 9%, but 35% of the respondents perceived that administrative expenses were 21% or more. Hence, many had inaccurate perceptions of administrative costs. In this group, production-line, clerical, sales, and professional-technical employees had more inaccurate perceptions than other groups.

The community perceptions study helped United Way of Greater Rochester to develop adjustments to its program and fund-raising activities. In this chapter, you will learn how a statistical test of independence, such as that described here, is conducted.

*The authors are indebted to Dr. Philip R. Tyler, Marketing Consultant to the United Way, for providing this Statistics in Practice.

Many statistical applications call for a comparison of population proportions. In Section 11.1 we describe statistical inferences concerning differences in the proportions for two populations. Two samples are required, one from each population, and the statistical inference is based on the two sample proportions. The second section looks at a hypothesis test comparing the proportions of a single multinomial population with the proportions selected in a null hypothesis. One sample from the multinomial population is used, and the

hypothesis test is based on comparing the sample proportions with those stated in the null hypothesis. In the last section of the chapter, we show how contingency tables can be used to test for the independence of two variables. One sample is used for the test of independence, but measures on two variables are required for each sampled element. Both Sections 11.2 and 11.3 rely on the use of a chi-square statistical test.

11.1 Inferences About the Difference Between Two Population Proportions

Letting p_1 denote the proportion for population 1 and p_2 denote the proportion for population 2, we consider inferences about the difference between the two population proportions: $p_1 - p_2$. To make an inference about this difference, we will select two independent random samples consisting of n_1 units from population 1 and n_2 units from population 2.

Interval Estimation of $p_1 - p_2$

In the following example, we show how to compute a margin of error and develop an interval estimate of the difference between two population proportions.

A tax preparation firm is interested in comparing the quality of work at two of its regional offices. By randomly selecting samples of tax returns prepared at each office and verifying the sample returns' accuracy, the firm will be able to estimate the proportion of erroneous returns prepared at each office. Of particular interest is the difference between these proportions.

p_1 = proportion of erroneous returns for population 1 (office 1)
p_2 = proportion of erroneous returns for population 2 (office 2)
$\bar{p}_1$ = sample proportion for a simple random sample from population 1
$\bar{p}_2$ = sample proportion for a simple random sample from population 2

The difference between the two population proportions is given by $p_1 - p_2$. The point estimator of $p_1 - p_2$ is as follows.

POINT ESTIMATOR OF THE DIFFERENCE BETWEEN TWO POPULATION PROPORTIONS

$$\bar{p}_1 - \bar{p}_2 \qquad (11.1)$$

Thus, the point estimator of the difference between two population proportions is the difference between the sample proportions of two independent simple random samples.

As with other point estimators, the point estimator $\bar{p}_1 - \bar{p}_2$ has a sampling distribution that reflects the possible values of $\bar{p}_1 - \bar{p}_2$ if we repeatedly took two independent random samples. The mean of this sampling distribution is $p_1 - p_2$ and the standard error is as follows:

$$\sigma_{\bar{p}_1 - \bar{p}_2} = \sqrt{\frac{p_1(1 - p_1)}{n_1} + \frac{p_2(1 - p_2)}{n_2}} \qquad (11.2)$$

Sample sizes involving proportions are usually large enough to use this approximation.

If the sample sizes are large enough that n_1p_1, $n_1(1 - p_1)$, n_2p_2, and $n_2(1 - p_2)$ are all greater than or equal to 5, the sampling distribution of $\bar{p}_1 - \bar{p}_2$ can be approximated by a normal distribution.

In the estimation of the difference between two population proportions, an interval estimate will take the following form:

$$\bar{p}_1 - \bar{p}_2 \pm \text{Margin of error}$$

With the sampling distribution of $\bar{p}_1 - \bar{p}_2$ approximated by a normal distribution, we would like to use $z_{\alpha/2}\sigma_{\bar{p}_1-\bar{p}_2}$ as the margin of error. However, $\sigma_{\bar{p}_1-\bar{p}_2}$ given by equation (11.2) involves the two unknown population proportions, p_1 and p_2. Thus, we use the sample proportion $\bar{p}_1$ to estimate p_1 and the sample proportion $\bar{p}_2$ to estimate p_2. The margin of error is thus:

$$\text{Margin of error} = z_{\alpha/2}\sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \frac{\bar{p}_2(1-\bar{p}_2)}{n_2}} \tag{11.3}$$

The general form of an interval estimate of the difference between two population proportions is as follows.

INTERVAL ESTIMATE OF THE DIFFERENCE BETWEEN TWO POPULATION PROPORTIONS

$$\bar{p}_1 - \bar{p}_2 \pm z_{\alpha/2}\sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \frac{\bar{p}_2(1-\bar{p}_2)}{n_2}} \tag{11.4}$$

where $1 - \alpha$ is the confidence coefficient.

Returning to the tax preparation example, we find that independent simple random samples from the two offices provide the following information.

Office 1	Office 2
$n_1 = 250$	$n_2 = 300$
Number of returns with errors = 35	Number of returns with errors = 27

The sample proportions for the two offices follow.

$$\bar{p}_1 = \frac{35}{250} = .14$$

$$\bar{p}_2 = \frac{27}{300} = .09$$

The point estimate of the difference between the proportions of erroneous tax returns for the two populations is $\bar{p}_1 - \bar{p}_2 = .14 - .09 = .05$. Thus, we estimate that office 1 has a .05, or 5%, greater error rate than office 2.

Expression (11.4) can now be used to provide a margin of error and interval estimate of the difference between the two population proportions. Using a 90% confidence interval with $z_{\alpha/2} = z_{.05} = 1.645$, we have

$$\bar{p}_1 - \bar{p}_2 \pm z_{\alpha/2}\sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \frac{\bar{p}_2(1-\bar{p}_2)}{n_2}}$$

$$.14 - .09 \pm 1.645\sqrt{\frac{.14(1-.14)}{250} + \frac{.09(1-.09)}{300}}$$

$$.05 \pm .045$$

Thus, the margin of error is .045, and the 90% confidence interval is .005 to .095.

Using Excel to Construct a Confidence Interval

We can create a worksheet for developing an interval estimate of the difference between population proportions. Let us illustrate by developing an interval estimate of the difference between the proportions of erroneous tax returns at the two offices of the tax preparation firm. Refer to Figure 11.1 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet appears in the foreground.

FIGURE 11.1 CONSTRUCTING A 90% CONFIDENCE INTERVAL FOR THE DIFFERENCE IN THE PROPORTION OF ERRONEOUS TAX RETURNS PREPARED BY TWO OFFICES

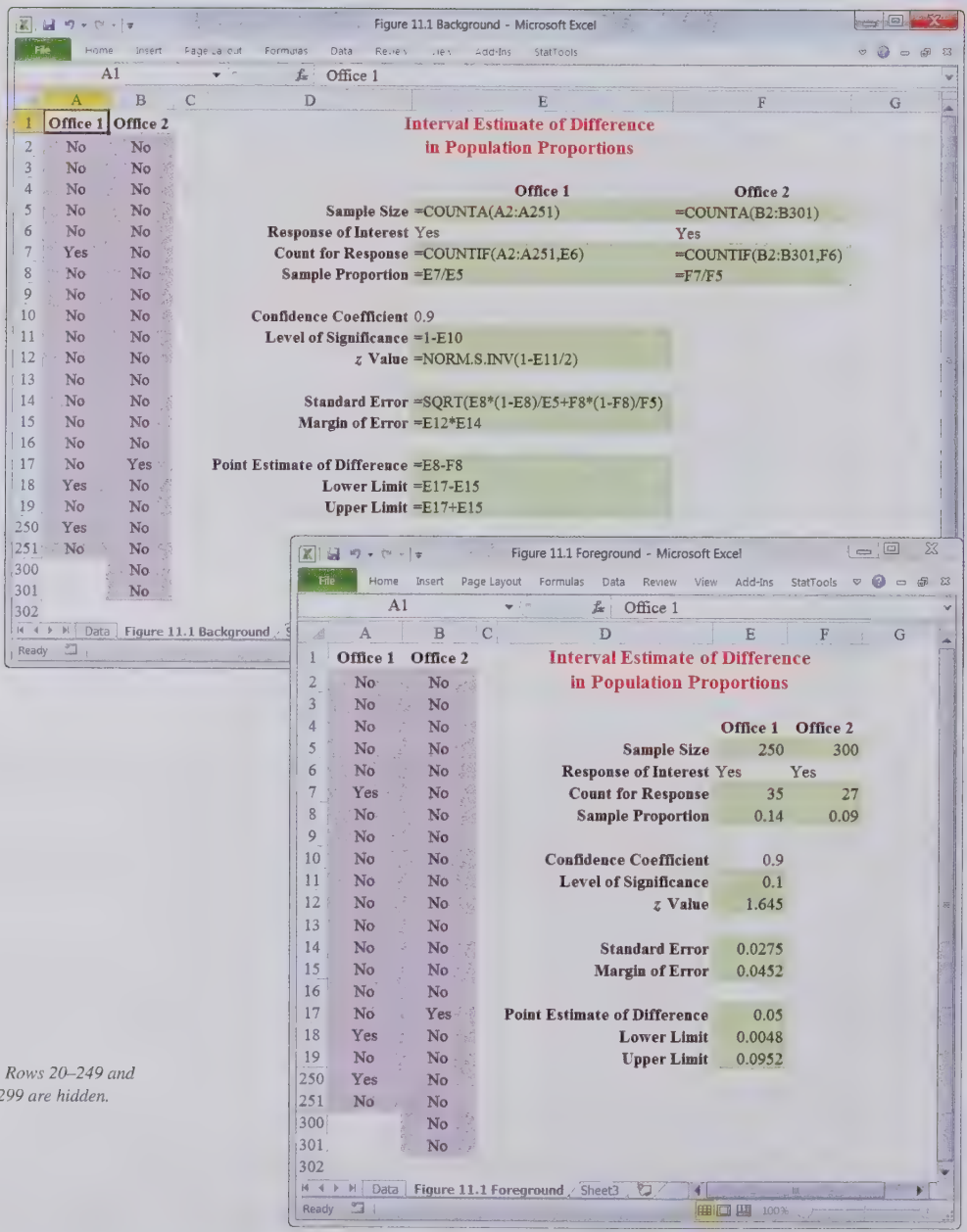


Figure 11.1 Background - Microsoft Excel
Interval Estimate of Difference in Population Proportions
Office 1
Office 2
Sample Size =COUNTA(A2:A251) =COUNTA(B2:B301)
Response of Interest Yes Yes
Count for Response =COUNTIF(A2:A251,E6) =COUNTIF(B2:B301,F6)
Sample Proportion =E7/E5 =F7/F5
Confidence Coefficient 0.9
Level of Significance =1-E10
z Value =NORM.S.INV(1-E11/2)
Standard Error =SQRT(E8*(1-E8)/E5+F8*(1-F8)/F5)
Margin of Error =E12*E14
Point Estimate of Difference =E8-F8
Lower Limit =E17-E15
Upper Limit =E17+E15
Figure 11.1 Foreground - Microsoft Excel
Interval Estimate of Difference in Population Proportions
Office 1 Office 2
Sample Size 250 300
Response of Interest Yes Yes
Count for Response 35 27
Sample Proportion 0.14 0.09
Confidence Coefficient 0.9
Level of Significance 0.1
z Value 1.645
Standard Error 0.0275
Margin of Error 0.0452
Point Estimate of Difference 0.05
Lower Limit 0.0048
Upper Limit 0.0952

Note: Rows 20–249 and 252–299 are hidden.

Enter Data: Columns A and B contain Yes or No labels that indicate which of the tax returns from each office contain an error.

Enter Functions and Formulas: The descriptive statistics needed are provided in cells E5:F5 and E7:F8. Note that Excel's COUNTA function is used in cells E5 and F5 to count the number of observations for each of the samples. The value worksheet indicates 250 returns in the sample from office 1 and 300 returns in the sample from office 2. In cells E6 and F6, we type Yes to indicate the response of interest (an erroneous return). Excel's COUNTIF function is used in cells E7 and F7 to count the number of Yes responses from each office. Formulas entered into cells E8 and F8 compute the sample proportions. The confidence coefficient entered into cell E10 (.9) is used to compute the corresponding level of significance ($\alpha = .10$) in cell E11. In cell E12 we use the NORM.S.INV function to compute the z value needed to compute the margin of error for the interval estimate.

In cell E14, a point estimate of $\sigma_{\bar{p}_1-\bar{p}_2}$, the standard error of the point estimator $\bar{p}_1 - \bar{p}_2$, is computed based on the two sample proportions (E8 and F8) and sample sizes (E5 and F5). The margin of error is then computed in cell E15 by multiplying the z value by the estimate of the standard error.

The point estimate of the difference in the two population proportions is computed in cell E17 as the difference in the sample proportions; the result, shown in the value worksheet, is .05. The lower limit of the confidence interval is computed in cell E18 by subtracting the margin of error from the point estimate. The upper limit is computed in cell E19 by adding the margin of error to the point estimate. The value worksheet shows that the 90% confidence interval estimate of the difference in the two population proportions is .0048 to .0952.

A template for other problems This worksheet can be used as a template for other problems requiring an interval estimate of the difference in population proportions. The new data must be entered in columns A and B. The data ranges in the cells used to compute the sample size (E5:F5) and the cells used to compute a count of the response of interest (E7:F7) must be changed to correctly indicate the location of the new data. The response of interest must be typed into cells E6:F6. The 90% confidence interval for the new data will then appear in cells E17:E19. If an interval estimate with a different confidence coefficient is desired, simply change the entry in cell E10.

This worksheet can also be used as a template for solving text exercises in which the sample data have already been summarized. No change in the data section is necessary. Simply type the values for the given sample sizes in cells E5:F5 and type the given values for the sample proportions in cells E8:F8. The 90% confidence interval will then appear in cells E17:E19. If an interval estimate with a different confidence coefficient is desired, simply change the entry in cell E10.

Hypothesis Tests About $p_1 - p_2$

Let us now consider hypothesis tests about the difference between the proportions of two populations. We focus on tests involving no difference between the two population proportions. In this case, the three forms for a hypothesis test are as follows:

All hypotheses considered use 0 as the difference of interest.

$$H_0: p_1 - p_2 \geq 0 \qquad H_0: p_1 - p_2 \leq 0 \qquad H_0: p_1 - p_2 = 0$$
$$H_a: p_1 - p_2 < 0 \qquad H_a: p_1 - p_2 > 0 \qquad H_a: p_1 - p_2 \neq 0$$

When we assume H_0 is true as an equality, we have $p_1 - p_2 = 0$, which is the same as saying that the population proportions are equal, $p_1 = p_2$.

We will base the test statistic on the sampling distribution of the point estimator $\bar{p}_1 - \bar{p}_2$. In equation (11.2), we showed that the standard error of $\bar{p}_1 - \bar{p}_2$ is given by

$$\sigma_{\bar{p}_1-\bar{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

Under the assumption H_0 is true as an equality, the population proportions are equal and $p_1 = p_2 = p$. In this case, $\sigma_{\bar{p}_1-\bar{p}_2}$ becomes as follows:

STANDARD ERROR OF $\bar{p}_1 - \bar{p}_2$ WHEN $p_1 = p_2 = p$

$$\sigma_{\bar{p}_1-\bar{p}_2} = \sqrt{\frac{p(1-p)}{n_1} + \frac{p(1-p)}{n_2}} = \sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \qquad (11.5)$$

With p unknown, we pool, or combine, the point estimators from the two samples ($\bar{p}_1$ and $\bar{p}_2$) to obtain a single point estimator of p as follows:

POOLED ESTIMATOR OF p WHEN $p_1 = p_2 = p$

$$\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2} \qquad (11.6)$$

This **pooled estimator of p** is a weighted average of $\bar{p}_1$ and $\bar{p}_2$.

Substituting $\bar{p}$ for p in equation (11.5), we obtain an estimate of the standard error of $\bar{p}_1 - \bar{p}_2$. This estimate of the standard error is used in computing the test statistic. The general form of the test statistic for hypothesis tests about the difference between two population proportions is the point estimator divided by the estimate of $\sigma_{\bar{p}_1-\bar{p}_2}$.

TEST STATISTIC FOR HYPOTHESIS TESTS ABOUT $p_1 - p_2$

$$z = \frac{(\bar{p}_1 - \bar{p}_2)}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \qquad (11.7)$$

This test statistic applies to large sample situations where n_1p_1, $n_1(1-p_1)$, n_2p_2, and $n_2(1-p_2)$ are all greater than or equal to 5.

Let us return to the tax preparation firm example and assume that the firm wants to use a hypothesis test to determine whether the error proportions differ between the two offices. A two-tailed test is required. The null and alternative hypotheses are as follows:

$$H_0: p_1 - p_2 = 0$$
$$H_a: p_1 - p_2 \neq 0$$

If H_0 is rejected, the firm can conclude that the error rates at the two offices differ. We will use $\alpha = .10$ as the level of significance.

The sample data previously collected showed $\bar{p}_1 = .14$ for the $n_1 = 250$ returns sampled at office 1 and $\bar{p}_2 = .09$ for the $n_2 = 300$ returns sampled at office 2. We continue by computing the pooled estimate of p.

$$\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2} = \frac{250(.14) + 300(.09)}{250 + 300} = .1127$$

Using this pooled estimate and the difference between the sample proportions, the value of the test statistic is as follows.

$$z = \frac{(\bar{p}_1 - \bar{p}_2)}{\sqrt{\bar{p}(1 - \bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(.14 - .09)}{\sqrt{.1127(1 - .1127)\left(\frac{1}{250} + \frac{1}{300}\right)}} = 1.85$$

In computing the p-value for this two-tailed test, we first note that $z = 1.85$ is in the upper tail of the standard normal distribution. Using $z = 1.85$ and the standard normal distribution table, we find the area in the upper tail is $1.0000 - .9678 = .0322$. Doubling this area for a two-tailed test, we find the p-value $= 2(.0322) = .0644$. With the p-value less than $\alpha = .10$, H_0 is rejected at the .10 level of significance. The firm can conclude that the error rates differ between the two offices. This hypothesis testing conclusion is consistent with the earlier 90% confidence interval results that showed the interval estimate of the difference between the population error rates at the two offices to be .005 to .095, with office 1 having the higher error rate.

Using Excel to Conduct a Hypothesis Test

We can create a worksheet for conducting a hypothesis test about the difference between population proportions. Let us illustrate by testing to see whether there is a significant difference between the proportions of erroneous tax returns at the two offices of the tax preparation firm. Refer to Figure 11.2 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet is in the foreground.

Enter Data: Columns A and B contain Yes or No labels that indicate which of the tax returns from each office contain an error.

Enter Functions and Formulas: The descriptive statistics needed to perform the hypothesis test are provided in cells E5:F6 and E7:F8. They are the same as the ones used for an interval estimate (see Figure 11.1). The hypothesized value of the difference between the two populations is zero; it is entered into cell E10. In cell E11, the difference in the sample proportions is used to compute a point estimate of the difference in the two population proportions. Using the two sample proportions and sample sizes, a pooled estimate of the population proportion p is computed in cell E13; its value is .1127. Then, in cell E14, an estimate of $\sigma_{\bar{p}_1 - \bar{p}_2}$ is computed using equation (11.5), with the pooled estimate of p and the sample sizes.

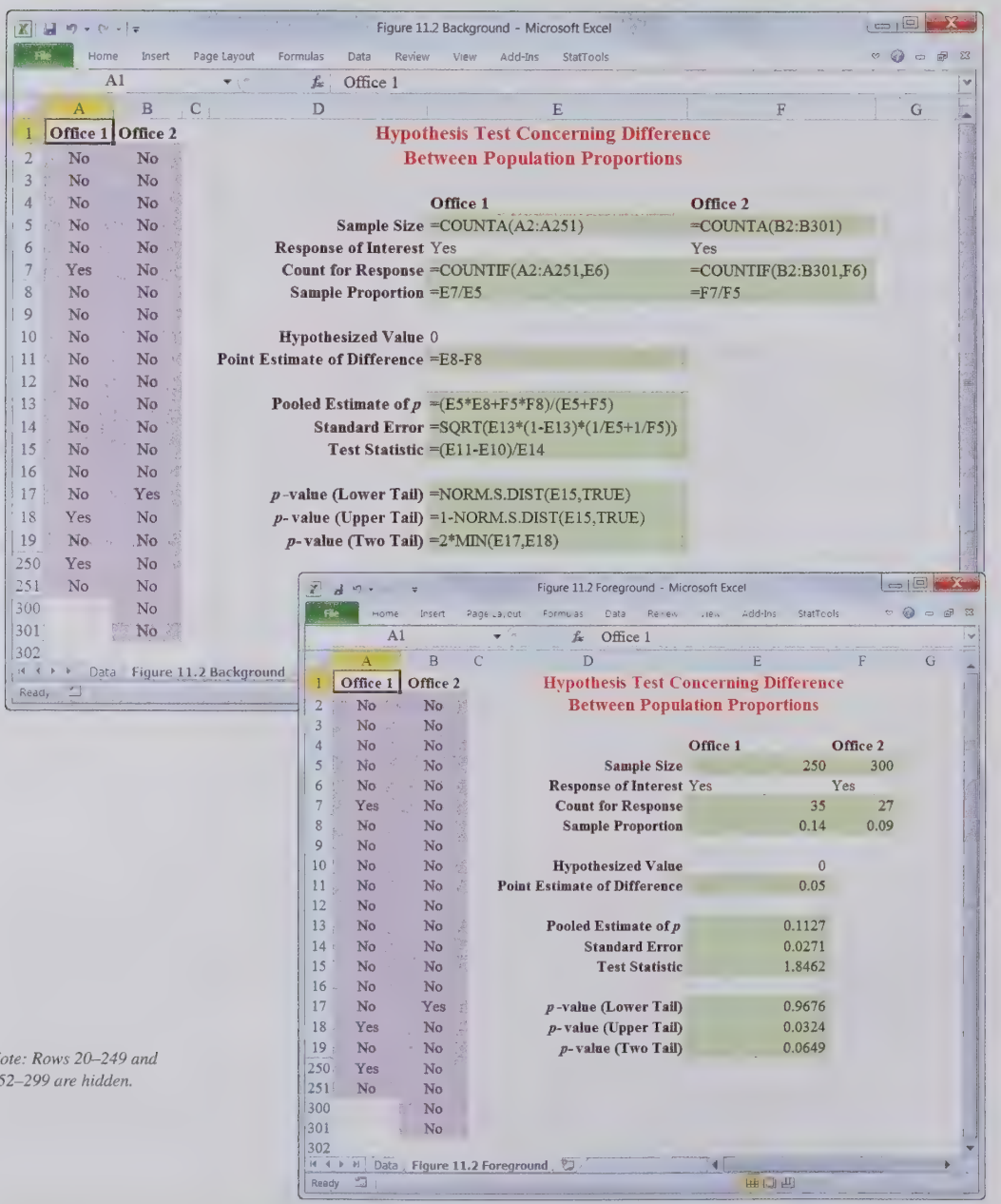
The formula =(E11-E10)/E14 entered into cell E15 computes the test statistic z (1.8462). The NORM.S.DIST function is then used to compute the p-value (Lower Tail) and the p-value (Upper Tail) in cells E17 and E18. The p-value (Two Tail) is computed in cell E19 as twice the minimum of the two one-tailed p-values. The value worksheet shows that p-value (Two Tail) = .0649. Because the p-value = .0649 is less than the level of significance, $\alpha = .10$, we have sufficient evidence to reject the null hypothesis and conclude that the population proportions are not equal.

The p-value here (.0649) differs from the one we found using the cumulative normal probability tables (.0644) due to rounding.

This worksheet can be used as a template for hypothesis testing problems involving differences between population proportions. The new data can be entered into columns A and B. The ranges for the new data and the response of interest need to be revised in cells E5:F7. The remainder of the worksheet will then be updated as needed to conduct the hypothesis test. If a hypothesized difference other than 0 is to be used, the new value must be entered in cell E10.

To use this worksheet for exercises in which the sample statistics are given, just type in the given values for cells E5:F5 and E7:F8. The remainder of the worksheet will then be updated to conduct the hypothesis test. If a hypothesized difference other than 0 is to be used, the new value must be entered in cell E10.

FIGURE 11.2 HYPOTHESIS TEST CONCERNING DIFFERENCE IN PROPORTION OF ERRONEOUS TAX RETURNS PREPARED BY TWO OFFICES



Note: Rows 20–249 and 252–299 are hidden.

Exercises

Methods

SELF test

1. Consider the following results for independent samples taken from two populations.

Sample 1	Sample 2
$n_1 = 400$	$n_2 = 300$
$\bar{p}_1 = .48$	$\bar{p}_2 = .36$

   a. What is the point estimate of the difference between the two population proportions?
   b. Develop a 90% confidence interval for the difference between the two population proportions.
   c. Develop a 95% confidence interval for the difference between the two population proportions.

SELF test

2. Consider the hypothesis test

$$H_0: p_1 - p_2 \leq 0$$
$$H_a: p_1 - p_2 > 0$$

The following results are for independent samples taken from the two populations.

Sample 1	Sample 2
$n_1 = 200$	$n_2 = 300$
$\bar{p}_1 = .22$	$\bar{p}_1 = .16$

   a. What is the p-value?
   b. With $\alpha = .05$, what is your hypothesis testing conclusion?

Applications

SELF test

3. The Professional Golf Association (PGA) measured the putting accuracy of professional golfers playing on the PGA Tour and the best amateur golfers playing in the World Amateur Championship (*Golf Magazine,* January 2007). A sample of 1075 6-foot putts by professional golfers found 688 made putts. A sample of 1200 6-foot putts by amateur golfers found 696 made putts.
   a. Estimate the proportion of made 6-foot putts by professional golfers. Estimate the proportion of made 6-foot putts by amateur golfers. Which group had a better putting accuracy?
   b. What is the point estimate of the difference between the proportions of the two populations? What does this estimate tell you about the percentage of putts made by the two groups of golfers?
   c. What is the 95% confidence interval for the difference between the two population proportions? Interpret this confidence interval in terms of the percentage of putts made by the two groups of golfers.

4. An American Automobile Association (AAA) study investigated the question of whether a man or a woman was more likely to stop and ask for directions (AAA, January 2006). The situation referred to in the study stated the following: "If you and your spouse are driving together and become lost, would you stop and ask for directions?" A sample representative of the data used by AAA showed 300 of 811 women said that they would stop and ask for directions, while 255 of 750 men said that they would stop and ask for directions.
   a. The AAA research hypothesis was that women would be more likely to say that they would stop and ask for directions. Formulate the null and alternative hypotheses for this study.

b. What is the percentage of women who indicated that they would stop and ask for directions?
c. What is the percentage of men who indicated that they would stop and ask for directions?
d. At $\alpha = .05$, test the hypothesis. What is the p-value, and what conclusion would you expect AAA to draw from this study?

5. In recent years, the number of people who use the Internet to obtain political news has grown. Often the political websites ask Internet users to register their opinions by participating in online surveys. Pew Research Center conducted a survey of its own to learn about the participation of Republicans and Democrats in online surveys (Associated Press, January 6, 2003). The following sample data apply.

Political Party	Sample Size	Participate in Online Surveys
Republican	250	115
Democrat	350	98

a. Compute the point estimate of the proportion of Republicans who indicate they would participate in online surveys. Compute the point estimate for the Democrats.
b. What is the point estimate of the difference between the two population proportions?
c. At 95% confidence, what is the margin of error?
d. Representatives of the scientific polling industry claim that the profusion of online surveys can confuse people about actual public opinion. Do you agree with this statement? Use the 95% confidence interval estimate of the difference between the Republican and Democrat population proportions to help justify your answer.

6. Chicago O'Hare and Atlanta Hartsfield-Jackson are the two busiest airports in the United States. The congestion often leads to delayed flight arrivals as well as delayed flight departures. The Bureau of Transportation tracks the on-time and delayed performance at major airports (*Travel & Leisure*, November 2006). A flight is considered delayed if it is more than 15 minutes behind schedule. The following sample data show the delayed departures at Chicago O'Hare and Atlanta Hartsfield-Jackson airports.

	Chicago O'Hare	Atlanta Hartsfield-Jackson
Flights	900	1200
Delayed Departures	252	312

a. State the hypothesis that can be used to determine whether the population proportions of delayed departures differ at these two airports.
b. What is the point estimate of the proportion of flights that have delayed departures at Chicago O'Hare?
c. What is the point estimate of the proportion of flights that have delayed departures at Atlanta Hartsfield-Jackson?
d. What is the p-value for the hypothesis test? What is your conclusion?

7. *BusinessWeek* reported that there seems to be a difference by age group in how well people like life in Russia (*BusinessWeek*, March 10, 2008). The following sample data are consistent with the *BusinessWeek* findings and show the responses by age group to the question: "Do you like life in Russia?"

	Russian Age Group	
	17–26	40 and over
Sample Size	300	260
Responded Yes	192	117

a. What is the point estimate of the proportion of Russians aged 17 to 26 who like life in Russia?
b. What is the point estimate of the proportion of Russians aged 40 and over who like life in Russia?
c. Provide a 95% confidence interval estimate of the difference between the proportion of young Russians aged 17 to 26 and older Russians aged 40 and over who like life in Russia.

8. During the 2003 Super Bowl, Miller Lite Beer's commercial referred to as "The Miller Lite Girls" ranked among the top three most effective advertisements aired during the Super Bowl (*USA Today,* December 29, 2003). The survey of advertising effectiveness, conducted by *USA Today*'s Ad Track poll, reported separate samples by respondent age group to learn about how the Super Bowl advertisement appealed to different age groups. The following sample data apply to the "The Miller Lite Girls" commercial.

Age Group	Sample Size	Liked the Ad a Lot
Under 30	100	49
30 to 49	150	54

a. Formulate a hypothesis test that can be used to determine whether there is a difference between the population proportions for the two age groups.
b. What is the point estimate of the difference between the two population proportions?
c. Conduct the hypothesis test and report the *p*-value. At $\alpha = .05$, what is your conclusion?
d. Discuss the appeal of the advertisements to the younger and the older age groups. Would the Miller Lite organization find the results of the *USA Today* Ad Track poll encouraging? Explain.

9. In a test of the quality of two television commercials, each commercial was shown in a separate test area six times over a one-week period. The following week a telephone survey was conducted to identify individuals who had seen the commercials. Those individuals were asked to state the primary message in the commercials. The following results were recorded.

	Commercial A	Commercial B
Number Who Saw Commercial	150	200
Number Who Recalled Message	63	60

a. Use $\alpha = .05$ and test the hypothesis that there is no difference in the recall proportions for the two commercials.
b. Compute a 95% confidence interval for the difference between the recall proportions for the two populations.

10. A 2003 *New York Times*/CBS News poll sampled 523 adults who were planning a vacation during the next six months and found that 141 were expecting to travel by airplane (*New York Times* News Service, March 2, 2003). A similar survey question in a May 1993 *New York Times*/CBS News poll found that of 477 adults who were planning a vacation in the next six months, 81 were expecting to travel by airplane.
a. State the hypotheses that can be used to determine whether a significant change occurred in the population proportion planning to travel by airplane over the 10-year period.
b. What is the sample proportion expecting to travel by airplane in 2003? In 1993?
c. Use $\alpha = .01$ and test for a significant difference. What is your conclusion?
d. Discuss reasons that might provide an explanation for this conclusion.

11.2 Hypothesis Test for Proportions of a Multinomial Population

In this section we consider the case in which each element of a population is assigned to one and only one of several classes or categories. Such a population is a **multinomial population**. The multinomial distribution can be thought of as an extension of the binomial distribution to the case of three or more categories of outcomes. On each trial of a multinomial experiment, one and only one of the outcomes occurs. Each trial of the experiment is assumed to be independent, and the probabilities of the outcomes remain the same for each trial.

The assumptions for the multinomial experiment parallel those for the binomial experiment with the exception that the multinomial has three or more outcomes per trial.

As an example, consider the market share study being conducted by Scott Marketing Research. Over the past year market shares stabilized at 30% for company A, 50% for company B, and 20% for company C. Recently company C developed a "new and improved" product to replace its current entry in the market. Company C retained Scott Marketing Research to determine whether the new product will alter market shares.

In this case, the population of interest is a multinomial population; each customer is classified as buying from company A, company B, or company C. Thus, we have a multinomial population with three outcomes. Let us use the following notation for the proportions:

p_A = market share for company A
p_B = market share for company B
p_C = market share for company C

Scott Marketing Research will conduct a sample survey and compute the proportion preferring each company's product. A hypothesis test will then be conducted to see whether the new product caused a change in market shares. For the null hypothesis, we assume that company C's new product will not alter the market shares. The null and alternative hypotheses are stated as follows:

H_0: $p_A = .30$, $p_B = .50$, and $p_C = .20$
H_a: The population proportions are not
$p_A = .30$, $p_B = .50$, and $p_C = .20$

If the sample results lead to the rejection of H_0, Scott Marketing Research will have evidence that the introduction of the new product affects market shares.

Let us assume that the market research firm has used a consumer panel of 200 customers for the study. Each individual was asked to specify a purchase preference among the three alternatives: company A's product, company B's product, and company C's new product. The 200 responses are summarized here.

The consumer panel of 200 customers in which each individual is asked to select one of three alternatives is equivalent to a multinomial experiment consisting of 200 trials.

	Observed Frequency	
Company A's Product	**Company B's Product**	**Company C's New Product**
48	98	54

We now can perform a **goodness of fit test** that will determine whether the sample of 200 customer purchase preferences is consistent with the null hypothesis. The goodness of fit test is based on a comparison of the sample of *observed* results with the *expected* results under the assumption that the null hypothesis is true. Hence, the next step is to compute expected purchase preferences for the 200 customers under the assumption that $p_A = .30$, $p_B = .50$, and $p_C = .20$. Doing so provides the expected results.

	Expected Frequency	
Company A's Product	**Company B's Product**	**Company C's New Product**
200(.30) = 60	200(.50) = 100	200(.20) = 40

Thus, we see that the expected frequency for each category is found by multiplying the sample size of 200 by the hypothesized proportion for the category.

The goodness of fit test now focuses on the differences between the observed frequencies and the expected frequencies. Large differences between observed and expected frequencies cast doubt on the assumption that the hypothesized proportions or market shares are correct. Whether the differences between the observed and expected frequencies are "large" or "small" is a question answered with the aid of the following chi-square test statistic.

TEST STATISTIC FOR GOODNESS OF FIT

$$\chi^2 = \sum_{i=1}^{k} \frac{(f_i - e_i)^2}{e_i} \tag{11.8}$$

where

f_i = observed frequency for category i
e_i = expected frequency for category i
k = the number of categories

Note: The test statistic has a chi-square distribution with $k - 1$ degrees of freedom provided that the expected frequencies are 5 *or more* for all categories.

Tables of areas or probabilities are readily available for the chi-square distribution. Since the test statistic for goodness of fit is known to have a chi-square distribution whenever the expected frequencies are 5 or more for all categories, we can use the chi-square distribution to conduct the hypothesis test. We will use the notation χ^2_α to denote the value of the chi-square distribution that provides an area or probability of α to the right of the χ^2_α value. For example, refer to Table 11.1 and verify that for a chi-square distribution with 2 degrees of freedom $\chi^2_{.10} = 4.605$, indicating that 10% of the chi-square values are to the right of 4.605.

Let us continue with the Scott Marketing Research example and use the sample data to test the hypothesis that the multinomial population retains the proportions $p_A = .30$, $p_B = .50$, and $p_C = .20$. We will use an $\alpha = .05$ level of significance. Let us proceed by using the observed and expected frequencies to compute the value of the test statistic. With the expected frequencies all 5 or more, the computation of the chi-square test statistic is shown in Table 11.2. At the bottom of the last column, we see that $\chi^2 = 7.34$.

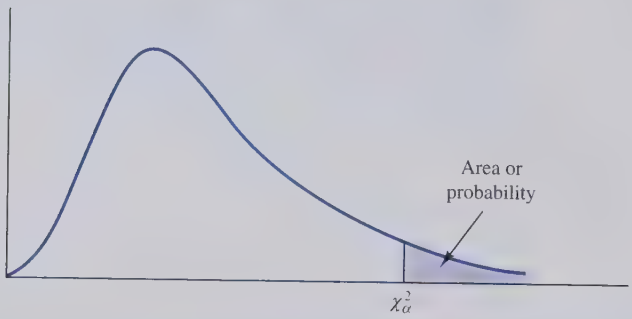
The test for goodness of fit is always a one-tailed test with the rejection occurring in the upper tail of the chi-square distribution.

We will reject the null hypothesis if the differences between the observed and expected frequencies are *large*. Large differences between the observed and expected frequencies will result in a large value for the test statistic. Thus the test of goodness of fit will always be an upper tail test. We can use the upper tail area for the test statistic and the p-value approach to determine whether the null hypothesis can be rejected. With $k - 1 = 3 - 1 = 2$ degrees of freedom, the chi-square table provides the following:

Area in Upper Tail	**.10**	**.05**	**.025**	**.01**
χ^2 Value (2 *df*)	4.605	5.991	7.378	9.210

$\chi^2 = 7.34$

TABLE 11.1 SELECTED VALUES FROM THE CHI-SQUARE DISTRIBUTION TABLE*



Degrees of Freedom	Area in Upper Tail							
	.99	.975	.95	.90	.10	.05	.025	.01
1	.000	.001	.004	.016	2.706	3.841	5.024	6.635
2	.020	.051	.103	.211	4.605	5.991	7.378	9.210
3	.115	.216	.352	.584	6.251	7.815	9.348	11.345
4	.297	.484	.711	1.064	7.779	9.488	11.143	13.277
5	.554	.831	1.145	1.610	9.236	11.070	12.832	15.086
6	.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812
7	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475
8	1.647	2.180	2.733	3.490	13.362	15.507	17.535	20.090
9	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666
10	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209
11	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725
12	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217
13	4.107	5.009	5.892	7.041	19.812	22.362	24.736	27.688
14	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141
15	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578
16	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000
17	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409
18	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805
19	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191
20	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566
21	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932
22	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289
23	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638
24	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980
25	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314
26	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642
27	12.878	14.573	16.151	18.114	36.741	40.113	43.195	46.963
28	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278
29	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588
30	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892
40	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691
60	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379
80	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329
100	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807

**Note:* A more extensive table is provided as Table 3 of Appendix B.

TABLE 11.2 COMPUTATION OF THE CHI-SQUARE TEST STATISTIC FOR THE SCOTT MARKETING RESEARCH MARKET SHARE STUDY

Category	Hypothesized Proportion	Observed Frequency (f_i)	Expected Frequency (e_i)	Difference ($f_i - e_i$)	Squared Difference $(f_i - e_i)^2$	Squared Difference Divided by Expected Frequency $(f_i - e_i)^2/e_i$
Company A	.30	48	60	−12	144	2.40
Company B	.50	98	100	−2	4	0.04
Company C	.20	54	40	14	196	4.90
Total		200				$\chi^2 = 7.34$

Using Excel, CHISQ.DIST.RT (7.34,2) = .0255.

The test statistic $\chi^2 = 7.34$ is between 5.991 and 7.378. Thus, the corresponding upper tail area or p-value must be between .05 and .025. Excel can be used to show that $\chi^2 = 7.34$ provides a p-value = .0255. With the p-value $\leq \alpha = .05$, we reject H_0 and conclude that the introduction of the new product by company C will alter the current market share structure.

Instead of using the p-value, we could use the critical value approach to draw the same conclusion. With $\alpha = .05$ and 2 degrees of freedom, the critical value for the test statistic is $\chi^2_{.05} = 5.991$. The upper tail rejection rule becomes

$$\text{Reject } H_0 \text{ if } \chi^2 \geq 5.991$$

With $7.34 > 5.991$, we reject H_0. The p-value approach and critical value approach provide the same hypothesis testing conclusion.

Although no further statistical conclusions can be made as a result of the test, we can compare the observed and expected frequencies informally to obtain an idea of how the market share structure may change. Considering company C, we find that the observed frequency of 54 is larger than the expected frequency of 40. Because the expected frequency was based on current market shares, the larger observed frequency suggests that the new product will have a positive effect on company C's market share. Comparisons of the observed and expected frequencies for the other two companies indicate that company C's gain in market share will hurt company A more than company B.

Let us summarize the general steps that can be used to conduct a goodness of fit test for a hypothesized multinomial population distribution.

MULTINOMIAL DISTRIBUTION GOODNESS OF FIT TEST: A SUMMARY

1. State the null and alternative hypotheses.

   H_0: The population follows a multinomial distribution with specified probabilities for each of the k categories

   H_a: The population does not follow a multinomial distribution with the specified probabilities for each of the k categories

2. Select a random sample and record the observed frequencies f_i for each category.
3. Assume the null hypothesis is true and determine the expected frequency e_i in each category by multiplying the category probability by the sample size.
4. Compute the value of the test statistic.

$$\chi^2 = \sum_{i=1}^{k} \frac{(f_i - e_i)^2}{e_i}$$

5. Rejection rule:

p-value approach: Reject H_0 if p-value $\leq \alpha$

Critical value approach: Reject H_0 if $\chi^2 \geq \chi^2_\alpha$

where α is the level of significance for the test and there are $k - 1$ degrees of freedom.

Using Excel to Conduct a Goodness of Fit Test

Excel can be used to conduct a goodness of fit test for the Scott Marketing Research Study. Refer to Figure 11.3 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet is in the foreground.

WEB file Research

Enter Data: Column A is used to identify each of the 200 customers who made up the consumer panel in the study. Column B shows the purchase preference (A, B, or C) for each customer. The hypothesized proportions, 0.3, 0.5, and 0.2, were entered into cells E4:E6.

FIGURE 11.3 EXCEL WORKSHEET FOR GOODNESS OF FIT TEST CONCERNING MARKET SHARE CHANGE

Figure 11.3 Background - Microsoft Excel

Category	Hyp. Proportion	Obs. Freq.	Exp. Freq.	Diff.	Squared Diff.	Squared Diff. Divided by Exp. Freq.
Company A	0.3	=COUNTIF(B2:B201,"A")	=E4*F7	=F4-G4	=H4^2	=I4/G4
Company B	0.5	=COUNTIF(B2:B201,"B")	=E5*F7	=F5-G5	=H5^2	=I5/G5
Company C	0.2	=COUNTIF(B2:B201,"C")	=E6*F7	=F6-G6	=H6^2	=I6/G6
Total		=SUM(F4:F6)				=SUM(J4:J6)

Number of Categories 3

Test Statistic =J7

Degrees of Freedom =F9-1

p-value =CHISQ.DIST.RT(F11,F12)

Figure 11.3 Foreground - Microsoft Excel

Category	Hyp. Proportion	Obs. Freq.	Exp. Freq.	Diff.	Squared Diff.	Squared Diff. Divided by Exp. Freq.
Company A	0.3	48	60	-12	144	2.4
Company B	0.5	98	100	-2	4	0.04
Company C	0.2	54	40	14	196	4.9
Total		200				7.34

Number of Categories 3

Test Statistic 7.34

Degrees of Freedom 2

p-value 0.0255

Note: Rows 16–199 are hidden.

Enter Functions and Formulas: The Excel formulas in cells F4:J7 are used to compute the chi-square test statistic in a fashion parallel to that shown in Table 11.2. The result is the chi-square test statistic shown in cell J7 (7.34). The number of categories, 3, was entered into cell F9 and the two inputs used by Excel's CHISQ.DIST.RT function (test statistic and degrees of freedom) are in cells F11 and F12. The value computed in cell F14 using Excel's CHISQ.DIST.RT function is the area in the upper tail of the chi-square distribution to the right of the test statistic. So, this value is the appropriate p-value for our upper tail hypothesis test.

The value worksheet shows that the resulting p-value is 0.0255. Thus, with $\alpha = .05$, we reject H_0 and conclude that the introduction of the new product by company C will alter the current market share structure.

Exercises

Methods

SELF test

11. Test the following hypotheses by using the χ^2 goodness of fit test.

$$H_0: p_A = .40, p_B = .40, \text{ and } p_C = .20$$
$$H_a: \text{The population proportions are not } p_A = .40, p_B = .40, \text{ and } p_C = .20$$

A sample of size 200 yielded 60 in category A, 120 in category B, and 20 in category C. Use $\alpha = .01$ and test to see whether the proportions are as stated in H_0.

a. Use the p-value approach.
b. Repeat the test using the critical value approach.

12. Suppose we have a multinomial population with four categories: A, B, C, and D. The null hypothesis is that the proportion of items is the same in every category. The null hypothesis is

$$H_0: p_A = p_B = p_C = p_D = .25$$

A sample of size 300 yielded the following results.

A: 85 B: 95 C: 50 D: 70

Use $\alpha = .05$ to determine whether H_0 should be rejected. What is the p-value?

Applications

SELF test

13. During the first 13 weeks of the television season, the Saturday evening 8:00 P.M. to 9:00 P.M. audience proportions were recorded as ABC 29%, CBS 28%, NBC 25%, and independents 18%. A sample of 300 homes two weeks after a Saturday night schedule revision yielded the following viewing audience data: ABC 95 homes, CBS 70 homes, NBC 89 homes, and independents 46 homes. Test with $\alpha = .05$ to determine whether the viewing audience proportions changed.

14. M&M/MARS, makers of M&M® Chocolate Candies, conducted a national poll in which consumers indicated their preference for colors. In the brochure "Colors," made available by M&M/MARS Consumer Affairs, the traditional distribution of colors for the plain candies is as follows:

Brown	Yellow	Red	Orange	Green	Blue
30%	20%	20%	10%	10%	10%

In a follow-up study, 1-pound bags were used to determine whether the reported percentages were valid. The following results were obtained for a sample of 506 plain candies.

Brown	Yellow	Red	Orange	Green	Blue
177	135	79	41	36	38

Use $\alpha = .05$ to determine whether these data support the percentages reported by the company.

15. Where do women most often buy casual clothing? Data from the U.S. Shopper Database provided the following percentages for women shopping at each of the various outlets (*The Wall Street Journal,* January 28, 2004).

Outlet	Percentage	Outlet	Percentage
Walmart	24	Kohl's	8
Traditional department stores	11	Mail order	12
JC Penney	8	Other	37

The other category included outlets such as Target, Kmart, and Sears as well as numerous smaller specialty outlets. No individual outlet in this group accounted for more than 5% of the women shoppers. A recent survey using a sample of 140 women shoppers in Atlanta, Georgia, found 42 Walmart, 20 traditional department store, 8 JC Penney, 10 Kohl's, 21 mail order, and 39 other outlet shoppers. Does this sample suggest that women shoppers in Atlanta differ from the shopping preferences expressed in the U.S. Shopper Database? What is the *p*-value? Use $\alpha = .05$. What is your conclusion?

16. The American Bankers Association collects data on the use of credit cards, debit cards, personal checks, and cash when consumers pay for in-store purchases (*The Wall Street Journal,* December 16, 2003). In 1999, the following usages were reported.

In-Store Purchase	Percentage
Credit card	22
Debit card	21
Personal check	18
Cash	39

A sample taken in 2003 found that for 220 in-stores purchases, 46 used a credit card, 67 used a debit card, 33 used a personal check, and 74 used cash.

a. At $\alpha = .01$, can we conclude that a change occurred in how customers paid for in-store purchases over the four-year period from 1999 to 2003? What is the *p*-value?
b. Compute the percentage of use for each method of payment using the 2003 sample data. What appears to have been the major change or changes over the four-year period?
c. In 2003, what percentage of payments was made using plastic (credit card or debit card)?

17. *The Wall Street Journal*'s Shareholder Scoreboard tracks the performance of 1000 major U.S. companies (*The Wall Street Journal,* March 10, 2003). The performance of each company is rated based on the annual total return, including stock price changes and the reinvestment of dividends. Ratings are assigned by dividing all 1000 companies into five groups from A (top

20%), B (next 20%), to E (bottom 20%). Shown here are the one-year ratings for a sample of 60 of the largest companies. Do the largest companies differ in performance from the performance of the 1000 companies in the Shareholder Scoreboard? Use $\alpha = .05$.

A	B	C	D	E
5	8	15	20	12

18. How well do airline companies serve their customers? A study showed the following customer ratings: 3% excellent, 28% good, 45% fair, and 24% poor (*BusinessWeek*, September 11, 2000). In a follow-up study of service by telephone companies, assume that a sample of 400 adults found the following customer ratings: 24 excellent, 124 good, 172 fair, and 80 poor. Is the distribution of the customer ratings for telephone companies different from the distribution of customer ratings for airline companies? Test with $\alpha = .01$. What is your conclusion?

11.3 Test of Independence

Another important application of the chi-square distribution involves using sample data to test for the independence of two variables. Let us illustrate the test of independence by considering the study conducted by the Alber's Brewery of Tucson, Arizona. Alber's manufactures and distributes three types of beer: light, regular, and dark. In an analysis of the market segments for the three beers, the firm's market research group raised the question of whether preferences for the three beers differ among male and female beer drinkers. If beer preference is independent of the gender of the beer drinker, one advertising campaign will be initiated for all of Alber's beers. However, if beer preference depends on the gender of the beer drinker, the firm will tailor its promotions to different target markets.

A test of independence addresses the question of whether the beer preference (light, regular, or dark) is independent of the gender of the beer drinker (male, female). The hypotheses for this test of independence are

H_0: Beer preference is independent of the gender of the beer drinker
H_a: Beer preference is not independent of the gender of the beer drinker

To test whether two variables are independent, one sample is selected and crosstabulation is used to summarize the data for the two variables simultaneously.

Table 11.3 can be used to describe the situation being studied. After identification of the population as all male and female beer drinkers, a sample can be selected and each individual asked to state his or her preference for the three Alber's beers. Every individual in the sample will be classified in one of the six cells in the table. For example, an individual may be a male preferring regular beer [cell (1,2)], a female preferring light beer [cell (2,1)], a female preferring dark beer [cell (2,3)], and so on. Because we have listed all possible

TABLE 11.3 CONTINGENCY TABLE FOR BEER PREFERENCE AND GENDER OF BEER DRINKER

		Beer Preference		
		Light	**Regular**	**Dark**
Gender	**Male**	cell(1,1)	cell(1,2)	cell(1,3)
	Female	cell(2,1)	cell(2,2)	cell(2,3)

TABLE 11.4 SAMPLE RESULTS FOR BEER PREFERENCES OF MALE AND FEMALE BEER DRINKERS (OBSERVED FREQUENCIES)

		Beer Preference			
		Light	Regular	Dark	Total
Gender	Male	20	40	20	80
	Female	30	30	10	70
	Total	50	70	30	150

combinations of beer preference and gender or, in other words, listed all possible contingencies, Table 11.3 is called a **contingency table**. The test of independence uses the contingency table format and for that reason is sometimes referred to as a *contingency table test.*

Suppose a simple random sample of 150 beer drinkers is selected. After tasting each beer, the individuals in the sample are asked to state their preference or first choice. The crosstabulation in Table 11.4 summarizes the responses for the study. As we see, the data for the test of independence are collected in terms of counts or frequencies for each cell or category. Of the 150 individuals in the sample, 20 were men who favored light beer, 40 were men who favored regular beer, 20 were men who favored dark beer, and so on.

The data in Table 11.4 are the observed frequencies for the six classes or categories. If we can determine the expected frequencies under the assumption of independence between beer preference and gender of the beer drinker, we can use the chi-square distribution to determine whether there is a significant difference between observed and expected frequencies.

Expected frequencies for the cells of the contingency table are based on the following rationale. First we assume that the null hypothesis of independence between beer preference and gender of the beer drinker is true. Then we note that in the entire sample of 150 beer drinkers, a total of 50 prefer light beer, 70 prefer regular beer, and 30 prefer dark beer. In terms of fractions we conclude that $^{50}/_{150} = 1/3$ of the beer drinkers prefer light beer, $^{70}/_{150} = 7/15$ prefer regular beer, and $^{30}/_{150} = 1/5$ prefer dark beer. If the *independence* assumption is valid, we argue that these fractions must be applicable to both male and female beer drinkers. Thus, under the assumption of independence, we would expect the sample of 80 male beer drinkers to show that $(1/3)80 = 26.67$ prefer light beer, $(7/15)80 = 37.33$ prefer regular beer, and $(1/5)80 = 16$ prefer dark beer. Application of the same fractions to the 70 female beer drinkers provides the expected frequencies shown in Table 11.5.

Let e_{ij} denote the expected frequency for the contingency table category in row i and column j. With this notation, let us reconsider the expected frequency calculation for males

TABLE 11.5 EXPECTED FREQUENCIES IF BEER PREFERENCE IS INDEPENDENT OF THE GENDER OF THE BEER DRINKER

		Beer Preference			
		Light	Regular	Dark	Total
Gender	Male	26.67	37.33	16.00	80
	Female	23.33	32.67	14.00	70
	Total	50.00	70.00	30.00	150

(row $i = 1$) who prefer regular beer (column $j = 2$); that is, expected frequency e_{12}. Following the preceding argument for the computation of expected frequencies, we can show that

$$e_{12} = (\tfrac{7}{15})80 = 37.33$$

This expression can be written slightly differently as

$$e_{12} = (\tfrac{7}{15})80 = (\tfrac{70}{150})80 = \frac{(80)(70)}{150} = 37.33$$

Note that 80 in the expression is the total number of males (row 1 total), 70 is the total number of individuals preferring regular beer (column 2 total), and 150 is the total sample size. Hence, we see that

$$e_{12} = \frac{(\text{Row 1 Total})(\text{Column 2 Total})}{\text{Sample Size}}$$

Generalization of the expression shows that the following formula provides the expected frequencies for a contingency table in the test of independence.

EXPECTED FREQUENCIES FOR CONTINGENCY TABLES UNDER THE ASSUMPTION OF INDEPENDENCE

$$e_{ij} = \frac{(\text{Row } i \text{ Total})(\text{Column } j \text{ Total})}{\text{Sample Size}} \qquad (11.9)$$

Using equation (11.9) for male beer drinkers who prefer dark beer, we find an expected frequency of $e_{13} = (80)(30)/150 = 16.00$, as shown in Table 11.5. Use equation (11.9) to verify the other expected frequencies shown in Table 11.5.

The test procedure for comparing the observed frequencies of Table 11.4 with the expected frequencies of Table 11.5 is similar to the goodness of fit calculations made in Section 11.2. Specifically, the χ^2 value based on the observed and expected frequencies is computed as follows.

TEST STATISTIC FOR INDEPENDENCE

$$\chi^2 = \sum_i \sum_j \frac{(f_{ij} - e_{ij})^2}{e_{ij}} \qquad (11.10)$$

where

f_{ij} = observed frequency for contingency table category in row i and column j
e_{ij} = expected frequency for contingency table category in row i and column j based on the assumption of independence

Note: With n rows and m columns in the contingency table, the test statistic has a chi-square distribution with $(n - 1)(m - 1)$ degrees of freedom provided that the expected frequencies are 5 or more for all categories.

The double summation in equation (11.10) is used to indicate that the calculation must be made for all the cells in the contingency table.

TABLE 11.6 COMPUTATION OF THE CHI-SQUARE TEST STATISTIC FOR DETERMINING WHETHER BEER PREFERENCE IS INDEPENDENT OF THE GENDER OF THE BEER DRINKER

Gender	Beer Preference	Observed Frequency (f_{ij})	Expected Frequency (e_{ij})	Difference ($f_{ij} - e_{ij}$)	Squared Difference $(f_{ij} - e_{ij})^2$	Squared Difference Divided by Expected Frequency $(f_{ij} - e_{ij})^2/e_{ij}$
Male	Light	20	26.67	−6.67	44.44	1.67
Male	Regular	40	37.33	2.67	7.11	0.19
Male	Dark	20	16.00	4.00	16.00	1.00
Female	Light	30	23.33	6.67	44.44	1.90
Female	Regular	30	32.67	−2.67	7.11	0.22
Female	Dark	10	14.00	−4.00	16.00	1.14
	Total	150				$\chi^2 = 6.12$

By reviewing the expected frequencies in Table 11.5, we see that the expected frequencies are 5 or more for each category. We therefore proceed with the computation of the chi-square test statistic. The calculations necessary to compute the chi-square test statistic for determining whether beer preference is independent of the gender of the beer drinker are shown in Table 11.6. We see that the value of the test statistic is $\chi^2 = 6.12$.

The number of degrees of freedom for the appropriate chi-square distribution is computed by multiplying the number of rows minus 1 by the number of columns minus 1. With two rows and three columns, we have $(2 - 1)(3 - 1) = 2$ degrees of freedom. Just like the test for goodness of fit, the test for independence rejects H_0 if the differences between observed and expected frequencies provide a large value for the test statistic. Thus the test for independence is also an upper tail test. Using the chi-square table (Table 3 of Appendix B), we conclude that the upper tail area or p-value at $\chi^2 = 6.12$ is between .025 and .05. Excel can be used to show that $\chi^2 = 6.12$ provides a p-value = .0468. At the .05 level of significance, p-value $\leq \alpha = .05$. We reject the null hypothesis of independence and conclude that beer preference is not independent of the gender of the beer drinker.

The test for independence is always a one-tailed test with the rejection region in the upper tail of the chi-square distribution.

Although no further statistical conclusions can be made as a result of the test, we can compare the observed and expected frequencies informally to obtain an idea about the dependence between beer preference and gender. Refer to Tables 11.4 and 11.5. We see that male beer drinkers have higher observed than expected frequencies for both regular and dark beers, whereas female beer drinkers have a higher observed than expected frequency only for light beer. These observations give us insight about the beer preference differences between male and female beer drinkers.

Let us summarize the steps in a contingency table test of independence.

TEST OF INDEPENDENCE: A SUMMARY

1. State the null and alternative hypotheses.

   H_0: The column variable is independent of the row variable

   H_a: The column variable is not independent of the row variable

2. Select a random sample and record the observed frequencies for each cell of the contingency table.
3. Use equation (11.9) to compute the expected frequency for each cell.
4. Use equation (11.10) to compute the value of the chi-square test statistic.

5. Rejection rule:

p-value approach:	Reject H_0 if p-value $\leq \alpha$
Critical value approach:	Reject H_0 if $\chi^2 \geq \chi^2_\alpha$

where α is the level of significance, with n rows and m columns providing $(n-1)(m-1)$ degrees of freedom.

In the chapter appendix we show how StatTools can be used for a test of independence.

Using Excel to Conduct a Test of Independence

Excel can be used to conduct a test of independence for the Alber's Brewery example. Refer to Figure 11.4 as we describe the tasks involved. The formula worksheet is in the background; the value worksheet is in the foreground.

WEB file
Alber's

Enter Data: Column A is used to identify each of the 150 individuals in the study. Column B shows the gender and column C shows the beer preference (Light, Regular, or Dark) for each individual.

FIGURE 11.4 EXCEL WORKSHEET FOR THE ALBER'S BREWERY TEST OF INDEPENDENCE

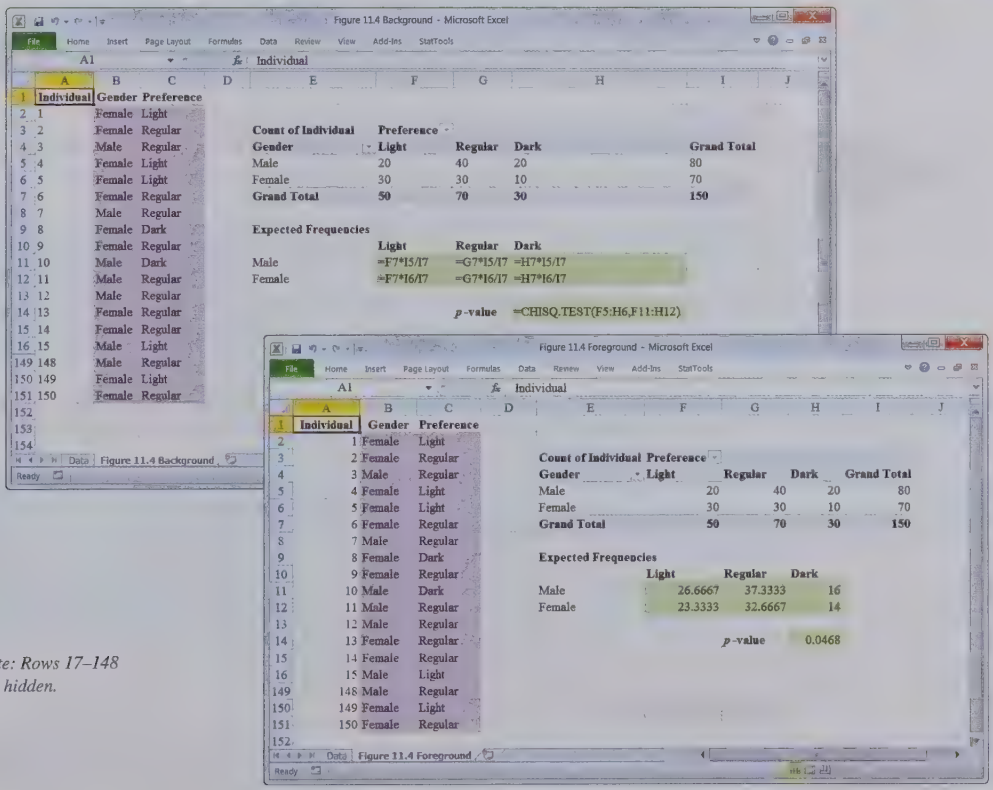


Figure 11.4 Background - Microsoft Excel
Count of Individual Preference
Gender Light Regular Dark Grand Total
Male 20 40 20 80
Female 30 30 10 70
Grand Total 50 70 30 150
Expected Frequencies
Light Regular Dark
Male =F7*I5/I7 =G7*I5/I7 =H7*I5/I7
Female =F7*I6/I7 =G7*I6/I7 =H7*I6/I7
p-value =CHISQ.TEST(F5:H6,F11:H12)
Figure 11.4 Foreground - Microsoft Excel
Count of Individual Preference
Gender Light Regular Dark Grand Total
Male 20 40 20 80
Female 30 30 10 70
Grand Total 50 70 30 150
Expected Frequencies
Light Regular Dark
Male 26.6667 37.3333 16
Female 23.3333 32.6667 14
p-value 0.0468

Note: Rows 17–148 are hidden.

Apply Tools: Using Excel's PivotTable tool (see Section 2.4 for details regarding how to use this tool), we developed the crosstabulation shown in cells E3:I7. The values in cells F5:H6 are the observed frequencies for the Alber's Brewery study.

Enter Functions and Formulas: To compute the expected frequencies for the Alber's Brewery contingency table under the assumption of independence, we use equation (11.9). The formulas in cells F11:H12 are used to compute these expected frequencies. Once the observed and expected frequencies have been computed, Excel's CHISQ.TEST function can be used to compute the p-value for a test of independence. The inputs to the CHISQ.TEST function are the range of values for the observed and expected frequencies. To compute the p-value for this test of independence, we entered the following function into cell H14:

=CHISQ.TEST(F5:H6,F11:H12)

The value worksheet shows that the resulting p-value is .0468. Thus, with $\alpha = .05$ we reject H_0 and conclude that beer preference is not independent of the gender of the beer drinker.

NOTES AND COMMENTS

The test statistic for the chi-square tests in this chapter requires an expected frequency of five for each category. When a category has fewer than five, it is often appropriate to combine two adjacent categories to obtain an expected frequency of five or more in each category.

Exercises

Methods

SELF test

19. The following 2×3 contingency table contains observed frequencies for a sample of 200. Test for independence of the row and column variables using the χ^2 test with $\alpha = .05$.

	Column Variable		
Row Variable	A	B	C
P	20	44	50
Q	30	26	30

20. The following 3×3 contingency table contains observed frequencies for a sample of 240. Test for independence of the row and column variables using the χ^2 test with $\alpha = .05$.

	Column Variable		
Row Variable	A	B	C
P	20	30	20
Q	30	60	25
R	10	15	30

Applications

SELF test

21. One of the questions on the *BusinessWeek* Subscriber Study was, "In the past 12 months, when traveling for business, what type of airline ticket did you purchase most often?" The data obtained are shown in the following contingency table.

	Type of Flight	
Type of Ticket	**Domestic Flights**	**International Flights**
First class	29	22
Business/executive class	95	121
Full fare economy/coach class	518	135

Use $\alpha = .05$ and test for the independence of type of flight and type of ticket. What is your conclusion?

22. Visa Card USA studied how frequently consumers of various age groups use plastic cards (debit and credit cards) when making purchases (Associated Press, January 16, 2006). Sample data for 300 customers shows the use of plastic cards by four age groups.

	Age Group			
Payment	**18–24**	**25–34**	**35–44**	**45 and over**
Plastic	21	27	27	36
Cash or Check	21	36	42	90

a. Test for the independence between method of payment and age group. What is the p-value? Using $\alpha = .05$, what is your conclusion?
b. If method of payment and age group are not independent, what observation can you make about how different age groups use plastic to make purchases?
c. What implications does this study have for companies such as Visa, MasterCard, and Discover?

23. With double-digit annual percentage increases in the cost of health insurance, more and more workers are likely to lack health insurance coverage (*USA Today,* January 23, 2004). The following sample data provide a comparison of workers with and without health insurance coverage for small, medium, and large companies. For the purposes of this study, small companies are companies that have fewer than 100 employees. Medium companies have 100 to 999 employees, and large companies have 1000 or more employees. Sample data are reported for 50 employees of small companies, 75 employees of medium companies, and 100 employees of large companies.

	Health Insurance		
Size of Company	**Yes**	**No**	**Total**
Small	36	14	50
Medium	65	10	75
Large	88	12	100

a. Conduct a test of independence to determine whether employee health insurance coverage is independent of the size of the company. Use $\alpha = .05$. What is the p-value, and what is your conclusion?
b. The *USA Today* article indicated employees of small companies are more likely to lack health insurance coverage. Use percentages based on the preceding data to support this conclusion.

24. *Consumer Reports* measures owner satisfaction of various automobiles by asking the survey question, "Considering factors such as price, performance, reliability, comfort and enjoyment, would you purchase this automobile if you had it to do all over again?"

(Consumer Reports website, January 2009). Sample data for 300 owners of four popular midsize sedans are as follows.

	Automobile				
Purchase Again	Chevrolet Impala	Ford Taurus	Honda Accord	Toyota Camry	Total
Yes	49	44	60	46	199
No	37	27	18	19	101

a. Conduct a test of independence to determine if the owner's intent to purchase again is independent of the automobile. Use a .05 level of significance. What is your conclusion?

b. *Consumer Reports* provides an owner satisfaction score for each automobile by reporting the percentage of owners who would purchase the same automobile if they could do it all over again. What are the *Consumer Reports* owner satisfaction scores for the Chevrolet Impala, Ford Taurus, Honda Accord, and Toyota Camry? Rank the four automobiles in terms of owner satisfaction.

c. Twenty-three different automobiles were reviewed in the *Consumer Reports* midsize sedan class. The overall owner satisfaction score for all automobiles in this class was 69. How do the United States manufactured automobiles (Impala and Taurus) compare to the Japanese manufactured automobiles (Accord and Camry) in terms of owner satisfaction? What is the implication of these findings on the future market share for these automobiles?

25. FlightStats, Inc., collects data on the number of flights scheduled and the number of flights flown at major airports throughout the United States. FlightStats data showed 56% of flights scheduled at Newark, La Guardia, and Kennedy airports were flown during a three-day snowstorm (*The Wall Street Journal,* February 21, 2006). All airlines say they always operate within set safety parameters—if conditions are too poor, they don't fly. The following data show a sample of 400 scheduled flights during the snowstorm.

	Airline				
Did It Fly?	American	Continental	Delta	United	Total
Yes	48	69	68	25	210
No	52	41	62	35	190

Use the chi-square test of independence with a .05 level of significance to analyze the data. What is your conclusion? Do you have a preference for which airline you would choose to fly during similar snowstorm conditions? Explain.

26. As the price of oil rises, there is increased worldwide interest in alternate sources of energy. A *Financial Times*/Harris Poll surveyed people in six countries to assess attitudes toward a variety of alternate forms of energy (*http://www.harrisinteractive.com*, February 27, 2008). The data in the table below are a portion of the poll's findings concerning whether people favor or oppose the building of new nuclear power plants.

	Great Britain	France	Italy	Spain	Germany	United States
Strongly favor	141	161	298	133	128	204
Favor more than oppose	348	366	309	222	272	326
Oppose more than favor	381	334	219	311	322	316
Strongly oppose	217	215	219	443	389	174

a. How large was the sample in this poll?
b. Conduct a hypothesis test to determine whether people's attitude toward building new nuclear power plants is independent of country. What is your conclusion?
c. Using the percentage of respondents who "strongly favor" and "favor more than oppose," which country has the most favorable attitude toward building new nuclear power plants? Which country has the least favorable attitude?

27. The National Sleep Foundation used a survey to determine whether hours of sleeping per night are independent of age (*Newsweek,* January 19, 2004). The following table shows the hours of sleep on weeknights for a sample of individuals age 49 and younger and for a sample of individuals age 50 and older.

	Hours of Sleep				
Age	**Fewer than 6**	**6 to 6.9**	**7 to 7.9**	**8 or more**	**Total**
49 or younger	38	60	77	65	240
50 or older	36	57	75	92	260

a. Conduct a test of independence to determine whether the hours of sleep on weeknights are independent of age. Use $\alpha = .05$. What is the p-value, and what is your conclusion?
b. What is your estimate of the percentage of people who sleep fewer than 6 hours, 6 to 6.9 hours, 7 to 7.9 hours, and 8 or more hours on weeknights?

28. Samples taken in three cities, Anchorage, Atlanta, and Minneapolis, were used to learn about the percentage of married couples with both the husband and the wife in the workforce (*USA Today,* January 15, 2006). Analyze the following data to see whether both the husband and wife being in the workforce is independent of location. Use a .05 level of significance. What is your conclusion? What is the overall estimate of the percentage of married couples with both the husband and the wife in the workforce?

	Location		
In Workforce	**Anchorage**	**Atlanta**	**Minneapolis**
Both	57	70	63
Only One	33	50	90

Summary

In this chapter we described statistical procedures for comparisons involving proportions and the contingency table test for independence of two variables. In the first section, we compared a proportion for one population with the same proportion from another population. We described how to construct an interval estimate for the difference between the proportions and how to conduct a hypothesis test to learn whether the difference between the proportions was statistically significant.

In the second section, we focused on a single multinomial population. There we saw how to conduct hypothesis texts to determine whether the sample proportions for the categories of the multinomial population were significantly different than the hypothesized values. The chi-square goodness of fit test was used to make the comparison.

The final section was concerned with a test of independence for two variables. A test of independence for two variables is an extension of the methodology employed in the goodness of fit test for a multinomial population. A contingency table is used to determine the observed and expected frequencies. Then a chi-square value is computed. Large chi-square values, caused by large differences between observed and expected frequencies, lead to the rejection of the null hypothesis of independence.

Glossary

Pooled estimator of p An estimator of a population proportion obtained by computing a weighted average of the sample proportions obtained from two independent samples.
Multinomial population A population in which each element is assigned to one and only one of several categories. The multinomial distribution extends the binomial distribution from two to three or more outcomes.
Goodness of fit test A statistical test conducted to determine whether to reject a hypothesized probability distribution for a population.
Contingency table A table used to summarize observed and expected frequencies for a test of independence.

Key Formulas

Point Estimator of the Difference Between Two Population Proportions

$$\bar{p}_1 - \bar{p}_2 \tag{11.1}$$

Standard Error of $\bar{p}_1 - \bar{p}_2$

$$\sigma_{\bar{p}_1 - \bar{p}_2} = \sqrt{\frac{p_1(1 - p_1)}{n_1} + \frac{p_2(1 - p_2)}{n_2}} \tag{11.2}$$

Interval Estimate of the Difference Between Two Population Proportions

$$\bar{p}_1 - \bar{p}_2 \pm z_{\alpha/2}\sqrt{\frac{\bar{p}_1(1 - \bar{p}_1)}{n_1} + \frac{\bar{p}_2(1 - \bar{p}_2)}{n_2}} \tag{11.4}$$

Standard Error of $\bar{p}_1 - \bar{p}_2$ when $p_1 = p_2 = p$

$$\sigma_{\bar{p}_1 - \bar{p}_2} = \sqrt{p(1 - p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \tag{11.5}$$

Pooled Estimator of p when $p_1 = p_2 = p$

$$\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2} \tag{11.6}$$

Test Statistic for Hypothesis Tests About $p_1 - p_2$

$$z = \frac{(\bar{p}_1 - \bar{p}_2)}{\sqrt{\bar{p}(1 - \bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \tag{11.7}$$

Test Statistic for Goodness of Fit

$$\chi^2 = \sum_{i=1}^{k} \frac{(f_i - e_i)^2}{e_i} \tag{11.8}$$

Expected Frequencies for Contingency Tables Under the Assumption of Independence

$$e_{ij} = \frac{(\text{Row } i \text{ Total})(\text{Column } j \text{ Total})}{\text{Sample Size}} \tag{11.9}$$

Test Statistic for Independence

$$\chi^2 = \sum_i \sum_j \frac{(f_{ij} - e_{ij})^2}{e_{ij}} \qquad (11.10)$$

Supplementary Exercises

29. Jupiter Media used a survey to determine how people use their free time. Watching television was the most popular activity selected by both men and women (*The Wall Street Journal,* January 26, 2004). The proportion of men and the proportion of women who selected watching television as their most popular leisure time activity can be estimated from the following sample data.

Gender	Sample Size	Watching Television
Men	800	248
Women	600	156

a. State the hypotheses that can be used to test for a difference between the proportion for the population of men and the proportion for the population of women who selected watching television as their most popular leisure time activity.
b. What is the sample proportion of men who selected watching television as their most popular leisure time activity? What is the sample proportion of women?
c. Conduct the hypothesis test and compute the *p*-value. At a .05 level of significance, what is your conclusion?
d. What is the margin of error and 95% confidence interval estimate of the difference between the population proportions?

30. A large automobile insurance company selected samples of single and married male policyholders and recorded the number who made an insurance claim over the preceding three-year period.

Single Policyholders	Married Policyholders
$n_1 = 400$	$n_2 = 900$
Number making claims = 76	Number making claims = 90

a. Use $\alpha = .05$. Test to determine whether the claim rates differ between single and married male policyholders.
b. Provide a 95% confidence interval for the difference between the proportions for the two populations.

31. Medical tests were conducted to learn about drug-resistant tuberculosis. Of 142 cases tested in New Jersey, 9 were found to be drug-resistant. Of 268 cases tested in Texas, 5 were found to be drug-resistant. Do these data suggest a statistically significant difference between the proportions of drug-resistant cases in the two states? Use a .02 level of significance. What is the *p*-value, and what is your conclusion?

WEB file
Occupancy

32. Vacation occupancy rates were expected to be up during March 2008 in Myrtle Beach, South Carolina (*The Sun News,* February 29, 2008). Data in the file Occupancy will allow you to replicate the findings presented in the newspaper. The data show units rented and not rented for a random sample of vacation properties during the first week of March 2007 and March 2008.
a. Estimate the proportion of units rented during the first week of March 2007 and the first week of March 2008.

b. Provide a 95% confidence interval for the difference in proportions.
c. On the basis of your findings, does it appear March rental rates for 2008 will be up from those a year earlier?

33. For the week ended January 15, 2009, the bullish sentiment of individual investors was 27.6% (*AAII Journal*, February 2009). The bullish sentiment was reported to be 48.7% one week earlier and 39.7% one month earlier. The sentiment measures are based on a poll conducted by the American Assocation of Individual Investors. Assume that each of the bullish sentiment measures was based on a sample size of 240.
a. Develop a 95% confidence interval for the difference between the bullish sentiment measures for the most recent two weeks.
b. Develop hypotheses so that rejection of the null hypothesis will allow us to conclude that the most recent bullish sentiment is weaker than that of one month ago.
c. Conduct a hypotheses test of part (b) using $\alpha = .01$. What is your conclusion?

34. In setting sales quotas, the marketing manager makes the assumption that order potentials are the same for each of four sales territories. A sample of 200 sales follows. Should the manager's assumption be rejected? Use $\alpha = .05$.

Sales Territories			
I	II	III	IV
60	45	59	36

35. Seven percent of mutual fund investors rate corporate stocks "very safe," 58% rate them "somewhat safe," 24% rate them "not very safe," 4% rate them "not at all safe," and 7% are "not sure." A *BusinessWeek*/Harris poll asked 529 mutual fund investors how they would rate corporate bonds on safety. The responses are as follows.

Safety Rating	Frequency
Very safe	48
Somewhat safe	323
Not very safe	79
Not at all safe	16
Not sure	63
Total	529

Do mutual fund investors' attitudes toward corporate bonds differ from their attitudes toward corporate stocks? Support your conclusion with a statistical test. Use $\alpha = .01$.

36. Since 2000, the Toyota Camry, Honda Accord, and Ford Taurus have been the three best selling passenger cars in the United States. Based on the 2003 sales data, the market shares among the top three are as follows: Toyota Camry 37%, Honda Accord 34%, and Ford Taurus 29% (*The World Almanac*, 2004). Assume a sample of 1200 sales of passenger cars during the first quarter of 2004 shows the following.

Passenger Car	Units Sold
Toyota Camry	480
Honda Accord	390
Ford Taurus	330

Can these data be used to conclude that the market shares among the top three passenger cars have changed during the first quarter of 2004? What is the *p*-value? Use a .05 level of significance. What is your conclusion?

37. A regional transit authority is concerned about the number of riders on one of its bus routes. In setting up the route, the assumption is that the number of riders is the same on every day from Monday through Friday. Using the following data, test with $\alpha = .05$ to determine whether the transit authority's assumption is correct.

Day	Number of Riders
Monday	13
Tuesday	16
Wednesday	28
Thursday	17
Friday	16

38. The results of *Computerworld*'s Annual Job Satisfaction Survey showed that 28% of information systems (IS) managers are very satisfied with their job, 46% are somewhat satisfied, 12% are neither satisfied nor dissatisfied, 10% are somewhat dissatisfied, and 4% are very dissatisfied. Suppose that a sample of 500 computer programmers yielded the following results.

Category	Number of Respondents
Very satisfied	105
Somewhat satisfied	235
Neither	55
Somewhat dissatisfied	90
Very dissatisfied	15

Use $\alpha = .05$ and test to determine whether the job satisfaction for computer programmers is different from the job satisfaction for IS managers.

39. A sample of parts provided the following contingency table data on part quality by production shift.

Shift	Number Good	Number Defective
First	368	32
Second	285	15
Third	176	24

Use $\alpha = .05$ and test the hypothesis that part quality is independent of the production shift. What is your conclusion?

40. *The Wall Street Journal* Subscriber Study showed data on the employment status of subscribers. Sample results corresponding to subscribers of the eastern and western editions are shown here.

	Region	
Employment Status	Eastern Edition	Western Edition
Full-time	1105	574
Part-time	31	15
Self-employed/consultant	229	186
Not employed	485	344

Use $\alpha = .05$ and test the hypothesis that employment status is independent of the region. What is your conclusion?

41. A lending institution supplied the following data on loan approvals by four loan officers. Use $\alpha = .05$ and test to determine whether the loan approval decision is independent of the loan officer reviewing the loan application.

	Loan Approval Decision	
Loan Officer	**Approved**	**Rejected**
Miller	24	16
McMahon	17	13
Games	35	15
Runk	11	9

42. A Pew Research Center survey asked respondents if they would rather live in a place with a slower pace of life or a place with a faster pace of life (*USA Today*, February 13, 2009). Consider the following data, which show a sample of preferences expressed by 150 men and 150 women.

	Preferred Pace of Life		
Respondent	**Slower**	**No Preference**	**Faster**
Men	102	9	39
Women	111	12	27

a. Combine the samples of men and women. What is the overall percentage of respondents who prefer to live in a place with a slower pace of life? What is the overall percentage of respondents who prefer to live in a place with a faster pace of life? What is your conclusion?

b. Is the preferred pace of life independent of the respondent? Use $\alpha = .05$. What is your conclusion? What is your recommendation?

43. Barna Research Group collected data showing church attendance by age group (*USA Today*, November 20, 2003). Use the sample data to determine whether attending church is independent of age. Use a .05 level of significance. What is your conclusion? What conclusion can you draw about church attendance as individuals grow older?

	Church Attendance		
Age	**Yes**	**No**	**Total**
20 to 29	31	69	100
30 to 39	63	87	150
40 to 49	94	106	200
50 to 59	72	78	150

44. The office occupancy rates were reported for four California metropolitan areas. Do the following data suggest that the office vacancies were independent of metropolitan area? Use a .05 level of significance. What is your conclusion?

Occupancy Status	**Los Angeles**	**San Diego**	**San Francisco**	**San Jose**
Occupied	160	116	192	174
Vacant	40	34	33	26

45. A salesperson makes four calls per day. A sample of 100 days gives the following frequencies of sales volumes.

Number of Sales	Observed Frequency (days)
0	30
1	32
2	25
3	10
4	3
Total	100

Records show sales are made to 30% of all sales calls. Assuming independent sales calls, the number of sales per day should follow a binomial distribution. The binomial probability function presented in Chapter 5 is

$$f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

For this exercise, assume that the population has a binomial distribution with $n = 4$, $p = .30$, and $x = 0, 1, 2, 3$, and 4.

a. Compute the expected frequencies for $x = 0, 1, 2, 3$, and 4 by using the binomial probability function. Combine categories if necessary to satisfy the requirement that the expected frequency is five or more for all categories.

b. Use the goodness of fit test to determine whether the assumption of a binomial distribution should be rejected. Use $\alpha = .05$. Because no parameters of the binomial distribution were estimated from the sample data, the degrees of freedom are $k - 1$ when k is the number of categories.

Case Problem A Bipartisan Agenda for Change

A survey of 100 individuals who live in the western region of New York State was conducted in order to obtain their views of how well the state government is functioning. The party affiliation (Democrat, Independent, Republican) of each individual surveyed was recorded, as well as their responses to the following three questions.

1. Should legislative pay be cut for every day the state budget is late?
   Yes ____ No ____
2. Should there be more restrictions on lobbyists?
   Yes ____ No ____
3. Should there be term limits requiring that legislators serve a fixed number of years?
   Yes ____ No ____

NYReform

The responses were coded using 1 for a Yes response and 2 for a No response. The complete data set is available on the website that accompanies the text in the data set named NYReform.

Managerial Report

1. Use descriptive statistics to summarize the data from this study. What are your preliminary conclusions about the independence of the response (Yes or No) and party affiliation for each of the three questions in the survey?
2. With regard to question 1, test for the independence of the response (Yes and No) and party affiliation. Use $\alpha = .05$.

3. With regard to question 2, test for the independence of the response (Yes and No) and party affiliation. Use $\alpha = .05$.
4. With regard to question 3, test for the independence of the response (Yes and No) and party affiliation. Use $\alpha = .05$.
5. Does it appear that there is broad support for change across all political lines? Explain.

Appendix Test of Independence Using StatTools

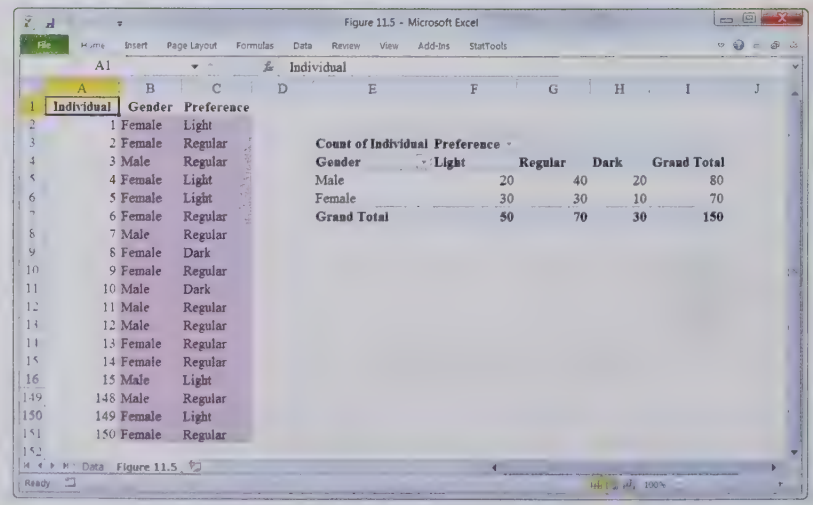
WEB file
Alber's

In this appendix we show how StatTools can be used to conduct a chi-square test of independence. We illustrate using the Alber's Brewery example from Section 11.3. Refer to Figure 11.5 as we describe the steps involved. We have created an Excel PivotTable for the data using the procedures described in Section 2.4. The row labels Male and Female appear in cells E5 and E6. The column labels Light, Regular, and Dark appear in cells F4, G4, and H4. The observed frequencies appear in cells F5:H6. Along with the row and column totals, this is the Alber's Brewery contingency table. The StatTools steps for conducting the test of independence follow.

Step 1. Click the **StatTools** tab on the Ribbon
Step 2. In the **Analyses Group**, click **Statistical Inference**
Step 3. Choose **Chi-square Independence Test**
Step 4. When the StatTools - Chi-square Test for Independence dialog box appears,
Enter E4:H6 in the **Contingency Table Range** box
Select **Table Includes Row and Column Headers**
Click **OK**

StatTools will create a report showing row and column percentages, expected counts, as well as the chi-square statistic and the p-value of .0468 for the test of independence.

FIGURE 11.5 CONTINGENCY TABLE FOR ALBER'S BREWERY TEST OF INDEPENDENCE



Note: Rows 17–148 are hidden.

CHAPTER 12

Simple Linear Regression

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STATISTICS in PRACTICE

ALLIANCE DATA SYSTEMS*

DALLAS, TEXAS

Alliance Data Systems (ADS) provides transaction processing, credit services, and marketing services for clients in the rapidly growing customer relationship management (CRM) industry. ADS clients are concentrated in four industries: retail, petroleum/convenience stores, utilities, and transportation. In 1983, Alliance began offering end-to-end credit processing services to the retail, petroleum, and casual dining industries; today Alliance employs more than 6500 employees who provide services to clients around the world. Operating more than 140,000 point-of-sale terminals in the United States alone, ADS processes in excess of 2.5 billion transactions annually. The company ranks second in the United States in private label credit services by representing 49 private label programs with nearly 72 million cardholders. In 2001, ADS made an initial public offering and is now listed on the New York Stock Exchange.

As one of its marketing services, ADS designs direct mail campaigns and promotions. With its database containing information on the spending habits of more than 100 million consumers, ADS can target those consumers most likely to benefit from a direct mail promotion. The Analytical Development Group uses regression analysis to build models that measure and predict the responsiveness of consumers to direct market campaigns. Some regression models predict the probability of purchase for individuals receiving a promotion, and others predict the amount spent by those consumers making a purchase.

For one particular campaign, a retail store chain wanted to attract new customers. To predict the effect of the campaign, ADS analysts selected a sample from the consumer database, sent the sampled individuals promotional materials, and then collected transaction data on the consumers' response. Sample data were collected on the amount of purchase made by the consumers responding to the campaign, as well as a variety of consumer-specific variables thought to be useful in predicting sales. The consumer-specific variable that contributed most to predicting the amount purchased was the total amount of credit purchases at related stores over the past 39 months. ADS analysts developed an estimated regression equation relating the amount of purchase to the amount spent at related stores:

© Courtesy of Alliance Data Systems.

Alliance Data Systems analysts discuss use of a regression model to predict sales for a direct marketing campaign.

$$\hat{y} = 26.7 + 0.00205x$$

where

$\hat{y}$ = amount of purchase
x = amount spent at related stores

Using this equation, we could predict that someone spending $10,000 over the past 39 months at related stores would spend $47.20 when responding to the direct mail promotion. In this chapter, you will learn how to develop this type of estimated regression equation.

The final model developed by ADS analysts also included several other variables that increased the predictive power of the preceding equation. Some of these variables included the absence/presence of a bank credit card, estimated income, and the average amount spent per trip at a selected store. In the following chapter, we will learn how such additional variables can be incorporated into a multiple regression model.

*The authors are indebted to Philip Clemance, Director of Analytical Development at Alliance Data Systems, for providing this Statistics in Practice.

Managerial decisions often are based on the relationship between two or more variables. For example, after considering the relationship between advertising expenditures and sales, a marketing manager might attempt to predict sales for a given level of advertising expenditures. In another case, a public utility might use the relationship between the daily high temperature and the demand for electricity to predict electricity usage on the basis of next month's anticipated daily high temperatures. Sometimes a manager will rely on intuition to judge how two variables are related. However, if data can be obtained, a statistical procedure called *regression analysis* can be used to develop an equation showing how the variables are related.

The statistical methods used in studying the relationship between two variables were first employed by Sir Francis Galton (1822–1911). Galton was interested in studying the relationship between a father's height and the son's height. Galton's disciple, Karl Pearson (1857–1936), analyzed the relationship between the father's height and the son's height for 1078 pairs of subjects.

In regression terminology, the variable being predicted is called the **dependent variable**. The variable or variables being used to predict the value of the dependent variable are called the **independent variables**. For example, in analyzing the effect of advertising expenditures on sales, a marketing manager's desire to predict sales would suggest making sales the dependent variable. Advertising expenditure would be the independent variable used to help predict sales. In statistical notation, y denotes the dependent variable and x denotes the independent variable.

In this chapter we consider the simplest type of regression analysis involving one independent variable and one dependent variable in which the relationship between the variables is approximated by a straight line. It is called **simple linear regression**. Regression analysis involving two or more independent variables is called multiple regression analysis; multiple regression and cases involving curvilinear relationships are covered in Chapter 13.

12.1 Simple Linear Regression Model

Armand's Pizza Parlors is a chain of Italian-food restaurants located in a five-state area. Armand's most successful locations are near college campuses. The managers believe that quarterly sales for these restaurants (denoted by y) are related positively to the size of the student population (denoted by x); that is, restaurants near campuses with a large student population tend to generate more sales than those located near campuses with a small student population. Using regression analysis, we can develop an equation showing how the dependent variable y is related to the independent variable x.

Regression Model and Regression Equation

In the Armand's Pizza Parlors example, the population consists of all the Armand's restaurants located near college campuses. For every restaurant in the population, there is a value of x (student population) and a corresponding value of y (quarterly sales). The equation that describes how y is related to x and an error term is called the **regression model**. The regression model used in simple linear regression follows.

SIMPLE LINEAR REGRESSION MODEL

$$y = \beta_0 + \beta_1 x + \epsilon \qquad (12.1)$$

β_0 and β_1 are referred to as the parameters of the model, and ϵ (the Greek letter epsilon) is a random variable referred to as the error term. The error term accounts for the variability in y that cannot be explained by the linear relationship between x and y.

The population of all Armand's restaurants can also be viewed as a collection of subpopulations, one for each distinct value of x. For example, one subpopulation consists of all Armand's restaurants located near college campuses with 8000 students; another subpopulation consists of all Armand's restaurants located near college campuses with 9000 students; and so on. Each subpopulation has a corresponding distribution of y values. Thus, a distribution of y values is associated with restaurants located near campuses with 8000 students; a distribution of y values is associated with restaurants located near campuses with 9000 students; and so on. Each distribution of y values has its own mean or expected value. The equation that describes how the expected value of y, denoted $E(y)$, is related to x is called the **regression equation**. The regression equation for simple linear regression follows.

SIMPLE LINEAR REGRESSION EQUATION

$$E(y) = \beta_0 + \beta_1 x \qquad (12.2)$$

The graph of the simple linear regression equation is a straight line; β_0 is the y-intercept of the regression line, β_1 is the slope, and $E(y)$ is the mean or expected value of y for a given value of x.

Examples of possible regression lines are shown in Figure 12.1. The regression line in Panel A shows that the mean value of y is related positively to x, with larger values of $E(y)$ associated with larger values of x. The regression line in Panel B shows the mean value of y is related negatively to x, with smaller values of $E(y)$ associated with larger values of x. The regression line in Panel C shows the case in which the mean value of y is not related to x; that is, the mean value of y is the same for every value of x.

Estimated Regression Equation

If the values of the population parameters β_0 and β_1 were known, we could use equation (12.2) to compute the mean value of y for a given value of x. In practice, the parameter values are not known and must be estimated using sample data. Sample statistics (denoted b_0 and b_1) are computed as estimates of the population parameters β_0 and β_1. Substituting the values of the sample statistics b_0 and b_1 for β_0 and β_1 in the regression equation, we obtain the

FIGURE 12.1 POSSIBLE REGRESSION LINES IN SIMPLE LINEAR REGRESSION

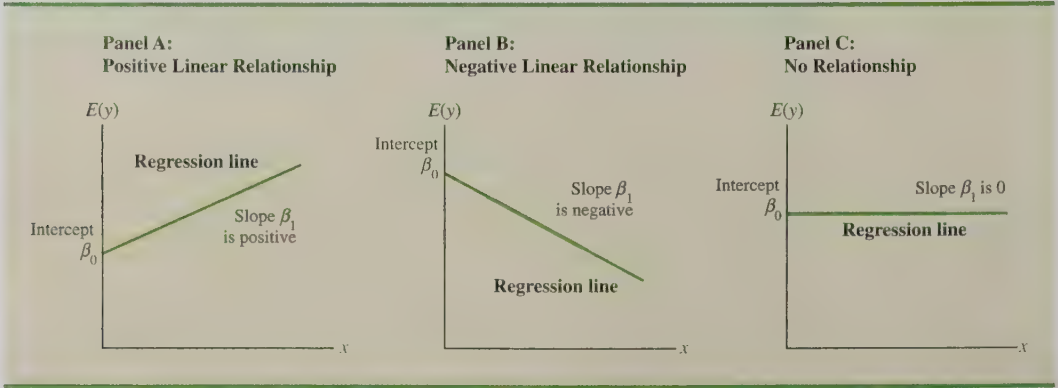
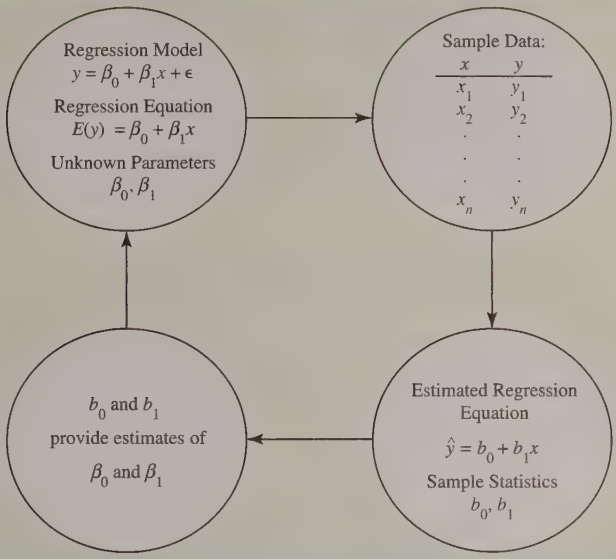


FIGURE 12.2 THE ESTIMATION PROCESS IN SIMPLE LINEAR REGRESSION



The estimation of β_0 and β_1 is a statistical process much like the estimation of μ discussed in Chapter 7. β_0 and β_1 are the unknown parameters of interest, and b_0 and b_1 are the sample statistics used to estimate the parameters.

estimated regression equation. The estimated regression equation for simple linear regression follows.

ESTIMATED SIMPLE LINEAR REGRESSION EQUATION

$$\hat{y} = b_0 + b_1 x \tag{12.3}$$

Figure 12.2 provides a summary of the estimation process for simple linear regression.

The graph of the estimated simple linear regression equation is called the *estimated regression line*; b_0 is the y intercept and b_1 is the slope. In the next section, we show how the least squares method can be used to compute the values of b_0 and b_1 in the estimated regression equation.

In general, $\hat{y}$ is the point estimator of $E(y)$, the mean value of y for a given value of x. Thus, to estimate the mean or expected value of quarterly sales for all restaurants located near campuses with 10,000 students, Armand's would substitute the value of 10,000 for x in equation (12.3). In some cases, however, Armand's may be more interested in predicting sales for one particular restaurant. For example, suppose Armand's would like to predict quarterly sales for a new restaurant they are considering building near Talbot College, a school with 10,000 students. As it turns out, the best predictor of y for a given value of x is also provided by $\hat{y}$. Thus, to predict quarterly sales for a new restaurant located near Talbot College, Armand's would also substitute the value of 10,000 for x in equation (12.3).

The value of $\hat{y}$ provides both a point estimate of $E(y)$ for a given value of x and a prediction of an individual value of y for a given value of x. In most cases, we will refer to $\hat{y}$ simply as the *predicted value of y*.

NOTES AND COMMENTS

1. Regression analysis cannot be interpreted as a procedure for establishing a cause-and-effect relationship between variables. It can only indicate how or to what extent variables are associated with each other. Any conclusions about cause and effect must be based upon the judgment of those individuals most knowledgeable about the application.
2. The regression equation in simple linear regression is $E(y) = \beta_0 + \beta_1 x$. More advanced texts in regression analysis often write the regression equation as $E(y|x) = \beta_0 + \beta_1 x$ to emphasize that the regression equation provides the mean value of y for a given value of x.

12.2 Least Squares Method

In simple linear regression, each observation consists of two values: one for the independent variable and one for the dependent variable.

The **least squares method** is a procedure for using sample data to find the estimated regression equation. To illustrate the least squares method, suppose data were collected from a sample of 10 Armand's Pizza Parlor restaurants located near college campuses. For the ith observation or restaurant in the sample, x_i is the size of the student population (in thousands) and y_i is the quarterly sales (in thousands of dollars). The values of x_i and y_i for the 10 restaurants in the sample are summarized in Table 12.1. We see that restaurant 1, with $x_1 = 2$ and $y_1 = 58$, is near a campus with 2000 students and has quarterly sales of $58,000. Restaurant 2, with $x_2 = 6$ and $y_2 = 105$, is near a campus with 6000 students and has quarterly sales of $105,000. The largest sales value is for restaurant 10, which is near a campus with 26,000 students and has quarterly sales of $202,000.

Figure 12.3 is a scatter diagram of the data in Table 12.1. Student population is shown on the horizontal axis and quarterly sales is shown on the vertical axis. **Scatter diagrams** for regression analysis are constructed with the independent variable x on the horizontal axis and the dependent variable y on the vertical axis. The scatter diagram enables us to observe the data graphically and to draw preliminary conclusions about the possible relationship between the variables.

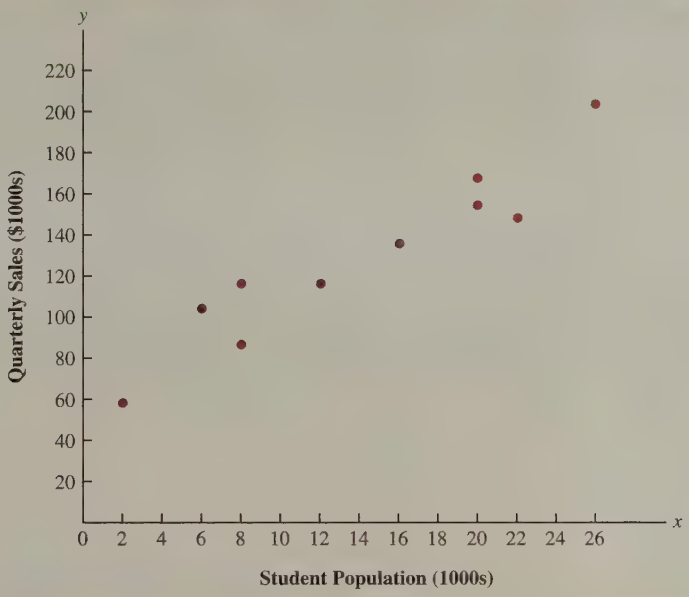
What preliminary conclusions can be drawn from Figure 12.3? Quarterly sales appear to be higher at campuses with larger student populations. In addition, for these data the relationship between the size of the student population and quarterly sales appears to be

TABLE 12.1 STUDENT POPULATION AND QUARTERLY SALES DATA FOR 10 ARMAND'S PIZZA PARLORS

WEB file Armand's

Restaurant i	Student Population (1000s) x_i	Quarterly Sales ($1000s) y_i
1	2	58
2	6	105
3	8	88
4	8	118
5	12	117
6	16	137
7	20	157
8	20	169
9	22	149
10	26	202

FIGURE 12.3 SCATTER DIAGRAM OF STUDENT POPULATION AND QUARTERLY SALES FOR ARMAND'S PIZZA PARLORS



approximated by a straight line; indeed, a positive linear relationship is indicated between x and y. We therefore choose the simple linear regression model to represent the relationship between quarterly sales and student population. Given that choice, our next task is to use the sample data in Table 12.1 to determine the values of b_0 and b_1 in the estimated simple linear regression equation. For the ith restaurant, the estimated regression equation provides

$$\hat{y}_i = b_0 + b_1 x_i \qquad \textbf{(12.4)}$$

where

$\hat{y}_i$ = predicted value of quarterly sales ($1000s) for the ith restaurant
b_0 = the y intercept of the estimated regression line
b_1 = the slope of the estimated regression line
x_i = size of the student population (1000s) for the ith restaurant

With y_i denoting the observed (actual) sales for restaurant i and $\hat{y}_i$ in equation (12.4) representing the predicted value of sales for restaurant i, every restaurant in the sample will have an observed value of sales y_i and a predicted value of sales $\hat{y}_i$. For the estimated regression line to provide a good fit to the data, we want the differences between the observed sales values and the predicted sales values to be small.

The least squares method uses the sample data to provide the values of b_0 and b_1 that minimize the *sum of the squares of the deviations* between the observed values of the dependent variable y_i and the predicted values of the dependent variable $\hat{y}_i$. The criterion for the least squares method is given by expression (12.5).

LEAST SQUARES CRITERION

$$\min \Sigma(y_i - \hat{y}_i)^2 \qquad (12.5)$$

where

y_i = observed value of the dependent variable for the ith observation
$\hat{y}_i$ = predicted value of the dependent variable for the ith observation

Carl Friedrich Gauss (1777–1855) proposed the least squares method.

Differential calculus can be used to show that the values of b_0 and b_1 that minimize expression (12.5) can be found by using equations (12.6) and (12.7).

SLOPE AND y-INTERCEPT FOR THE ESTIMATED REGRESSION EQUATION[1]

$$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} \qquad (12.6)$$

$$b_0 = \bar{y} - b_1\bar{x} \qquad (12.7)$$

where

x_i = value of the independent variable for the ith observation
y_i = value of the dependent variable for the ith observation
$\bar{x}$ = mean value for the independent variable
$\bar{y}$ = mean value for the dependent variable
n = total number of observations

In computing b_1 with a calculator, carry as many significant digits as possible in the intermediate calculations. We recommend carrying at least four significant digits.

Some of the calculations necessary to develop the least squares estimated regression equation for Armand's Pizza Parlors are shown in Table 12.2. With the sample of 10 restaurants, we have $n = 10$ observations. Because equations (12.6) and (12.7) require $\bar{x}$ and $\bar{y}$, we begin the calculations by computing $\bar{x}$ and $\bar{y}$.

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{140}{10} = 14$$

$$\bar{y} = \frac{\Sigma y_i}{n} = \frac{1300}{10} = 130$$

Using equations (12.6) and (12.7) and the information in Table 12.2, we can compute the slope and intercept of the estimated regression equation for Armand's Pizza Parlors. The calculation of the slope (b_1) proceeds as follows.

[1] An alternate formula for b_1 is

$$b_1 = \frac{\Sigma x_i y_i - (\Sigma x_i \Sigma y_i)/n}{\Sigma x_i^2 - (\Sigma x_i)^2/n}$$

This form of equation (12.6) is often recommended when using a calculator to compute b_1.

TABLE 12.2 CALCULATIONS FOR THE LEAST SQUARES ESTIMATED REGRESSION EQUATION FOR ARMAND PIZZA PARLORS

Restaurant i	x_i	y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})(y_i - \bar{y})$	$(x_i - \bar{x})^2$
1	2	58	−12	−72	864	144
2	6	105	−8	−25	200	64
3	8	88	−6	−42	252	36
4	8	118	−6	−12	72	36
5	12	117	−2	−13	26	4
6	16	137	2	7	14	4
7	20	157	6	27	162	36
8	20	169	6	39	234	36
9	22	149	8	19	152	64
10	26	202	12	72	864	144
Totals	140	1300			2840	568
	Σx_i	Σy_i			$\Sigma(x_i - \bar{x})(y_i - \bar{y})$	$\Sigma(x_i - \bar{x})^2$

$$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{2840}{568} = 5$$

The calculation of the y intercept (b_0) follows.

$$b_0 = \bar{y} - b_1\bar{x} = 130 - 5(14) = 60$$

Thus, the estimated regression equation is

$$\hat{y} = 60 + 5x$$

Figure 12.4 shows the graph of this equation on the scatter diagram.

The slope of the estimated regression equation ($b_1 = 5$) is positive, implying that as student population increases, sales increase. In fact, we can conclude (based on sales measured in \$1000s and student population in 1000s) that an increase in the student population of 1000 is associated with an increase of \$5000 in expected sales; that is, quarterly sales are expected to increase by \$5 per student.

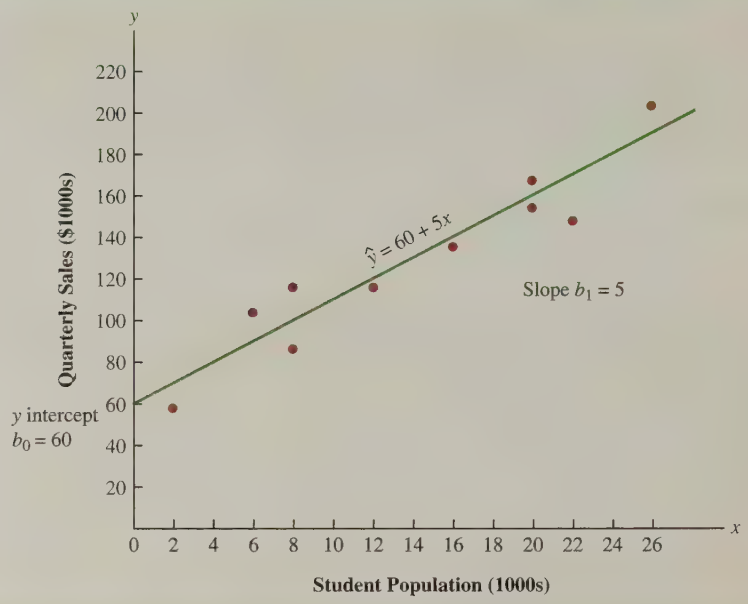
Using the estimated regression equation to make predictions outside the range of the values of the independent variable should be done with caution because outside that range we cannot be sure that the same relationship is valid.

If we believe the least squares estimated regression equation adequately describes the relationship between x and y, it would seem reasonable to use the estimated regression equation to predict the value of y for a given value of x. For example, if we wanted to predict quarterly sales for a restaurant to be located near a campus with 16,000 students, we would compute

$$\hat{y} = 60 + 5(16) = 140$$

Hence, we would predict quarterly sales of \$140,000 for this restaurant. In the following sections we will discuss methods for assessing the appropriateness of using the estimated regression equation for estimation and prediction.

FIGURE 12.4 GRAPH OF THE ESTIMATED REGRESSION EQUATION FOR ARMAND'S PIZZA PARLORS: $\hat{y} = 60 + 5x$



Using Excel's Chart Tools to Construct a Scatter Diagram and Compute the Estimated Regression Equation

We can use Excel's chart tools to construct a scatter diagram and compute the estimated regression equation for the Armand's Pizza Parlors data appearing in Table 12.1. Refer to Figure 12.5 as we describe the tasks involved.

Enter Data: Appropriate labels and the sample data have been entered into cells A1:C11 of the worksheet shown in Figure 12.5.

Apply Tools: The following steps describe how to use Excel's chart tools to construct a scatter diagram and compute the estimated regression equation using the data in the worksheet.

Step 1. Select cells B2:C11
Step 2. Click the **Insert** tab on the Ribbon
Step 3. In the **Charts** group, click **Scatter**
Step 4. When the list of scatter diagram subtypes appears,
Click **Scatter with only Markers** (the chart in the upper left corner)
Step 5. In the **Chart Layouts** group, click **Layout 1**
Step 6. Right-click on the **Chart Title** to display a list of options; choose **Delete**
Step 7. Select the **Horizontal (Value) Axis Title** and replace it with **Student Population (1000s)**
Step 8. Select the **Vertical (Value) Axis Title** and replace it with **Quarterly Sales ($1000s)**

FIGURE 12.5 SCATTER DIAGRAM, ESTIMATED REGRESSION EQUATION, AND THE ESTIMATED REGRESSION LINE FOR ARMAND'S PIZZA PARLORS

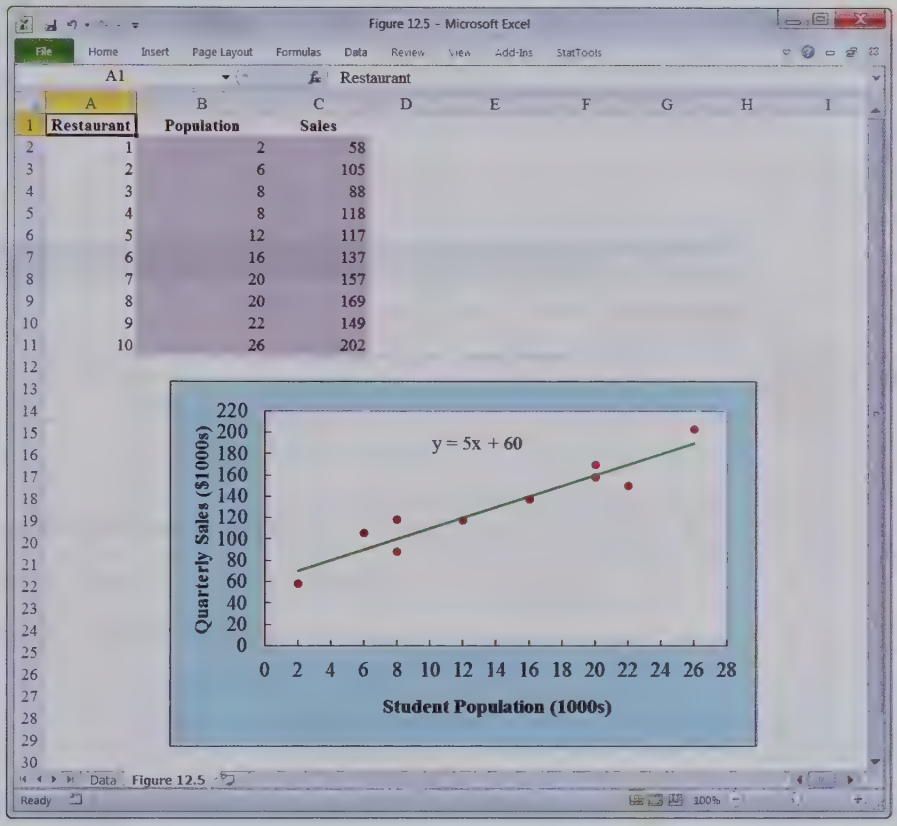


Figure 12.5 - Microsoft Excel
File Home Insert Page Layout Formulas Data Review View Add-Ins StatTools
A1 Restaurant
Restaurant Population Sales
1 2 58
2 6 105
3 8 88
4 8 118
5 12 117
6 16 137
7 8 157
8 20 169
9 22 149
10 26 202
y = 5x + 60
Quarterly Sales ($1000s)
Student Population (1000s)
0 20 40 60 80 100 120 140 160 180 200 220
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28
Data Figure 12.5
Ready

Step 9. Right-click on the **Series 1 Legend Entry** to display a list of options; choose **Delete**

Step 10. Position the mouse pointer over any **Vertical (Value) Axis Major Gridline** in the scatter diagram and right-click to display a list of options; choose **Delete**

Step 11. Position the mouse pointer over any data point in the scatter diagram and right-click to display a list of options; choose **Add Trendline**

Step 12. When the **Format Trendline** dialog box appears,
Select **Trendline Options** and then
Choose **Linear** from the **Trend/Regression Type** list
Choose **Display Equation on chart**
Click **Close**

The worksheet displayed in Figure 12.5 shows the scatter diagram, the estimated regression line, and the estimated regression equation.

NOTES AND COMMENTS

The least squares method provides an estimated regression equation that minimizes the sum of squared deviations between the observed values of the dependent variable y_i and the predicted values of the dependent variable $\hat{y}_i$. This least squares criterion is used to choose the equation that provides the best fit. If some other criterion were used, such as minimizing the sum of the absolute deviations between y_i and $\hat{y}_i$, a different equation would be obtained. In practice, the least squares method is the most widely used.

Exercises

Methods

SELF test

1. Given are five observations for two variables, x and y.

x_i	1	2	3	4	5
y_i	3	7	5	11	14

   a. Develop a scatter diagram for these data.
   b. What does the scatter diagram developed in part (a) indicate about the relationship between the two variables?
   c. Try to approximate the relationship between x and y by drawing a straight line through the data.
   d. Develop the estimated regression equation by computing the values of b_0 and b_1 using equations (12.6) and (12.7).
   e. Use the estimated regression equation to predict the value of y when $x = 4$.

2. Given are five observations for two variables, x and y.

x_i	3	12	6	20	14
y_i	55	40	55	10	15

   a. Develop a scatter diagram for these data.
   b. What does the scatter diagram developed in part (a) indicate about the relationship between the two variables?
   c. Try to approximate the relationship between x and y by drawing a straight line through the data.
   d. Develop the estimated regression equation by computing the values of b_0 and b_1 using equations (12.6) and (12.7).
   e. Use the estimated regression equation to predict the value of y when $x = 10$.

3. Given are five observations collected in a regression study on two variables.

x_i	2	6	9	13	20
y_i	7	18	9	26	23

   a. Develop a scatter diagram for these data.
   b. Develop the estimated regression equation for these data.
   c. Use the estimated regression equation to predict the value of y when $x = 6$.

Applications

SELF test

4. The following data give the percentage of women working in five companies in the retail and trade industry. The percentage of management jobs held by women in each company is also shown.

% Working	67	45	73	54	61
% Management	49	21	65	47	33

   a. Develop a scatter diagram for these data with the percentage of women working in the company as the independent variable.
   b. What does the scatter diagram developed in part (a) indicate about the relationship between the two variables?
   c. Try to approximate the relationship between the percentage of women working in the company and the percentage of management jobs held by women in that company.
   d. Develop the estimated regression equation by computing the values of b_0 and b_1.
   e. Predict the percentage of management jobs held by women in a company that has 60% women employees.

5. Elliptical trainers are becoming one of the more popular exercise machines. Their smooth and steady low-impact motion makes them a preferred choice for individuals with knee and ankle problems. But selecting the right trainer can be a difficult process. Price and quality are two important factors in any purchase decision. Are higher prices generally associated with higher quality elliptical trainers? *Consumer Reports* conducted extensive tests to develop an overall rating based on ease of use, ergonomics, construction, and exercise range. The following data show the price and rating for eight elliptical trainers tested (*Consumer Reports,* February 2008).

WEB file
Ellipticals

Brand and Model	Price ($)	Rating
Precor 5.31	3700	87
Keys Fitness CG2	2500	84
Octane Fitness Q37e	2800	82
LifeFitness X1 Basic	1900	74
NordicTrack AudioStrider 990	1000	73
Schwinn 430	800	69
Vision Fitness X6100	1700	68
ProForm XP 520 Razor	600	55

   a. Develop a scatter diagram with price as the independent variable.
   b. An exercise equipment store that sells primarily higher priced equipment has a sign over the display area that says "Quality: You Get What You Pay For." Based upon your analysis of the data for ellipical trainers, do you think this sign fairly reflects the price-quality relationship for elliptical trainers?
   c. Use the least squares method to develop the estimated regression equation.
   d. Use the estimated regression equation to predict the rating for an ellipitical trainer with a price of $1500.

6. The cost of a previously owned car depends upon factors such as make and model, model year, mileage, condition, and whether the car is purchased from a dealer or from a private

seller. To investigate the relationship between the car's mileage and the sales price, data were collected on the mileage and the sale price for 10 private sales of model year 2000 Honda Accords (PriceHub website, October 2008).

Miles (1000s)	Price ($1000s)
90	7.0
59	7.5
66	6.6
87	7.2
90	7.0
106	5.4
94	6.4
57	7.0
138	5.1
87	7.2

a. Develop a scatter diagram with miles as the independent variable.
b. What does the scatter diagram developed in part (a) indicate about the relationship between the two variables?
c. Use the least squares method to develop the estimated regression equation.
d. Provide an interpretation for the slope of the estimated regression equation.
e. Predict the sales price for a 2000 Honda Accord with 100,000 miles.

7. A sales manager collected the following data on annual sales and years of experience.

Salesperson	Years of Experience	Annual Sales ($1000s)
1	1	80
2	3	97
3	4	92
4	4	102
5	6	103
6	8	111
7	10	119
8	10	123
9	11	117
10	13	136

a. Develop a scatter diagram for these data with years of experience as the independent variable.
b. Develop an estimated regression equation that can be used to predict annual sales given the years of experience.
c. Use the estimated regression equation to predict annual sales for a salesperson with 9 years of experience.

8. Bergans of Norway has been making outdoor gear since 1908. The following data show the temperature rating (F°) and the price ($) for 11 models of sleeping bags produced by Bergans (*Backpacker* 2006 Gear Guide).

WEB file
SleepingBags

Model	Temperature Rating (F°)	Price ($)
Ranger 3-Seasons	12	319
Ranger Spring	24	289
Ranger Winter	3	389
Rondane 3-Seasons	13	239
Rondane Summer	38	149
Rondane Winter	4	289
Senja Ice	5	359
Senja Snow	15	259
Senja Zero	25	229
Super Light	45	129
Tight & Light	25	199

a. Develop a scatter diagram for these data with temperature rating (F°) as the independent variable.
b. What does the scatter diagram developed in part (a) indicate about the relationship between temperature rating (F°) and price?
c. Use the least squares method to develop the estimated regression equation.
d. Predict the price for a sleeping bag with a temperature rating (F°) of 20.

9. To avoid extra checked-bag fees, airline travelers often pack as much as they can into their suitcase. Finding a rolling suitcase that is durable, has good capacity, and is easy to pull can be difficult. The following table shows the results of tests conducted by *Consumer Reports* for 10 rolling suitcases; higher scores indicate better overall test results (Consumer Reports website, October 2008).

WEB file
Suitcases

Brand	Price ($)	Score
Briggs & Riley	325	72
Hartman	350	74
Heys	67	54
Kenneth Cole Reaction	120	54
Liz Claiborne	85	64
Samsonite	180	57
Titan	360	66
TravelPro	156	67
Tumi	595	87
Victorinox	400	77

a. Develop a scatter diagram with price as the independent variable.
b. What does the scatter diagram developed in part (a) indicate about the relationship between the two variables?
c. Use the least squares method to develop the estimated regression equation.
d. Provide an interpretation for the slope of the estimated regression equation.
e. The Eagle Creek Hovercraft suitcase has a price of $225. Predict the score for this suitcase using the estimated regression equation developed in part (c).

10. According to *Advertising Age*'s annual salary review, Mark Hurd, the 49-year-old chairman, president, and CEO of Hewlett-Packard Co., received an annual salary of $817,000, a bonus of more than $5 million, and other compensation exceeding $17 million. His total compensation was slightly better than the average CEO total pay of $12.4 million. The following table shows the age and annual salary (in thousands of dollars) for Mark Hurd and 14 other executives who led publicly held companies (*Advertising Age,* December 5, 2006).

WEB file
ExecSalary

Executive	Title	Company	Age	Salary ($1000s)
Charles Prince	Chmn/CEO	Citigroup	56	1000
Harold McGraw III	Chmn/Pres/CEO	McGraw-Hill Cos.	57	1172
James Dimon	Pres/CEO	JP Morgan Chase & Co.	50	1000
K. Rupert Murdoch	Chmn/CEO	News Corp.	75	4509
Kenneth D. Lewis	Chmn/Pres/CEO	Bank of America	58	1500
Kenneth I. Chenault	Chmn/CEO	American Express Co.	54	1092
Louis C. Camilleri	Chmn/CEO	Altria Group	51	1663
Mark V. Hurd	Chmn/Pres/CEO	Hewlett-Packard Co.	49	817
Martin S. Sorrell	CEO	WPP Group	61	1562
Robert L. Nardelli	Chmn/Pres/CEO	Home Depot	57	2164
Samuel J. Palmisano	Chmn/Pres/CEO	IBM Corp.	55	1680
David C. Novak	Chmn/Pres/CEO	Yum Brands	53	1173
Henry R. Silverman	Chmn/CEO	Cendant Corp.	65	3300
Robert C. Wright	Chmn/CEO	NBC Universal	62	2500
Sumner Redstone	Exec Chmn/Founder	Viacom	82	5807

a. Develop a scatter diagram for these data with the age of the executive as the independent variable.
b. What does the scatter diagram developed in part (a) indicate about the relationship between the two variables?
c. Develop the least squares estimated regression equation.
d. Suppose Bill Gustin is the 72-year-old chairman, president, and CEO of a major electronics company. Predict the annual salary for Bill Gustin.

11. Sporty cars are designed to provide better handling, acceleration, and a more responsive driving experience than a typical sedan. But, even within this select group of cars, performance as well as price can vary. *Consumer Reports* provided road-test scores and prices for the following 12 sporty cars (Consumer Reports website, October 2008). Prices are in thousands of dollars and road-test scores are based on a 0–100 rating scale, with higher values indicating better performance.

WEB file
SportyCars

Car	Price ($1000s)	Road-Test Score
Chevrolet Cobalt SS	24.5	78
Dodge Caliber SRT4	24.9	56
Ford Mustang GT (V8)	29.0	73
Honda Civic Si	21.7	78
Mazda RX-8	31.3	86
Mini Cooper S	26.4	74
Mitsubishi Lancer Evolution GSR	38.1	83
Nissan Sentra SE-R Spec V	23.3	66
Suburu Impreza WRX	25.2	81
Suburu Impreza WRX Sti	37.6	89
Volkswagen GTI	24.0	83
Volkswagen R32	33.6	83

a. Develop a scatter diagram with price as the independent variable.
b. What does the scatter diagram developed in part (a) indicate about the relationship between the two variables?
c. Use the least squares method to develop the estimated regression equation.
d. Provide an interpretation for the slope of the estimated regression equation.

e. Another sporty car that *Consumer Reports* tested is the BMW 135i; the price for this car was $36,700. Predict the road-test score for the BMW 135i using the estimated regression equation developed in part (c).

12. A personal watercraft (PWC) is a vessel propelled by water jets, designed to be operated by a person sitting, standing, or kneeling on the vessel. In the early 1970s, Kawasaki Motors Corp. U.S.A. introduced the JET SKI® watercraft, the first commercially successful PWC. Today, *jet ski* is commonly used as a generic term for personal watercraft. The following data show the weight (rounded to the nearest 10 lbs.) and the price (rounded to the nearest $50) for 10 three-seater personal watercraft (Jetski News website, 2006).

WEB file
JetSki

Make and Model	Weight (lbs.)	Price ($)
Honda AquaTrax F-12	750	9500
Honda AquaTrax F-12X	790	10500
Honda AquaTrax F-12X GPScape	800	11200
Kawasaki STX-12F Jetski	740	8500
Yamaha FX Cruiser Waverunner	830	10000
Yamaha FX High Output Waverunner	770	10000
Yamaha FX Waverunner	830	9300
Yamaha VX110 Deluxe Waverunner	720	7700
Yamaha VX110 Sport Waverunner	720	7000
Yamaha XLT1200 Waverunner	780	8500

a. Develop a scatter diagram for these data with weight as the independent variable.
b. What does the scatter diagram developed in part (a) indicate about the relationship between weight and price?
c. Use the least squares method to develop the estimated regression equation.
d. Predict the price for a three-seater PWC with a weight of 750 pounds.
e. The Honda AquaTrax F-12 weighs 750 pounds and has a price of $9500. Shouldn't the predicted price you developed in part (d) for a PWC with a weight of 750 pounds also be $9500?
f. The Kawasaki SX-R 800 Jetski has a seating capacity of one and weighs 350 pounds. Do you think the estimated regression equation developed in part (c) should be used to predict the price for this model?

13. To the Internal Revenue Service, the reasonableness of total itemized deductions depends on the taxpayer's adjusted gross income. Large deductions, which include charity and medical deductions, are more reasonable for taxpayers with large adjusted gross incomes. If a taxpayer claims larger than average itemized deductions for a given level of income, the chances of an IRS audit are increased. Data (in thousands of dollars) on adjusted gross income and the average or reasonable amount of total itemized deductions follow.

Adjusted Gross Income ($1000s)	Reasonable Amount of Itemized Deductions ($1000s)
22	9.6
27	9.6
32	10.1
48	11.1
65	13.5
85	17.7
120	25.5

a. Develop a scatter diagram for these data with adjusted gross income as the independent variable.
b. Use the least squares method to develop the estimated regression equation.
c. Predict a reasonable level of total itemized deductions for a taxpayer with an adjusted gross income of $52,500. If this taxpayer claimed itemized deductions of $20,400, would the IRS agent's request for an audit appear justified? Explain.

14. *PCWorld* rated four component characteristics for 10 ultraportable laptop computers: features, performance, design, and price. Each characteristic was rated using a 0–100 point scale. An overall rating, referred to as the *PC World* Rating, was then developed for each laptop. The following table shows the features rating and the *PC World* Rating for the 10 laptop computers (PC World website, February 5, 2009).

WEB file
Laptop

Model	Features Rating	*PC World* Rating
Thinkpad X200	87	83
VGN-Z598U	85	82
U6V	80	81
Elitebook 2530P	75	78
X360	80	78
Thinkpad X300	76	78
Ideapad U110	81	77
Micro Express JFT2500	73	75
Toughbook W7	79	73
HP Voodoo Envy133	68	72

a. Develop a scatter diagram with the features rating as the independent variable.
b. What does the scatter diagram developed in part (a) indicate about the relationship between the two variables?
c. Use the least squares method to develop the estimated regression equation.
d. Predict the *PC World* Rating for a new laptop computer that has a features rating of 70.

12.3 Coefficient of Determination

For the Armand's Pizza Parlors example, we developed the estimated regression equation $\hat{y} = 60 + 5x$ to approximate the linear relationship between the size of the student population x and quarterly sales y. A question now is: How well does the estimated regression equation fit the data? In this section, we show that the **coefficient of determination** provides a measure of the goodness of fit for the estimated regression equation.

For the ith observation, the difference between the observed value of the dependent variable, y_i, and the predicted value of the dependent variable, $\hat{y}_i$, is called the ***i*th residual**. The ith residual represents the error in using $\hat{y}_i$ to predict y_i. Thus, for the ith observation, the residual is $y_i - \hat{y}_i$. The sum of squares of these residuals or errors is the quantity that is minimized by the least squares method. This quantity, also known as the *sum of squares due to error*, is denoted by SSE.

SUM OF SQUARES DUE TO ERROR

$$\text{SSE} = \Sigma(y_i - \hat{y}_i)^2 \qquad (12.8)$$

TABLE 12.3 CALCULATION OF SSE FOR ARMAND'S PIZZA PARLORS

Restaurant i	x_i = Student Population (1000s)	y_i = Quarterly Sales ($1000s)	Predicted Sales $\hat{y}_i = 60 + 5x_i$	Error $y_i - \hat{y}_i$	Squared Error $(y_i - \hat{y}_i)^2$
1	2	58	70	−12	144
2	6	105	90	15	225
3	8	88	100	−12	144
4	8	118	100	18	324
5	12	117	120	−3	9
6	16	137	140	−3	9
7	20	157	160	−3	9
8	20	169	160	9	81
9	22	149	170	−21	441
10	26	202	190	12	144
				SSE =	1530

The value of SSE is a measure of the error in using the estimated regression equation to predict the values of the dependent variable in the sample.

In Table 12.3 we show the calculations required to compute the sum of squares due to error for the Armand's Pizza Parlors example. For instance, for restaurant 1 the values of the independent and dependent variables are $x_1 = 2$ and $y_1 = 58$. Using the estimated regression equation, we find that the predicted value of quarterly sales for restaurant 1 is $\hat{y}_1 = 60 + 5(2) = 70$. Thus, the error in using $\hat{y}_1$ to estimate y_1 for restaurant 1 is $y_1 - \hat{y}_1 = 58 - 70 = -12$. The squared error, $(-12)^2 = 144$, is shown in the last column of Table 12.3. After computing and squaring the residuals for each restaurant in the sample, we sum them to obtain SSE = 1530. Thus, SSE = 1530 measures the error in using the estimated regression equation $\hat{y} = 60 + 5x$ to predict sales.

Now suppose we are asked to predict quarterly sales without knowledge of the size of the student population. Without knowledge of any related variables, we would use the sample mean as a predictor of quarterly sales at any given restaurant. Table 12.2 showed that for the sales data, $\Sigma y_i = 1300$. Hence, the mean value of quarterly sales for the sample of 10 Armand's restaurants is $\bar{y} = \Sigma y_i/n = 1300/10 = 130$. In Table 12.4 we show the sum of squared deviations obtained by using the sample mean $\bar{y} = 130$ to predict the value of quarterly sales for each restaurant in the sample. For the ith restaurant in the sample, the difference $y_i - \bar{y}$ provides a measure of the error involved in using $\bar{y}$ to predict sales. The corresponding sum of squares, called the *total sum of squares,* is denoted SST.

TOTAL SUM OF SQUARES

$$\text{SST} = \Sigma(y_i - \bar{y})^2 \qquad (12.9)$$

The sum at the bottom of the last column in Table 12.4 is the total sum of squares for Armand's Pizza Parlors; it is SST = 15,730.

In Figure 12.6 we show the estimated regression line $\hat{y} = 60 + 5x$ and the line corresponding to $\bar{y} = 130$. Note that the points cluster more closely around the estimated

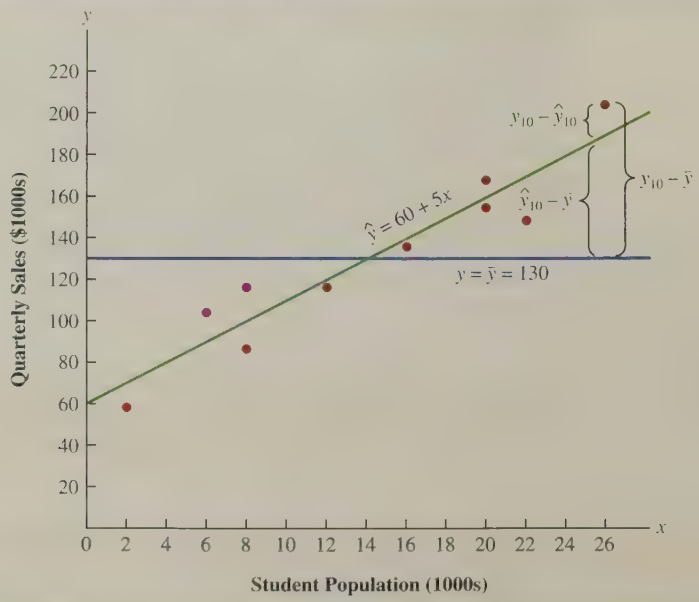
TABLE 12.4 COMPUTATION OF THE TOTAL SUM OF SQUARES FOR ARMAND'S PIZZA PARLORS

Restaurant i	x_i = Student Population (1000s)	y_i = Quarterly Sales ($1000s)	Deviation $y_i - \bar{y}$	Squared Deviation $(y_i - \bar{y})^2$
1	2	58	−72	5,184
2	6	105	−25	625
3	8	88	−42	1,764
4	8	118	−12	144
5	12	117	−13	169
6	16	137	7	49
7	20	157	27	729
8	20	169	39	1,521
9	22	149	19	361
10	26	202	72	5,184
				SST = 15,730

With SST = 15,730 and SSE = 1530, the estimated regression line provides a much better fit to the data than the line $y = \bar{y}$.

regression line than they do about the line $\bar{y} = 130$. For example, for the 10th restaurant in the sample we see that the error is much larger when $\bar{y} = 130$ is used to predict y_{10} than when $\hat{y}_{10} = 60 + 5(26) = 190$ is used. We can think of SST as a measure of how well the observations cluster about the $\bar{y}$ line and SSE as a measure of how well the observations cluster about the $\hat{y}$ line.

FIGURE 12.6 DEVIATIONS ABOUT THE ESTIMATED REGRESSION LINE AND THE LINE $y = \bar{y}$ FOR ARMAND'S PIZZA PARLORS



To measure how much the $\hat{y}$ values on the estimated regression line deviate from $\bar{y}$, another sum of squares is computed. This sum of squares, called the *sum of squares due to regression,* is denoted SSR.

SUM OF SQUARES DUE TO REGRESSION

$$\text{SSR} = \Sigma(\hat{y}_i - \bar{y})^2 \qquad \textbf{(12.10)}$$

From the preceding discussion, we should expect that SST, SSR, and SSE are related. Indeed, the relationship among these three sums of squares provides one of the most important results in statistics.

RELATIONSHIP AMONG SST, SSR, AND SSE

$$\text{SST} = \text{SSR} + \text{SSE} \qquad \textbf{(12.11)}$$

SSR can be thought of as the explained portion of SST, and SSE can be thought of as the unexplained portion of SST.

where

SST = total sum of squares
SSR = sum of squares due to regression
SSE = sum of squares due to error

Equation (12.11) shows that the total sum of squares can be partitioned into two components, the sum of squares due to regression and the sum of squares due to error. Hence, if the values of any two of these sum of squares are known, the third sum of squares can be computed easily. For instance, in the Armand's Pizza Parlors example, we already know that SSE = 1530 and SST = 15,730; therefore, solving for SSR in equation (12.11), we find that the sum of squares due to regression is

$$\text{SSR} = \text{SST} - \text{SSE} = 15{,}730 - 1530 = 14{,}200$$

Now let us see how the three sums of squares, SST, SSR, and SSE, can be used to provide a measure of the goodness of fit for the estimated regression equation. The estimated regression equation would provide a perfect fit if every value of the dependent variable y_i happened to lie on the estimated regression line. In this case, $y_i - \hat{y}_i$ would be zero for each observation, resulting in SSE = 0. Because SST = SSR + SSE, we see that for a perfect fit SSR must equal SST, and the ratio (SSR/SST) must equal 1. Poorer fits will result in larger values for SSE. Solving for SSE in equation (12.11), we see that SSE = SST − SSR. Hence, the largest value for SSE (and hence the poorest fit) occurs when SSR = 0 and SSE = SST.

The ratio SSR/SST, which will take values between 0 and 1, is used to evaluate the goodness of fit for the estimated regression equation. This ratio is called the *coefficient of determination* and is denoted by r^2.

COEFFICIENT OF DETERMINATION

$$r^2 = \frac{\text{SSR}}{\text{SST}} \qquad \textbf{(12.12)}$$

For the Armand's Pizza Parlors example, the value of the coefficient of determination is

$$r^2 = \frac{\text{SSR}}{\text{SST}} = \frac{14{,}200}{15{,}730} = .9027$$

When we express the coefficient of determination as a percentage, r^2 can be interpreted as the percentage of the total sum of squares that can be explained by using the estimated regression equation. For Armand's Pizza Parlors, we can conclude that 90.27% of the total sum of squares can be explained by using the estimated regression equation $\hat{y} = 60 + 5x$ to predict quarterly sales. In other words, 90.27% of the variability in sales can be explained by the linear relationship between the size of the student population and sales. We should be pleased to find such a good fit for the estimated regression equation.

Using Excel to Compute the Coefficient of Determination

In Section 12.2 we used Excel's chart tools to construct a scatter diagram and compute the estimated regression equation for the Armand's Pizza Parlors data. We will now describe how to compute the coefficient of determination using the scatter diagram in Figure 12.5.

Step 1. Position the mouse pointer over any data point in the scatter diagram and right-click to display a list of options
Step 2. Choose **Add Trendline**
Step 3. When the Format Trendline dialog box appears,
Select **Trendline Options** and then
Choose **Display R-squared value on chart**
Click **Close**

Figure 12.7 displays the scatter diagram, the estimated regression equation, the graph of the estimated regression equation, and the coefficient of determination for the Armand's Pizza Parlors data. We see that $r^2 = .9027$.

Correlation Coefficient

In Chapter 3 we introduced the **correlation coefficient** as a descriptive measure of the strength of linear association between two variables, x and y. Values of the correlation coefficient are always between -1 and $+1$. A value of $+1$ indicates that the two variables x and y are perfectly related in a positive linear sense. That is, all data points are on a straight line that has a positive slope. A value of -1 indicates that x and y are perfectly related in a negative linear sense, with all data points on a straight line that has a negative slope. Values of the correlation coefficient close to zero indicate that x and y are not linearly related.

In Section 3.5 we presented the equation for computing the sample correlation coefficient. If a regression analysis has already been performed and the coefficient of determination r^2 computed, the sample correlation coefficient can be computed as follows.

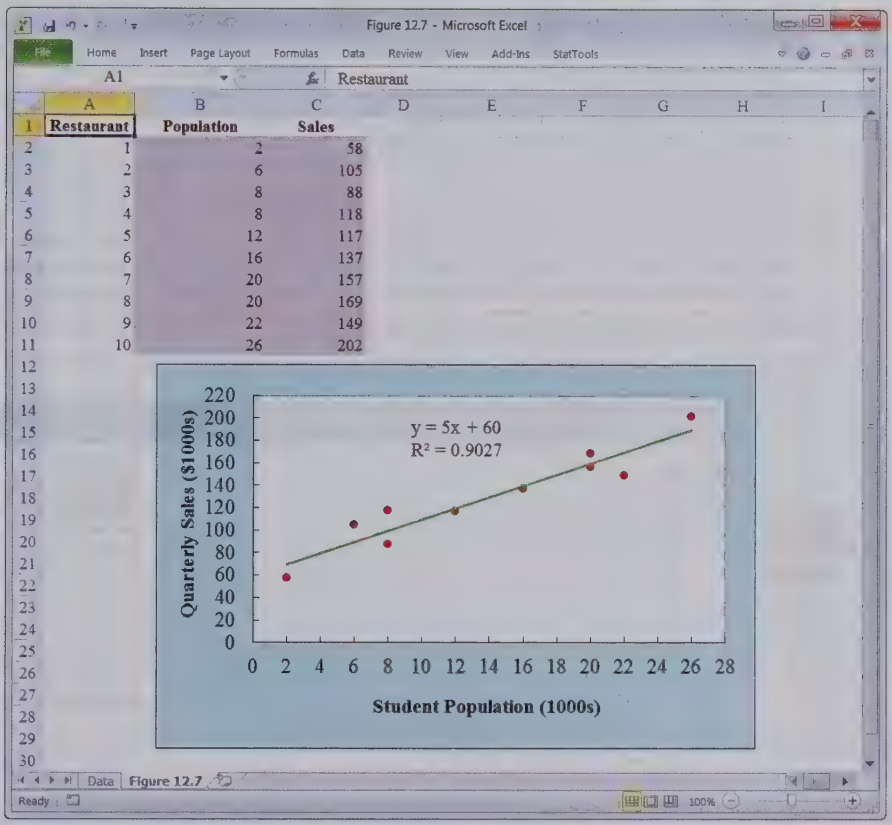
SAMPLE CORRELATION COEFFICIENT

$$r_{xy} = (\text{sign of } b_1)\sqrt{\text{Coefficient of determination}} = (\text{sign of } b_1)\sqrt{r^2} \qquad (12.13)$$

where

b_1 = the slope of the estimated regression equation $\hat{y} = b_0 + b_1 x$

FIGURE 12.7 USING EXCEL TO COMPUTE THE COEFFICIENT OF DETERMINATION



The sign for the sample correlation coefficient is positive if the estimated regression equation has a positive slope ($b_1 > 0$) and negative if the estimated regression equation has a negative slope ($b_1 < 0$).

For the Armand's Pizza Parlor example, the value of the coefficient of determination corresponding to the estimated regression equation $\hat{y} = 60 + 5x$ is .9027. Because the slope of the estimated regression equation is positive, equation (12.13) shows that the sample correlation coefficient is $+\sqrt{.9027} = +.9501$. With a sample correlation coefficient of $r_{xy} = +.9501$, we would conclude that a strong positive linear association exists between x and y.

In the case of a linear relationship between two variables, both the coefficient of determination and the sample correlation coefficient provide measures of the strength of the relationship. The coefficient of determination provides a measure between 0 and 1, whereas the sample correlation coefficient provides a measure between -1 and $+1$. Although the sample correlation coefficient is restricted to a linear relationship between two variables, the coefficient of determination can be used for nonlinear relationships and for relationships that have two or more independent variables. Thus, the coefficient of determination provides a wider range of applicability.

NOTES AND COMMENTS

1. In developing the least squares estimated regression equation and computing the coefficient of determination, we made no probabilistic assumptions about the error term ϵ, and no statistical tests for significance of the relationship between x and y were conducted. Larger values of r^2 imply that the least squares line provides a better fit to the data; that is, the observations are more closely grouped about the least squares line. But, using only r^2, we can draw no conclusion about whether the relationship between x and y is statistically significant. Such a conclusion must be based on considerations that involve the sample size and the properties of the appropriate sampling distributions of the least squares estimators.
2. As a practical matter, for typical data found in the social sciences, values of r^2 as low as .25 are often considered useful. For data in the physical and life sciences, r^2 values of .60 or greater are often found; in fact, in some cases, r^2 values greater than .90 can be found. In business applications, r^2 values vary greatly, depending on the unique characteristics of each application.

Exercises

Methods

SELF test

15. The data from exercise 1 follow.

x_i	1	2	3	4	5
y_i	3	7	5	11	14

The estimated regression equation for these data is $\hat{y} = .20 + 2.60x$.

a. Compute SSE, SST, and SSR using equations (12.8), (12.9), and (12.10).
b. Compute the coefficient of determination r^2. Comment on the goodness of fit.
c. Compute the sample correlation coefficient.

16. The data from exercise 2 follow.

x_i	3	12	6	20	14
y_i	55	40	55	10	15

The estimated regression equation for these data is $\hat{y} = 68 - 3x$.

a. Compute SSE, SST, and SSR.
b. Compute the coefficient of determination r^2. Comment on the goodness of fit.
c. Compute the sample correlation coefficient.

17. The data from exercise 3 follow.

x_i	2	6	9	13	20
y_i	7	18	9	26	23

The estimated regression equation for these data is $\hat{y} = 7.6 + .9x$. What percentage of the total sum of squares can be accounted for by the estimated regression equation? What is the value of the sample correlation coefficient?

Applications

SELF test

18. The following data are the monthly salaries y and the grade point averages x for students who obtained a bachelor's degree in business administration with a major in information systems. The estimated regression equation for these data is $\hat{y} = 1790.5 + 581.1x$.

GPA	Monthly Salary ($)
2.6	3300
3.4	3600
3.6	4000
3.2	3500
3.5	3900
2.9	3600

a. Compute SST, SSR, and SSE.
b. Compute the coefficient of determination r^2. Comment on the goodness of fit.
c. What is the value of the sample correlation coefficient?

19. In exercise 7 a sales manager collected the following data on x = annual sales and y = years of experience. The estimated regression equation for these data is $\hat{y} = 80 + 4x$.

WEB file
Sales

Salesperson	Years of Experience	Annual Sales ($1000s)
1	1	80
2	3	97
3	4	92
4	4	102
5	6	103
6	8	111
7	10	119
8	10	123
9	11	117
10	13	136

a. Compute SST, SSR, and SSE.
b. Compute the coefficient of determination r^2. Comment on the goodness of fit.
c. What is the value of the sample correlation coefficient?

20. *Consumer Reports* provided extensive testing and ratings for more than 100 HDTVs. An overall score, based primarily on picture quality, was developed for each model. In general, a higher overall score indicates better performance. The following data show the price and overall score for the ten 42-inch plasma televisions (*Consumer Reports*, March 2006).

WEB file
PlasmaTV

Brand	Price	Score
Dell	2800	62
Hisense	2800	53
Hitachi	2700	44
JVC	3500	50
LG	3300	54
Maxent	2000	39
Panasonic	4000	66
Phillips	3000	55
Proview	2500	34
Samsung	3000	39

a. Use these data to develop an estimated regression equation that could be used to predict the overall score for a 42-inch plasma television given the price.
b. Compute r^2. Did the estimated regression equation provide a good fit?
c. Estimate the overall score for a 42-inch plasma television with a price of $3200.

21. An important application of regression analysis in accounting is in the estimation of cost. By collecting data on volume and cost and using the least squares method to develop an estimated regression equation relating volume and cost, an accountant can estimate the cost associated with a particular manufacturing volume. Consider the following sample of production volumes and total cost data for a manufacturing operation.

Production Volume (units)	Total Cost ($)
400	4000
450	5000
550	5400
600	5900
700	6400
750	7000

a. Use these data to develop an estimated regression equation that could be used to predict the total cost for a given production volume.
b. What is the variable cost per unit produced?
c. Compute the coefficient of determination. What percentage of the variation in total cost can be explained by production volume?
d. The company's production schedule shows 500 units must be produced next month. Predict the total cost for this operation.

22. Refer to exercise 5, where the following data were used to investigate whether higher prices are generally associated with higher ratings for elliptical trainers (*Consumer Reports*, February 2008).

WEB file
Ellipticals

Brand and Model	Price ($)	Rating
Precor 5.31	3700	87
Keys Fitness CG2	2500	84
Octane Fitness Q37e	2800	82
LifeFitness X1 Basic	1900	74
NordicTrack AudioStrider 990	1000	73
Schwinn 430	800	69
Vision Fitness X6100	1700	68
ProForm XP 520 Razor	600	55

With x = price ($) and y = rating, the estimated regression equation is $\hat{y} = 58.158 + .008449x$. For these data, SSE = 173.88.
a. Compute the coefficient of determination r^2.
b. Did the estimated regression equation provide a good fit? Explain.
c. What is the value of the sample correlation coefficient? Does it reflect a strong or weak relationship between price and rating?

12.4 Model Assumptions

In conducting a regression analysis, we begin by making an assumption about the appropriate model for the relationship between the dependent and independent variable(s). For the case of simple linear regression, the assumed regression model is

$$y = \beta_0 + \beta_1 x + \epsilon$$

Then the least squares method is used to develop values for b_0 and b_1, the estimates of the model parameters β_0 and β_1, respectively. The resulting estimated regression equation is

$$\hat{y} = b_0 + b_1 x$$

We saw that the value of the coefficient of determination (r^2) is a measure of the goodness of fit of the estimated regression equation. However, even with a large value of r^2, the estimated regression equation should not be used until further analysis of the appropriateness of the assumed model has been conducted. An important step in determining whether the assumed model is appropriate involves testing for the significance of the relationship. The tests of significance in regression analysis are based on the following assumptions about the error term ϵ.

ASSUMPTIONS ABOUT THE ERROR TERM ϵ IN THE REGRESSION MODEL

$$y = \beta_0 + \beta_1 x + \epsilon$$

1. The error term ϵ is a random variable with a mean or expected value of zero; that is, $E(\epsilon) = 0$.
   Implication: β_0 and β_1 are constants, therefore $E(\beta_0) = \beta_0$ and $E(\beta_1) = \beta_1$; thus, for a given value of x, the expected value of y is

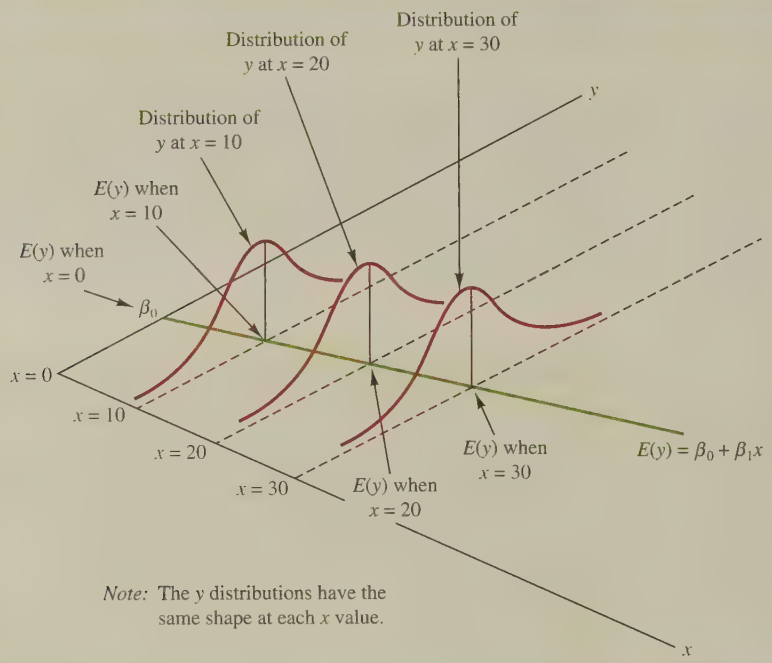
   $$E(y) = \beta_0 + \beta_1 x \qquad (12.14)$$

   As we indicated previously, equation (12.14) is referred to as the regression equation.
2. The variance of ϵ, denoted by σ^2, is the same for all values of x.
   Implication: The variance of y about the regression line equals σ^2 and is the same for all values of x.
3. The values of ϵ are independent.
   Implication: The value of ϵ for a particular value of x is not related to the value of ϵ for any other value of x; thus, the value of y for a particular value of x is not related to the value of y for any other value of x.
4. The error term ϵ is a normally distributed random variable.
   Implication: Because y is a linear function of ϵ, y is also a normally distributed random variable.

Figure 12.8 illustrates the model assumptions and their implications; note that in this graphical interpretation, the value of $E(y)$ changes according to the specific value of x considered. However, regardless of the x value, the probability distribution of ϵ and hence the probability distributions of y are normally distributed, each with the same variance. The specific value of the error ϵ at any particular point depends on whether the actual value of y is greater than or less than $E(y)$.

At this point, we must keep in mind that we are also making an assumption or hypothesis about the form of the relationship between x and y. That is, we assume that a straight line represented by $\beta_0 + \beta_1 x$ is the basis for the relationship between the variables. We must not lose sight of the fact that some other model, for instance $y = \beta_0 + \beta_1 x^2 + \epsilon$, may turn out to be a better model for the underlying relationship.

FIGURE 12.8 ASSUMPTIONS FOR THE REGRESSION MODEL


Distribution of y at x = 20
Distribution of y at x = 30
Distribution of y at x = 10
y
E(y) when x = 10
E(y) when x = 0
β0
x = 0
x = 10
x = 20
x = 30
E(y) when x = 30
E(y) = β0 + β1x
E(y) when x = 20
Note: The y distributions have the same shape at each x value.
x

12.5 Testing for Significance

In a simple linear regression equation, the mean or expected value of y is a linear function of x: $E(y) = \beta_0 + \beta_1 x$. If the value of β_1 is zero, $E(y) = \beta_0 + (0)x = \beta_0$. In this case, the mean value of y does not depend on the value of x and hence we would conclude that x and y are not linearly related. Alternatively, if the value of β_1 is not equal to zero, we would conclude that the two variables are related. Thus, to test for a significant regression relationship, we must conduct a hypothesis test to determine whether the value of β_1 is zero. Two tests are commonly used. Both require an estimate of σ^2, the variance of ϵ in the regression model.

Estimate of σ^2

From the regression model and its assumptions we can conclude that σ^2, the variance of ϵ, also represents the variance of the y values about the regression line. Recall that the deviations of the y values about the estimated regression line are called residuals. Thus, SSE, the sum of squared residuals, is a measure of the variability of the actual observations about the estimated regression line. The **mean square error** (MSE) provides the estimate of σ^2; it is SSE divided by its degrees of freedom.

With $\hat{y}_i = b_0 + b_1 x_i$, SSE can be written as

$$\text{SSE} = \Sigma(y_i - \hat{y}_i)^2 = \Sigma(y_i - b_0 - b_1 x_i)^2$$

Every sum of squares has associated with it a number called its degrees of freedom. Statisticians have shown that SSE has $n - 2$ degrees of freedom because two parameters (β_0 and β_1) must be estimated to compute SSE. Thus, the mean square error is computed by dividing SSE by $n - 2$. MSE provides an unbiased estimator of σ^2. Because the value of MSE provides an estimate of σ^2, the notation s^2 is also used.

MEAN SQUARE ERROR (ESTIMATE OF σ^2)

$$s^2 = \text{MSE} = \frac{\text{SSE}}{n - 2} \tag{12.15}$$

In Section 12.3 we showed that for the Armand's Pizza Parlors example, SSE = 1530; hence,

$$s^2 = \text{MSE} = \frac{1530}{8} = 191.25$$

provides an unbiased estimate of σ^2.

To estimate σ we take the square root of s^2. The resulting value, s, is referred to as the **standard error of the estimate**.

STANDARD ERROR OF THE ESTIMATE

$$s = \sqrt{\text{MSE}} = \sqrt{\frac{\text{SSE}}{n - 2}} \tag{12.16}$$

For the Armand's Pizza Parlors example, $s = \sqrt{\text{MSE}} = \sqrt{191.25} = 13.829$. In the following discussion, we use the standard error of the estimate in the tests for a significant relationship between x and y.

t Test

The simple linear regression model is $y = \beta_0 + \beta_1 x + \epsilon$. If x and y are linearly related, we must have $\beta_1 \neq 0$. The purpose of the t test is to see whether we can conclude that $\beta_1 \neq 0$. We will use the sample data to test the following hypotheses about the parameter β_1.

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

If H_0 is rejected, we will conclude that $\beta_1 \neq 0$ and that a statistically significant relationship exists between the two variables. However, if H_0 cannot be rejected, we will have insufficient evidence to conclude that a significant relationship exists. The properties of the sampling distribution of b_1, the least squares estimator of β_1, provide the basis for the hypothesis test.

First, let us consider what would happen if we used a different random sample for the same regression study. For example, suppose that Armand's Pizza Parlors used the sales

records of a different sample of 10 restaurants. A regression analysis of this new sample might result in an estimated regression equation similar to our previous estimated regression equation $\hat{y} = 60 + 5x$. However, it is doubtful that we would obtain exactly the same equation (with an intercept of exactly 60 and a slope of exactly 5). Indeed, b_0 and b_1, the least squares estimators, are sample statistics with their own sampling distributions. The properties of the sampling distribution of b_1 follow.

SAMPLING DISTRIBUTION OF b_1

Expected Value

$$E(b_1) = \beta_1$$

Standard Deviation

$$\sigma_{b_1} = \frac{\sigma}{\sqrt{\Sigma(x_i - \bar{x})^2}} \qquad \textbf{(12.17)}$$

Distribution Form

Normal

Note that the expected value of b_1 is equal to β_1, so b_1 is an unbiased estimator of β_1.

Because we do not know the value of σ, we develop an estimate of σ_{b_1}, denoted s_{b_1}, by estimating σ with s in equation (12.17). Thus, we obtain the following estimate of σ_{b_1}.

The standard deviation of b_1 is also referred to as the standard error of b_1. Thus, s_{b_1} provides an estimate of the standard error of b_1.

ESTIMATED STANDARD DEVIATION OF b_1

$$s_{b_1} = \frac{s}{\sqrt{\Sigma(x_i - \bar{x})^2}} \qquad \textbf{(12.18)}$$

For Armand's Pizza Parlors, $s = 13.829$. Hence, using $\Sigma(x_i - \bar{x})^2 = 568$ as shown in Table 12.2, we have

$$s_{b_1} = \frac{13.829}{\sqrt{568}} = .5803$$

as the estimated standard deviation of b_1.

The t test for a significant relationship is based on the fact that the test statistic

$$\frac{b_1 - \beta_1}{s_{b_1}}$$

follows a t distribution with $n - 2$ degrees of freedom. If the null hypothesis is true, then $\beta_1 = 0$ and $t = b_1/s_{b_1}$.

Let us conduct this test of significance for Armand's Pizza Parlors at the $\alpha = .01$ level of significance. The test statistic is

$$t = \frac{b_1}{s_{b_1}} = \frac{5}{.5803} = 8.62$$

The t distribution table shows that with $n - 2 = 10 - 2 = 8$ degrees of freedom, $t = 3.355$ provides an area of .005 in the upper tail. Thus, the area in the upper tail of the t distribution corresponding to the test statistic $t = 8.62$ must be less than .005. Because this test is a two-tailed test, we double this value to conclude that the p-value associated with $t = 8.62$ must be less than 2(.005) = .01. Excel shows the p-value = .000. Because the p-value is less than $\alpha = .01$, we reject H_0 and conclude that β_1 is not equal to zero. This evidence is sufficient to conclude that a significant relationship exists between student population and quarterly sales. A summary of the t test for significance in simple linear regression follows.

Using Excel, p-value = T.DIST.2T(8.62,8) = .000.

t TEST FOR SIGNIFICANCE IN SIMPLE LINEAR REGRESSION

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

TEST STATISTIC

$$t = \frac{b_1}{s_{b_1}} \tag{12.19}$$

REJECTION RULE

p-value approach:	Reject H_0 if p-value $\leq \alpha$
Critical value approach:	Reject H_0 if $t \leq -t_{\alpha/2}$ or if $t \geq t_{\alpha/2}$

where $t_{\alpha/2}$ is based on a t distribution with $n - 2$ degrees of freedom.

Confidence Interval for β_1

The form of a confidence interval for β_1 is as follows:

$$b_1 \pm t_{\alpha/2} s_{b_1}$$

The point estimator is b_1 and the margin of error is $t_{\alpha/2} s_{b_1}$. The confidence coefficient associated with this interval is $1 - \alpha$, and $t_{\alpha/2}$ is the t value providing an area of $\alpha/2$ in the upper tail of a t distribution with $n - 2$ degrees of freedom. For example, suppose that we wanted to develop a 99% confidence interval estimate of β_1 for Armand's Pizza Parlors. From Table 2 of Appendix B we find that the t value corresponding to $\alpha = .01$ and $n - 2 = 10 - 2 = 8$ degrees of freedom is $t_{.005} = 3.355$. Thus, the 99% confidence interval estimate of β_1 is

$$b_1 \pm t_{\alpha/2} s_{b_1} = 5 \pm 3.355(.5803) = 5 \pm 1.95$$

or 3.05 to 6.95.

In using the t test for significance, the hypotheses tested were

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

At the $\alpha = .01$ level of significance, we can use the 99% confidence interval as an alternative for drawing the hypothesis testing conclusion for the Armand's data. Because 0, the hypothesized value of β_1, is not included in the confidence interval (3.05 to 6.95), we can

reject H_0 and conclude that a significant statistical relationship exists between the size of the student population and quarterly sales. In general, a confidence interval can be used to test any two-sided hypothesis about β_1. If the hypothesized value of β_1 is contained in the confidence interval, do not reject H_0. Otherwise, reject H_0.

F Test

An *F* test, based on the *F* probability distribution, can also be used to test for significance in regression. With only one independent variable, the *F* test will provide the same conclusion as the *t* test; that is, if the *t* test indicates $\beta_1 \neq 0$ and hence a significant relationship, the *F* test will also indicate a significant relationship. But with more than one independent variable, only the *F* test can be used to test for an overall significant relationship.

The logic behind the use of the *F* test for determining whether the regression relationship is statistically significant is based on the development of two independent estimates of σ^2. We explained how MSE provides an estimate of σ^2. If the null hypothesis H_0: $\beta_1 = 0$ is true, the sum of squares due to regression, SSR, divided by its degrees of freedom provides another independent estimate of σ^2. This estimate is called the *mean square due to regression,* or simply the *mean square regression,* and is denoted MSR. In general,

$$\text{MSR} = \frac{\text{SSR}}{\text{Regression degrees of freedom}}$$

For the models we consider in this text, the regression degrees of freedom is always equal to the number of independent variables in the model:

$$\text{MSR} = \frac{\text{SSR}}{\text{Number of independent variables}} \qquad \textbf{(12.20)}$$

Because we consider only regression models with one independent variable in this chapter, we have MSR = SSR/1 = SSR. Hence, for Armand's Pizza Parlors, MSR = SSR = 14,200.

If the null hypothesis (H_0: $\beta_1 = 0$) is true, MSR and MSE are two independent estimates of σ^2 and the sampling distribution of MSR/MSE follows an *F* distribution with numerator degrees of freedom equal to one and denominator degrees of freedom equal to $n - 2$. Therefore, when $\beta_1 = 0$, the value of MSR/MSE should be close to one. However, if the null hypothesis is false ($\beta_1 \neq 0$), MSR will overestimate σ^2 and the value of MSR/MSE will be inflated; thus, large values of MSR/MSE lead to the rejection of H_0 and the conclusion that the relationship between *x* and *y* is statistically significant.

Let us conduct the *F* test for the Armand's Pizza Parlors example. The test statistic is

The F test and the t test provide identical results for simple linear regression.

$$F = \frac{\text{MSR}}{\text{MSE}} = \frac{14{,}200}{191.25} = 74.25$$

The *F* distribution table (Table 4 of Appendix B) shows that with one degree of freedom in the numerator and $n - 2 = 10 - 2 = 8$ degrees of freedom in the denominator, $F = 11.26$ provides an area of .01 in the upper tail. Thus, the area in the upper tail of the *F* distribution corresponding to the test statistic $F = 74.25$ must be less than .01. Thus, we conclude that the *p*-value must be less than .01. Excel shows the *p*-value = .000. Because the *p*-value is less than $\alpha = .01$, we reject H_0 and conclude that a significant relationship exists between the size of the student population and quarterly sales. A summary of the *F* test for significance in simple linear regression follows.

Using Excel, p-value = F.DIST.RT(74.25,1,8) = .000

If H_0 is false, MSE *still provides an unbiased estimate of* σ^2 *and* MSR *overestimates* σ^2. *If* H_0 *is true, both* MSE *and* MSR *provide unbiased estimates of* σ^2; *in this case the value of* MSR/MSE *should be close to 1.*

F TEST FOR SIGNIFICANCE IN SIMPLE LINEAR REGRESSION

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

TEST STATISTIC

$$F = \frac{\text{MSR}}{\text{MSE}} \tag{12.21}$$

REJECTION RULE

p-value approach: Reject H_0 if *p*-value $\leq \alpha$

Critical value approach: Reject H_0 if $F \geq F_\alpha$

where F_α is based on an *F* distribution with 1 degree of freedom in the numerator and $n - 2$ degrees of freedom in the denominator.

In Chapter 10 we covered analysis of variance (ANOVA) and showed how an **ANOVA table** could be used to provide a convenient summary of the computational aspects of analysis of variance. A similar ANOVA table can be used to summarize the results of the *F* test for significance in regression. Table 12.5 is the general form of the ANOVA table for simple linear regression. Table 12.6 is the ANOVA table with the *F* test computations performed for Armand's Pizza Parlors. Regression, Error, and Total are the labels for the three sources of variation, with SSR, SSE, and SST appearing as the corresponding sum of squares in column 2. The degrees of freedom, 1 for SSR, $n - 2$ for SSE, and $n - 1$ for SST, are shown in column 3. Column 4 contains the values of MSR and MSE, column 5 contains the value of $F = \text{MSR/MSE}$, and column 6 contains the *p*-value corresponding to the *F* value in column 5. Almost all computer printouts of regression analysis include an ANOVA table summary of the *F* test for significance.

Some Cautions About the Interpretation of Significance Tests

Rejecting the null hypothesis $H_0: \beta_1 = 0$ and concluding that the relationship between *x* and *y* is significant does not enable us to conclude that a cause-and-effect relationship is present between *x* and *y*. Concluding a cause-and-effect relationship is warranted only if the analyst can provide some type of theoretical justification that the relationship is in fact causal.

TABLE 12.5 GENERAL FORM OF THE ANOVA TABLE FOR SIMPLE LINEAR REGRESSION

In every analysis of variance table the total sum of squares is the sum of the regression sum of squares and the error sum of squares; in addition, the total degrees of freedom is the sum of the regression degrees of freedom and the error degrees of freedom.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	*F*	*p*-value
Regression	SSR	1	$\text{MSR} = \frac{\text{SSR}}{1}$	$F = \frac{\text{MSR}}{\text{MSE}}$	
Error	SSE	$n - 2$	$\text{MSE} = \frac{\text{SSE}}{n-2}$		
Total	SST	$n - 1$			

TABLE 12.6 ANOVA TABLE FOR THE ARMAND'S PIZZA PARLORS PROBLEM

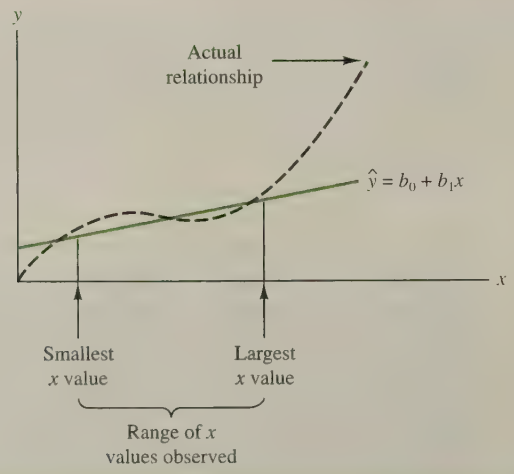
Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	*F*	*p*-value
Regression	14,200	1	$\frac{14,200}{1} = 14,200$	$\frac{14,200}{191.25} = 74.25$	.000
Error	1,530	8	$\frac{1530}{8} = 191.25$		
Total	15,730	9			

Regression analysis, which can be used to identify how variables are associated with one another, cannot be used as evidence of a cause-and-effect relationship.

In the Armand's Pizza Parlors example, we can conclude that there is a significant relationship between the size of the student population x and quarterly sales y; moreover, the estimated regression equation $\hat{y} = 60 + 5x$ provides the least squares estimate of the relationship. We cannot, however, conclude that changes in student population x *cause* changes in quarterly sales y just because we identified a statistically significant relationship. The appropriateness of such a cause-and-effect conclusion is left to supporting theoretical justification and to good judgment on the part of the analyst. Armand's managers felt that increases in the student population were a likely cause of increased quarterly sales. Thus, the result of the significance test enabled them to conclude that a cause-and-effect relationship was present.

In addition, just because we are able to reject H_0: $\beta_1 = 0$ and demonstrate statistical significance does not enable us to conclude that the relationship between x and y is linear. We can state only that x and y are related and that a linear relationship explains a significant portion of the variability in y over the range of values for x observed in the sample. Figure 12.9 illustrates this situation. The test for significance calls for the rejection of the null hypothesis H_0: $\beta_1 = 0$ and leads to the conclusion that x and y are significantly related, but the figure shows that the actual relationship between x and y is not linear. Although the

FIGURE 12.9 EXAMPLE OF A LINEAR APPROXIMATION OF A NONLINEAR RELATIONSHIP



linear approximation provided by $\hat{y} = b_0 + b_1 x$ is good over the range of x values observed in the sample, it becomes poor for x values outside that range.

Given a significant relationship, we should feel confident in using the estimated regression equation for predictions corresponding to x values within the range of the x values observed in the sample. For Armand's Pizza Parlors, this range corresponds to values of x between 2 and 26. Unless other reasons indicate that the model is valid beyond this range, predictions outside the range of the independent variable should be made with caution. For Armand's Pizza Parlors, because the regression relationship has been found significant at the .01 level, we should feel confident using it to predict sales for restaurants where the associated student population is between 2000 and 26,000.

NOTES AND COMMENTS

1. The assumptions made about the error term (Section 12.4) are what allow the tests of statistical significance in this section. The properties of the sampling distribution of b_1 and the subsequent t and F tests follow directly from these assumptions.
2. Do not confuse statistical significance with practical significance. With very large sample sizes, statistically significant results can be obtained for small values of b_1; in such cases, one must exercise care in concluding that the relationship has practical significance.
3. A test of significance for a linear relationship between x and y can also be performed by using the sample correlation coefficient r_{xy}. With ρ_{xy} denoting the population correlation coefficient, the hypotheses are as follows.

$$H_0: \rho_{xy} = 0$$
$$H_a: \rho_{xy} \neq 0$$

A significant relationship can be concluded if H_0 is rejected. However, the t and F tests presented previously in this section give the same result as the test for significance using the correlation coefficient. Conducting a test for significance using the correlation coefficient therefore is not necessary if a t or F test has already been conducted.

Exercises

Methods

SELF test

23. The data from exercise 1 follow.

x_i	1	2	3	4	5
y_i	3	7	5	11	14

a. Compute the mean square error using equation (12.15).
b. Compute the standard error of the estimate using equation (12.16).
c. Compute the estimated standard deviation of b_1 using equation (12.18).
d. Use the t test to test the following hypotheses ($\alpha = .05$):

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

e. Use the F test to test the hypotheses in part (d) at a .05 level of significance. Present the results in the analysis of variance table format.

24. The data from exercise 2 follow.

x_i	3	12	6	20	14
y_i	55	40	55	10	15

a. Compute the mean square error using equation (12.15).
b. Compute the standard error of the estimate using equation (12.16).
c. Compute the estimated standard deviation of b_1 using equation (12.18).
d. Use the t test to test the following hypotheses ($\alpha = .05$):

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

e. Use the F test to test the hypotheses in part (d) at a .05 level of significance. Present the results in the analysis of variance table format.

25. The data from exercise 3 follow.

x_i	2	6	9	13	20
y_i	7	18	9	26	23

a. What is the value of the standard error of the estimate?
b. Test for a significant relationship by using the t test. Use $\alpha = .05$.
c. Use the F test to test for a significant relationship. Use $\alpha = .05$. What is your conclusion?

Applications

SELF test

26. In exercise 18 the data on grade point average and monthly salary were as follows.

GPA	Monthly Salary ($)	GPA	Monthly Salary ($)
2.6	3300	3.2	3500
3.4	3600	3.5	3900
3.6	4000	2.9	3600

a. Does the t test indicate a significant relationship between grade point average and monthly salary? What is your conclusion? Use $\alpha = .05$.
b. Test for a significant relationship using the F test. What is your conclusion? Use $\alpha = .05$.
c. Show the ANOVA table.

27. The number of megapixels in a digital camera is one of the most important factors in determining picture quality. But, do digital cameras with more megapixels cost more? The following data show the number of megapixels and the price ($) for 10 digital cameras (*Consumer Reports*, March 2009).

WEB file
DigitalCameras

Brand and Model	Megapixels	Price ($)
Canon PowerShot SD1100 IS	8	180
Casio Exilim Card EX-510	10	200
Sony Cyber-shot DSC-T70	7	230
Pentax Optio M50	8	120
Canon PowerShot G10	15	470
Canon PowerShot A590 IS	8	140
Canon PowerShot E1	10	180
Fujifilm FinePix F00FD	12	310
Sony Cyber-shot DSC-W170	10	250
Canon PowerShot A470	7	110

a. Use these data to develop an estimated regression equation that can be used to predict the price of a digital camera given the number of megapixels.
b. At the .05 level of significance, are the number of megapixels and the price related? Explain.
c. Would you feel comfortable using the estimated regression equation developed in part (a) to predict the price of a digital camera given the number of megapixels? Explain.
d. The Kodak EasyShare Z1012 IS digital camera has 10 megapixels. Predict the price of this camera using the estimated regression equation developed in part (a).

SleepingBags

28. In exercise 8, data on x = temperature rating (F°) and y = price ($) for 11 sleeping bags manufactured by Bergans of Norway provided the estimated regression equation $\hat{y} = 359.2668 - 5.2772x$. At the .05 level of significance, test whether temperature rating and price are related. Show the ANOVA table. What is your conclusion?

29. Refer to exercise 21, where data on production volume and cost were used to develop an estimated regression equation relating production volume and cost for a particular manufacturing operation. Use $\alpha = .05$ to test whether the production volume is significantly related to the total cost. Show the ANOVA table. What is your conclusion?

30. Refer to excercise 5, where the following data were used to investigate whether higher prices are generally associated with higher ratings for elliptical trainers (*Consumer Reports*, February 2008).

Ellipticals

Brand and Model	Price ($)	Rating
Precor 5.31	3700	87
Keys Fitness CG2	2500	84
Octane Fitness Q37e	2800	82
LifeFitness X1 Basic	1900	74
NordicTrack AudioStrider 990	1000	73
Schwinn 430	800	69
Vision Fitness X6100	1700	68
ProForm XP 520 Razor	600	55

With x = price ($) and y = rating, the estimated regression equation is $\hat{y} = 58.158 + .008449x$. For these data, SSE = 173.88 and SST = 756. Does the evidence indicate a significant relationship between price and rating?

31. In exercise 20, data on x = price ($) and y = overall score for ten 42-inch plasma televisions tested by *Consumer Reports* provided the estimated regression equation $\hat{y} = 12.0169 + .0127x$. For these data SSE = 540.04 and SST = 982.40. Use the F test to determine whether the price for a 42-inch plasma television and the overall score are related at the .05 level of significance.

12.6 Using the Estimated Regression Equation for Estimation and Prediction

When using the simple linear regression model, we are making an assumption about the relationship between x and y. We then use the least squares method to obtain the estimated simple linear regression equation. If a significant relationship exists between x and y and the coefficient of determination shows that the fit is good, the estimated regression equation should be useful for estimation and prediction.

For the Armand's Pizza Parlors example, the estimated regression equation is $\hat{y} = 60 + 5x$. At the end of Section 12.1 we stated that $\hat{y}$ can be used as a *point estimator* of $E(y)$, the mean or expected value of y for a given value of x, and as a predictor of an individual value of y. For example, suppose Armand's managers want to estimate the mean quarterly sales for *all* restaurants located near college campuses with 10,000 students. Using the estimated regression equation $\hat{y} = 60 + 5x$, we see that for $x = 10$ (10,000 students), $\hat{y} = 60 + 5(10) = 110$. Thus, a *point estimate* of the mean quarterly sales for all restaurant locations near campuses with 10,000 students is \$110,000. In this case we are using $\hat{y}$ as the point estimator of the mean value of y when $x = 10$.

We can also use the estimated regression equation to *predict* an individual value of y for a given value of x. For example, to predict quarterly sales for a new restaurant Armand's is considering building near Talbot College, a campus with 10,000 students, we would compute $\hat{y} = 60 + 5(10) = 110$. Hence, we would predict quarterly sales of \$110,000 for such a new restaurant. In this case, we are using $\hat{y}$ as the *predictor* of y for a new observation when $x = 10$.

When we are using the estimated regression equation to estimate the mean value of y or to predict an individual value of y, it is clear that the estimate or prediction depends on the given value of x. For this reason, as we discuss in more depth the issues concerning estimation and prediction, the following notation will help clarify matters.

x^* = the given value of the independent variable x

y^* = the random variable denoting the possible values of the dependent variable y when $x = x^*$

$E(y^*)$ = the mean or expected value of the dependent variable y when $x = x^*$

$\hat{y}^* = b_0 + b_1 x^*$ = the point estimator of $E(y^*)$ and the predictor of an individual value of y^* when $x = x^*$

To illustrate the use of this notation, suppose we want to estimate the mean value of quarterly sales for *all* Armand's restaurants located near a campus with 10,000 students. For this case, $x^* = 10$ and $E(y^*)$ denotes the unknown mean value of quarterly sales for all restaurants where $x^* = 10$. Thus, the point estimate of $E(y^*)$ is provided by $\hat{y}^* = 60 + 5(10) = 110$, or \$110,000. But, using this notation, $\hat{y}^* = 110$ is also the predictor of quarterly sales for the new restaurant located near Talbot College, a school with 10,000 students.

Interval Estimation

Confidence intervals and prediction intervals show the precision of the regression results. Narrower intervals provide a higher degree of precision.

Point estimators and predictors do not provide any information about the precision associated with the estimate and/or prediction. For that we must develop confidence intervals and prediction intervals. A **confidence interval** is an interval estimate of the *mean value of y* for a given value of x. A **prediction interval** is used whenever we want to *predict an individual value of y* for a new observation corresponding to a given value of x. Although the predictor of y for a given value of x is the same as the point estimator of the mean value of y for a given value of x, the interval estimates we obtain for the two cases are different. As we will show, the margin of error is larger for a prediction interval. We begin by showing how to develop an interval estimate of the mean value of y.

Confidence Interval for the Mean Value of y

In general, we cannot expect $\hat{y}^*$ to equal $E(y^*)$ exactly. If we want to make an inference about how close $\hat{y}^*$ is to the true mean value $E(y^*)$, we will have to estimate the variance of $\hat{y}^*$. The formula for estimating the variance of $\hat{y}^*$, denoted by $s^2_{\hat{y}^*}$, is

$$s_{\hat{y}^*}^2 = s^2\left[\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}\right] \tag{12.22}$$

The estimate of the standard deviation of $\hat{y}^*$ is given by the square root of equation (12.22).

$$s_{\hat{y}^*} = s\sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}} \tag{12.23}$$

The computational results for Armand's Pizza Parlors in Section 12.5 provided $s = 13.829$. With $x^* = 10$, $\bar{x} = 14$, and $\Sigma(x_i - \bar{x})^2 = 568$, we can use equation (12.23) to obtain

$$s_{\hat{y}^*} = 13.829\sqrt{\frac{1}{10} + \frac{(10 - 14)^2}{568}}$$

$$= 13.829\sqrt{.1282} = 4.95$$

The general expression for a confidence interval follows.

CONFIDENCE INTERVAL FOR $E(y^*)$

$$\hat{y}^* \pm t_{\alpha/2}s_{\hat{y}^*} \tag{12.24}$$

where the confidence coefficient is $1 - \alpha$ and $t_{\alpha/2}$ is based on the t distribution with $n - 2$ degrees of freedom.

The margin of error associated with this confidence interval is $t_{\alpha/2}s_{\hat{y}^*}$.

Using expression (12.24) to develop a 95% confidence interval of the mean quarterly sales for all Armand's restaurants located near campuses with 10,000 students, we need the value of t for $\alpha/2 = .025$ and $n - 2 = 10 - 2 = 8$ degrees of freedom. Using Table 2 of Appendix B, we have $t_{.025} = 2.306$. Thus, with $\hat{y}^* = 110$ and a margin of error of $t_{\alpha/2}s_{\hat{y}^*} = 2.306(4.95) = 11.415$, the 95% confidence interval estimate is

$$110 \pm 11.415$$

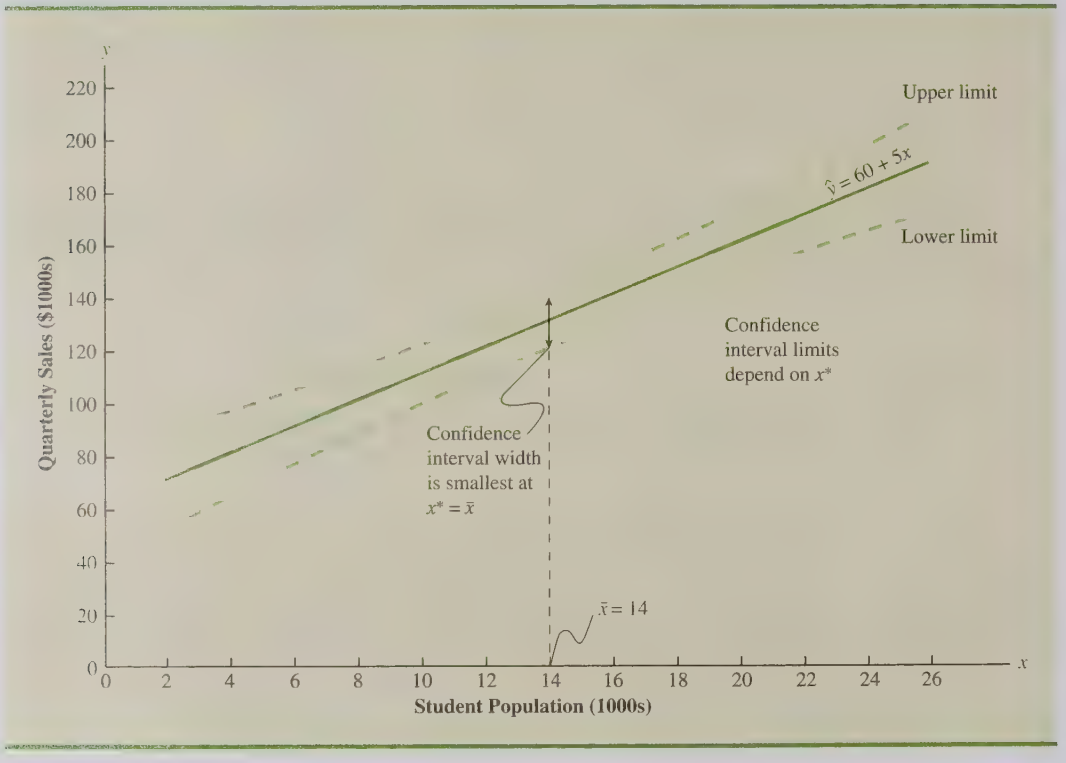
In dollars, the 95% confidence interval for the mean quarterly sales of all restaurants near campuses with 10,000 students is \$110,000 ± \$11,415. Therefore, the 95% confidence interval for the mean quarterly sales when the student population is 10,000 is \$98,585 to \$121,415.

Note that the estimated standard deviation of $\hat{y}^*$ given by equation (12.23) is smallest when $x^* - \bar{x} = 0$. In this case the estimated standard deviation of $\hat{y}^*$ becomes

$$s_{\hat{y}^*} = s\sqrt{\frac{1}{n} + \frac{(\bar{x} - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}} = s\sqrt{\frac{1}{n}}$$

This result implies that we can make the best or most precise estimate of the mean value of y whenever $x^* = \bar{x}$. In fact, the further x^* is from $\bar{x}$, the larger $x^* - \bar{x}$ becomes. As a result, the confidence interval for the mean value of y will become wider as x^* deviates more from $\bar{x}$. This pattern is shown graphically in Figure 12.10.

FIGURE 12.10 CONFIDENCE INTERVALS FOR THE MEAN SALES y AT GIVEN VALUES OF STUDENT POPULATION x



Prediction Interval for an Individual Value of y

Instead of estimating the mean value of quarterly sales for all Armand's restaurants located near campuses with 10,000 students, suppose we want to predict quarterly sales for a new restaurant Armand's is considering building near Talbot College, a campus with 10,000 students. As noted previously, the predictor of y^*, the value of y corresponding to the given x^*, is $\hat{y}^* = b_0 + b_1 x^*$. For the new restaurant located near Talbot College, $x^* = 10$ and the prediction of quarterly sales is $\hat{y}^* = 60 + 5(10) = 110$, or \$110,000. Note that the prediction of quarterly sales for the new Armand's restaurant near Talbot College is the same as the point estimate of the mean sales for all Armand's restaurants located near campuses with 10,000 students.

To develop a prediction interval, let us first determine the variance associated with using $\hat{y}^*$ as a predictor of y when $x = x^*$. This variance is made up of the sum of the following two components.

1. The variance of the y^* values about the mean $E(y^*)$, an estimate of which is given by s^2
2. The variance associated with using $\hat{y}^*$ to estimate $E(y^*)$, an estimate of which is given by $s_{\hat{y}}^2$.

The formula for estimating the variance corresponding to the prediction of the value of y when $x = x^*$, denoted s^2_{pred}, is

$$\begin{aligned} s^2_{\text{pred}} &= s^2 + s^2_{\hat{y}^*} \\ &= s^2 + s^2\left[\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}\right] \\ &= s^2\left[1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}\right] \end{aligned} \tag{12.25}$$

Hence, an estimate of the standard deviation corresponding to the prediction of the value of y^* is

$$s_{\text{pred}} = s\sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}} \tag{12.26}$$

For Armand's Pizza Parlors, the estimated standard deviation corresponding to the prediction of quarterly sales for a new restaurant located near Talbot College, a campus with 10,000 students, is computed as follows.

$$\begin{aligned} s_{\text{pred}} &= 13.829\sqrt{1 + \frac{1}{10} + \frac{(10 - 14)^2}{568}} \\ &= 13.829\sqrt{1.282} \\ &= 14.69 \end{aligned}$$

The general expression for a prediction interval follows.

PREDICTION INTERVAL FOR y^*

$$\hat{y}^* \pm t_{\alpha/2}s_{\text{pred}} \tag{12.27}$$

where the confidence coefficient is $1 - \alpha$ and $t_{\alpha/2}$ is based on a t distribution with $n - 2$ degrees of freedom.

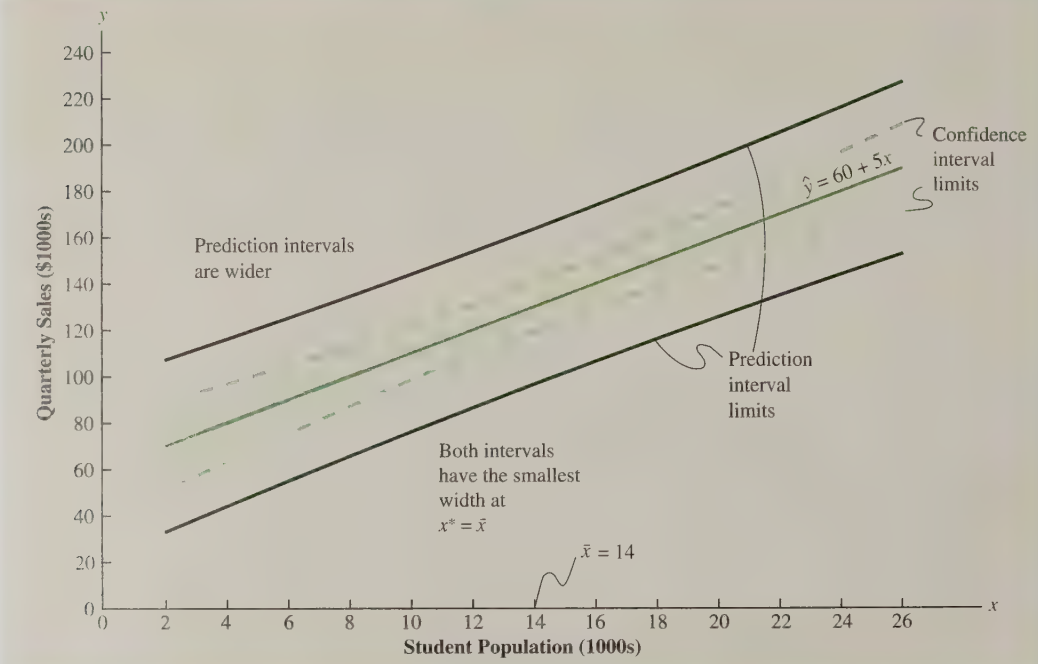
The margin of error associated with this prediction interval is $t_{\alpha/2}s_{\text{pred}}$.

The 95% prediction interval for quarterly sales for the new Armand's restaurant located near Talbot College can be found using $t_{\alpha/2} = t_{.025} = 2.306$ and $s_{\text{pred}} = 14.69$. Thus, with $\hat{y}^* = 110$ and a margin of error of $t_{.025}s_{\text{pred}} = 2.306(14.69) = 33.875$, the 95% prediction interval is

$$110 \pm 33.875$$

In dollars, this prediction interval is \$110,000 ± \$33,875 or \$76,125 to \$143,875. Note that the prediction interval for the new restaurant located near Talbot College, a campus with 10,000 students, is wider than the confidence interval for the mean quarterly sales of all restaurants located near campuses with 10,000 students. The difference reflects the fact that we are able to estimate the mean value of y more precisely than we can predict an individual value of y.

FIGURE 12.11 CONFIDENCE AND PREDICTION INTERVALS FOR SALES y AT GIVEN VALUES OF STUDENT POPULATION x



In general, the lines for the confidence interval limits and the prediction interval limits both have curvature.

Confidence intervals and prediction intervals are both more precise when the value of the independent variable x^* is closer to $\bar{x}$. The general shapes of confidence intervals and the wider prediction intervals are shown together in Figure 12.11.

NOTES AND COMMENTS

A prediction interval is used to predict the value of the dependent variable y for a *new observation*. As an illustration, we showed how to develop a prediction interval of quarterly sales for a new restaurant that Armand's is considering building near Talbot College, a campus with 10,000 students. The fact that the value of $x = 10$ is not one of the values of student population for the Armand's sample data in Table 12.1 is not meant to imply that prediction intervals cannot be developed for values of x in the sample data. But, for the ten restaurants that make up the data in Table 12.1, developing a prediction interval for quarterly sales for *one of these restaurants* does not make any sense because we already know the value of quarterly sales for each of these restaurants. In other words, a prediction interval only has meaning for something new, in this case a new observation corresponding to a particular value of x that may or may not equal one of the values of x in the sample.

Exercises

Methods

SELF test

32. The data from exercise 1 follow.

x_i	1	2	3	4	5
y_i	3	7	5	11	14

a. Use equation (12.23) to compute an estimate of the standard deviation of $\hat{y}^*$ when $x = 4$.
b. Use expression (12.24) to develop a 95% confidence interval for the expected value of y when $x = 4$.
c. Use equation (12.26) to estimate the standard deviation of an individual value of y when $x = 4$.
d. Use expression (12.27) to develop a 95% prediction interval for y when $x = 4$.

33. The data from exercise 2 follow.

x_i	3	12	6	20	14
y_i	55	40	55	10	15

a. Compute an estimate of the standard deviation of $\hat{y}^*$ when $x = 8$.
b. Develop a 95% confidence interval for the expected value of y when $x = 8$.
c. Estimate the standard deviation of an individual value of y when $x = 8$.
d. Develop a 95% prediction interval for y when $x = 8$.

34. The data from exercise 3 follow.

x_i	2	6	9	13	20
y_i	7	18	9	26	23

Develop the 95% confidence and prediction intervals when $x = 12$. Explain why these two intervals are different.

Applications

SELF test

35. In exercise 18, the data on grade point average x and monthly salary y provided the estimated regression equation $\hat{y} = 1790.5 + 581.1x$.
a. Develop a 95% confidence interval for the mean starting salary for all students with a 3.0 GPA.
b. Develop a 95% prediction interval for the starting salary for Joe Heller, a student with a GPA of 3.0.

WEB file
SleepingBags

36. In exercise 8, data on x = temperature rating (F°) and y = price ($) for 11 sleeping bags manufactured by Bergans of Norway provided the estimated regression equation $\hat{y} = 359.2668 - 5.2772x$. For these data $s = 37.9372$.
a. Develop a point estimate of the mean price for all sleeping bags with a temperature rating of 30.
b. Develop a 95% confidence interval for the mean price for all sleeping bags with a temperature rating of 30.
c. Suppose that Bergans developed a new model with a temperature rating of 30. Develop a 95% prediction interval for the price of this new model.
d. Discuss the differences in your answers to parts (b) and (c).

37. In exercise 13, data were given on the adjusted gross income x and the average or reasonable amount of total itemized deductions taken by taxpayers. Data were reported in thousands of dollars. With the estimated regression equation $\hat{y} = 4.68 + .16x$, the point

estimate of the mean reasonable level of total itemized deductions for a taxpayer with an adjusted gross income of $52,500 is $13,080.

a. Develop a 95% confidence interval for the mean amount of total itemized deductions for all taxpayers with an adjusted gross income of $52,500.
b. Develop a 95% prediction interval estimate for the amount of total itemized deductions for a particular taxpayer with an adjusted gross income of $52,500.
c. If the particular taxpayer referred to in part (b) claimed total itemized deductions of $20,400, would the IRS agent's request for an audit appear to be justified?
d. Use your answer to part (b) to give the IRS agent a guideline as to the amount of total itemized deductions a taxpayer with an adjusted gross income of $52,500 should claim before an audit is recommended.

38. Refer to exercise 21, where data on the production volume x and total cost y for a particular manufacturing operation were used to develop the estimated regression equation $\hat{y} = 1246.67 + 7.6x$.
    a. The company's production schedule shows that 500 units must be produced next month. What is the point estimate of the mean total cost for next month?
    b. Develop a 99% prediction interval for the total cost for next month.
    c. If an accounting cost report at the end of next month shows that the actual production cost during the month was $6000, should managers be concerned about incurring such a high total cost for the month? Discuss.

39. Almost all U.S. light-rail systems use electric cars that run on tracks built at street level. The Federal Transit Administration claims light-rail is one of the safest modes of travel, with an accident rate of .99 accidents per million passenger miles as compared to 2.29 for buses. The following data show the miles of track and the weekday ridership in thousands of passengers for six light-rail systems (*USA Today*, January 7, 2003).

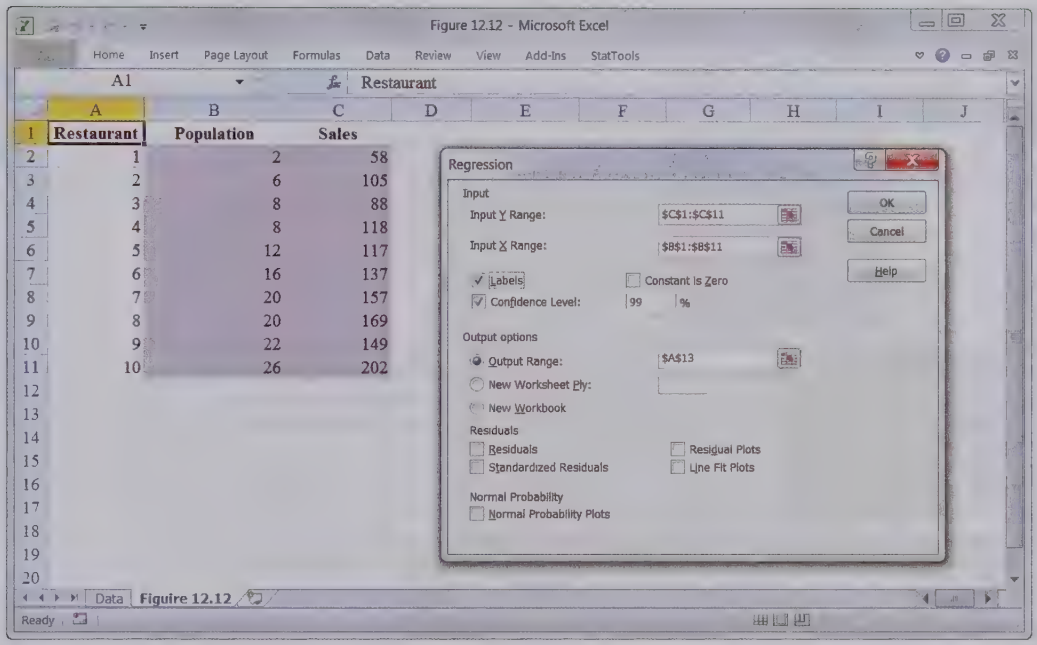
City	Miles of Track	Ridership (1000s)
Cleveland	15	15
Denver	17	35
Portland	38	81
Sacramento	21	31
San Diego	47	75
San Jose	31	30
St. Louis	34	42

    a. Use these data to develop an estimated regression equation that could be used to predict the ridership given the miles of track.
    b. Did the estimated regression equation provide a good fit? Explain.
    c. Develop a 95% confidence interval for the mean weekday ridership for all light-rail systems with 30 miles of track.
    d. Suppose that Charlotte is considering construction of a light-rail system with 30 miles of track. Develop a 95% prediction interval for the weekday ridership for the Charlotte system. Do you think that the prediction interval you developed would be of value to Charlotte planners in anticipating the number of weekday riders for their new light-rail system? Explain.

12.7 Excel's Regression Tool

In previous sections of this chapter we have shown how Excel's chart tools can be used for various tasks in a regression analysis. Excel also has a more comprehensive Regression tool. In this section we will illustrate how Excel's Regression tool can be used to perform a complete regression analysis, including statistical tests of significance for the Armand's Pizza Parlors data in Table 12.1. We also show how StatTools can be used to develop prediction interval estimates.

FIGURE 12.12 REGRESSION TOOL DIALOG BOX FOR THE ARMAND'S PIZZA PARLORS EXAMPLE



In the chapter appendix we show how to use StatTools to perform the regression analysis computations for the Armand's Pizza Parlors data. The regression analysis capabilities of StatTools are more comprehensive than those available using Excel's Regression tool.

Using Excel's Regression Tool for the Armand's Pizza Parlors Example

Refer to Figures 12.12 and 12.13 as we describe the tasks involved to use Excel's Regression tool to perform the regression analysis computations for the Armand's data.

Enter Data: The labels Restaurant, Population, and Sales are entered into cells A1:C1 of the worksheet. To identify each of the 10 observations, we entered the numbers 1 through 10 into cells A2:A11. The sample data are entered into cells B2:C11.

Apply Tools: The following steps describe how to use Excel's Regression tool to perform the regression analysis computations performed in Sections 12.2–12.5.

Step 1. Click the **Data** tab on the Ribbon
Step 2. In the **Analysis** group, click **Data Analysis**
Step 3. Choose **Regression** from the list of Analysis Tools
Step 4. When the Regression dialog box appears (see Figure 12.12),
Enter C1:C11 in the **Input Y Range** box
Enter B1:B11 in the **Input X Range** box
Select **Labels**
Select **Confidence Level**
Enter 99 in the **Confidence Level** box
Select **Output Range**
Enter A13 in the **Output Range** box (to identify the upper left corner of the section of the worksheet where the output will appear)
Click **OK**

FIGURE 12.13 REGRESSION TOOL OUTPUT FOR ARMAND'S PIZZA PARLORS

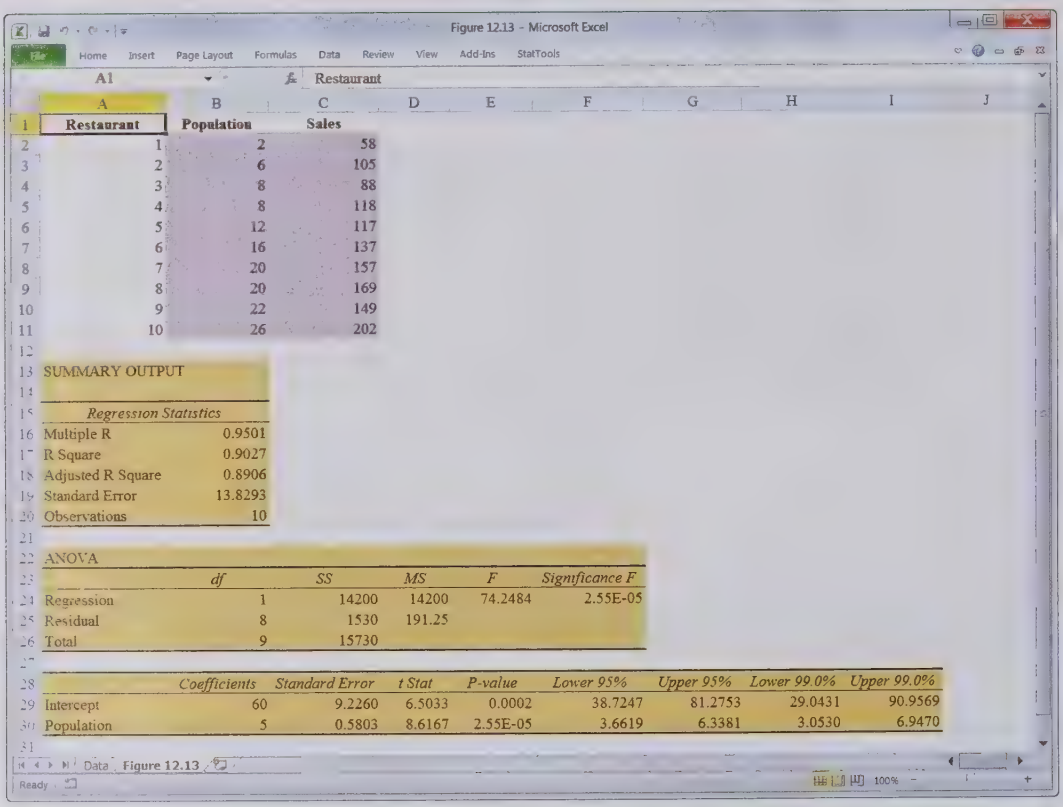


Figure 12.13 - Microsoft Excel

Home Insert Page Layout Formulas Data Review View Add-Ins StatTools

A1 Restaurant

	A	B	C
1	Restaurant	Population	Sales
2	1	2	58
3	2	6	105
4	3	8	88
5	4	8	118
6	5	12	117
7	6	16	137
8	7	20	157
9	8	20	169
10	9	22	149
11	10	26	202

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.9501
R Square	0.9027
Adjusted R Square	0.8906
Standard Error	13.8293
Observations	10

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	14200	14200	74.2484	2.55E-05
Residual	8	1530	191.25		
Total	9	15730			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 99.0%*	*Upper 99.0%*
Intercept	60	9.2260	6.5033	0.0002	38.7247	81.2753	29.0431	90.9569
Population	5	0.5803	8.6167	2.55E-05	3.6619	6.3381	3.0530	6.9470

Data Figure 12.13

Ready 100%

The Excel output can be reformatted to improve readability.

The regression output, titled SUMMARY OUTPUT, begins with row 13 in Figure 12.13. Because Excel initially displays the output using standard column widths, many of the row and column labels are unreadable. In several places we have reformatted to improve readability. We have also reformatted cells displaying numerical values to a maximum of four decimal places. Numbers displayed using scientific notation have not been modified. Regression output in future figures will be similarly reformatted to improve readability.

The first section of the summary output, titled *Regression Statistics,* contains summary statistics such as the coefficient of determination (R Square). The second section of the output, titled ANOVA, contains the analysis of variance table. The last section of the output, which is not titled, contains the estimated regression coefficients and related information. Let us begin our interpretation of the regression output with the information contained in rows 29 and 30.

Interpretation of Estimated Regression Equation Output

Row 29 contains information about the y-intercept of the estimated regression line. Row 30 contains information about the slope of the estimated regression line. The y-intercept of the estimated regression line, $b_0 = 60$, is shown in cell B29, and the slope of the estimated

regression line, $b_1 = 5$, is shown in cell B30. The label Intercept in cell A29 and the label Population in cell A30 are used to identify these two values.

In Section 12.5 we showed that the estimated standard deviation of b_1 is $s_{b_1} = .5803$. Cell C30 contains the estimated standard deviation of b_1. As we indicated previously, the standard deviation of b_1 is also referred to as the standard error of b_1. Thus, s_{b_1} provides an estimate of the standard error of b_1. The label Standard Error in cell C28 is Excel's way of indicating that the value in cell C30 is the estimate of the standard error, or standard deviation, of b_1.

In Section 12.5 we stated that the form of the null and alternative hypotheses needed to test for a significant relationship between population and sales are as follows:

$$H_0: \beta_1 = 0$$
$$H_a: \beta_1 \neq 0$$

Recall that the t test for a significant relationship required the computation of the t statistic, $t = b_1/s_{b_1}$. For the Armand's data, the value of t that we computed was $t = 5/.5803 = 8.62$. Note that after rounding, the value in cell D30 is 8.62. The label in cell D28, *t Stat,* reminds us that cell D30 contains the value of the t test statistic.

***t* Test** The information in cell E30 provides a means for conducting a test of significance. The value in cell E30 is the p-value associated with the t test for significance. Excel has displayed the p-value using scientific notation. To obtain the decimal equivalent, we move the decimal point 5 places to the left; we obtain a p-value of .0000255. Thus, the p-value associated with the t test for significance is .0000255. Given the level of significance α, the decision of whether to reject H_0 can be made as follows:

$$\text{Reject } H_0 \text{ if } p\text{-value} \leq \alpha$$

Suppose the level of significance is $\alpha = .01$. Because the p-value = .0000255 $< \alpha = .01$, we can reject H_0 and conclude that we have a significant relationship between student population and sales. Because p-values are provided as part of the computer output for regression analysis, the p-value approach is most often used for hypothesis tests in regression analysis.

The information in cells F28:I30 can be used to develop confidence interval estimates of the y-intercept and slope of the estimated regression equation. Excel always provides the lower and upper limits for a 95% confidence interval. Recall that in the Regression dialog box (see Figure 12.12) we selected Confidence Level and entered 99 in the Confidence Level box. As a result, Excel's Regression tool also provides the lower and upper limits for a 99% confidence interval. For instance, the value in cell H30 is the lower limit for the 99% confidence interval estimate of β_1 and the value in cell I30 is the upper limit. Thus, after rounding, the 99% confidence interval estimate of β_1 is 3.05 to 6.95. The values in cells F30 and G30 provide the lower and upper limits for the 95% confidence interval. Thus, the 95% confidence interval is 3.66 to 6.34.

Interpretation of ANOVA Output

Excel refers to the error sum of squares as the residual sum of squares.

The information in cells A22:F26 summarizes the analysis of variance computations for the Armand's data. The three sources of variation are labeled Regression, Residual, and Total. The label *df* in cell B23 stands for degrees of freedom, the label *SS* in cell C23 stands for sum of squares, and the label *MS* in cell D23 stands for mean square. Looking at cells C24:C26, we see that the regression sum of squares is 14200, the residual or error sum

of squares is 1530, and the total sum of squares is 15730. The values in cells B24:B26 are the degrees of freedom corresponding to each sum of squares. Thus, the regression sum of squares has 1 degree of freedom, the residual or error sum of squares has 8 degrees of freedom, and the total sum of squares has 9 degrees of freedom. As we discussed previously, the regression degrees of freedom plus the residual degrees of freedom are equal to the total degrees of freedom, and the regression sum of squares plus the residual sum of squares are equal to the total sum of squares.

In Section 12.5 we stated that the mean square error, obtained by dividing the error or residual sum of squares by its degrees of freedom, provides an estimate of σ^2. The value in cell D25, 191.25, is the mean square error for the Armand's regression output. We also stated that the mean square regression is the sum of squares due to regression divided by the regression degrees of freedom. The value in cell D24, 14200, is the mean square regression.

***F* Test** In Section 12.5 we showed that an F test, based upon the F probability distribution, could also be used to test for significance in regression. The value in cell F24, .0000255, is the p-value associated with the F test for significance. Suppose the level of significance is $\alpha = .01$. Because the p-value $= .0000255 < \alpha = .01$, we can reject H_0 and conclude that we have a significant relationship between student population and sales. Note that it is the same conclusion that we obtained using the p-value approach for the t test for significance. In fact, because the t test for significance is equivalent to the F test for significance in simple linear regression, the p-values provided by both approaches are identical. The label Excel uses to identify the p-value for the F test for significance, shown in cell F23, is *Significance F*. In Chapter 9 we also stated that the p-value is often referred to as the observed level of significance. Thus, the label *Significance F* may be more meaningful if you think of the value in cell F24 as the observed level of significance for the F test.

Interpretation of Regression Statistics Output

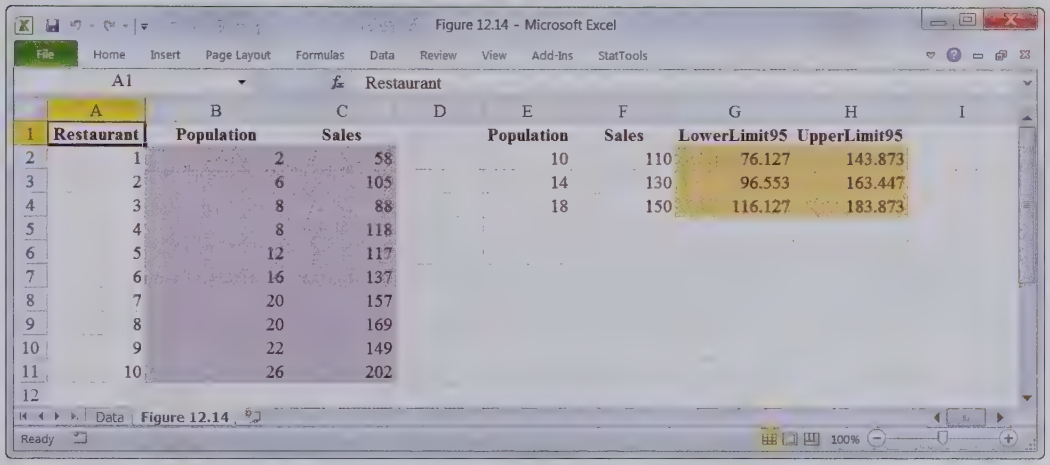
The output in cells A15:B20 summarizes the regression statistics. The number of observations in the data set, 10, is shown in cell B20. The coefficient of determination, .9027, appears in cell B17; the corresponding label, R Square, is shown in cell A17. The square root of the coefficient of determination provides the sample correlation coefficient of 0.9501 shown in cell B16. Note that Excel uses the label Multiple R (cell A16) to identify this value. In cell A19, the label Standard Error is used to identify the value of s, the estimate of σ. Cell B19 shows that the value of s is 13.8293. We caution the reader to keep in mind that in the Excel output, the label Standard Error appears in two different places. In the Regression Statistics section of the output the label Standard Error refers to s, the estimate of σ. In the Estimated Regression Equation section of the output, the label Standard Error refers to s_{b_1}, the estimated standard deviation of the sampling distribution of b_1.

Using StatTools to Compute Prediction Intervals

In the chapter appendix we show how to use StatTools to compute prediction intervals.

In Section 12.6 we showed how to estimate quarterly sales for an individual Armand's restaurant located near Talbot College, a school with 10,000 students. With $x = 10$, the 95% prediction interval is \$76,125 to \$143,875. Hand computation of such intervals is not practical. Unfortunately, Excel's Regression Tool does not have an option for computing prediction intervals. However, StatTools can be used to compute prediction intervals. To illustrate, suppose we would like to compute prediction intervals corresponding to three particular Armand's restaurants: the proposed restaurant located near Talbot College, a

FIGURE 12.14 USING STATTOOLS TO COMPUTE PREDICTION INTERVALS



	A	B	C	D	E	F	G	H	I
1	Restaurant	Population	Sales		Population	Sales	LowerLimit95	UpperLimit95	
2	1	2	58		10	110	76.127	143.873	
3	2	6	105		14	130	96.553	163.447	
4	3	8	88		18	150	116.127	183.873	
5	4	8	118						
6	5	12	117						
7	6	16	137						
8	7	20	157						
9	8	20	169						
10	9	22	149						
11	10	26	202						
12									

school with 10,000 students; another Armand's restaurant located near a school with 14,000 students; and a third Armand's restaurant located near a school with 18,000 students. The chapter appendix describes how StatTools can be used to compute prediction intervals for these three cases; the results are shown in Figure 12.14.

Refer to Figure 12.14. The usual Armand's data appears in cells A1:C11 of the worksheet. In addition, we also entered the label Population in cell E1 and the values of Population for the three Armand's restaurants for which we want to compute prediction intervals in cells E2:E4. The StatTools prediction interval output appears in cells F1:H4. The predicted values of Sales corresponding to the three values of Population are shown in cells F2:F4 in the worksheet, and the lower and upper limits for the three prediction intervals are shown in cells G2:H4. For instance, the value in cell F2 shows that the predicted value of sales for the Armand's restaurant located near Talbot college, a school with 10,000 students, is 110 or \$110,000. The value in cell G2 shows that the lower limit for the 95% prediction interval for this Armand's restaurant is 76.127 or \$76,127. And the value in cell H2 shows that the upper limit is 143.873 or \$143,875. The slight differences between these limits and the limits shown previously using hand calculation are due to rounding. Note that for the Armand's restaurant located near a school with 14,000 students, the 95% prediction interval is \$96,553 to \$163,447. And, for the Armand's restaurant located near a school with 18,000 students, the 95% prediction interval is \$116,127 to \$183,873.

Exercises

Applications

SELF test

40. The commercial division of a real estate firm conducted a study to determine the extent of the relationship between annual gross rents (\$1000s) and the selling price (\$1000s) for apartment buildings. Data were collected on several properties sold, and Excel's Regression tool was used to develop an estimated regression equation. A portion of the regression output follows.

ANOVA

	df	*SS*	*MS*	*F*
Regression	1	41587.3		
Residual	7			
Total	8	51984.1		

	Coefficients	*Standard Error*	*t Stat*
Intercept	20.000	3.2213	6.21
Annual Gross Rents	7.210	1.3626	5.29

a. How many apartment buildings were in the sample?
b. Write the estimated regression equation.
c. Use the t test to determine whether the selling price is related to annual gross rents. Use $\alpha = .05$.
d. Use the F test to determine whether the selling price is related to annual gross rents. Use $\alpha = .05$.
e. Predict the selling price of an apartment building with gross annual rents of $50,000.

41. Following is a portion of the regression output for an application relating maintenance expense (dollars per month) to usage (hours per week) for a particular brand of computer terminal.

ANOVA

	df	*SS*
Regression	1	1575.76
Residual	8	349.14
Total	9	1924.90

	Coefficients	*Standard Error*	*t Stat*
Intercept	6.1092	.9361	
Usage	0.8951	.149	

a. Write the estimated regression equation.
b. Use a t test to determine whether monthly maintenance expense is related to usage at the .05 level of significance.
c. Did the estimated regression equation provide a good fit? Explain.

42. A regression model relating the number of salespersons at a branch office to annual sales at the office (in thousands of dollars) provided the following regression output.

ANOVA

	df	*SS*	*MS*	*F*
Regression		6828.6		
Residual				
Total		9127.4		

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	80.0	11.333		
Number of Salespersons	50.0	5.482		

a. Write the estimated regression equation.
b. Compute the F statistic and test the significance of the relationship at the .05 level of significance.

c. Compute the t statistic and test the significance of the relationship at the .05 level of significance.
d. Predict the annual sales at the Memphis branch office. This branch employs 12 salespersons.

43. Out-of-state tuition and fees at the top graduate schools of business can be very expensive, but the starting salary and bonus paid to graduates from many of these schools can be substantial. The following data show the out-of-state tuition and fees (rounded to the nearest $1000) and the average starting salary and bonus paid to recent graduates (rounded to the nearest $1000) for a sample of 20 graduate schools of business (*U.S. News & World Report 2009 Edition America's Best Graduate Schools*).

WEB file
BusinessSchools

School	Tuition & Fees ($1000s)	Salary & Bonus ($1000s)
Arizona State University	28	98
Babson College	35	94
Cornell University	44	119
Georgetown University	40	109
Georgia Institute of Technology	30	88
Indiana University–Bloomington	35	105
Michigan State University	26	99
Northwestern University	44	123
Ohio State University	35	97
Purdue University–West Lafayette	33	96
Rice University	36	102
Stanford University	46	135
University of California–Davis	35	89
University of Florida	23	71
University of Iowa	25	78
University of Minnesota–Twin Cities	37	100
University of Notre Dame	36	95
University of Rochester	38	99
University of Washington	30	94
University of Wisconsin–Madison	27	93

a. Develop a scatter diagram with salary and bonus as the dependent variable.
b. Does there appear to be any relationship between these variables? Explain.
c. Develop an estimated regression equation that can be used to predict the starting salary and bonus paid to graduates given the cost of out-of-state tuition and fees at the school.
d. Test for a significant relationship at the .05 level of significance. What is your conclusion?
e. Did the estimated regression equation provide a good fit? Explain.
f. Suppose that we randomly select a recent graduate of the University of Virginia graduate school of business. The school has an out-of-state tuition and fees of $43,000. Predict the starting salary and bonus for this graduate.

44. Automobile racing, high-performance driving schools, and driver education programs run by automobile clubs continue to grow in popularity. All these activities require the participant to wear a helmet that is certified by the Snell Memorial Foundation, a not-for-profit organization dedicated to research, education, testing, and development of helmet safety standards. Snell "SA" (Sports Application) rated professional helmets are designed for auto racing and provide extreme impact resistance and high fire protection. One of the key factors in selecting a helmet is weight, since lower weight helmets tend to place less stress on the neck. The following data show the weight and price for 18 SA helmets (SoloRacer website, April 20, 2008).

Helmet	Weight (oz)	Price ($)
Pyrotect Pro Airflow	64	248
Pyrotect Pro Airflow Graphics	64	278
RCi Full Face	64	200
RaceQuip RidgeLine	64	200
HJC AR-10	58	300
HJC Si-12	47	700
HJC HX-10	49	900
Impact Racing Super Sport	59	340
Zamp FSA-1	66	199
Zamp RZ-2	58	299
Zamp RZ-2 Ferrari	58	299
Zamp RZ-3 Sport	52	479
Zamp RZ-3 Sport Painted	52	479
Bell M2	63	369
Bell M4	62	369
Bell M4 Pro	54	559
G Force Pro Force 1	63	250
G Force Pro Force 1 Grafx	63	280

WEB file
RaceHelmets

a. Develop a scatter diagram with weight as the independent variable.
b. Does there appear to be any relationship between these two variables?
c. Develop the estimated regression equation that could be used to predict the price given the weight.
d. Test for the significance of the relationship at the .05 level of significance.
e. Did the estimated regression equation provide a good fit? Explain.

12.8 Residual Analysis: Validating Model Assumptions

Residual analysis *is the primary tool for determining whether the assumed regression model is appropriate.*

As we noted previously, the *residual* for observation i is the difference between the observed value of the dependent variable (y_i) and the estimated value of the dependent variable ($\hat{y}_i$).

RESIDUAL FOR OBSERVATION i

$$y_i - \hat{y}_i \tag{12.28}$$

where

y_i is the observed value of the dependent variable
$\hat{y}_i$ is the predicted value of the dependent variable

In other words, the ith residual is the error resulting from using the estimated regression equation to predict the value of the dependent variable. The residuals for the Armand's Pizza Parlors example are computed in Table 12.7. The observed values of the dependent variable are in the second column and the estimated values of the dependent variable, obtained using the estimated regression equation $\hat{y} = 60 + 5x$, are in the third column. An

TABLE 12.7 RESIDUALS FOR ARMAND'S PIZZA PARLORS

Student Population x_i	Sales y_i	Predicted Sales $\hat{y}_i = 60 + 5x_i$	Residuals $y_i - \hat{y}_i$
2	58	70	−12
6	105	90	15
8	88	100	−12
8	118	100	18
12	117	120	−3
16	137	140	−3
20	157	160	−3
20	169	160	9
22	149	170	−21
26	202	190	12

analysis of the corresponding residuals in the fourth column will help determine whether the assumptions made about the regression model are appropriate.

Let us now review the regression assumptions for the Armand's Pizza Parlors example. A simple linear regression model was assumed.

$$y = \beta_0 + \beta_1 x + \epsilon \tag{12.29}$$

This model indicates that we assumed quarterly sales (y) to be a linear function of the size of the student population (x) plus an error term ϵ. In Section 12.4 we made the following assumptions about the error term ϵ.

1. $E(\epsilon) = 0$.
2. The variance of ϵ, denoted by σ^2, is the same for all values of x.
3. The values of ϵ are independent.
4. The error term ϵ has a normal distribution.

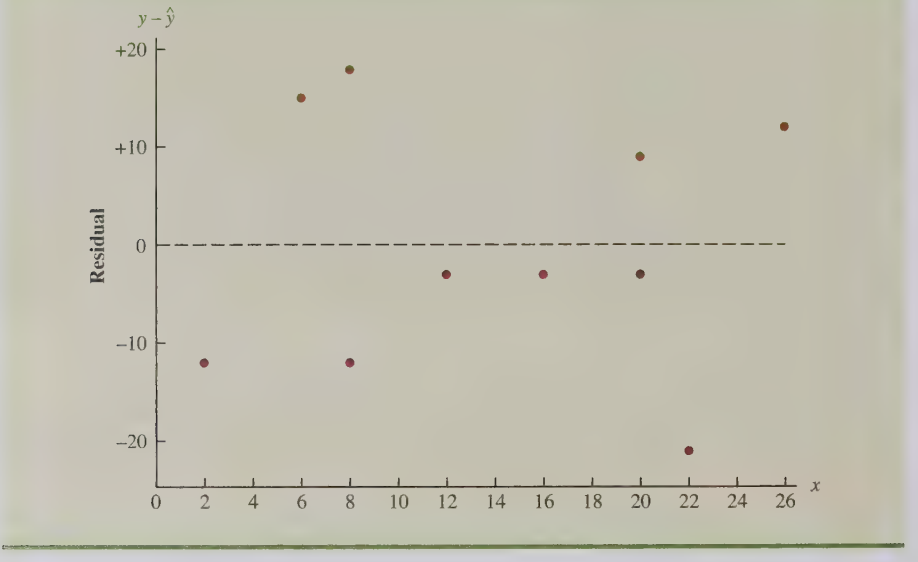
These assumptions provide the theoretical basis for the t test and the F test used to determine whether the relationship between x and y is significant, and for the confidence and prediction interval estimates presented in Section 12.6. If the assumptions about the error term ϵ appear questionable, the hypothesis tests about the significance of the regression relationship and the interval estimation results may not be valid.

The residuals provide the best information about ϵ; hence an analysis of the residuals is an important step in determining whether the assumptions for ϵ are appropriate. Much of residual analysis is based on an examination of graphical plots. In this section, we discuss the following residual plots: a plot of the residuals against values of the independent variable x, and a plot of residuals against the predicted values of the dependent variable $\hat{y}$.

Residual Plot Against x

A **residual plot** against the independent variable x is a graph in which the values of the independent variable are represented by the horizontal axis and the corresponding residual values are represented by the vertical axis. A point is plotted for each residual. The first coordinate for each point is given by the value of x_i and the second coordinate is given by the corresponding value of the residual $y_i - \hat{y}_i$. For a residual plot against x with the Armand's Pizza Parlors data from Table 12.7, the coordinates of the first point are $(2, -12)$, corresponding to

FIGURE 12.15 PLOT OF THE RESIDUALS AGAINST THE INDEPENDENT VARIABLE x FOR ARMAND'S PIZZA PARLORS



$x_1 = 2$ and $y_1 - \hat{y}_1 = -12$; the coordinates of the second point are (6, 15), corresponding to $x_2 = 6$ and $y_2 - \hat{y}_2 = 15$; and so on. Figure 12.15 shows the resulting residual plot.

Before interpreting the results for this residual plot, let us consider some general patterns that might be observed in any residual plot. Three examples appear in Figure 12.16. If the assumption that the variance of ϵ is the same for all values of x and the assumed regression model is an adequate representation of the relationship between the variables, the residual plot should give an overall impression of a horizontal band of points such as the one in Panel A of Figure 12.16. However, if the variance of ϵ is not the same for all values of x—for example, if variability about the regression line is greater for larger values of x—a pattern such as the one in Panel B of Figure 12.16 could be observed. In this case, the assumption of a constant variance of ϵ is violated. Another possible residual plot is shown in Panel C. In this case, we would conclude that the assumed regression model is not an adequate representation of the relationship between the variables. A curvilinear regression model or multiple regression model should be considered.

Now let us return to the residual plot for Armand's Pizza Parlors shown in Figure 12.15. The residuals appear to approximate the horizontal pattern in Panel A of Figure 12.16. Hence, we conclude that the residual plot does not provide evidence that the assumptions made for Armand's regression model should be challenged. At this point, we are confident in the conclusion that Armand's simple linear regression model is valid.

Experience and good judgment are always factors in the effective interpretation of residual plots. Seldom does a residual plot conform precisely to one of the patterns in Figure 12.16. Yet analysts who frequently conduct regression studies and frequently review residual plots become adept at understanding the differences between patterns that are reasonable and patterns that indicate the assumptions of the model should be questioned. A residual plot provides one technique to assess the validity of the assumptions for a regression model.

FIGURE 12.16 RESIDUAL PLOTS FROM THREE REGRESSION STUDIES

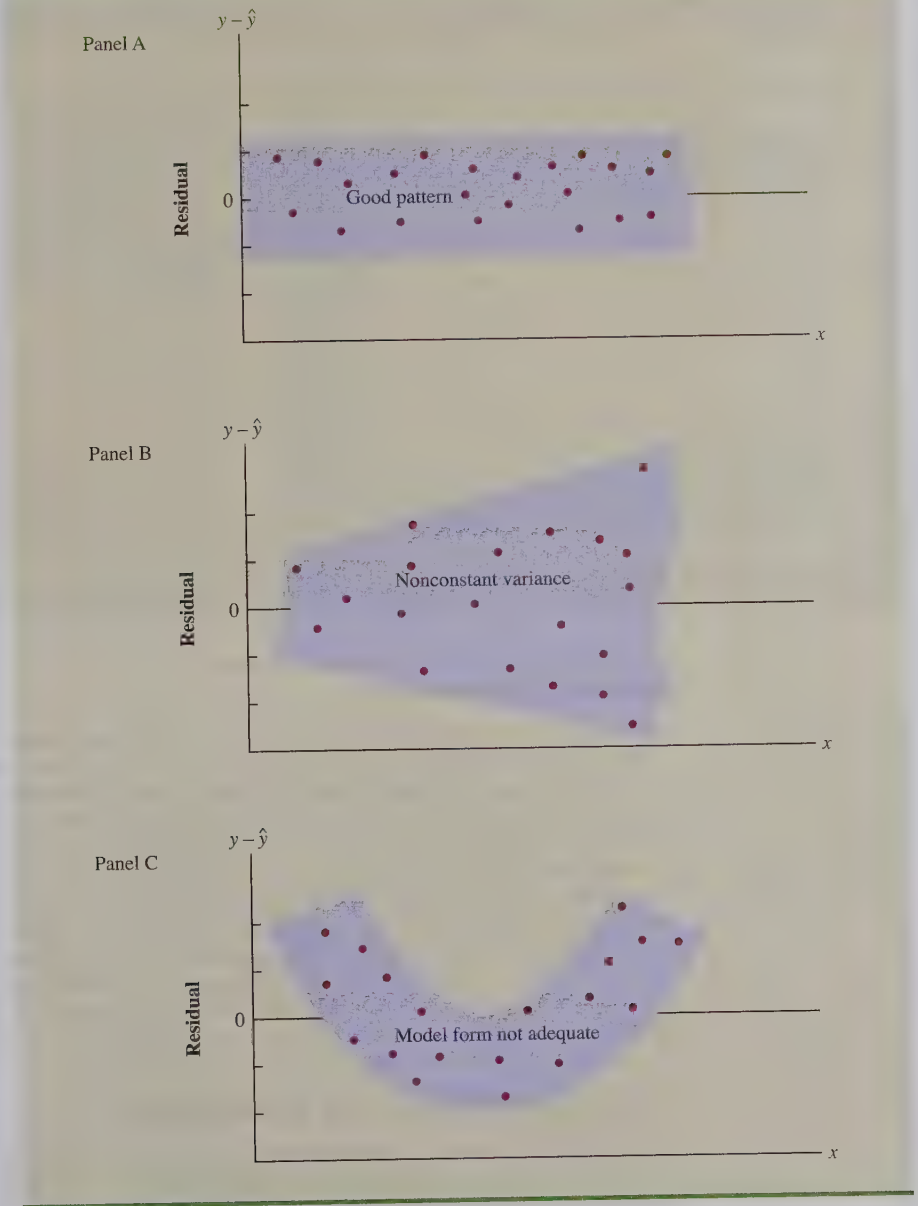
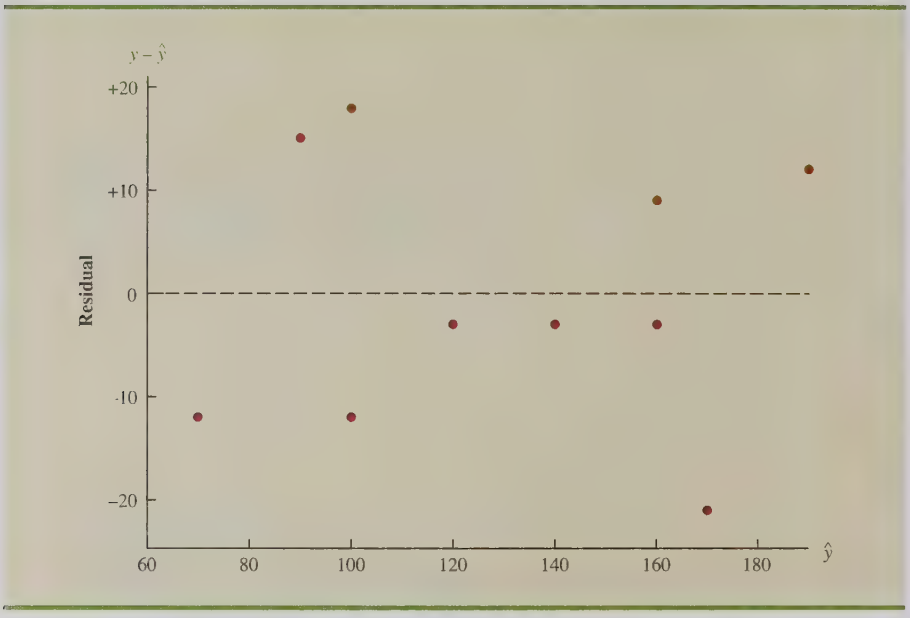


FIGURE 12.17 PLOT OF THE RESIDUALS AGAINST THE PREDICTED VALUES $\hat{y}$ FOR ARMAND'S PIZZA PARLORS



Residual Plot Against $\hat{y}$

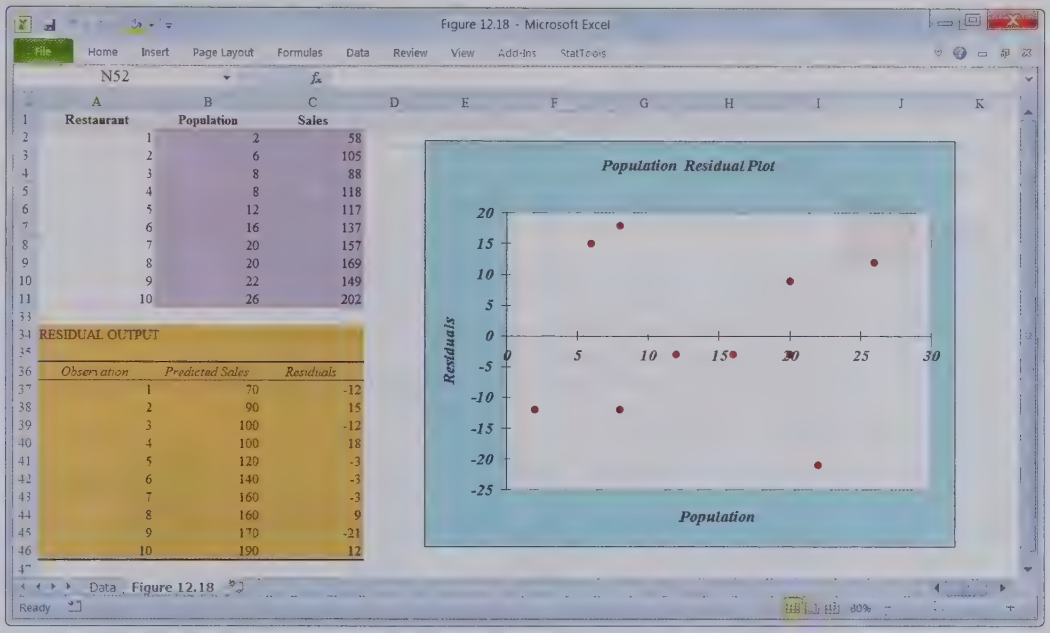
Another residual plot represents the predicted value of the dependent variable $\hat{y}$ on the horizontal axis and the residual values on the vertical axis. A point is plotted for each residual. The first coordinate for each point is given by $\hat{y}_i$ and the second coordinate is given by the corresponding value of the ith residual $y_i - \hat{y}_i$. With the Armand's data from Table 12.7, the coordinates of the first point are $(70, -12)$, corresponding to $\hat{y}_1 = 70$ and $y_1 - \hat{y}_1 = -12$; the coordinates of the second point are (90, 15); and so on. Figure 12.17 provides the residual plot. Note that the pattern of this residual plot is the same as the pattern of the residual plot against the independent variable x. It is not a pattern that would lead us to question the model assumptions. For simple linear regression, both the residual plot against x and the residual plot against $\hat{y}$ provide the same pattern. For multiple regression analysis, the residual plot against $\hat{y}$ is more widely used because of the presence of more than one independent variable.

Using Excel to Construct a Residual Plot

In Section 12.7 we showed how Excel's Regression tool could be used for regression analysis. The Regression tool also provides the capability to obtain a residual plot against the independent variable x and, when used with Excel's Chart tool, the Regression tool residual output can also be used to construct a residual plot against $\hat{y}$.

Residual plot against x To obtain a residual plot against x, the steps that we describe in Section 12.7 in order to obtain the regression output are performed with one change. When the Regression tool dialog box appears (see Figure 12.12), we must also select the Residual

FIGURE 12.18 REGRESSION TOOL RESIDUAL OUTPUT FOR THE ARMAND'S PIZZA PARLORS PROBLEM



Restaurant	Population	Sales
1	2	58
2	6	105
3	8	88
4	8	118
5	12	117
6	16	137
7	20	157
8	20	169
9	22	149
10	26	202

Observation	Predicted Sales	Residuals
1	70	-12
2	90	15
3	100	-12
4	100	18
5	120	-3
6	140	-3
7	160	-3
8	160	9
9	170	-21
10	190	12

Note: Rows 12–32 are hidden.

Plots option in the Residual section. The regression output will appear as described previously, and the worksheet will also contain a chart showing a plot of the residuals against the independent variable Population. In addition, a list of predicted values of y and the corresponding residual values are provided below the regression output. Figure 12.18 shows the residual output for the Armand's Pizza Parlors problem; note that rows 12–32, containing the standard Regression tool output, have been hidden to better focus on the residual portion of the output. We see that the shape of this plot is the same as shown previously in Figure 12.15.

Residual plot against $\hat{y}$ Using Excel's chart tools and the residual output provided in Figure 12.18, we can easily construct a residual plot against $\hat{y}$. The following steps describe how to use Excel's chart tools to construct the residual plot using the regression tool output in the worksheet.

Step 1. Select cells B37:C46
Step 2. Click the **Insert** tab on the Ribbon
Step 3. In the **Chart** group, click **Scatter**
Step 4. When the list of scatter diagram subtypes appears,
Click **Scatter with only Markers** (the chart in the upper left corner)

The resulting chart will look similar to the residual plot shown in Figure 12.17. Adding a chart title, labels for the horizontal and vertical axes, as well as other formatting options

can be easily done. Note that except for using different data to construct the chart as well as different labels for the chart output, the steps describing how to use Excel's chart tools to construct a residual plot against $\hat{y}$ are the same as the steps we used to construct a scatter diagram in Section 12.2.

Exercises

Methods

SELF test

45. Given are data for two variables, x and y.

x_i	6	11	15	18	20
y_i	6	8	12	20	30

a. Develop an estimated regression equation for these data.
b. Compute the residuals.
c. Develop a plot of the residuals against the independent variable x. Do the assumptions about the error term seem to be satisfied?

46. The following data were used in a regression study.

Observation	x_i	y_i	Observation	x_i	y_i
1	2	4	6	7	6
2	3	5	7	7	9
3	4	4	8	8	5
4	5	6	9	9	11
5	7	4			

a. Develop an estimated regression equation for these data.
b. Construct a plot of the residuals. Do the assumptions about the error term seem to be satisfied?

Applications

SELF test

47. Data on advertising expenditures and revenue (in thousands of dollars) for the Four Seasons Restaurant follow.

Advertising Expenditures	Revenue
1	19
2	32
4	44
6	40
10	52
14	53
20	54

a. Let x equal advertising expenditures and y equal revenue. Use the method of least squares to develop a straight line approximation of the relationship between the two variables.
b. Test whether revenue and advertising expenditures are related at a .05 level of significance.

c. Prepare a residual plot of $y - \hat{y}$ versus $\hat{y}$. Use the result from part (a) to obtain the values of $\hat{y}$.
d. What conclusions can you draw from residual analysis? Should this model be used, or should we look for a better one?

48. Refer to exercise 9, where an estimated regression equation relating years of experience and annual sales was developed.
a. Compute the residuals and construct a residual plot for this problem.
b. Do the assumptions about the error terms seem reasonable in light of the residual plot?

49. Recent family home sales in San Antonio provided the following data (San Antonio Realty Watch website, November 2008).

WEB file
HomePrices

Square Footage	Price ($)
1580	142,500
1572	145,000
1352	115,000
2224	155,900
1556	95,000
1435	128,000
1438	100,000
1089	55,000
1941	142,000
1698	115,000
1539	115,000
1364	105,000
1979	155,000
2183	132,000
2096	140,000
1400	85,000
2372	145,000
1752	155,000
1386	80,000
1163	100,000

a. Develop the estimated regression equation that can be used to predict the sales prices given the square footage.
b. Construct a residual plot against the independent variable.
c. Do the assumptions about the error term and model form seem reasonable in light of the residual plot?

Summary

In this chapter we showed how regression analysis can be used to determine how a dependent variable y is related to an independent variable x. In simple linear regression, the regression model is $y = \beta_0 + \beta_1 x + \epsilon$. The simple linear regression equation $E(y) = \beta_0 + \beta_1 x$ describes how the mean or expected value of y is related to x. We used sample data and the least squares method to develop the estimated regression equation $\hat{y} = b_0 + b_1 x$. In effect, b_0 and b_1 are the sample statistics used to estimate the unknown model parameters β_0 and β_1.

The coefficient of determination was presented as a measure of the goodness of fit for the estimated regression equation; it can be interpreted as the proportion of the variation in the dependent variable y that can be explained by the estimated regression equation. We reviewed correlation as a descriptive measure of the strength of a linear relationship between two variables.

The assumptions about the regression model and its associated error term ϵ were discussed, and t and F tests, based on those assumptions, were presented as a means for determining whether the relationship between two variables is statistically significant. We showed how to use the estimated regression equation to develop confidence interval estimates of the mean value of y and prediction interval estimates of individual values of y.

The chapter concluded with a section on the computer solution of regression problems and a section on the use of residual analysis to validate the model assumptions.

Glossary

Dependent variable The variable that is being predicted or explained. It is denoted by y.

Independent variable The variable that is doing the predicting or explaining. It is denoted by x.

Simple linear regression Regression analysis involving one independent variable and one dependent variable in which the relationship between the variables is approximated by a straight line.

Regression model The equation that describes how y is related to x and an error term; in simple linear regression, the regression model is $y = \beta_0 + \beta_1 x + \epsilon$.

Regression equation The equation that describes how the mean or expected value of the dependent variable is related to the independent variable; in simple linear regression, $E(y) = \beta_0 + \beta_1 x$.

Estimated regression equation The estimate of the regression equation developed from sample data by using the least squares method. For simple linear regression, the estimated regression equation is $\hat{y} = b_0 + b_1 x$.

Least squares method A procedure used to develop the estimated regression equation. The objective is to minimize $\Sigma(y_i - \hat{y}_i)^2$.

Scatter diagram A graph of bivariate data in which the independent variable is on the horizontal axis and the dependent variable is on the vertical axis.

Coefficient of determination A measure of the goodness of fit of the estimated regression equation. It can be interpreted as the proportion of the variability in the dependent variable y that is explained by the estimated regression equation.

***i*th residual** The difference between the observed value of the dependent variable and the value predicted using the estimated regression equation; for the ith observation the ith residual is $y_i - \hat{y}_i$.

Correlation coefficient A measure of the strength of the linear relationship between two variables (previously discussed in Chapter 3).

Mean square error The unbiased estimate of the variance of the error term σ^2. It is denoted by MSE or s^2.

Standard error of the estimate The square root of the mean square error, denoted by s. It is the estimate of σ, the standard deviation of the error term ϵ.

ANOVA table The analysis of variance table used to summarize the computations associated with the F test for significance.

Confidence interval The interval estimate of the mean value of y for a given value of x.

Prediction interval The interval estimate of an individual value of y for a given value of x.

Residual analysis The analysis of the residuals used to determine whether the assumptions made about the regression model appear to be valid. Residual analysis is also used to identify outliers and influential observations.

Residual plot Graphical representation of the residuals that can be used to determine whether the assumptions made about the regression model appear to be valid.

Key Formulas

Simple Linear Regression Model

$$y = \beta_0 + \beta_1 x + \epsilon \tag{12.1}$$

Simple Linear Regression Equation

$$E(y) = \beta_0 + \beta_1 x \tag{12.2}$$

Estimated Simple Linear Regression Equation

$$\hat{y} = b_0 + b_1 x \tag{12.3}$$

Least Squares Criterion

$$\min \Sigma(y_i - \hat{y}_i)^2 \tag{12.5}$$

Slope and y-Intercept for the Estimated Regression Equation

$$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} \tag{12.6}$$

$$b_0 = \bar{y} - b_1 \bar{x} \tag{12.7}$$

Sum of Squares Due to Error

$$\text{SSE} = \Sigma(y_i - \hat{y}_i)^2 \tag{12.8}$$

Total Sum of Squares

$$\text{SST} = \Sigma(y_i - \bar{y})^2 \tag{12.9}$$

Sum of Squares Due to Regression

$$\text{SSR} = \Sigma(\hat{y}_i - \bar{y})^2 \tag{12.10}$$

Relationship Among SST, SSR, and SSE

$$\text{SST} = \text{SSR} + \text{SSE} \tag{12.11}$$

Coefficient of Determination

$$r^2 = \frac{\text{SSR}}{\text{SST}} \tag{12.12}$$

Sample Correlation Coefficient

$$\begin{aligned} r_{xy} &= (\text{sign of } b_1)\sqrt{\text{Coefficient of determination}} \\ &= (\text{sign of } b_1)\sqrt{r^2} \end{aligned} \tag{12.13}$$

Mean Square Error (Estimate of σ^2)

$$s^2 = \text{MSE} = \frac{\text{SSE}}{n - 2} \tag{12.15}$$

Standard Error of the Estimate

$$s = \sqrt{\text{MSE}} = \sqrt{\frac{\text{SSE}}{n-2}} \tag{12.16}$$

Standard Deviation of b_1

$$\sigma_{b_1} = \frac{\sigma}{\sqrt{\Sigma(x_i - \bar{x})^2}} \tag{12.17}$$

Estimated Standard Deviation of b_1

$$s_{b_1} = \frac{s}{\sqrt{\Sigma(x_i - \bar{x})^2}} \tag{12.18}$$

***t* Test Statistic**

$$t = \frac{b_1}{s_{b_1}} \tag{12.19}$$

Mean Square Regression

$$\text{MSR} = \frac{\text{SSR}}{\text{Number of independent variables}} \tag{12.20}$$

***F* Test Statistic**

$$F = \frac{\text{MSR}}{\text{MSE}} \tag{12.21}$$

Estimated Standard Deviation of $\hat{y}^*$

$$s_{\hat{y}} = s\sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}} \tag{12.23}$$

Confidence Interval for $E(y^*)$

$$\hat{y}^* \pm t_{\alpha/2} s_{\hat{y}^*} \tag{12.24}$$

Estimated Standard Deviation of an Individual Value

$$s_{\text{pred}} = s\sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}} \tag{12.26}$$

Prediction Interval for y^*

$$\hat{y}^* \pm t_{\alpha/2} s_{\text{pred}} \tag{12.27}$$

Residual for Observation *i*

$$y_i - \hat{y}_i \tag{12.28}$$

Supplementary Exercises

50. The data in the following table show the number of shares selling (millions) and the expected price (average of projected low price and projected high price) for 10 selected initial public stock offerings.

WEB file
IPO

Company	Shares Selling (millions)	Expected Price ($)
American Physician	5.0	15
Apex Silver Mines	9.0	14
Dan River	6.7	15
Franchise Mortgage	8.75	17
Gene Logic	3.0	11
International Home Foods	13.6	19
PRT Group	4.6	13
Rayovac	6.7	14
RealNetworks	3.0	10
Software AG Systems	7.7	13

a. Develop an estimated regression equation with the number of shares selling as the independent variable and the expected price as the dependent variable.
b. At the .05 level of significance, is there a significant relationship between the two variables?
c. Did the estimated regression equation provide a good fit? Explain.
d. Use the estimated regression equation to predict the expected price for a firm considering an initial public offering of 6 million shares.

51. The following data show Morningstar's Fair Value estimate and the Share Price for 28 companies. Fair Value is an estimate of a company's value per share that takes into account estimates of the company's growth, profitability, riskiness, and other factors over the next five years (*Morningstar Stocks 500*, 2008 edition).

WEB file
Stocks500

Company	Fair Value ($)	Share Price ($)
Air Products and Chemicals	80	98.63
Allied Waste Industries	17	11.02
America Mobile	83	61.39
AT&T	35	41.56
Bank of America	70	41.26
Barclays PLC	68	40.37
Citigroup	53	29.44
Costco Wholesale Corp.	75	69.76
Covidien, Ltd.	58	44.29
Darden Restaurants	52	27.71
Dun & Bradstreet	87	88.63
Equifax	42	36.36
Gannett Co.	38	39.00
Genuine Parts	48	46.30
GlaxoSmithKline PLC	57	50.39
Iron Mountain	33	37.02
ITT Corporation	83	66.04
Johnson & Johnson	80	66.70
Las Vegas Sands	98	103.05
Macrovision	23	18.33
Marriott International	39	34.18
Nalco Holding Company	29	24.18
National Interstate	25	33.10
Portugal Telecom	15	13.02
Qualcomm	48	39.35
Royal Dutch Shell Ltd.	87	84.20
SanDisk	60	33.17
Time Warner	42	27.60

a. Develop the estimated regression equation that could be used to estimate the Share Price given the Fair Value.
b. At the .05 level of significance, is there a significant relationship between the two variables?
c. Use the estimated regression equation to predict the Share Price for a company that has a Fair Value of $50.
d. Do you believe the estimated regression equation would provide a good prediction of the share price? Use r^2 to support your answer.

52. One of the biggest changes in higher education in recent years has been the growth of online universities. The Online Education Database is an independent organization whose mission is to build a comprehensive list of the top accredited online colleges. The following table shows the retention rate (%) and the graduation rate (%) for 29 online colleges (Online Education Database website, January 2009).

College	Retention Rate (%)	Graduation Rate (%)
Western International University	7	25
South University	51	25
University of Phoenix	4	28
American InterContinental University	29	32
Franklin University	33	33
Devry University	47	33
Tiffin University	63	34
Post University	45	36
Peirce College	60	36
Everest University	62	36
Upper Iowa University	67	36
Dickinson State University	65	37
Western Governors University	78	37
Kaplan University	75	38
Salem International University	54	39
Ashford University	45	41
ITT Technical Institute	38	44
Berkeley College	51	45
Grand Canyon University	69	46
Nova Southeastern University	60	47
Westwood College	37	48
Everglades University	63	50
Liberty University	73	51
LeTourneau University	78	52
Rasmussen College	48	53
Keiser University	95	55
Herzing College	68	56
National University	100	57
Florida National College	100	61

a. Develop a scatter diagram with retention rate as the independent variable. What does the scatter diagram indicate about the relationship between the two variables?
b. Develop the estimated regression equation.
c. Test for a significant relationship. Use $\alpha = .05$.
d. Did the estimated regression equation provide a good fit?
e. Suppose you were the president of South University. After reviewing the results, would you have any concerns about the performance of your university as compared to other online universities?

f. Suppose you were the president of the University of Phoenix. After reviewing the results, would you have any concerns about the performance of your university as compared to other online universities?

53. Jensen Tire & Auto is in the process of deciding whether to purchase a maintenance contract for its new computer wheel alignment and balancing machine. Managers feel that maintenance expense should be related to usage, and they collected the following information on weekly usage (hours) and annual maintenance expense (in hundreds of dollars).

Jensen

Weekly Usage (hours)	Annual Maintenance Expense
13	17.0
10	22.0
20	30.0
28	37.0
32	47.0
17	30.5
24	32.5
31	39.0
40	51.5
38	40.0

a. Develop the estimated regression equation that relates annual maintenance expense to weekly usage.
b. Test the significance of the relationship in part (a) at a .05 level of significance.
c. Jensen expects to use the new machine 30 hours per week. Develop a 95% prediction interval for the company's annual maintenance expense.
d. If the maintenance contract costs $3000 per year, would you recommend purchasing it? Why or why not?

54. In a manufacturing process the assembly line speed (feet per minute) was thought to affect the number of defective parts found during the inspection process. To test this theory, managers devised a situation in which the same batch of parts was inspected visually at a variety of line speeds. They collected the following data.

Line Speed	Number of Defective Parts Found
20	21
20	19
40	15
30	16
60	14
40	17

a. Develop the estimated regression equation that relates line speed to the number of defective parts found.
b. At a .05 level of significance, determine whether line speed and number of defective parts found are related.
c. Did the estimated regression equation provide a good fit to the data?
d. Develop a 95% confidence interval to predict the mean number of defective parts for a line speed of 50 feet per minute.

55. A sociologist was hired by a large city hospital to investigate the relationship between the number of unauthorized days that employees are absent per year and the distance (miles) between home and work for the employees. A sample of 10 employees was chosen, and the following data were collected.

WEB file
Absent

Distance to Work (miles)	Number of Days Absent
1	8
3	5
4	8
6	7
8	6
10	3
12	5
14	2
14	4
18	2

a. Develop a scatter diagram for these data. Does a linear relationship appear reasonable? Explain.
b. Develop the least squares estimated regression equation.
c. Is there a significant relationship between the two variables? Use $\alpha = .05$.
d. Did the estimated regression equation provide a good fit? Explain.
e. Use the estimated regression equation developed in part (b) to develop a 95% confidence interval for the expected number of days absent for employees living 5 miles from the company.

56. The regional transit authority for a major metropolitan area wants to determine whether there is any relationship between the age of a bus and the annual maintenance cost. A sample of 10 buses resulted in the following data.

WEB file
AgeCost

Age of Bus (years)	Maintenance Cost ($)
1	350
2	370
2	480
2	520
2	590
3	550
4	750
4	800
5	790
5	950

a. Develop the least squares estimated regression equation.
b. Test to see whether the two variables are significantly related with $\alpha = .05$.
c. Did the least squares line provide a good fit to the observed data? Explain.
d. Develop a 95% prediction interval for the maintenance cost for a specific bus that is 4 years old.

57. A marketing professor at Givens College is interested in the relationship between hours spent studying and total points earned in a course. Data collected on 10 students who took the course last quarter follow.

WEB file
HoursPts

Hours Spent Studying	Total Points Earned
45	40
30	35
90	75
60	65
105	90
65	50
90	90
80	80
55	45
75	65

a. Develop an estimated regression equation showing how total points earned is related to hours spent studying.
b. Test the significance of the model with $\alpha = .05$.
c. Predict the total points earned by Mark Sweeney. He spent 95 hours studying.
d. Develop a 95% prediction interval for the total points earned by Mark Sweeney.

58. Reuters reported the market beta for Xerox was 1.22 (Reuters website, January 30, 2009). Market betas for individual stocks are determined by simple linear regression. For each stock, the dependent variable is its quarterly percentage return (capital appreciation plus dividends) minus the percentage return that could be obtained from a risk-free investment (the Treasury bill rate is used as the risk-free rate). The independent variable is the quarterly percentage return (capital appreciation plus dividends) for the stock market (S&P 500) minus the percentage return from a risk-free investment. An estimated regression equation is developed with quarterly data; the market beta for the stock is the slope of the estimated regression equation (b_1). The value of the market beta is often interpreted as a measure of the risk associated with the stock. Market betas greater than 1 indicate that the stock is more volatile than the market average; market betas less than 1 indicate that the stock is less volatile than the market average. Suppose that the following figures are the differences between the percentage return and the risk-free return for 10 quarters for the S&P 500 and Horizon Technology.

WEB file
MktBeta

S&P 500	Horizon
1.2	−0.7
−2.5	−2.0
−3.0	−5.5
2.0	4.7
5.0	1.8
1.2	4.1
3.0	2.6
−1.0	2.0
.5	−1.3
2.5	5.5

a. Develop an estimated regression equation that can be used to determine the market beta for Horizon Technology. What is Horizon Technology's market beta?
b. Test for a significant relationship at the .05 level of significance.
c. Did the estimated regression equation provide a good fit? Explain.
d. Use the market betas of Xerox and Horizon Technology to compare the risk associated with the two stocks.

59. The Transactional Records Access Clearinghouse at Syracuse University reported data showing the odds of an Internal Revenue Service audit. The following table shows the average adjusted gross income reported and the percent of the returns that were audited for 20 selected IRS districts.

WEB file
IRSAudit

District	Adjusted Gross Income ($)	Percent Audited
Los Angeles	36,664	1.3
Sacramento	38,845	1.1
Atlanta	34,886	1.1
Boise	32,512	1.1
Dallas	34,531	1.0
Providence	35,995	1.0
San Jose	37,799	0.9
Cheyenne	33,876	0.9
Fargo	30,513	0.9
New Orleans	30,174	0.9
Oklahoma City	30,060	0.8
Houston	37,153	0.8
Portland	34,918	0.7
Phoenix	33,291	0.7
Augusta	31,504	0.7
Albuquerque	29,199	0.6
Greensboro	33,072	0.6
Columbia	30,859	0.5
Nashville	32,566	0.5
Buffalo	34,296	0.5

a. Develop the estimated regression equation that could be used to predict the percent audited given the average adjusted gross income reported.
b. At the .05 level of significance, determine whether the adjusted gross income and the percent audited are related.
c. Did the estimated regression equation provide a good fit? Explain.
d. Use the estimated regression equation developed in part (a) to calculate a 95% confidence interval for the expected percent audited for districts with an average adjusted gross income of $35,000.

60. The Australian Public Service Commission's State of the Service Report 2002–2003 reported job satisfaction ratings for employees. One of the survey questions asked employees to choose the five most important workplace factors (from a list of factors) that most affected how satisfied they were with their job. Respondents were then asked to indicate their level of satisfaction with their top five factors. The following data show the percentage of employees who nominated the factor in their top five, and a corresponding satisfaction rating measured using the percentage of employees who nominated the factor in the top five and who were "very satisfied" or "satisfied" with the factor in their current workplace (*www.apsc.gov.au/stateoftheservice*).

WEB file
JobSat

Workplace Factor	Top Five (%)	Satisfaction Rating (%)
Appropriate workload	30	49
Chance to be creative/innovative	38	64
Chance to make a useful contribution to society	40	67
Duties/expectations made clear	40	69
Flexible working arrangements	55	86
Good working relationships	60	85
Interesting work provided	48	74

Workplace Factor	Top Five (%)	Satisfaction Rating (%)
Opportunities for career development	33	43
Opportunities to develop my skills	46	66
Opportunities to utilize my skills	50	70
Regular feedback/recognition for effort	42	53
Salary	47	62
Seeing tangible results from my work	42	69

a. Develop a scatter diagram with Top Five (%) on the horizontal axis and Satisfaction Rating (%) on the vertical axis.
b. What does the scatter diagram developed in part (a) indicate about the relationship between the two variables?
c. Develop the estimated regression equation that could be used to predict the Satisfaction Rating (%) given the Top Five (%).
d. Test for a significant relationship at the .05 level of significance.
e. Did the estimated regression equation provide a good fit? Explain.
f. What is the value of the sample correlation coefficient?

Case Problem 1 Measuring Stock Market Risk

One measure of the risk or volatility of an individual stock is the standard deviation of the total return (capital appreciation plus dividends) over several periods of time. Although the standard deviation is easy to compute, it does not take into account the extent to which the price of a given stock varies as a function of a standard market index, such as the S&P 500. As a result, many financial analysts prefer to use another measure of risk referred to as *beta*.

Betas for individual stocks are determined by simple linear regression. The dependent variable is the total return for the stock and the independent variable is the total return for the stock market.* For this case problem we will use the S&P 500 index as the measure of the total return for the stock market, and an estimated regression equation will be developed using monthly data. The beta for the stock is the slope of the estimated regression equation (b_1). The data contained in the file named Beta provides the total return (capital appreciation plus dividends) over 36 months for eight widely traded common stocks and the S&P 500.

WEB file
Beta

The value of beta for the stock market will always be 1; thus, stocks that tend to rise and fall with the stock market will also have a beta close to 1. Betas greater than 1 indicate that the stock is more volatile than the market, and betas less than 1 indicate that the stock is less volatile than the market. For instance, if a stock has a beta of 1.4, it is 40% *more* volatile than the market, and if a stock has a beta of .4, it is 60% *less* volatile than the market.

Managerial Report

You have been assigned to analyze the risk characteristics of these stocks. Prepare a report that includes but is not limited to the following items.

a. Compute descriptive statistics for each stock and the S&P 500. Comment on your results. Which stocks are the most volatile?
b. Compute the value of beta for each stock. Which of these stocks would you expect to perform best in an up market? Which would you expect to hold their value best in a down market?
c. Comment on how much of the return for the individual stocks is explained by the market.

*Various sources use different approaches for computing betas. For instance, some sources subtract the return that could be obtained from a risk-free investment (e.g., T-bills) from the dependent variable and the independent variable before computing the estimated regression equation. Some also use different indexes for the total return of the stock market; for instance, *Value Line* computes betas using the New York Stock Exchange composite index.

Case Problem 2 U.S. Department of Transportation

As part of a study on transportation safety, the U.S. Department of Transportation collected data on the number of fatal accidents per 1000 licenses and the percentage of licensed drivers under the age of 21 in a sample of 42 cities. Data collected over a one-year period follow. These data are contained in the file named Safety.

WEB file
Safety

Percent Under 21	Fatal Accidents per 1000 Licenses	Percent Under 21	Fatal Accidents per 1000 Licenses
13	2.962	17	4.100
12	0.708	8	2.190
8	0.885	16	3.623
12	1.652	15	2.623
11	2.091	9	0.835
17	2.627	8	0.820
18	3.830	14	2.890
8	0.368	8	1.267
13	1.142	15	3.224
8	0.645	10	1.014
9	1.028	10	0.493
16	2.801	14	1.443
12	1.405	18	3.614
9	1.433	10	1.926
10	0.039	14	1.643
9	0.338	16	2.943
11	1.849	12	1.913
12	2.246	15	2.814
14	2.855	13	2.634
14	2.352	9	0.926
11	1.294	17	3.256

Managerial Report

1. Develop numerical and graphical summaries of the data.
2. Use regression analysis to investigate the relationship between the number of fatal accidents and the percentage of drivers under the age of 21. Discuss your findings.
3. What conclusion and recommendations can you derive from your analysis?

Case Problem 3 Alumni Giving

Alumni donations are an important source of revenue for colleges and universities. If administrators could determine the factors that influence increases in the percentage of alumni who make a donation, they might be able to implement policies that could lead to increased revenues. Research shows that students who are more satisfied with their contact with teachers are more likely to graduate. As a result, one might suspect that smaller class sizes and lower student-faculty ratios might lead to a higher percentage of satisfied graduates, which in turn might lead to increases in the percentage of alumni who make a donation. Table 12.8 shows data for 48 national universities (*America's Best Colleges*, Year 2000 ed.). The column labeled % of Classes Under 20 shows the percentage of classes offered with

TABLE 12.8 DATA FOR 48 NATIONAL UNIVERSITIES

WEB file
Alumni

	% of Classes Under 20	Student/Faculty Ratio	Alumni Giving Rate
Boston College	39	13	25
Brandeis University	68	8	33
Brown University	60	8	40
California Institute of Technology	65	3	46
Carnegie Mellon University	67	10	28
Case Western Reserve Univ.	52	8	31
College of William and Mary	45	12	27
Columbia University	69	7	31
Cornell University	72	13	35
Dartmouth College	61	10	53
Duke University	68	8	45
Emory University	65	7	37
Georgetown University	54	10	29
Harvard University	73	8	46
Johns Hopkins University	64	9	27
Lehigh University	55	11	40
Massachusetts Inst. of Technology	65	6	44
New York University	63	13	13
Northwestern University	66	8	30
Pennsylvania State Univ.	32	19	21
Princeton University	68	5	67
Rice University	62	8	40
Stanford University	69	7	34
Tufts University	67	9	29
Tulane University	56	12	17
U. of California–Berkeley	58	17	18
U. of California–Davis	32	19	7
U. of California–Irvine	42	20	9
U. of California–Los Angeles	41	18	13
U. of California–San Diego	48	19	8
U. of California–Santa Barbara	45	20	12
U. of Chicago	65	4	36
U. of Florida	31	23	19
U. of Illinois–Urbana Champaign	29	15	23
U. of Michigan–Ann Arbor	51	15	13
U. of North Carolina–Chapel Hill	40	16	26
U. of Notre Dame	53	13	49
U. of Pennsylvania	65	7	41
U. of Rochester	63	10	23
U. of Southern California	53	13	22
U. of Texas–Austin	39	21	13
U. of Virginia	44	13	28
U. of Washington	37	12	12
U. of Wisconsin–Madison	37	13	13
Vanderbilt University	68	9	31
Wake Forest University	59	11	38
Washington University–St. Louis	73	7	33
Yale University	77	7	50

fewer than 20 students. The column labeled Student/Faculty Ratio is the number of students enrolled divided by the total number of faculty. Finally, the column labeled Alumni Giving Rate is the percentage of alumni that made a donation to the university.

Managerial Report

1. Develop numerical and graphical summaries of the data.
2. Use regression analysis to develop an estimated regression equation that could be used to predict the alumni giving rate given the percentage of classes with fewer than 20 students.
3. Use regression analysis to develop an estimated regression equation that could be used to predict the alumni giving rate given the student-faculty ratio.
4. Which of the two estimated regression equations provides the best fit? For this estimated regression equation, perform an analysis of the residuals and discuss your findings and conclusions.
5. What conclusions and recommendations can you derive from your analysis?

Case Problem 4 PGA Tour Statistics

The Professional Golfers Association (PGA) maintains data on performance and earnings for members of the PGA Tour. The top 125 players based on total earnings in PGA Tour events are exempt for the following season. Making the top 125 money list is important because a player who is "exempt" has qualified to be a full-time member of the PGA tour for the following season.

During recent years on the PGA Tour there have been significant advances in the technology of golf balls and golf clubs, and this technology has been one of the major reasons for the increase in the average driving distance of PGA Tour players. In 1992, the average driving distance was 260 yards, but in 2003 this increased to 286 yards. PGA Tour pros are hitting the ball farther than ever before, but how important is driving distance in terms of a player's performance? And what effect has this increased distance had on the players' accuracy? To investigate these issues, year-end performance data for the 125 players who had the highest total earnings in PGA Tour events for 2008 are contained in the file named PGA-Tour (PGA Tour website, 2009). Each row of the data set corresponds to a PGA Tour player, and the data have been sorted based upon total earnings. Descriptions for the data follow.

Money: Total earnings in PGA Tour events.

Scoring Average: The average number of strokes per completed round.

DrDist (Driving Distance): DrDist is the average number of yards per measured drive. On the PGA Tour, driving distance is measured on two holes per round. Care is taken to select two holes which face in opposite directions to counteract the effect of wind. Drives are measured to the point at which they come to rest regardless of whether they are in the fairway or not.

DrAccu (Driving Accuracy): The percentage of time a tee shot comes to rest in the fairway (regardless of club). Driving accuracy is measured on every hole, excluding par 3's.

GIR (Greens in Regulation): The percentage of time a player was able to hit the green in regulation. A green is considered hit in regulation if any portion of the ball is touching the putting surface after the GIR stroke has been taken. The GIR stroke is determined by subtracting 2 from par (first stroke on a par 3, second on a par 4, third on a par 5). In other words, a green is considered hit in regulation if the player has reached the putting surface in par minus two strokes.

Managerial Report

1. Develop numerical and graphical summaries of the data.
2. Use regression analysis to investigate the relationship between Scoring Average and DrDist. Does it appear that players who drive the ball farther have lower average scores?
3. Use regression analysis to investigate the relationship between Scoring Average and DrAccu. Does it appear that players who are more accurate in hitting the fairway have lower average scores?
4. Use regression analysis to investigate the relationship between Scoring Average and GIR. Does it appear that players who are more accurate in hitting greens in regulation have lower average scores?
5. Which of the three variables (DrDist, DrAccu, and GIR) appears to be the most significant factor in terms of a player's average score?
6. Treating DrDist as the independent variable and DrAccu as the dependent variable, investigate the relationship between driving distance and driving accuracy.

Appendix Regression Analysis Using StatTools

In this appendix we show how StatTools can be used to perform a regression analysis and compute prediction intervals for the Armand's Pizza Parlors problem. We will begin by showing how to use StatTools to perform the regression analysis computations for the Armand's Pizza Parlors problem. We will then illustrate how to use the prediction interval option.

Regression Analysis for the Armand's Pizza Parlors Problem

WEB file Armand's

To perform a regression analysis for the Armand's Pizza Parlors problem we begin by using the Data Set Manager to create a StatTools data set for the Armand's data using the procedure described in the appendix in Chapter 1. The label for the independent variable in the Excel worksheet is "Population" and the label for the dependent variable is "Sales." The default name StatTools uses for the Armand's data set is "Data Set #1." The following steps describe how StatTools can be used to provide the regression results.

Step 1. Click the **StatTools** tab on the Ribbon
Step 2. In the **Analyses** group, click **Regression and Classification**
Step 3. Choose the **Regression** option
Step 4. When the StatTools - Regression dialog box appears,
Select **Multiple** in the **Regression Type** box
In the **Variables** section,
Click the **Format button** and select **Unstacked**
In the column labeled **I**, select **Population** as the independent variable
In the column labeled **D**, select **Sales** as the dependent variable
Click **OK**

Multiple is used for both simple linear and multiple regression.

The regression output will appear.

The StatTools Help facility provides information on using all of the options shown in the StatTools - Regression dialog box.

The StatTools - Regression dialog box contains a number of options that can be used to construct several types of residual plots as well as compute prediction intervals. Let us see how the prediction interval option can be used to compute prediction interval estimates for the Armand's Pizza Parlor problem.

Computing Prediction Intervals for the Armand's Pizza Parlors Problem

In Section 12.7 we showed the results of using StatTools to compute prediction intervals corresponding to three values of the independent variable Population: 10, 14, and 18. Refer to Figure 12.14 as we describe how the StatTools regression procedure can be extended to perform both a regression analysis *and* compute the prediction intervals corresponding to these three values.

In the preceding subsection, we created a StatTools data set for the Armand's data shown in cells A1:C11 and the StatTools name for these data is "Data Set #1." To use the StatTools regression option to compute prediction intervals for values of Population equal to 10, 14, and 18, we must also create a new StatTools data set that contains the label "Population" and the values 10, 14, and 18. As in Figure 12.14, we enter the label "Population" into cell E1 and the values 10, 14, and 18 into cells E1:E4. Using the StatTools Data Set Manager, we now create a new StatTools data set corresponding to the values in cells E1:E4. The default name StatTools uses for this new data set is "Data Set #2."

The following steps describe how StatTools can be used to provide the regression results (as shown in the previous subsection) *and* compute the prediction intervals corresponding to three the three values (10, 14, and 18) of the independent variable.

Step 1. Click the **StatTools** tab on the Ribbon
Step 2. In the **Analyses** group, click **Regression and Classification**
Step 3. Choose the **Regression** option
Step 4. When the StatTools - Regression dialog box appears,
Select **Multiple** in the **Regression Type** box
In the **Variables** section,
Click the **Format** button and select **Unstacked**
In the column labeled **I**, select **Population**
In the column labeled **D**, select **Sales**
In the **Advanced Options** section,
Select **Include Prediction for Data Set** and choose **Data Set #2** in the corresponding box
Choose **95%** in the **Confidence Level** box
Click **OK**

After clicking OK in step 4, StatTools will display a series of three dialog boxes, each of which contains a question involving the prediction interval output; click Yes in response to each of these questions.

> Matching variable not found in prediction data set for dependent variable Sales. Do you want to insert new variable in data set?
>
> Matching variable not found in prediction data set for lower limit variable. Do you want to insert new variable in data set?
>
> Matching variable not found in prediction data set for upper limit variable. Do you want to insert new variable in data set?

The StatTools prediction interval output is shown in cells F1:H8 of the worksheet shown in Figure 12.14. In addition, the standard regression output for the Armand's Pizza Parlors problem is provided in a new worksheet.

CHAPTER 13

Multiple Regression

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- Note on Interpretation of Coefficients

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STATISTICS in PRACTICE

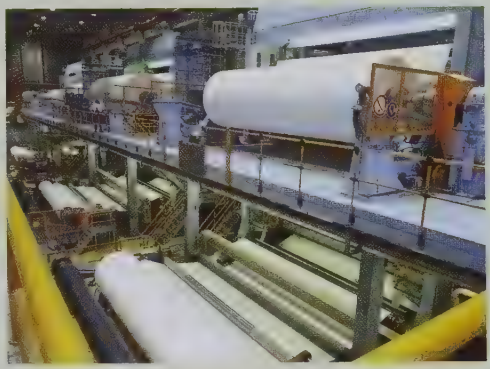
INTERNATIONAL PAPER*

PURCHASE, NEW YORK

International Paper is the world's largest paper and forest products company. The company employs more than 117,000 people in its operations in nearly 50 countries, and exports its products to more than 130 nations. International Paper produces building materials such as lumber and plywood; consumer packaging materials such as disposable cups and containers; industrial packaging materials such as corrugated boxes and shipping containers; and a variety of papers for use in photocopiers, printers, books, and advertising materials.

To make paper products, pulp mills process wood chips and chemicals to produce wood pulp. The wood pulp is then used at a paper mill to produce paper products. In the production of white paper products, the pulp must be bleached to remove any discoloration. A key bleaching agent used in the process is chlorine dioxide, which, because of its combustible nature, is usually produced at a pulp mill facility and then piped in solution form into the bleaching tower of the pulp mill. To improve one of the processes used to produce chlorine dioxide, researchers studied the process's control and efficiency. One aspect of the study looked at the chemical feed rate for chlorine dioxide production.

To produce the chlorine dioxide, four chemicals flow at metered rates into the chlorine dioxide generator. The chlorine dioxide produced in the generator flows to an absorber where chilled water absorbs the chlorine dioxide gas to form a chlorine dioxide solution. The solution is then piped into the paper mill. A key part of controlling the process involves the chemical feed rates. Historically, experienced operators set the chemical feed rates, but this approach led to overcontrol by the operators. Consequently, chemical engineers at the mill requested that a set of control equations, one for each chemical feed, be developed to aid the operators in setting the rates.



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Multiple regression analysis assisted in the development of a better bleaching process for making white paper products.

Using multiple regression analysis, statistical analysts developed an estimated multiple regression equation for each of the four chemicals used in the process. Each equation related the production of chlorine dioxide to the amount of chemical used and the concentration level of the chlorine dioxide solution. The resulting set of four equations was programmed into a microcomputer at each mill. In the new system, operators enter the concentration of the chlorine dioxide solution and the desired production rate; the computer software then calculates the chemical feed needed to achieve the desired production rate. After the operators began using the control equations, the chlorine dioxide generator efficiency increased, and the number of times the concentrations fell within acceptable ranges increased significantly.

This example shows how multiple regression analysis can be used to develop a better bleaching process for producing white paper products. In this chapter we will show how Excel can be used for such purposes. Most of the concepts introduced in Chapter 12 for simple linear regression can be directly extended to the multiple regression case.

*The authors are indebted to Marian Williams and Bill Griggs for providing this Statistics in Practice. This application was originally developed at Champion International Corporation, which became part of International Paper in 2000.

In Chapter 12 we presented simple linear regression and demonstrated its use in developing an estimated regression equation that describes the relationship between two variables. Recall that the variable being predicted or explained is called the dependent variable and the variable being used to predict or explain the dependent variable is called the independent variable. In this chapter we continue our study of regression analysis by considering situations involving two or more independent variables. This subject area, called **multiple regression analysis**, enables us to consider more factors and thus obtain better estimates than are possible with simple linear regression.

13.1 Multiple Regression Model

Multiple regression analysis is the study of how a dependent variable y is related to two or more independent variables. In the general case, we will use p to denote the number of independent variables.

Regression Model and Regression Equation

The concepts of a regression model and a regression equation introduced in the preceding chapter are applicable in the multiple regression case. The equation that describes how the dependent variable y is related to the independent variables $x_1, x_2, \ldots, x_p$ and an error term is called the **multiple regression model**. We begin with the assumption that the multiple regression model takes the following form.

MULTIPLE REGRESSION MODEL

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p + \epsilon \qquad (13.1)$$

In the multiple regression model, $\beta_0, \beta_1, \beta_2, \ldots, \beta_p$ are the parameters and the error term ϵ (the Greek letter epsilon) is a random variable. A close examination of this model reveals that y is a linear function of $x_1, x_2, \ldots, x_p$ (the $\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p$ part) plus the error term ϵ. The error term accounts for the variability in y that cannot be explained by the linear effect of the p independent variables.

In Section 13.4 we will discuss the assumptions for the multiple regression model and ϵ. One of the assumptions is that the mean or expected value of ϵ is zero. A consequence of this assumption is that the mean or expected value of y, denoted $E(y)$, is equal to $\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p$. The equation that describes how the mean value of y is related to $x_1, x_2, \ldots, x_p$ is called the **multiple regression equation**.

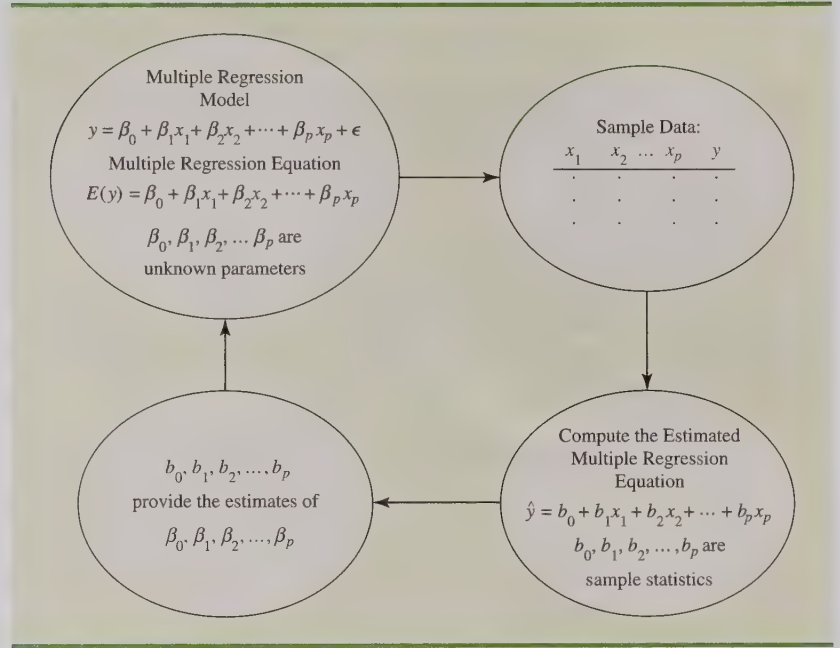
MULTIPLE REGRESSION EQUATION

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p \qquad (13.2)$$

Estimated Multiple Regression Equation

If the values of $\beta_0, \beta_1, \beta_2, \ldots, \beta_p$ were known, equation (13.2) could be used to compute the mean value of y at given values of $x_1, x_2, \ldots, x_p$. Unfortunately, these parameter values will not, in general, be known and must be estimated from sample data. A simple random sample is used to compute sample statistics $b_0, b_1, b_2, \ldots, b_p$ that are used as the point

FIGURE 13.1 THE ESTIMATION PROCESS FOR MULTIPLE REGRESSION


Multiple Regression Model
$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p + \epsilon$
Multiple Regression Equation
$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p$
$\beta_0, \beta_1, \beta_2, \ldots \beta_p$ are unknown parameters
Sample Data:
x_1 x_2 ... x_p y
Compute the Estimated Multiple Regression Equation
$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \cdots + b_p x_p$
$b_0, b_1, b_2, \ldots, b_p$ are sample statistics
$b_0, b_1, b_2, \ldots, b_p$ provide the estimates of $\beta_0, \beta_1, \beta_2, \ldots, \beta_p$

In simple linear regression, b_0 and b_1 were the sample statistics used to estimate the parameters β_0 and β_1. Multiple regression parallels this statistical inference process, with b_0, b_1, b_2, . . . , b_p denoting the sample statistics used to estimate the parameters β_0, β_1, β_2, . . . , β_p.

estimators of the parameters $\beta_0, \beta_1, \beta_2, \ldots, \beta_p$. These sample statistics provide the following **estimated multiple regression equation**.

ESTIMATED MULTIPLE REGRESSION EQUATION

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \cdots + b_p x_p \qquad (13.3)$$

where

$b_0, b_1, b_2, \ldots, b_p$ are the estimates of $\beta_0, \beta_1, \beta_2, \ldots, \beta_p$
$\hat{y}$ = predicted value of the dependent variable

The estimation process for multiple regression is shown in Figure 13.1.

13.2 Least Squares Method

In Chapter 12 we used the **least squares method** to develop the estimated regression equation that best approximated the straight-line relationship between the dependent and independent variables. This same approach is used to develop the estimated multiple regression equation. The least squares criterion is restated as follows.

LEAST SQUARES CRITERION

$$\min \Sigma(y_i - \hat{y}_i)^2 \qquad (13.4)$$

where

y_i = observed value of the dependent variable for the ith observation
$\hat{y}_i$ = predicted value of the dependent variable for the ith observation

The predicted values of the dependent variable are computed by using the estimated multiple regression equation,

$$\hat{y} = b_0 + b_1x_1 + b_2x_2 + \cdots + b_px_p$$

As expression (13.4) shows, the least squares method uses sample data to provide the values of $b_0, b_1, b_2, \ldots, b_p$ that make the sum of squared residuals [the deviations between the observed values of the dependent variable (y_i) and the predicted values of the dependent variable ($\hat{y}_i$)] a minimum.

In Chapter 12 we presented formulas for computing the least squares estimators b_0 and b_1 for the estimated simple linear regression equation $\hat{y} = b_0 + b_1x$. With relatively small data sets, we were able to use those formulas to compute b_0 and b_1 by manual calculations. In multiple regression, however, the presentation of the formulas for the regression coefficients $b_0, b_1, b_2, \ldots, b_p$ involves the use of matrix algebra and is beyond the scope of this text. Therefore, in presenting multiple regression, we focus on how Excel's Regression tool can be used to obtain the estimated regression equation and other information. The emphasis will be on how to interpret the Excel output rather than on how to make the multiple regression computations.

An Example: Butler Trucking Company

As an illustration of multiple regression analysis, we will consider a problem faced by the Butler Trucking Company, an independent trucking company in southern California. A major portion of Butler's business involves deliveries throughout its local area. To develop better work schedules, the managers want to estimate the total daily travel time for their drivers.

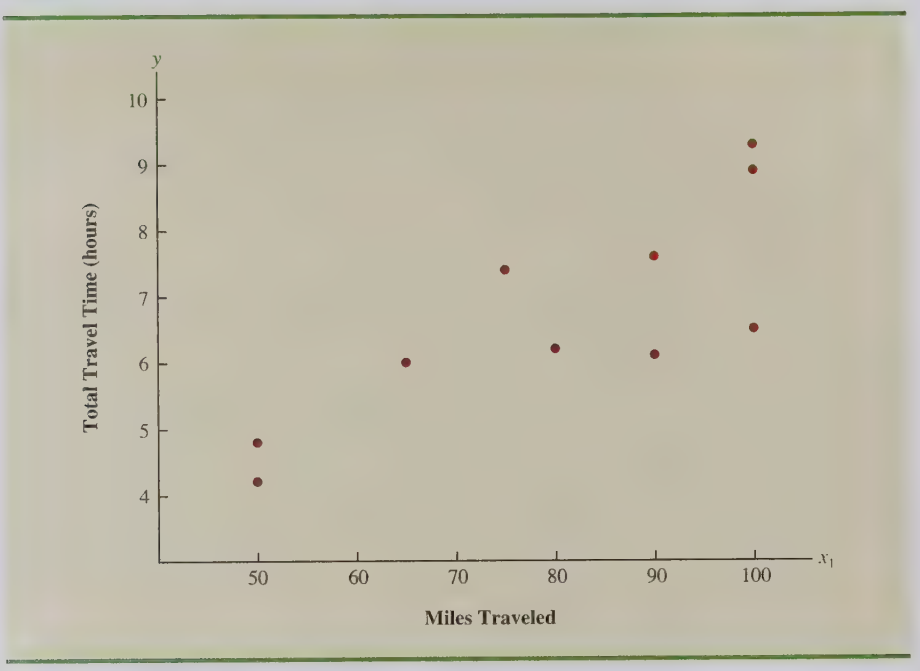
Initially the managers believed that the total daily travel time would be closely related to the number of miles traveled in making the daily deliveries. A simple random sample of 10 driving assignments provided the data shown in Table 13.1 and the scatter diagram shown in Figure 13.2. After reviewing this scatter diagram, the managers hypothesized that

WEB file
Butler

TABLE 13.1 PRELIMINARY DATA FOR BUTLER TRUCKING

Driving Assignment	x_1 = Miles Traveled	y = Travel Time (hours)
1	100	9.3
2	50	4.8
3	100	8.9
4	100	6.5
5	50	4.2
6	80	6.2
7	75	7.4
8	65	6.0
9	90	7.6
10	90	6.1

FIGURE 13.2 SCATTER DIAGRAM OF PRELIMINARY DATA FOR BUTLER TRUCKING



the simple linear regression model $y = \beta_0 + \beta_1 x_1 + \epsilon$ could be used to describe the relationship between the total travel time (y) and the number of miles traveled (x_1). To estimate the parameters β_0 and β_1, the least squares method was used to develop the estimated regression equation.

$$\hat{y} = b_0 + b_1 x_1 \tag{13.5}$$

In Figure 13.3, we show the Excel Regression tool output[1] from applying simple linear regression to the data in Table 13.1. The estimated regression equation is

$$\hat{y} = 1.2739 + .0678x_1$$

At the .05 level of significance, the F value of 15.8146 and its corresponding p-value of .0041 indicate that the relationship is significant; that is, we can reject H_0: $\beta_1 = 0$ because the p-value is less than $\alpha = .05$. Note that the same conclusion is obtained from the t value of 3.9768 and its associated p-value of .0041. Thus, we can conclude that the relationship between the total travel time and the number of miles traveled is significant; longer travel times are associated with more miles traveled. With a coefficient of determination of R Square = .6641, we see that 66.41% of the variability in travel time can be explained by the linear effect of the number of miles traveled. This finding is fairly good, but the managers might want to consider adding a second independent variable to explain some of the remaining variability in the dependent variable.

[1]Excel's Regression tool was used to obtain the output. Section 12.7 describes how to use Excel's Regression tool for simple linear regression.

FIGURE 13.3 REGRESSION TOOL OUTPUT FOR BUTLER TRUCKING WITH ONE INDEPENDENT VARIABLE

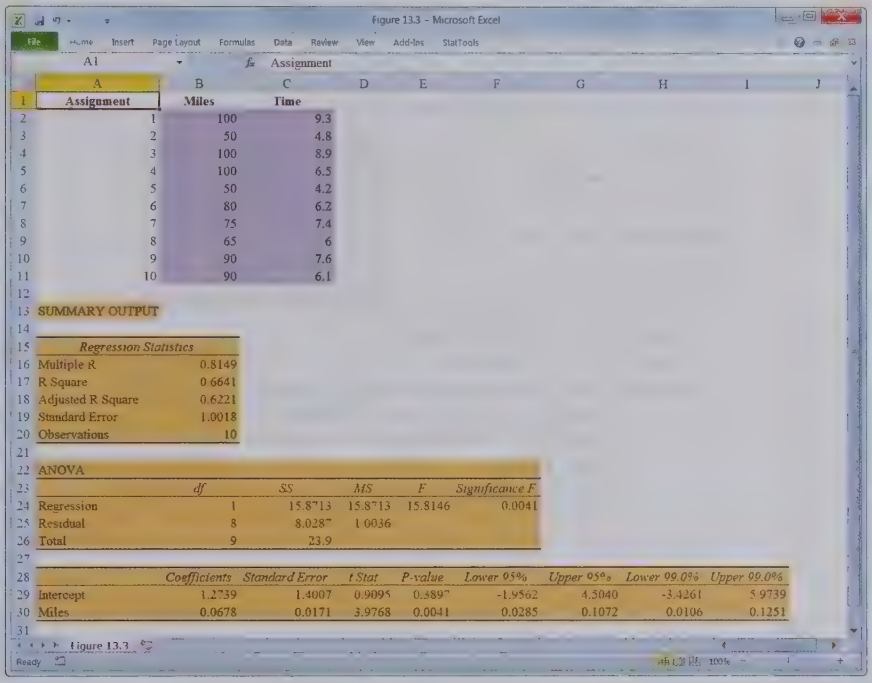


Figure 13.3 – Microsoft Excel

	A	B	C	D	E	F	G	H	I	J
1	Assignment	Miles	Time							
2	1	100	9.3							
3	2	50	4.8							
4	3	100	8.9							
5	4	100	6.5							
6	5	50	4.2							
7	6	80	6.2							
8	7	75	7.4							
9	8	65	6							
10	9	90	7.6							
11	10	90	6.1							
12										
13	SUMMARY OUTPUT									
14										
15	*Regression Statistics*									
16	Multiple R	0.8149								
17	R Square	0.6641								
18	Adjusted R Square	0.6221								
19	Standard Error	1.0018								
20	Observations	10								
21										
22	ANOVA									
23		*df*	*SS*	*MS*	*F*	*Significance F*				
24	Regression	1	15.8713	15.8713	15.8146	0.0041				
25	Residual	8	8.0287	1.0036						
26	Total	9	23.9							
27										
28		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 99.0%*	*Upper 99.0%*	
29	Intercept	1.2739	1.4007	0.9095	0.3897	-1.9562	4.5040	-3.4261	5.9739	
30	Miles	0.0678	0.0171	3.9768	0.0041	0.0285	0.1072	0.0106	0.1251	
31										

Figure 13.3

Ready

In attempting to identify another independent variable, the managers felt that the number of deliveries could also contribute to the total travel time. The Butler Trucking data, with the number of deliveries added, are shown in Table 13.2. To develop the estimated multiple regression equation with both miles traveled (x_1) and number of deliveries (x_2) as independent variables, we will use Excel's Regression tool.

TABLE 13.2 DATA FOR BUTLER TRUCKING WITH MILES TRAVELED (x_1) AND NUMBER OF DELIVERIES (x_2) AS THE INDEPENDENT VARIABLES

WEB file Butler

Driving Assignment	x_1 = Miles Traveled	x_2 = Number of Deliveries	y = Travel Time (hours)
1	100	4	9.3
2	50	3	4.8
3	100	4	8.9
4	100	2	6.5
5	50	2	4.2
6	80	2	6.2
7	75	3	7.4
8	65	4	6.0
9	90	3	7.6
10	90	2	6.1

Using Excel's Regression Tool to Develop the Estimated Multiple Regression Equation

In Section 12.7 we showed how Excel's Regression tool could be used to determine the estimated regression equation for Armand's Pizza Parlors. We can use the same procedure with minor modifications to develop the estimated multiple regression equation for Butler Trucking. Refer to Figures 13.4 and 13.5 as we describe the tasks involved.

Enter Data: The labels Assignment, Miles, Deliveries, and Time are entered into cells A1:D1 of the worksheet, and the sample data are entered into cells B2:D11. The numbers 1–10 are entered into cells A2:A11 to identify each observation.

Apply Tools: The following steps describe how to use Excel's Regression tool for multiple regression analysis.

Step 1. Select the **Tools** menu
Step 2. Choose the **Data Analysis** option
Step 3. Choose **Regression** from the list of Analysis Tools
Step 4. When the Regression dialog box appears (see Figure 13.4),
Enter D1:D11 in the **Input Y Range** box
Enter B1:C11 in the **Input X Range** box
Select **Labels**
Select **Confidence Level**

FIGURE 13.4 REGRESSION TOOL DIALOG BOX FOR THE BUTLER TRUCKING EXAMPLE

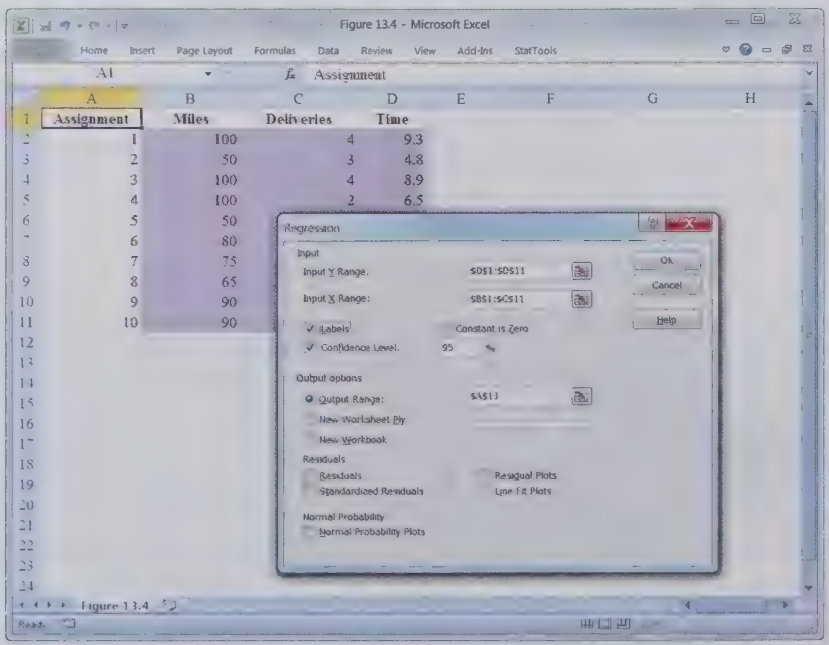


FIGURE 13.5 REGRESSION TOOL OUTPUT FOR BUTLER TRUCKING WITH TWO INDEPENDENT VARIABLES

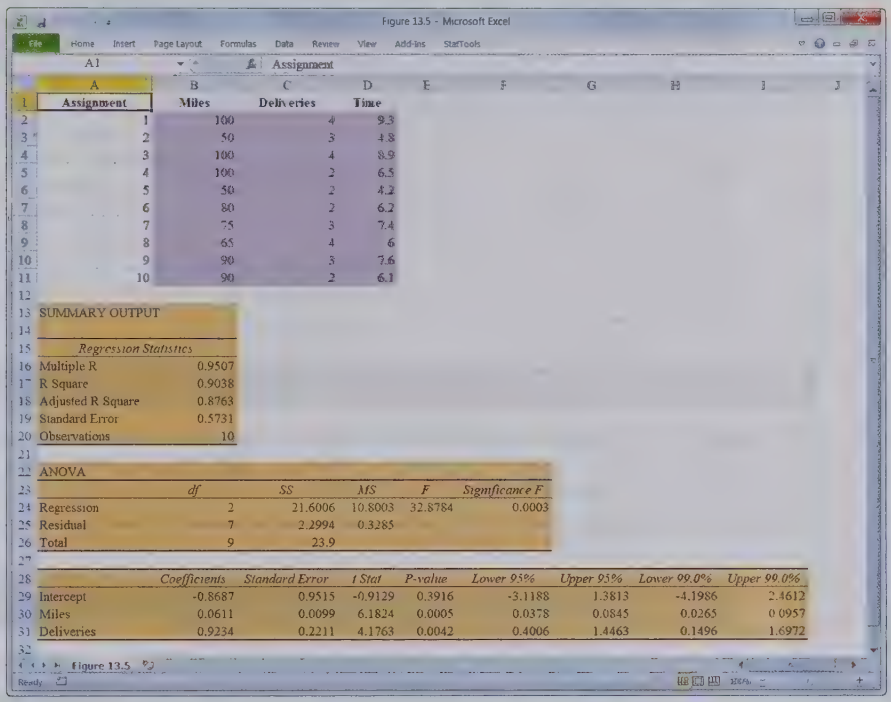


Figure 13.5 - Microsoft Excel

Assignment	Miles	Deliveries	Time
1	100	4	9.3
2	50	3	4.8
3	100	4	8.9
4	100	2	6.5
5	50	2	4.2
6	80	2	6.2
7	75	3	7.4
8	65	4	6
9	90	3	7.6
10	90	2	6.1

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.9507
R Square	0.9038
Adjusted R Square	0.8763
Standard Error	0.5731
Observations	10

ANOVA

	df	SS	MS	F	Significance F
Regression	2	21.6006	10.8003	32.8784	0.0003
Residual	7	2.2994	0.3285		
Total	9	23.9			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 99.0%	Upper 99.0%
Intercept	-0.8687	0.9515	-0.9129	0.3916	-3.1188	1.3813	-4.1986	2.4612
Miles	0.0611	0.0099	6.1824	0.0005	0.0378	0.0845	0.0265	0.0957
Deliveries	0.9234	0.2211	4.1763	0.0042	0.4006	1.4463	0.1496	1.6972

Figure 13.5

Enter 99 in the **Confidence Level** box
Select **Output Range**
Enter A13 in the **Output Range** box (to identify the upper left corner of the section of the worksheet where the output will appear)
Click **OK**

In the Excel output shown in Figure 13.5 the label for the independent variable x_1 is Miles (see cell A30), and the label for the independent variable x_2 is Deliveries (see cell A31). The estimated regression equation is

$$\hat{y} = -.8687 + .0611x_1 + .9234x_2 \qquad (13.6)$$

Note that using Excel's Regression tool for multiple regression is almost the same as using it for simple linear regression. The major difference is that in the multiple regression case a larger range of cells has to be provided in order to identify the independent variables.

In the next section we will discuss the use of the coefficient of multiple determination in measuring how good a fit is provided by this estimated regression equation. Before doing so, let us examine more carefully the values of $b_1 = .0611$ and $b_2 = .9234$ in equation (13.6).

Note on Interpretation of Coefficients

One observation can be made at this point about the relationship between the estimated regression equation with only the miles traveled as an independent variable and the equation that includes the number of deliveries as a second independent variable. The value of b_1 is not the same in both cases. In simple linear regression, we interpret b_1 as an estimate of the change in y for a one-unit change in the independent variable. In multiple regression analysis, this interpretation must be modified somewhat. That is, in multiple regression analysis, we interpret each regression coefficient as follows: b_i represents an estimate of the change in y corresponding to a one-unit change in x_i when all other independent variables are held constant. In the Butler Trucking example involving two independent variables, $b_1 = .0611$. Thus, .0611 hours is an estimate of the expected increase in travel time corresponding to an increase of one mile in the distance traveled when the number of deliveries is held constant. Similarly, because $b_2 = .9234$, an estimate of the expected increase in travel time corresponding to an increase of one delivery when the number of miles traveled is held constant is .9234 hours.

NOTES AND COMMENTS

In the appendix to this chapter we show how to use StatTools to perform multiple regression analysis for the Butler Trucking data. The regression analysis capabilities of StatTools are more comprehensive than those available using Excel's Regression tool.

Exercises

Note to student: The exercises involving data in this and subsequent sections were designed to be solved using Excel.

Methods

1. The estimated regression equation for a model involving two independent variables and 10 observations follows.

$$\hat{y} = 29.1270 + .5906x_1 + .4980x_2$$

   a. Interpret b_1 and b_2 in this estimated regression equation.
   b. Predict y when $x_1 = 180$ and $x_2 = 310$.

SELF test

2. Consider the following data for a dependent variable y and two independent variables, x_1 and x_2.

WEB file
Exer2

x_1	x_2	y
30	12	94
47	10	108
25	17	112
51	16	178
40	5	94
51	19	175
74	7	170
36	12	117
59	13	142
76	16	211

a. Develop an estimated regression equation relating y to x_1. Estimate y if $x_1 = 45$.
b. Develop an estimated regression equation relating y to x_2. Estimate y if $x_2 = 15$.
c. Develop an estimated regression equation relating y to x_1 and x_2. Estimate y if $x_1 = 45$ and $x_2 = 15$.

3. In a regression analysis involving 30 observations, the following estimated regression equation was obtained.

$$\hat{y} = 17.6 + 3.8x_1 - 2.3x_2 + 7.6x_3 + 2.7x_4$$

a. Interpret b_1, b_2, b_3, and b_4 in this estimated regression equation.
b. Predict y when $x_1 = 10$, $x_2 = 5$, $x_3 = 1$, and $x_4 = 2$.

Applications

4. A shoe store developed the following estimated regression equation relating sales to inventory investment and advertising expenditures.

$$\hat{y} = 25 + 10x_1 + 8x_2$$

where

x_1 = inventory investment ($1000s)
x_2 = advertising expenditures ($1000s)
y = sales ($1000s)

a. Predict the sales resulting from a $15,000 investment in inventory and an advertising budget of $10,000.
b. Interpret b_1 and b_2 in this estimated regression equation.

SELF test

5. The owner of Showtime Movie Theaters, Inc., would like to estimate weekly gross revenue as a function of advertising expenditures. Historical data for a sample of eight weeks follow.

WEB file
Showtime

Weekly Gross Revenue ($1000s)	Television Advertising ($1000s)	Newspaper Advertising ($1000s)
96	5.0	1.5
90	2.0	2.0
95	4.0	1.5
92	2.5	2.5
95	3.0	3.3
94	3.5	2.3
94	2.5	4.2
94	3.0	2.5

a. Develop an estimated regression equation with the amount of television advertising as the independent variable.
b. Develop an estimated regression equation with both television advertising and newspaper advertising as the independent variables.
c. Is the estimated regression equation coefficient for television advertising expenditures the same in part (a) and in part (b)? Interpret the coefficient in each case.

d. Predict the weekly gross revenue for a week when $3500 is spent on television advertising and $1800 is spent on newspaper advertising.

6. Out-of-state tuition and fees at the top graduate schools of business can be very expensive. However, the starting salary and bonus paid to graduates from many of these schools can be very high. The following data show the recruiter assessment score (highest = 5), the out-of-state tuition and fees (rounded to the nearest $1000), and the average starting salary and bonus paid to recent graduates (rounded to the nearest $1000) for a sample of 20 graduate schools of business (*U.S. News & World Report 2009 Edition America's Best Graduate Schools*).

BusinessSchools2

School	Recruiter Score	Tuition & Fees ($)	Salary & Bonus ($)
Arizona State University	3.4	28	98
Babson College	3.5	35	94
Cornell University	3.8	44	119
Georgetown University	3.7	40	109
Georgia Institute of Technology	3.6	30	88
Indiana University–Bloomington	3.7	35	105
Michigan State University	3.2	26	99
Northwestern University	4.3	44	123
Ohio State University	3.1	35	97
Purdue University–West Lafayette	3.6	33	96
Rice University	3.4	36	102
Stanford University	4.4	46	135
University of California–Davis	3.3	35	89
University of Florida	3.5	23	71
University of Iowa	3.1	25	78
University of Minnesota–Twin Cities	3.5	37	100
University of Notre Dame	3.4	36	95
University of Rochester	3.5	38	99
University of Washington	3.4	30	94
University of Wisconsin–Madison	3.3	27	93

a. Develop an estimated regression equation that can be used to predict the starting salary and bonus paid to graduates given the recruiter assessment score.
b. Develop an estimated regression equation that can be used to predict the starting salary and bonus paid to graduates given both the recruiter assessment score and the out-of-state tuition and fees.
c. Is the estimated regression coefficient for the recruiter score the same in part (a) and in part (b)? Interpret the coefficient in each case.
d. Suppose that we randomly select a recent graduate of the University of Virginia graduate school of business. This school has a recruiter assessment score of 4.1 and an out-of-state tuition and fees of $43,000. Predict the starting salary and bonus for this graduate.

7. *PC World* rated four component characteristics for 10 ultraportable laptop computers: features; performance; design; and price. Each characteristic was rated using a 0–100 point scale. An overall rating, referred to as the *PC World* Rating, was then developed for each laptop. The following table shows the performance rating, features rating, and the *PC World* Rating for the 10 laptop computers (PC World website, February 5, 2009).

WEB file Laptop

Model	Performance	Features	*PC* Rating
Thinkpad X200	77	87	83
VGN-Z598U	97	85	82
U6V	83	80	81
Elitebook 2530P	77	75	78
X360	64	80	78
Thinkpad X300	56	76	78
Ideapad U110	55	81	77
Micro Express JFT2500	76	73	75
Toughbook W7	46	79	73
HP Voodoo Envy133	54	68	72

a. Determine the estimated regression equation that can be used to predict the *PC World* Rating using the performance rating as the independent variable.
b. Determine the estimated regression equation that can be used to predict the *PC World* Rating using both the performance rating and the features rating.
c. Predict the *PC World* Rating for a laptop computer that has a performance rating of 80 and a features rating of 70.

8. Would you expect more reliable and better performing cars to cost more? *Consumer Reports* provided reliability ratings, overall road-test scores, and prices for affordable family sedans, midpriced family sedans, and large sedans (*Consumer Reports*, February 2008). A portion of the data follows. Reliability was rated on a 5-point scale from poor (1) to excellent (5). The road-test score was rated on a 100-point scale, with higher values indicating better performance. The complete data set is contained in the file named Sedans.

WEB file Sedans

Make and Model	Road-Test Score	Reliability	Price ($)
Nissan Altima 2.5 S	85	4	22,705
Honda Accord LX-P	79	4	22,795
Kia Optima EX (4-cyl.)	78	4	22,795
Toyota Camry LE	77	4	21,080
Hyundai Sonata SE	76	3	22,995
.	.	.	.
.	.	.	.
.	.	.	.
Chrysler 300 Touring	60	2	30,255
Dodge Charger SXT	58	4	28,860

a. Develop an estimated regression equation that can be used to predict the price of the car given the reliability rating. Test for significance using $\alpha = .05$.
b. Consider the addition of the independent variable overall road-test score. Develop the estimated regression equation that can be used to predict the price of the car given the road-test score and the reliability rating.
c. Predict the price for a car with a road-test score of 80 and a reliability rating of 4.

9. Waterskiing and wakeboarding are two popular water-sports. Finding a model that best suits your intended needs, whether it is waterskiing, wakeboading, or general boating, can be a difficult task. *WaterSki* magazine did extensive testing for 88 boats and provided a wide variety of information to help consumers select the best boat. A portion of the data they reported for 20 boats with a length of between 20 and 22 feet follows (*WaterSki*, January/February 2006). Beam is the maximum width of the boat in inches, HP is the horsepower of the boat's engine, and TopSpeed is the top speed in miles per hour (mph).

Make and Model	Beam	HP	TopSpeed
Calabria Cal Air Pro V-2	100	330	45.3
Correct Craft Air Nautique 210	91	330	47.3
Correct Craft Air Nautique SV-211	93	375	46.9
Correct Craft Ski Nautique 206 Limited	91	330	46.7
Gekko GTR 22	96	375	50.1
Gekko GTS 20	83	375	52.2
Malibu Response LXi	93.5	340	47.2
Malibu Sunsetter LXi	98	400	46
Malibu Sunsetter 21 XTi	98	340	44
Malibu Sunscape 21 LSV	98	400	47.5
Malibu Wakesetter 21 XTi	98	340	44.9
Malibu Wakesetter VLX	98	400	47.3
Malibu vRide	93.5	340	44.5
Malibu Ride XTi	93.5	320	44.5
Mastercraft ProStar 209	96	350	42.5
Mastercraft X-1	90	310	45.8
Mastercraft X-2	94	310	42.8
Mastercraft X-9	96	350	43.2
MB Sports 190 Plus	92	330	45.3
Svfara SVONE	91	330	47.7

WEB file Boats

a. Using these data, develop an estimated regression equation relating the top speed with the boat's beam and horsepower rating.

b. The Svfara SV609 has a beam of 85 inches and an engine with a 330 horsepower rating. Use the estimated regression equation developed in part (a) to estimate the mean top speed for the Svfara SV609.

10. The National Basketball Association (NBA) records a variety of statistics for each team. Four of these statistics are the proportion of games won (PCT), the proportion of field goals made by the team (FG%), the proportion of three-point shots made by the team's opponent (Opp 3 Pt%), and the number of turnovers committed by the team's opponent (Opp TO). The following data show the values of these statistics for the 29 teams in the NBA for a portion of the 2004 season (NBA website, January 3, 2004).

Team	PCT	FG%	Opp 3 Pt%	Opp TO	Team	PCT	FG%	Opp 3 Pt%	Opp TO
Atlanta	0.265	0.435	0.346	13.206	Minnesota	0.677	0.473	0.348	13.839
Boston	0.471	0.449	0.369	16.176	New Jersey	0.563	0.435	0.338	17.063
Chicago	0.313	0.417	0.372	15.031	New Orleans	0.636	0.421	0.330	16.909
Cleveland	0.303	0.438	0.345	12.515	New York	0.412	0.442	0.330	13.588
Dallas	0.581	0.439	0.332	15.000	Orlando	0.242	0.417	0.360	14.242
Denver	0.606	0.431	0.366	17.818	Philadelphia	0.438	0.428	0.364	16.938
Detroit	0.606	0.423	0.262	15.788	Phoenix	0.364	0.438	0.326	16.515
Golden State	0.452	0.445	0.384	14.290	Portland	0.484	0.447	0.367	12.548
Houston	0.548	0.426	0.324	13.161	Sacramento	0.724	0.466	0.327	15.207
Indiana	0.706	0.428	0.317	15.647	San Antonio	0.688	0.429	0.293	15.344
L.A. Clippers	0.464	0.424	0.326	14.357	Seattle	0.533	0.436	0.350	16.767
L.A. Lakers	0.724	0.465	0.323	16.000	Toronto	0.516	0.424	0.314	14.129
Memphis	0.485	0.432	0.358	17.848	Utah	0.531	0.456	0.368	15.469
Miami	0.424	0.410	0.369	14.970	Washington	0.300	0.411	0.341	16.133
Milwaukee	0.500	0.438	0.349	14.750					

WEB file NBA

a. Determine the estimated regression equation that can be used to predict the proportion of games won given the proportion of field goals made by the team.
b. Provide an interpretation for the slope of the estimated regression equation developed in part (a).
c. Determine the estimated regression equation that can be used to predict the proportion of games won given the proportion of field goals made by the team, the proportion of three-point shots made by the team's opponent, and the number of turnovers committed by the team's opponent.
d. Discuss the practical implications of the estimated regression equation developed in part (c).
e. Predict the proportion of games won for a team with the following values for the three independent variables: FG% = .45, Opp 3 Pt% = .34, and Opp TO = 17.

13.3 Multiple Coefficient of Determination

In simple linear regression we showed that the total sum of squares can be partitioned into two components: the sum of squares due to regression and the sum of squares due to error. The same procedure applies to the sum of squares in multiple regression.

RELATIONSHIP AMONG SST, SSR, AND SSE

$$\text{SST} = \text{SSR} + \text{SSE} \tag{13.7}$$

where

$$\text{SST} = \text{total sum of squares} = \Sigma(y_i - \bar{y})^2$$
$$\text{SSR} = \text{sum of squares due to regression} = \Sigma(\hat{y}_i - \bar{y})^2$$
$$\text{SSE} = \text{sum of squares due to error} = \Sigma(y_i - \hat{y}_i)^2$$

Because of the computational difficulty in computing the three sums of squares, we rely on computer packages to determine those values. The analysis of variance part of the Excel output in Figure 13.5 shows the three values for the Butler Trucking problem with two independent variables: SST = 23.9, SSR = 21.6006, and SSE = 2.2994. With only one independent variable (number of miles traveled), the Excel output in Figure 13.3 shows that SST = 23.9, SSR = 15.8713, and SSE = 8.0287. The value of SST is the same in both cases because it does not depend on $\hat{y}$, but SSR increases and SSE decreases when a second independent variable (number of deliveries) is added. The implication is that the estimated multiple regression equation provides a better fit for the observed data.

In Chapter 12 we used the coefficient of determination, $r^2 = \text{SSR/SST}$, to measure the goodness of fit for the estimated regression equation. The same concept applies to multiple regression. The term **multiple coefficient of determination** indicates that we are measuring the goodness of fit for the estimated multiple regression equation. The multiple coefficient of determination, denoted R^2, is computed as follows.

In the Excel Regression tool output the label R Square is used to identify the value of R^2.

MULTIPLE COEFFICIENT OF DETERMINATION

$$R^2 = \frac{\text{SSR}}{\text{SST}} \tag{13.8}$$

The multiple coefficient of determination can be interpreted as the proportion of the variability in the dependent variable that can be explained by the estimated multiple regression equation. Hence, when multiplied by 100, it can be interpreted as the percentage of the variability in y that can be explained by the estimated regression equation.

In the two-independent-variable Butler Trucking example, with SSR = 21.6006 and SST = 23.9, we have

$$R^2 = \frac{21.6006}{23.9} = .9038$$

Therefore, 90.38% of the variability in travel time y is explained by the estimated multiple regression equation with miles traveled and number of deliveries as the independent variables. In Figure 13.5, we see that the multiple coefficient of determination is also provided by the Excel output; it is denoted by R Square = .9038 (see cell B17).

Adding independent variables causes the prediction errors (residuals) to become smaller, thus reducing the sum of squares due to error, SSE. Because SSR = SST − SSE, when SSE becomes smaller, SSR becomes larger, causing R^2 = SSR/SST to increase.

Figure 13.3 shows that the R Square value for the estimated regression equation with only one independent variable, number of miles traveled (x_1), is .6641. Thus, the percentage of the variability in travel time that is explained by the estimated regression equation increases from 66.41% to 90.38% when number of deliveries is added as a second independent variable. In general, R^2 always increases as independent variables are added to the model.

Many analysts prefer adjusting R^2 for the number of independent variables to avoid overestimating the impact of adding an independent variable on the amount of variability explained by the estimated regression equation. With n denoting the number of observations and p denoting the number of independent variables, the **adjusted multiple coefficient of determination** is computed as follows.

If a variable is added to the model, R^2 becomes larger even if the variable added is not statistically significant. The adjusted multiple coefficient of determination compensates for the number of independent variables in the model.

ADJUSTED MULTIPLE COEFFICIENT OF DETERMINATION

$$R_a^2 = 1 - (1 - R^2)\frac{n - 1}{n - p - 1} \qquad (13.9)$$

For the Butler Trucking example with $n = 10$ and $p = 2$, we have

$$R_a^2 = 1 - (1 - .9038)\frac{10 - 1}{10 - 2 - 1} = .8763$$

Thus, after adjusting for the two independent variables, we have an adjusted multiple coefficient of determination of .8763. This value is provided by the Excel output in Figure 13.5 as Adjusted R Square = .8763 (see cell B18).

Exercises

Methods

11. In exercise 1, the following estimated regression equation based on 10 observations was presented.

$$\hat{y} = 29.1270 + .5906x_1 + .4980x_2$$

The values of SST and SSR are 6724.125 and 6216.375, respectively.

a. Find SSE.
b. Compute R^2.
c. Compute R_a^2.
d. Comment on the goodness of fit.

SELF test

12. In exercise 2, 10 observations were provided for a dependent variable y and two independent variables x_1 and x_2; for these data SST = 15,182.9 and SSR = 14,052.2.
    a. Compute R^2.
    b. Compute R_a^2.
    c. Does the estimated regression equation explain a large amount of the variability in the data? Explain.

13. In exercise 3, the following estimated regression equation based on 30 observations was presented.

$$\hat{y} = 17.6 + 3.8x_1 - 2.3x_2 + 7.6x_3 + 2.7x_4$$

The values of SST and SSR are 1805 and 1760, respectively.

a. Compute R^2.
b. Compute R_a^2.
c. Comment on the goodness of fit.

Applications

14. In exercise 4, the following estimated regression equation relating sales to inventory investment and advertising expenditures was given.

$$\hat{y} = 25 + 10x_1 + 8x_2$$

The data used to develop the model came from a survey of 10 stores; for those data, SST = 16,000 and SSR = 12,000.

a. For the estimated regression equation given, compute R^2.
b. Compute R_a^2.
c. Does the model appear to explain a large amount of variability in the data? Explain.

SELF test

15. In exercise 5, the owner of Showtime Movie Theaters, Inc., used multiple regression analysis to predict gross revenue (y) as a function of television advertising (x_1) and newspaper advertising (x_2). The estimated regression equation was

$$\hat{y} = 83.2 + 2.29x_1 + 1.30x_2$$

WEB file Showtime

The computer solution provided SST = 25.5 and SSR = 23.435.

a. Compute and interpret R^2 and R_a^2.
b. When television advertising was the only independent variable, $R^2 = .653$ and $R_a^2 = .595$. Do you prefer the multiple regression results? Explain.

16. In exercise 6, data were given on the recruiter assessment score (highest = 5), the out-of-state tuition and fees (rounded to the nearest $1000), and the average starting salary and bonus (rounded to the nearest $1000) for a sample of 20 graduate schools of business (*U.S. News & World Report 2009 Edition America's Best Graduate Schools*).

WEB file BusinessSchools2

    a. Did the estimated regression equation that uses only the recruiter assessment score as the independent variable to predict the starting salary and bonus provide a good fit? Explain.
    b. Discuss the benefits of using both the recruiter assessment score and the out-of-state tuition and fees to predict the starting salary and bonus.

17. In exercise 9, an estimated regression equation was developed relating the top speed for a boat to the boat's beam and horsepower rating.

WEB file Boats

    a. Compute and interpret and R^2 and R_a^2.
    b. Does the estimated regression equation provide a good fit to the data? Explain.

WEB file NBA

18. Refer to exercise 10, where data were reported on a variety of statistics for the 29 teams in the National Basketball Association for a portion of the 2004 season (NBA website, January 3, 2004).
    a. In part (c) of exercise 10, an estimated regression equation was developed relating the proportion of games won given the percentage of field goals made by the team, the proportion of three-point shots made by the team's opponent, and the number of turnovers committed by the team's opponent. What are the values of R^2 and R_a^2?
    b. Does the estimated regression equation provide a good fit to the data? Explain.

13.4 Model Assumptions

In Section 13.1 we introduced the following multiple regression model.

MULTIPLE REGRESSION MODEL

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p + \epsilon \qquad \textbf{(13.10)}$$

The assumptions about the error term ϵ in the multiple regression model parallel those for the simple linear regression model.

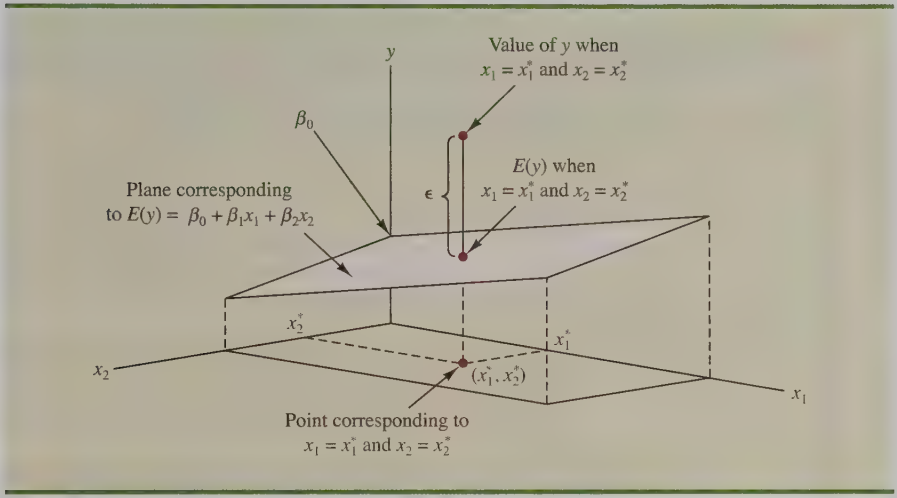
ASSUMPTIONS ABOUT THE ERROR TERM ϵ IN THE MULTIPLE REGRESSION MODEL $y = \beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p + \epsilon$

1. The error term ϵ is a random variable with mean or expected value of zero; that is, $E(\epsilon) = 0$.
   Implication: For given values of $x_1, x_2, \ldots, x_p$, the expected, or average, value of y is given by
   $$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p \qquad \textbf{(13.11)}$$
   Equation (13.11) is the multiple regression equation we introduced in Section 13.1. In this equation, $E(y)$ represents the average of all possible values of y that might occur for the given values of $x_1, x_2, \ldots, x_p$.
2. The variance of ϵ is denoted by σ^2 and is the same for all values of the independent variables $x_1, x_2, \ldots, x_p$.
   Implication: The variance of y about the regression line equals σ^2 and is the same for all values of $x_1, x_2, \ldots, x_p$.
3. The values of ϵ are independent.
   Implication: The value of ϵ for a particular set of values for the independent variables is not related to the value of ϵ for any other set of values.
4. The error term ϵ is a normally distributed random variable reflecting the deviation between the y value and the expected value of y given by $\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p$.
   Implication: Because $\beta_0, \beta_1, \ldots, \beta_p$ are constants for the given values of $x_1, x_2, \ldots, x_p$, the dependent variable y is also a normally distributed random variable.

To obtain more insight about the form of the relationship given by equation (13.11), consider the following two-independent-variable multiple regression equation:

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

FIGURE 13.6 GRAPH OF THE REGRESSION EQUATION FOR MULTIPLE REGRESSION ANALYSIS WITH TWO INDEPENDENT VARIABLES



The graph of this equation is a plane in three-dimensional space. Figure 13.6 provides an example of such a graph. Note that the value of ϵ shown is the difference between the actual y value and the expected value of y, $E(y)$, when $x_1 = x_1^*$ and $x_2 = x_2^*$.

In regression analysis, the term *response variable* is often used in place of the term *dependent variable*. Furthermore, since the multiple regression equation generates a plane or surface, its graph is called a *response surface*.

13.5 Testing for Significance

In this section we show how to conduct significance tests for a multiple regression relationship. The significance tests we used in simple linear regression were a t test and an F test. In simple linear regression, both tests provide the same conclusion; that is, if the null hypothesis is rejected, we conclude that $\beta_1 \neq 0$. In multiple regression, the t test and the F test have different purposes.

1. The F test is used to determine whether a significant relationship exists between the dependent variable and the set of all the independent variables; we will refer to the F test as the test for *overall significance*.
2. If the F test shows an overall significance, the t test is used to determine whether each of the individual independent variables is significant. A separate t test is conducted for each of the independent variables in the model; we refer to each of these t tests as a test for *individual significance*.

In the material that follows, we will explain the F test and the t test and apply each to the Butler Trucking Company example.

F Test

The multiple regression model as defined in Section 13.4 is

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p + \epsilon$$

The hypotheses for the F test involve the parameters of the multiple regression model.

H_0: $\beta_1 = \beta_2 = \cdots = \beta_p = 0$

H_a: One or more of the parameters is not equal to zero

If H_0 is rejected, the test gives us sufficient statistical evidence to conclude that one or more of the parameters is not equal to zero and that the overall relationship between y and the set of independent variables $x_1, x_2, \ldots, x_p$ is significant. However, if H_0 cannot be rejected, we do not have sufficient evidence to conclude that a significant relationship is present.

Before describing the steps of the F test, we need to review the concept of *mean square*. A mean square is a sum of squares divided by its corresponding degrees of freedom. In the multiple regression case, the total sum of squares has $n - 1$ degrees of freedom, the sum of squares due to regression has p degrees of freedom, and the sum of squares due to error has $n - p - 1$ degrees of freedom. Hence, the mean square due to regression (MSR) is SSR/p and the mean square due to error (MSE) is SSE/$(n - p - 1)$.

$$\text{MSR} = \frac{\text{SSR}}{p} \tag{13.12}$$

and

$$\text{MSE} = \frac{\text{SSE}}{n - p - 1} \tag{13.13}$$

As discussed in Chapter 12, MSE provides an unbiased estimate of σ^2, the variance of the error term ϵ. If H_0: $\beta_1 = \beta_2 = \cdots = \beta_p = 0$ is true, MSR also provides an unbiased estimate of σ^2, and the value of MSR/MSE should be close to 1. However, if H_0 is false, MSR overestimates σ^2 and the value of MSR/MSE becomes larger. To determine how large the value of MSR/MSE must be to reject H_0, we make use of the fact that if H_0 is true and the assumptions about the multiple regression model are valid, the sampling distribution of MSR/MSE is an F distribution with p degrees of freedom in the numerator and $n - p - 1$ degrees of freedom in the denominator. A summary of the F test for significance in multiple regression follows.

F TEST FOR OVERALL SIGNIFICANCE

H_0: $\beta_1 = \beta_2 = \cdots = \beta_p = 0$

H_a: One or more of the parameters is not equal to zero

TEST STATISTIC

$$F = \frac{\text{MSR}}{\text{MSE}} \tag{13.14}$$

REJECTION RULE

p-value approach:	Reject H_0 if p-value $\leq \alpha$
Critical value approach:	Reject H_0 if $F \geq F_\alpha$

where F_α is based on an F distribution with p degrees of freedom in the numerator and $n - p - 1$ degrees of freedom in the denominator.

Let us apply the F test to the Butler Trucking Company multiple regression problem. With two independent variables, the hypotheses are written as follows:

H_0: $\beta_1 = \beta_2 = 0$

H_a: β_1 and/or β_2 is not equal to zero

FIGURE 13.7 REGRESSION TOOL OUTPUT FOR THE BUTLER TRUCKING EXAMPLE WITH TWO INDEPENDENT VARIABLES

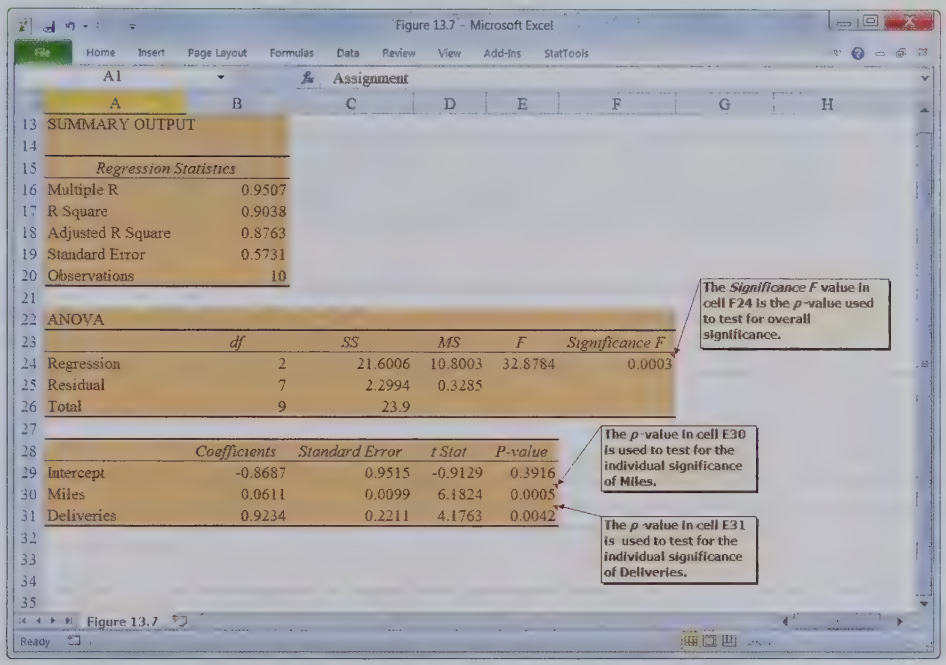


Figure 13.7 - Microsoft Excel
File Home Insert Page Layout Formulas Data Review View Add-Ins StatTools
A1 Assignment
SUMMARY OUTPUT
Regression Statistics
Multiple R 0.9507
R Square 0.9038
Adjusted R Square 0.8763
Standard Error 0.5731
Observations 10
ANOVA
df SS MS F Significance F
Regression 2 21.6006 10.8003 32.8784 0.0003
Residual 7 2.2994 0.3285
Total 9 23.9
Coefficients Standard Error t Stat P-value
Intercept -0.8687 0.9515 -0.9129 0.3916
Miles 0.0611 0.0099 6.1824 0.0005
Deliveries 0.9234 0.2211 4.1763 0.0042
The Significance F value in cell F24 is the p-value used to test for overall significance.
The p-value in cell E30 is used to test for the individual significance of Miles.
The p-value in cell E31 is used to test for the individual significance of Deliveries.
Figure 13.7
Ready

Note: Rows 1–12 are hidden.

Figure 13.7 shows a portion of the Excel Regression tool output shown previously in Figure 13.5 with miles traveled (x_1) and number of deliveries (x_2) as the two independent variables. In the analysis of variance part of the output, we see that MSR = 10.8003 and MSE = .3285. Using equation (13.14), we obtain the test statistic.

$$F = \frac{10.8003}{.3285} = 32.9$$

The label Significance F in cell F23 is used to identify the p-value in cell F24.

Note that the F value in the Excel output is $F = 32.8784$; the value we calculated differs because we used rounded values for MSR and MSE in the calculation. Using $\alpha = .01$, the p-value = 0.0003 in cell F24 indicates that we can reject $H_0: \beta_1 = \beta_2 = 0$ because the p-value is less than $\alpha = .01$. Alternatively, Table 4 of Appendix B shows that with two degrees of freedom in the numerator and seven degrees of freedom in the denominator, $F_{.01} = 9.55$. With $32.9 > 9.55$, we reject $H_0: \beta_1 = \beta_2 = 0$ and conclude that a significant relationship is present between travel time y and the two independent variables, miles traveled and number of deliveries.

As noted previously, MSE provides an unbiased estimate of σ^2, the variance of the error term ϵ. Thus, the estimate of σ^2 is MSE = .3285. The square root of MSE is the estimate of the standard deviation of the error term. As defined in Section 12.5, this standard deviation is called the standard error of the estimate and is denoted s. Hence, we have $s = \sqrt{\text{MSE}} = \sqrt{.3285} = .5731$. Note that the value of the standard error of the estimate appears in cell B19 of Figure 13.7.

TABLE 13.3 GENERAL FORM OF THE ANOVA TABLE FOR MULTIPLE REGRESSION WITH p INDEPENDENT VARIABLES

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Regression	SSR	p	$\text{MSR} = \dfrac{\text{SSR}}{p}$	$F = \dfrac{\text{MSR}}{\text{MSE}}$	
Error	SSE	$n - p - 1$	$\text{MSE} = \dfrac{\text{SSE}}{n - p - 1}$		
Total	SST	$n - 1$			

Table 13.3 is the general form of the ANOVA table for multiple regression. The value of the F test statistic and its corresponding p-value in the last column can be used to make the hypothesis test conclusion. By reviewing the Excel output for Butler Trucking Company in Figure 13.7, we see that Excel's analysis of variance table contains this information.

t Test

If the F test shows that the multiple regression relationship is significant, a t test can be conducted to determine the significance of each of the individual parameters. The t test for individual significance follows.

t TEST FOR INDIVIDUAL SIGNIFICANCE

For any parameter β_i

$$H_0: \beta_i = 0$$
$$H_a: \beta_i \neq 0$$

TEST STATISTIC

$$t = \frac{b_i}{s_{b_i}} \tag{13.15}$$

REJECTION RULE

p-value approach: Reject H_0 if p-value $\leq \alpha$

Critical value approach: Reject H_0 if $t \leq -t_{\alpha/2}$ or if $t \geq t_{\alpha/2}$

where $t_{\alpha/2}$ is based on a t distribution with $n - p - 1$ degrees of freedom.

In the test statistic, s_{b_i} is the estimate of the standard deviation of b_i. The value of s_{b_i} will be provided by the computer software package.

Let us conduct the t test for the Butler Trucking regression problem. Refer to the section of Figure 13.7 that shows the Excel output for the t-ratio calculations. Values of b_1, b_2, s_{b_1}, and s_{b_2} are as follows.

$$b_1 = .0611 \quad s_{b_1} = .0099$$
$$b_2 = .9234 \quad s_{b_2} = .2211$$

Using equation (13.15), we obtain the test statistic for the hypotheses involving parameters β_1 and β_2.

$$t = .0611/.0099 = 6.1717$$
$$t = .9234/.2211 = 4.1764$$

The t values in the Regression tool output are 6.1824 and 4.1763. The difference is due to rounding.

Note that both of these t-ratio values and the corresponding p-values are provided by the Excel Regression tool output in Figure 13.7. Using $\alpha = .01$, the p-values of .0005 and .0042 on the Excel output indicate that we can reject H_0: $\beta_1 = 0$ and H_0: $\beta_2 = 0$. Hence, both parameters are statistically significant. Alternatively, Table 2 of Appendix B shows that with $n - p - 1 = 10 - 2 - 1 = 7$ degrees of freedom, $t_{.005} = 3.499$. Because $6.1717 > 3.499$, we reject H_0: $\beta_1 = 0$. Similarly, with $4.1763 > 3.499$, we reject H_0: $\beta_2 = 0$.

Multicollinearity

We use the term *independent variable* in regression analysis to refer to any variable being used to predict or explain the value of the dependent variable. The term does not mean, however, that the independent variables themselves are independent in any statistical sense. On the contrary, most independent variables in a multiple regression problem are correlated to some degree with one another. For example, in the Butler Trucking example involving the two independent variables x_1 (miles traveled) and x_2 (number of deliveries), we could treat the miles traveled as the dependent variable and the number of deliveries as the independent variable to determine whether those two variables are themselves related. We could then compute the sample correlation coefficient $r_{x_1x_2}$ to determine the extent to which the variables are related. Doing so yields $r_{x_1x_2} = .16$. Thus, we find some degree of linear association between the two independent variables. In multiple regression analysis, **multicollinearity** refers to the correlation among the independent variables.

To provide a better perspective of the potential problems of multicollinearity, let us consider a modification of the Butler Trucking example. Instead of x_2 being the number of deliveries, let x_2 denote the number of gallons of gasoline consumed. Clearly, x_1 (the miles traveled) and x_2 are related; that is, we know that the number of gallons of gasoline used depends on the number of miles traveled. Hence, we would conclude logically that x_1 and x_2 are highly correlated independent variables.

When the independent variables are highly correlated, it is not possible to determine the separate effect of any particular independent variable on the dependent variable.

Assume that we obtain the equation $\hat{y} = b_0 + b_1x_1 + b_2x_2$ and find that the F test shows the relationship to be significant. Then suppose we conduct a t test on β_1 to determine whether $\beta_1 \neq 0$, and we cannot reject H_0: $\beta_1 = 0$. Does this result mean that travel time is not related to miles traveled? Not necessarily. What it probably means is that with x_2 already in the model, x_1 does not make a significant contribution to determining the value of y. This interpretation makes sense in our example; if we know the amount of gasoline consumed, we do not gain much additional information useful in predicting y by knowing the miles traveled. Similarly, a t test might lead us to conclude $\beta_2 = 0$ on the grounds that, with x_1 in the model, knowledge of the amount of gasoline consumed does not add much.

A sample correlation coefficient greater than +.7 or less than −.7 for two independent variables is a rule of thumb warning of potential problems with multicollinearity.

To summarize, in t tests for the significance of individual parameters, the difficulty caused by multicollinearity is that it is possible to conclude that none of the individual parameters are significantly different from zero when an F test on the overall multiple regression equation indicates a significant relationship. This problem is avoided when there is little correlation among the independent variables.

Statisticians have developed several tests for determining whether multicollinearity is high enough to cause problems. According to the rule of thumb test, multicollinearity is a potential problem if the absolute value of the sample correlation coefficient exceeds .7 for

any two of the independent variables. The other types of tests are more advanced and beyond the scope of this text.

If possible, every attempt should be made to avoid including independent variables that are highly correlated. In practice, however, strict adherence to this policy is rarely possible. When decision makers have reason to believe substantial multicollinearity is present, they must realize that separating the effects of the individual independent variables on the dependent variable is difficult.

NOTES AND COMMENTS

Ordinarily, multicollinearity does not affect the way in which we perform our regression analysis or interpret the output from a study. However, when multicollinearity is severe—that is, when two or more of the independent variables are highly correlated with one another—we can have difficulty interpreting the results of t tests on the individual parameters. In addition to the type of problem illustrated in this section, severe cases of multicollinearity have been shown to result in least squares estimates that have the wrong sign. That is, in simulated studies where researchers created the underlying regression model and then applied the least squares technique to develop estimates of β_0, β_1, β_2, and so on, it has been shown that under conditions of high multicollinearity the least squares estimates can have a sign opposite that of the parameter being estimated. For example, β_2 might actually be $+10$ and b_2, its estimate, might turn out to be -2. Thus, little faith can be placed in the individual coefficients if multicollinearity is present to a high degree.

Exercises

Methods

SELF test

19. In exercise 1, the following estimated regression equation based on 10 observations was presented.

$$\hat{y} = 29.1270 + .5906x_1 + .4980x_2$$

Here SST = 6724.125, SSR = 6216.375, $s_{b_1} = .0813$, and $s_{b_2} = .0567$.

a. Compute MSR and MSE.
b. Compute F and perform the appropriate F test. Use $\alpha = .05$.
c. Perform a t test for the significance of β_1. Use $\alpha = .05$.
d. Perform a t test for the significance of β_2. Use $\alpha = .05$.

20. Refer to the data presented in exercise 2. The estimated regression equation for these data is

$$\hat{y} = -18.4 + 2.01x_1 + 4.74x_2$$

Here SST = 15,182.9, SSR = 14,052.2, $s_{b_1} = .2471$, and $s_{b_2} = .9484$.

a. Test for a significant relationship among x_1, x_2, and y. Use $\alpha = .05$.
b. Is β_1 significant? Use $\alpha = .05$.
c. Is β_2 significant? Use $\alpha = .05$.

21. The following estimated regression equation was developed for a model involving two independent variables.

$$\hat{y} = 40.7 + 8.63x_1 + 2.71x_2$$

After x_2 was dropped from the model, the least squares method was used to obtain an estimated regression equation involving only x_1 as an independent variable.

$$\hat{y} = 42.0 + 9.01x_1$$

a. Give an interpretation of the coefficient of x_1 in both models.
b. Could multicollinearity explain why the coefficient of x_1 differs in the two models? If so, how?

Applications

22. In exercise 4, the following estimated regression equation relating sales to inventory investment and advertising expenditures was given.

$$\hat{y} = 25 + 10x_1 + 8x_2$$

The data used to develop the model came from a survey of 10 stores; for these data SST = 16,000 and SSR = 12,000.
a. Compute SSE, MSE, and MSR.
b. Use an F test and a .05 level of significance to determine whether there is a relationship among the variables.

SELF test

23. Refer to exercise 5.
a. Use $\alpha = .01$ to test the hypotheses

$$H_0: \beta_1 = \beta_2 = 0$$
$$H_a: \beta_1 \text{ and/or } \beta_2 \text{ is not equal to zero}$$

for the model $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$, where

x_1 = television advertising ($1000s)
x_2 = newspaper advertising ($1000s)

b. Use $\alpha = .05$ to test the significance of β_1. Should x_1 be dropped from the model?
c. Use $\alpha = .05$ to test the significance of β_2. Should x_2 be dropped from the model?

24. *The Wall Street Journal* conducted a study of basketball spending at top colleges. A portion of the data showing the revenue ($ millions), percentage of wins, and the coach's salary ($ millions) for 39 of the country's top basketball programs follows (*The Wall Street Journal,* March 11–12, 2006).

WEB file
Basketball

School	Revenue	%Wins	Salary
Alabama	6.5	61	1.00
Arizona	16.6	63	0.70
Arkansas	11.1	72	0.80
Boston College	3.4	80	0.53
.	.	.	.
.	.	.	.
.	.	.	.
Washington	5.0	83	0.89
West Virginia	4.9	67	0.70
Wichita State	3.1	75	0.41
Wisconsin	12.0	66	0.70

a. Develop the estimated regression equation that can be used to predict the coach's salary given the revenue generated by the program and the percentage of wins.
b. Use the F test to determine the overall significance of the relationship. What is your conclusion at the .05 level of significance?
c. Use the t test to determine the significance of each independent variable. What is your conclusion at the .05 level of significance?

25. Small cars offer higher fuel efficiency, are easy to maneuver and park, and have generally performed well in road tests. The following data show the overall fuel efficiency rating (miles per gallon), predicted reliability (highest = 5), safety rating (highest = 5), and the price ($) for 20 small cars (*Consumer Reports*, March 2009).

WEB file SmallCars

Make & Model	MPG	Reliability	Safety	Price ($)
Chevrolet Aveo LT	25	2	2	16,205
Chevrolet Cobalt LT	24	3	3	17,450
Ford Focus SES	26	4	3	18,490
Ford Focus SES (MT)	29	4	3	17,440
Honda Civic EX	28	5	4	19,610
Honda Civic EX (MT)	31	5	4	18,810
Honda Fit Sport (MT)	33	5	4	16,730
Hyundai Accent GLS	28	4	2	14,230
Hyundai Elantra GLS	27	5	3	17,555
Hyundai Elantra SE	27	5	4	17,980
Kia Spectra EX	25	4	2	16,185
Kia Spectra EX (MT)	28	4	2	15,185
Nissan Versa 1.8 SL	28	3	3	16,675
Nissan Versa 1.85	27	1	3	16,130
Scion xD	29	5	3	16,620
Scion xD (MT)	34	5	3	15,820
Suzuki SX4 LE	26	5	3	17,378
Toyota Corolla (base, MT)	32	4	4	16,419
Toyota Corolla LE	32	4	4	18,404
Toyota Yaris Hatchback	30	5	3	16,095

a. Determine the estimated regression equation that can be used to predict the price of a small car given its overall fuel efficiency rating, predicted reliability, and safety rating.
b. Use the F test to determine the overall significance of the relationship. What is the conclusion at the .05 level of significance?
c. Use the t test to determine the significance of each independent variable. What is your conclusion at the .05 level of significance?
d. Remove any independent variable that is not significant from the estimated regression equation. What is your recommended estimated regression equation? Compare the value of R^2 for this estimated regression equation with the value of R^2 for the estimated regression equation from part (a). Discuss the differences.

WEB file NBA

26. In exercise 10 an estimated regression equation was developed relating the proportion of games won given the proportion of field goals made by the team, the proportion of three-point shots made by the team's opponent, and the number of turnovers committed by the team's opponent.
a. Use the F test to determine the overall significance of the relationship. What is your conclusion at the .05 level of significance?
b. Use the t test to determine the significance of each independent variable. What is your conclusion at the .05 level of significance?

13.6 Using the Estimated Regression Equation for Estimation and Prediction

The procedures for estimating the mean value of y and predicting an individual value of y in multiple regression are similar to those in regression analysis involving one independent variable. First, recall that in Chapter 12 we showed that the estimated regression equation $\hat{y} = b_0 + b_1x$ can be used to estimate the mean value of y for a given value of x as well as to predict an individual value of y for a given value of x. In multiple regression we use the same procedure. That is, we substitute the value of $x_1, x_2, \ldots, x_p$ into the estimated regression equation and use the corresponding value of $\hat{y}$ to estimate the mean value of y given $x_1, x_2, \ldots, x_p$ as well as to predict an individual of y given $x_1, x_2, \ldots, x_p$.

To illustrate the procedure in multiple regression, suppose that for the Butler Trucking example we want to use the estimated regression equation involving x_1 (miles traveled) and x_2 (number of deliveries) to develop two interval estimates:

1. A *confidence interval* of the mean travel time for all trucks that travel 100 miles and make two deliveries
2. A *prediction interval* of the travel time for *one specific* truck that travels 100 miles and makes two deliveries

The Excel output in Figure 13.5 showed the estimated regression equation is

$$\hat{y} = -.8687 + .0611x_1 + .9234x_2$$

With $x_1 = 100$ and $x_2 = 2$, we obtain the following value of $\hat{y}$:

$$\hat{y} = -.8687 + .0611(100) + .9234(2) = 7.09$$

Hence a point estimate of the mean travel time for all trucks that travel 100 miles and make two deliveries is approximately seven hours. And a prediction of the travel time for one specific truck that travels 100 miles and makes two deliveries is also seven hours.

In the appendix to Chapter 12 we showed how to compute prediction intervals using StatTools.

The formulas required to develop confidence and prediction intervals for multiple regression are beyond the scope of the text. And in multiple regression hand computation is simply not practical. Although Excel's Regression Tool does not have an option for computing interval estimates, StatTools—the Excel add-in provided with the text—can be used to compute prediction intervals. Table 13.4 shows the 95% prediction intervals for the

TABLE 13.4 THE 95% PREDICTION INTERVALS FOR BUTLER TRUCKING

Value of x_1	Value of x_2	Prediction Interval	
		Lower Limit	Upper Limit
50	2	2.414	5.656
50	3	3.368	6.548
50	4	4.157	7.607
100	2	5.500	8.683
100	3	6.520	9.510
100	4	7.362	10.515

Butler Trucking problem for selected values of x_1 and x_2. We see that the prediction interval of the travel time for *one specific* truck that travels 100 miles and makes two deliveries is approximately 5.5 to 8.7 hours.

Exercises

Methods

27. In exercise 1, the following estimated regression equation based on 10 observations was presented.

$$\hat{y} = 29.1270 + .5906x_1 + .4980x_2$$

a. Develop a point estimate of the mean value of y when $x_1 = 180$ and $x_2 = 310$.
b. Predict an individual value of y when $x_1 = 180$ and $x_2 = 310$.

SELF test

28. Refer to the data in exercise 2. The estimated regression equation for those data is

$$\hat{y} = -18.4 + 2.01x_1 + 4.74x_2$$

a. Develop a point estimate of the mean value of y when $x_1 = 45$ and $x_2 = 15$.
b. Develop a 95% prediction interval for y when $x_1 = 45$ and $x_2 = 15$.

Applications

SELF test

29. In exercise 5, the owner of Showtime Movie Theaters, Inc., used multiple regression analysis to predict gross revenue (y) as a function of television advertising (x_1) and newspaper advertising (x_2). The estimated regression equation was

$$\hat{y} = 83.2 + 2.29x_1 + 1.30x_2$$

a. What is the gross revenue expected for a week when \$3500 is spent on television advertising ($x_1 = 3.5$) and \$1800 is spent on newspaper advertising ($x_2 = 1.8$)?
b. Provide a 95% prediction interval for next week's revenue, assuming that the advertising expenditures will be allocated as in part (a).

WEB file
Boats

30. In exercise 9 an estimated regression equation was developed relating the top speed for a boat to the boat's beam and horsepower rating.
a. Develop a point estimate for the mean top speed of a boat with a beam of 85 inches and an engine with a 330 horsepower rating.
b. The Svfara SV609 has a beam of 85 inches and an engine with a 330 horsepower rating. Develop a 95% prediction interval for the mean top speed for the Svfara SV609.

31. The Buyer's Guide section of the website for *Car and Driver* magazine provides reviews and road tests for cars, trucks, SUVs, and vans. The average ratings of overall quality, vehicle styling, braking, handling, fuel economy, interior comfort, acceleration, dependability, fit and finish, transmission, and ride are summarized for each vehicle using a scale ranging from 1 (worst) to 10 (best). A portion of the data for 14 Sports/GT cars is shown here (Car and Driver website, January 7, 2004).

WEB file SportsCar

Sports/GT	Overall	Handling	Dependability	Fit and Finish
Acura 3.2CL	7.80	7.83	8.17	7.67
Acura RSX	9.02	9.46	9.35	8.97
Audi TT	9.00	9.58	8.74	9.38
BMW 3-Series/M3	8.39	9.52	8.39	8.55
Chevrolet Corvette	8.82	9.64	8.54	7.87
Ford Mustang	8.34	8.85	8.70	7.34
Honda Civic Si	8.92	9.31	9.50	7.93
Infiniti G35	8.70	9.34	8.96	8.07
Mazda RX-8	8.58	9.79	8.96	8.12
Mini Cooper	8.76	10.00	8.69	8.33
Mitsubishi Eclipse	8.17	8.95	8.25	7.36
Nissan 350Z	8.07	9.35	7.56	8.21
Porsche 911	9.55	9.91	8.86	9.55
Toyota Celica	8.77	9.29	9.04	7.97

a. Develop an estimated regression equation using handling, dependability, and fit and finish to predict overall quality.
b. Another Sports/GT car rated by *Car and Driver* is the Honda Accord. The ratings for handling, dependability, and fit and finish for the Honda Accord were 8.28, 9.06, and 8.07, respectively. Estimate the overall rating for this car.
c. Provide a 95% prediction interval for overall quality for the Honda Accord described in part (b).
d. The overall rating reported by *Car and Driver* for the Honda Accord was 8.65. How does this rating compare to the estimates you developed in parts (b) and (d)?

13.7 Categorical Independent Variables

The independent variables may be categorical or quantitative.

Thus far, the examples we have considered involved quantitative independent variables such as student population, distance traveled, and number of deliveries. In many situations, however, we must work with **categorical independent variables** such as gender (male, female), method of payment (cash, credit card, check), and so on. The purpose of this section is to show how categorical variables are handled in regression analysis. To illustrate the use and interpretation of a categorical independent variable, we will consider a problem facing the managers of Johnson Filtration, Inc.

An Example: Johnson Filtration, Inc.

Johnson Filtration, Inc., provides maintenance service for water-filtration systems throughout southern Florida. Customers contact Johnson with requests for maintenance service on their water-filtration systems. To estimate the service time and the service cost, Johnson's managers want to predict the repair time necessary for each maintenance request. Hence, repair time in hours is the dependent variable. Repair time is believed to be related to two factors, the number of months since the last maintenance service and the type of repair problem (mechanical or electrical). Data for a sample of 10 service calls are reported in Table 13.5.

Let y denote the repair time in hours and x_1 denote the number of months since the last maintenance service. The regression model that uses only x_1 to predict y is

$$y = \beta_0 + \beta_1 x_1 + \epsilon$$

TABLE 13.5 DATA FOR THE JOHNSON FILTRATION EXAMPLE

Service Call	Months Since Last Service	Type of Repair	Repair Time in Hours
1	2	electrical	2.9
2	6	mechanical	3.0
3	8	electrical	4.8
4	3	mechanical	1.8
5	2	electrical	2.9
6	7	electrical	4.9
7	9	mechanical	4.2
8	8	mechanical	4.8
9	4	electrical	4.4
10	6	electrical	4.5

Using Excel's Regression tool to develop the estimated regression equation, we obtained the Excel output shown in Figure 13.8. The estimated regression equation is

$$\hat{y} = 2.1473 + .3041x_1 \qquad \textbf{(13.16)}$$

At the .05 level of significance, the p-value of .0163 for the t (or F) test indicates that the number of months since the last service is significantly related to repair time. R Square = .5342 indicates that x_1 alone explains 53.42% of the variability in repair time.

FIGURE 13.8 REGRESSION TOOL OUTPUT FOR THE JOHNSON FILTRATION EXAMPLE WITH MONTHS SINCE LAST SERVICE CALL AS THE INDEPENDENT VARIABLE

The Excel Regression tool output appears in a new worksheet because we selected New Worksheet Ply as the Output option in the Regression dialog box.

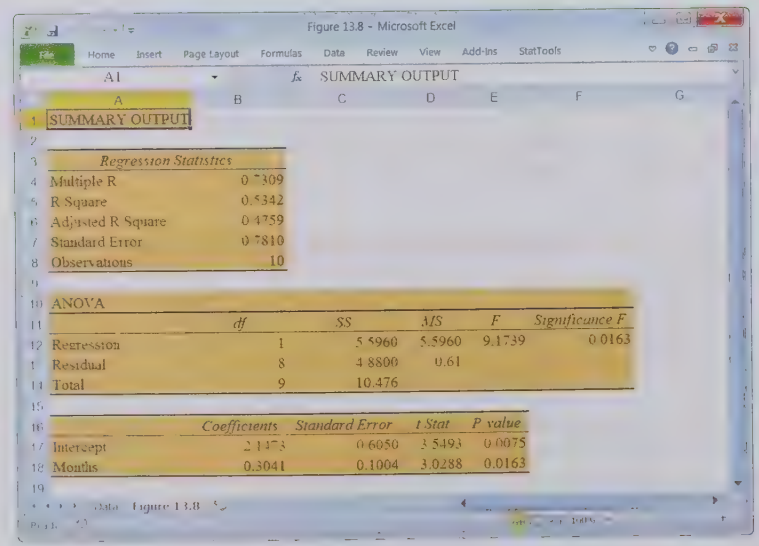


Figure 13.8 - Microsoft Excel
File Home Insert Page Layout Formulas Data Review View Add-Ins StatTools
A1 SUMMARY OUTPUT
SUMMARY OUTPUT
Regression Statistics
Multiple R 0.7309
R Square 0.5342
Adjusted R Square 0.4759
Standard Error 0.7810
Observations 10
ANOVA
df SS MS F Significance F
Regression 1 5.5960 5.5960 9.1739 0.0163
Residual 8 4.8800 0.61
Total 9 10.476
Coefficients Standard Error t Stat P value
Intercept 2.1473 0.6050 3.5493 0.0075
Months 0.3041 0.1004 3.0288 0.0163

TABLE 13.6 DATA FOR THE JOHNSON FILTRATION EXAMPLE WITH TYPE OF REPAIR INDICATED BY A DUMMY VARIABLE ($x_2 = 0$ FOR MECHANICAL; $x_2 = 1$ FOR ELECTRICAL)

Customer	Months Since Last Service (x_1)	Type of Repair (x_2)	Repair Time in Hours (y)
1	2	1	2.9
2	6	0	3.0
3	8	1	4.8
4	3	0	1.8
5	2	1	2.9
6	7	1	4.9
7	9	0	4.2
8	8	0	4.8
9	4	1	4.4
10	6	1	4.5

WEB file Johnson

To incorporate the type of repair into the regression model, we define the following variable.

$$x_2 = \begin{cases} 0 \text{ if the type of repair is mechanical} \\ 1 \text{ if the type of repair is electrical} \end{cases}$$

In regression analysis x_2 is called a **dummy** or *indicator* **variable**. Using this dummy variable, we can write the multiple regression model as

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$$

Table 13.6 is the revised data set that includes the values of the dummy variable. Using Excel and the data in Table 13.6, we can develop estimates of the model parameters. The Excel Regression tool output in Figure 13.9 shows that the estimated multiple regression equation is

$$\hat{y} = .9305 + .3876x_1 + 1.2627x_2 \qquad \textbf{(13.17)}$$

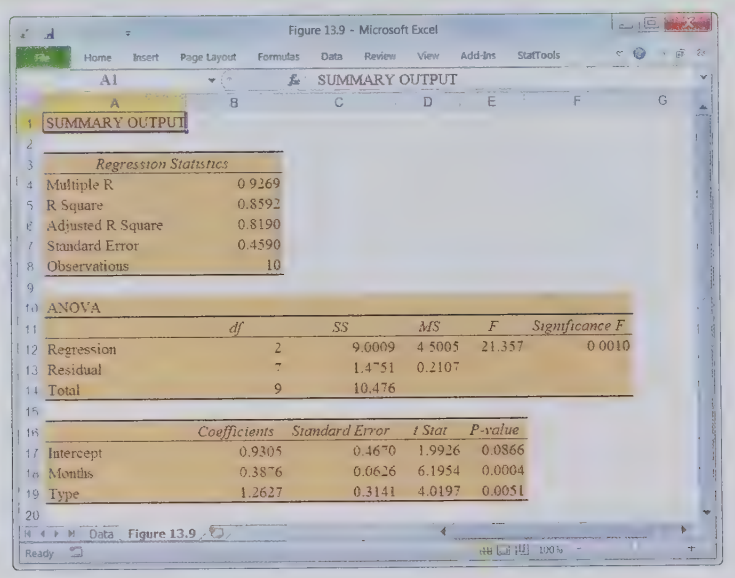
At the .05 level of significance, the p-value of .0010 associated with the F test ($F = 21.357$) indicates that the regression relationship is significant. The t test part of the printout in Figure 13.9 shows that both months since last service (p-value = .0004) and type of repair (p-value = .0051) are statistically significant. In addition, R Square = 0.8952 and Adjusted R Square = 0.8190 indicate that the estimated regression equation does a good job of explaining the variability in repair times. Thus, equation (13.17) should prove helpful in estimating the repair time necessary for the various service calls.

Interpreting the Parameters

The multiple regression equation for the Johnson Filtration example is

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 \qquad \textbf{(13.18)}$$

FIGURE 13.9 REGRESSION TOOL OUTPUT FOR THE JOHNSON FILTRATION EXAMPLE WITH MONTHS SINCE LAST SERVICE CALL AND TYPE OF REPAIR AS THE INDEPENDENT VARIABLES



To understand how to interpret the parameters β_0, β_1, and β_2 when a categorical variable is present, consider the case when $x_2 = 0$ (mechanical repair). Using $E(y \mid \text{mechanical})$ to denote the mean or expected value of repair time *given* a mechanical repair, we have

$$E(y \mid \text{mechanical}) = \beta_0 + \beta_1 x_1 + \beta_2(0) = \beta_0 + \beta_1 x_1 \qquad \textbf{(13.19)}$$

Similarly, for an electrical repair ($x_2 = 1$), we have

$$\begin{aligned} E(y \mid \text{electrical}) &= \beta_0 + \beta_1 x_1 + \beta_2(1) = \beta_0 + \beta_1 x_1 + \beta_2 \\ &= (\beta_0 + \beta_2) + \beta_1 x_1 \end{aligned} \qquad \textbf{(13.20)}$$

Comparing equations (13.19) and (13.20), we see that the mean repair time is a linear function of x_1 for both mechanical and electrical repairs. The slope of both equations is β_1, but the y-intercept differs. The y-intercept is β_0 in equation (13.19) for mechanical repairs and $(\beta_0 + \beta_2)$ in equation (13.20) for electrical repairs. The interpretation of β_2 is that it indicates the difference between the mean repair time for an electrical repair and the mean repair time for a mechanical repair.

If β_2 is positive, the mean repair time for an electrical repair will be greater than that for a mechanical repair; if β_2 is negative, the mean repair time for an electrical repair will be less than that for a mechanical repair. Finally, if $\beta_2 = 0$, there is no difference in the mean repair time between electrical and mechanical repairs and the type of repair is not related to the repair time.

Using the estimated multiple regression equation $\hat{y} = .9305 + .3876x_1 + 1.2627x_2$, we see that .9305 is the estimate of β_0, .3876 is the estimate of β_1, and 1.2627 is the estimate of β_2. Thus, when $x_2 = 0$ (mechanical repair)

$$\hat{y} = .9305 + .3876x_1 \tag{13.21}$$

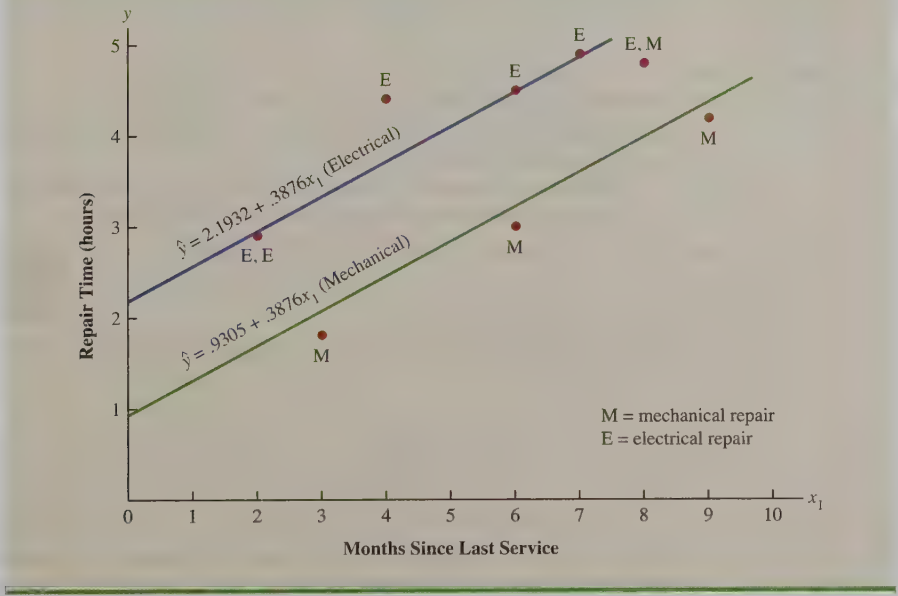
and when $x_2 = 1$ (electrical repair)

$$\begin{aligned}\hat{y} &= .9305 + .3876x_1 + 1.2627(1) \\ &= 2.1932 + .3876x_1\end{aligned} \tag{13.22}$$

In effect, the use of a dummy variable for type of repair provides two estimated regression equations that can be used to predict the repair time, one corresponding to mechanical repairs and one corresponding to electrical repairs. In addition, with $b_2 = 1.2627$, we learn that, on average, electrical repairs require 1.2627 hours longer than mechanical repairs.

Figure 13.10 is the plot of the Johnson data from Table 13.6. Repair time in hours (y) is represented by the vertical axis and months since last service (x_1) is represented by the horizontal axis. A data point for a mechanical repair is indicated by an M and a data point for an electrical repair is indicated by an E. Equations (13.21) and (13.22) are plotted on the graph to show graphically the two equations that can be used to predict the repair time, one corresponding to mechanical repairs and one corresponding to electrical repairs.

FIGURE 13.10 SCATTER DIAGRAM FOR THE JOHNSON FILTRATION REPAIR DATA FROM TABLE 13.6



More Complex Categorical Variables

A categorical variable with k levels must be modeled using k − 1 dummy variables. Care must be taken in defining and interpreting the dummy variables.

Because the categorical variable for the Johnson Filtration example had two levels (mechanical and electrical), defining a dummy variable with zero indicating a mechanical repair and one indicating an electrical repair was easy. However, when a categorical variable has more than two levels, care must be taken in both defining and interpreting the dummy variables. As we will show, if a categorical variable has k levels, $k - 1$ dummy variables are required, with each dummy variable being coded as 0 or 1.

For example, suppose a manufacturer of copy machines organized the sales territories for a particular state into three regions: A, B, and C. The managers want to use regression analysis to help predict the number of copiers sold per week. With the number of units sold as the dependent variable, they are considering several independent variables (the number of sales personnel, advertising expenditures, and so on). Suppose the managers believe sales region is also an important factor in predicting the number of copiers sold. Because sales region is a categorical variable with three levels, A, B and C, we will need $3 - 1 = 2$ dummy variables to represent the sales region. Each variable can be coded 0 or 1 as follows.

$$x_1 = \begin{cases} 1 \text{ if sales region B} \\ 0 \text{ otherwise} \end{cases}$$

$$x_2 = \begin{cases} 1 \text{ if sales region C} \\ 0 \text{ otherwise} \end{cases}$$

With this definition, we have the following values of x_1 and x_2.

Region	x_1	x_2
A	0	0
B	1	0
C	0	1

Observations corresponding to region A would be coded $x_1 = 0, x_2 = 0$; observations corresponding to region B would be coded $x_1 = 1, x_2 = 0$; and observations corresponding to region C would be coded $x_1 = 0, x_2 = 1$.

The regression equation relating the expected value of the number of units sold, $E(y)$, to the dummy variables would be written as

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

To help us interpret the parameters β_0, β_1, and β_2, consider the following three variations of the regression equation.

$$E(y \mid \text{region A}) = \beta_0 + \beta_1(0) + \beta_2(0) = \beta_0$$
$$E(y \mid \text{region B}) = \beta_0 + \beta_1(1) + \beta_2(0) = \beta_0 + \beta_1$$
$$E(y \mid \text{region C}) = \beta_0 + \beta_1(0) + \beta_2(1) = \beta_0 + \beta_2$$

Thus, β_0 is the mean or expected value of sales for region A; β_1 is the difference between the mean number of units sold in region B and the mean number of units sold in region A;

and β_2 is the difference between the mean number of units sold in region C and the mean number of units sold in region A.

Two dummy variables were required because sales region is a categorical variable with three levels. But the assignment of $x_1 = 0$, $x_2 = 0$ to indicate region A, $x_1 = 1$, $x_2 = 0$ to indicate region B, and $x_1 = 0$, $x_2 = 1$ to indicate region C was arbitrary. For example, we could have chosen $x_1 = 1$, $x_2 = 0$ to indicate region A, $x_1 = 0$, $x_2 = 0$ to indicate region B, and $x_1 = 0$, $x_2 = 1$ to indicate region C. In that case, β_1 would have been interpreted as the mean difference between regions A and B and β_2 as the mean difference between regions C and B.

The important point to remember is that when a categorical variable has k levels, $k - 1$ dummy variables are required in the multiple regression analysis. Thus, if the sales region example had a fourth region, labeled D, three dummy variables would be necessary. For example, the three dummy variables can be coded as follows.

$$x_1 = \begin{cases} 1 \text{ if sales region B} \\ 0 \text{ otherwise} \end{cases} \qquad x_2 = \begin{cases} 1 \text{ if sales region C} \\ 0 \text{ otherwise} \end{cases} \qquad x_3 = \begin{cases} 1 \text{ if sales region D} \\ 0 \text{ otherwise} \end{cases}$$

Exercises

Methods

SELF test

32. Consider a regression study involving a dependent variable y, a quantitative independent variable x_1, and a categorical variable with two levels (level 1 and level 2).
    a. Write a multiple regression equation relating x_1 and the categorical variable to y.
    b. What is the expected value of y corresponding to level 1 of the categorical variable?
    c. What is the expected value of y corresponding to level 2 of the categorical variable?
    d. Interpret the parameters in your regression equation.

33. Consider a regression study involving a dependent variable y, a quantitative independent variable x_1, and a categorical independent variable with three possible levels (level 1, level 2, and level 3).
    a. How many dummy variables are required to represent the categorical variable?
    b. Write a multiple regression equation relating x_1 and the categorical variable to y.
    c. Interpret the parameters in your regression equation.

Applications

SELF test

34. Management proposed the following regression model to predict sales at a fast-food outlet.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \epsilon$$

where

x_1 = number of competitors within one mile
x_2 = population within one mile (1000s)
$x_3 = \begin{cases} 1 \text{ if drive-up window present} \\ 0 \text{ otherwise} \end{cases}$
y = sales ($1000s)

The following estimated regression equation was developed after 20 outlets were surveyed.

$$\hat{y} = 10.1 - 4.2x_1 + 6.8x_2 + 15.3x_3$$

a. What is the expected amount of sales attributable to the drive-up window?
b. Predict sales for a store with two competitors, a population of 8000 within one mile, and no drive-up window.
c. Predict sales for a store with one competitor, a population of 3000 within one mile, and a drive-up window.

35. Refer to the Johnson Filtration problem introduced in this section. Suppose that in addition to information on the number of months since the machine was serviced and whether a mechanical or an electrical repair was necessary, the managers obtained a list showing which repairperson performed the service. The revised data follow.

WEB file
Repair

Repair Time in Hours	Months Since Last Service	Type of Repair	Repairperson
2.9	2	Electrical	Dave Newton
3.0	6	Mechanical	Dave Newton
4.8	8	Electrical	Bob Jones
1.8	3	Mechanical	Dave Newton
2.9	2	Electrical	Dave Newton
4.9	7	Electrical	Bob Jones
4.2	9	Mechanical	Bob Jones
4.8	8	Mechanical	Bob Jones
4.4	4	Electrical	Bob Jones
4.5	6	Electrical	Dave Newton

a. Ignore for now the months since the last maintenance service (x_1) and the repairperson who performed the service. Develop the estimated simple linear regression equation to predict the repair time (y) given the type of repair (x_2). Recall that $x_2 = 0$ if the type of repair is mechanical and 1 if the type of repair is electrical.
b. Does the equation that you developed in part (a) provide a good fit for the observed data? Explain.
c. Ignore for now the months since the last maintenance service and the type of repair associated with the machine. Develop the estimated simple linear regression equation to predict the repair time given the repairperson who performed the service. Let $x_3 = 0$ if Bob Jones performed the service and $x_3 = 1$ if Dave Newton performed the service.
d. Does the equation that you developed in part (c) provide a good fit for the observed data? Explain.

36. This problem is an extension of the situation described in exercise 35.
a. Develop the estimated regression equation to predict the repair time given the number of months since the last maintenance service, the type of repair, and the repairperson who performed the service.
b. At the .05 level of significance, test whether the estimated regression equation developed in part (a) represents a significant relationship between the independent variables and the dependent variable.
c. Is the addition of the independent variable x_3, the repairperson who performed the service, statistically significant? Use $\alpha = .05$. What explanation can you give for the results observed?

37. The *Consumer Reports* Restaurant Customer Satisfaction Survey is based upon 148,599 visits to full-service restaurant chains (Consumer Reports website, February 11, 2009). Assume the following data are representative of the results reported. The variable Type indicates whether the restaurant is an Italian restaurant or a seafood/steakhouse. Price indicates the average amount paid per person for dinner and drinks, minus the tip. Score reflects diners' overall satisfaction, with higher values indicating greater overall satisfaction. A score of 80 can be interpreted as very satisfied.

RestaurantRatings

Restaurant	Type	Price ($)	Score
Bertucci's	Italian	16	77
Black Angus Steakhouse	Seafood/Steakhouse	24	79
Bonefish Grill	Seafood/Steakhouse	26	85
Bravo! Cucina Italiana	Italian	18	84
Buca di Beppo	Italian	17	81
Bugaboo Creek Steak House	Seafood/Steakhouse	18	77
Carrabba's Italian Grill	Italian	23	86
Charlie Brown's Steakhouse	Seafood/Steakhouse	17	75
Il Fornaio	Italian	28	83
Joe's Crab Shack	Seafood/Steakhouse	15	71
Johnny Carino's Italian	Italian	17	81
Lone Star Steakhouse & Saloon	Seafood/Steakhouse	17	76
LongHorn Steakhouse	Seafood/Steakhouse	19	81
Maggiano's Little Italy	Italian	22	83
McGrath's Fish House	Seafood/Steakhouse	16	81
Olive Garden	Italian	19	81
Outback Steakhouse	Seafood/Steakhouse	20	80
Red Lobster	Seafood/Steakhouse	18	78
Romano's Macaroni Grill	Italian	18	82
The Old Spaghetti Factory	Italian	12	79
Uno Chicago Grill	Italian	16	76

a. Develop the estimated regression equation to show how overall customer satisfaction is related to the independent variable average meal price.
b. At the .05 level of significance, test whether the estimated regression equation developed in part (a) indicates a significant relationship between overall customer satisfaction and average meal price.
c. Develop a dummy variable that will account for the type of restaurant (Italian or seafood/steakhouse).
d. Develop the estimated regression equation to show how overall customer satisfaction is related to the average meal price and the type of restaurant.
e. Is type of restaurant a significant factor in overall customer satisfaction?
f. Predict the *Consumer Reports* customer satisfaction score for a seafood/steakhouse that has an average meal price of $20. How much would the predicted score have changed for an Italian restaurant?

38. A 10-year study conducted by the American Heart Association provided data on how age, blood pressure, and smoking relate to the risk of strokes. Assume that the following data are from a portion of this study. Risk is interpreted as the probability (times 100) that the patient will have a stroke over the next 10-year period. For the smoking variable, define a dummy variable with 1 indicating a smoker and 0 indicating a nonsmoker.

WEB file
Stroke

Risk	Age	Pressure	Smoker
12	57	152	No
24	67	163	No
13	58	155	No
56	86	177	Yes
28	59	196	No
51	76	189	Yes
18	56	155	Yes
31	78	120	No
37	80	135	Yes
15	78	98	No
22	71	152	No
36	70	173	Yes
15	67	135	Yes
48	77	209	Yes
15	60	199	No
36	82	119	Yes
8	66	166	No
34	80	125	Yes
3	62	117	No
37	59	207	Yes

a. Develop an estimated regression equation that relates risk of a stroke to the person's age, blood pressure, and whether the person is a smoker.
b. Is smoking a significant factor in the risk of a stroke? Explain. Use $\alpha = .05$.
c. What is the probability of a stroke over the next 10 years for Art Speen, a 68-year-old smoker who has blood pressure of 175? What action might the physician recommend for this patient?

13.8 Modeling Curvilinear Relationships

Curvilinear relationships can be easily handled using a multiple regression model. To illustrate let us consider the problem facing Reynolds, Inc., a manufacturer of industrial scales and laboratory equipment. Mangers at Reynolds want to investigate the relationship between length of employment of their salespeople and the number of electronic laboratory scales sold. Table 13.7 gives the number of months each salesperson has been employed by the firm (x) and the number of scales sold (y) by 15 randomly selected salespeople for the most recent sales period. In Chapter 12 we showed how Excel's chart tools can be used to construct a scatter diagram and compute the estimated regression equation and the coefficient of determination for simple linear regression. Figure 13.11 shows the results of using Excel's chart tools to fit a line to the Reynolds data. The chart tools output shows that the estimated regression equation is

$$\hat{y} = 111.23 + 2.3768x$$

where

$\hat{y}$ = predicted number of electronic laboratory scales sold

x = the number of months the salesperson has been employed

Although the chart tools output shows that a linear relationship explains a high percentage of the variability in sales ($r^2 = 0.7812$), the scatter diagram indicates a possible curvilinear relationship between the length of time employed and the number of units sold.

TABLE 13.7 DATA FOR THE REYNOLDS EXAMPLE

Months Employed (x)	Scales Sold (y)
41	275
106	296
76	317
104	376
22	162
12	150
85	367
111	308
40	189
51	235
9	83
12	112
6	67
56	325
19	189

WEB file
Reynolds

FIGURE 13.11 CHART TOOLS OUTPUT FOR THE REYNOLDS EXAMPLE: SIMPLE LINEAR REGRESSION

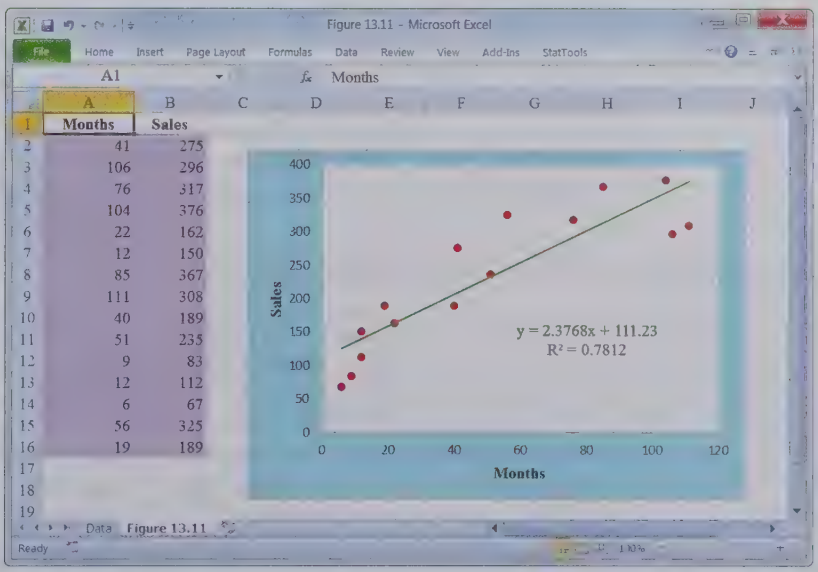
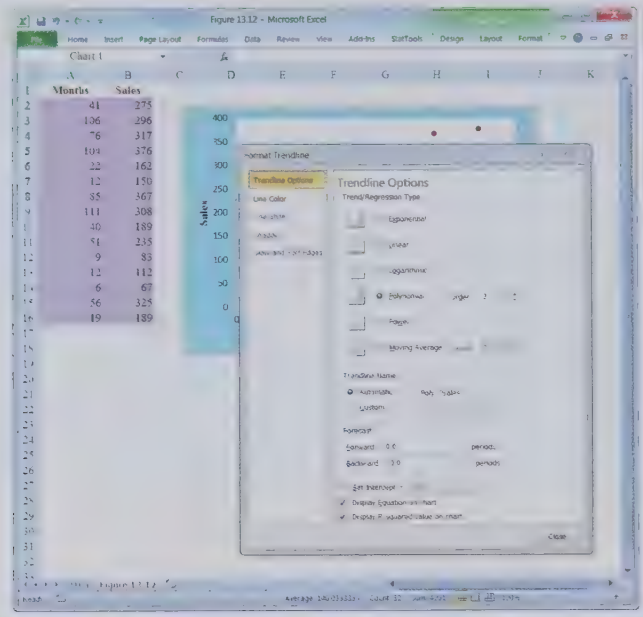


FIGURE 13.12 CHART TOOLS FORMAT TRENDLINE DIALOG BOX FOR THE REYNOLDS EXAMPLE: SECOND-ORDER MODEL



To account for the curvilinear relationship, we will use a multiple regression model with two independent variables: x and x^2.

$$y = \beta_0 + \beta_1 x + \beta_1 x^2 + \epsilon$$

This model is commonly referred to as a second-order polynomial or a quadratic model. Refer to Figure 13.12 as we describe how to use Excel's chart tools to fit a polynomial curve to the data. The estimated multiple regression equation and multiple coefficient of determination for this second-order model are also obtained.

Step 1. Position the mouse pointer over any data point in the scatter diagram and right-click to display a list of options; choose **Add Trendline**

Step 2. When the Format Trendline dialog box appears (Figure 13.12),
Select **Trendline Options** and then
Choose **Polynomial** from the Trend/Regression Type list and enter 2 in the **Order** box
Choose **Display Equation on chart**
Choose **Display R-squared value on chart**
Click **Close**

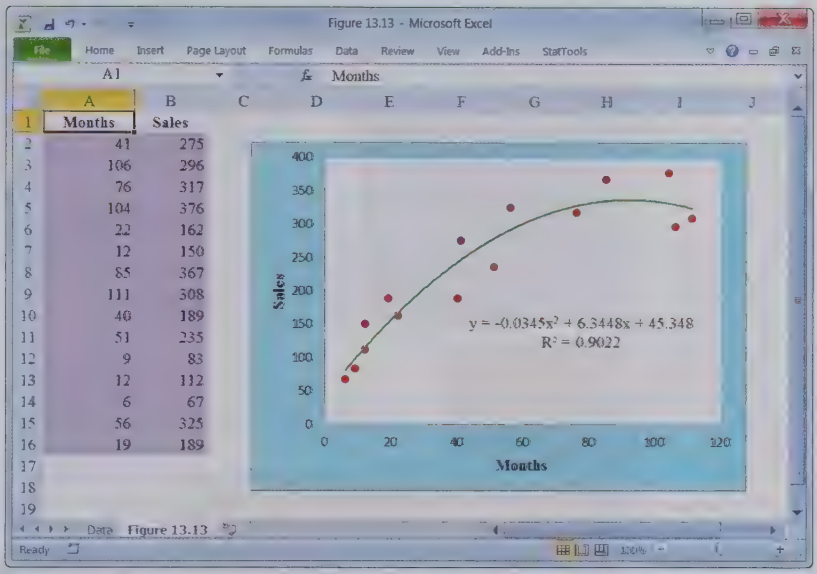
The results are shown in Figure 13.13. The estimated multiple regression equation is

$$\hat{y} = 45.348 + 6.3448x - 0.0345x^2$$

and the multiple coefficient of determination is $R^2 = 0.9022$.

Excel's chart tools make fitting curvilinear functions very easy. But the output does not provide any means for testing the significance of the results. To do so, we need to use Excel's Regression tool.

FIGURE 13.13 CHART TOOLS OUTPUT FOR THE REYNOLDS EXAMPLE: SECOND-ORDER MODEL



To use Excel's Regression tool for this polynomial model we treat the values of x^2 as a second independent variable. In Figure 13.14 we show the values of x as Month and the values of x^2 as MonthsSq. Then we run the Regression tool just as if Month and MonthSq are two separate independent variables. The Regression tool output is shown in Figure 13.14. The estimated multiple regression equation is

$$\hat{y} = 45.3476 + 6.3448 \text{ Months} - .0345 \text{ MonthsSq}$$

where

MonthsSq = the square of the number of months the salesperson has been employed

At the .05 level of significance, the Excel output shows that the overall model is significant (p-value for the F test is 8.75E-07). Note also that the p-value = .0023, corresponding to the tStat value for MonthsSq, is less than .05. Thus, we can conclude that adding MonthsSq to the model involving Months is significant. With an Adjusted R Square value of .8859, we should be pleased with the fit provided by this estimated multiple regression equation.

Many types of curvilinear relationships can be modeled in a similar fashion. The regression techniques with which we have been working are definitely not limited to linear, or straight-line, relationships. In multiple regression analysis the word *linear* refers only to the fact that $\beta_0, \beta_1, \ldots, \beta_p$ all have exponents of 1; it does not imply that the relationship between y and the underlying x_i variables is linear. Indeed, in this section we have seen one example of how regression analysis can be used to model a curvilinear relationship.

FIGURE 13.14 REGRESSION TOOL OUTPUT FOR THE REYNOLDS EXAMPLE: SECOND-ORDER MODEL

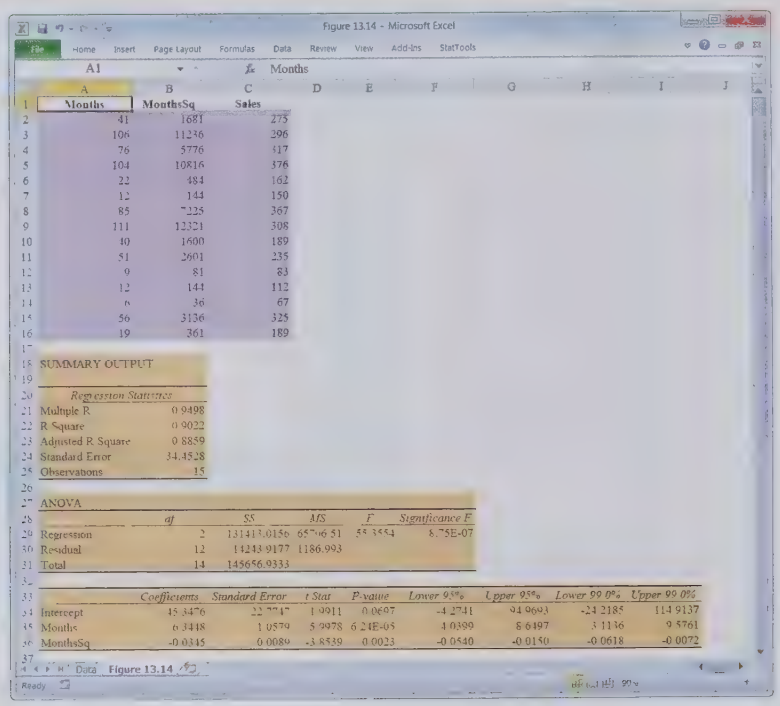


Figure 13.14 - Microsoft Excel

	A	B	C
1	Months	MonthsSq	Sales
2	41	1681	275
3	106	11236	296
4	76	5776	317
5	104	10816	376
6	22	484	162
7	12	144	150
8	85	7225	367
9	111	12321	308
10	40	1600	189
11	51	2601	235
12	0	81	83
13	12	144	112
14	6	36	67
15	56	3136	325
16	19	361	189

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.9498
R Square	0.9022
Adjusted R Square	0.8859
Standard Error	34.4528
Observations	15

ANOVA

	df	SS	MS	F	Significance F
Regression	2	131413.0156	65706.51	55.3554	8.75E-07
Residual	12	14243.9177	1186.993		
Total	14	145656.9333			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 99.0%	Upper 99.0%
Intercept	45.3476	22.7747	1.9911	0.0607	-4.2741	94.9693	-24.2185	114.9137
Months	6.3418	1.0579	5.9978	6.24E-05	4.0390	8.6447	3.1136	9.5761
MonthsSq	-0.0345	0.0089	-3.8839	0.0023	-0.0540	-0.0150	-0.0618	-0.0072

Data | Figure 13.14 | Ready

Exercises

Methods

SELF test

39. Consider the following data for two variables, x and y.

x	22	24	26	30	35	40
y	12	21	33	35	40	36

a. Develop an estimated regression equation for the data of the form $\hat{y} = b_0 + b_1 x$.
b. Using the results from part (a), test for a significant relationship between x and y; use $\alpha = .05$.
c. Develop a scatter diagram for the data. Does the scatter diagram suggest an estimated regression equation of the form $\hat{y} = b_0 + b_1 x + b_2 x^2$? Explain.
d. Develop an estimated regression equation for the data of the form $\hat{y} = b_0 + b_1 x + b_2 x^2$.
e. Refer to part (d). Is the relationship between x, x^2, and y significant? Use $\alpha = .05$.
f. Predict the value of y when $x = 25$.

40. Consider the following data for two variables, x and y.

x	9	32	18	15	26
y	10	20	21	16	22

a. Develop an estimated regression equation for the data of the form $\hat{y} = b_0 + b_1x$. Comment on the adequacy of this equation for predicting y.
b. Develop an estimated regression equation for the data of the form $\hat{y} = b_0 + b_1x + b_2x^2$. Comment on the adequacy of this equation for predicting y.
c. Predict the value of y when $x = 20$.

Applications

41. A highway department is studying the relationship between traffic flow and speed. The following model has been hypothesized.

$$y = \beta_0 + \beta_1 x + \epsilon$$

where

y = traffic flow in vehicles per hour
x = vehicle speed in miles per hour

The following data were collected during rush hour for six highways leading out of the city.

Traffic Flow (y)	Vehicle Speed (x)
1256	35
1329	40
1226	30
1335	45
1349	50
1124	25

SELF test

a. Develop an estimated regression equation for the data of the form $\hat{y} = b_0 + b_1x$ and test for a significant relationship using $\alpha = .01$.
b. Develop an estimated multiple regression equation of the form $\hat{y} = b_0 + b_1x + b_2x^2$ and test for a significant relationship using $\alpha = .01$.
c. Predict the traffic flow in vehicles per hour at a speed of 38 miles per hour.

42. A study of emergency service facilities investigated the relationship between the number of facilities and the average distance traveled to provide the emergency service. The following table gives the data collected.

Number of Facilities	Average Distance (miles)
9	1.66
11	1.12
16	.83
21	.62
27	.51
30	.47

a. Develop a scatter diagram for these data, treating average distance traveled as the dependent variable.
b. Does a simple linear model appear to be appropriate? Explain.
c. Develop an estimated multiple regression equation for the data of the form $\hat{y} = b_0 + b_1 x + b_1 x^2$.

43. Corvette, Ferrari, and Jaguar produced a variety of classic cars that continue to increase in value. The following data, based upon the Martin Rating System for Collectible Cars, show the rarity rating (1–20) and the high price ($1000) for 15 classic cars (BusinessWeek website, February 2006).

WEB file
ClassicCars

Year	Make	Model	Rating	Price ($1000)
1984	Chevrolet	Corvette	18	1600
1956	Chevrolet	Corvette 265/225-hp	19	4000
1963	Chevrolet	Corvette coupe (340-bhp 4-speed)	18	1000
1978	Chevrolet	Corvette coupe Silver Anniversary	19	1300
1960–1963	Ferrari	250 GTE 2+2	16	350
1962–1964	Ferrari	250 GTL Lusso	19	2650
1962	Ferrari	250 GTO	18	375
1967–1968	Ferrari	275 GTB/4 NART Spyder	17	450
1968–1973	Ferrari	365 GTB/4 Daytona	17	140
1962–1967	Jaguar	E-type OTS	15	77.5
1969–1971	Jaguar	E-type Series II OTS	14	62
1971–1974	Jaguar	E-type Series III OTS	16	125
1951–1954	Jaguar	XK 120 roadster (steel)	17	400
1950–1953	Jaguar	XK C-type	16	250
1956–1957	Jaguar	XKSS	13	70

a. Develop a scatter diagram of the data using the rarity rating as the independent variable and price as the independent variable. Does a simple linear regression model appear to be appropriate?
b. Develop an estimated multiple regression equation with x = rarity rating and x^2 as the two independent variables.

Summary

In this chapter, we introduced multiple regression analysis as an extension of simple linear regression analysis presented in Chapter 12. Multiple regression analysis enables us to understand how a dependent variable is related to two or more independent variables. The regression equation $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p$ shows that the expected value or mean value of the dependent variable y is related to the values of the independent variables $x_1, x_2, \ldots, x_p$. Sample data and the least squares method are used to develop the estimated regression equation $\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \cdots + b_p x_p$. In effect $b_0, b_1, b_2, \ldots, b_p$ are sample statistics used to estimate the unknown model parameters $\beta_0, \beta_1, \beta_2, \ldots, \beta_p$. Excel output was used throughout the chapter to emphasize the fact that computer software packages are the only realistic means of performing the numerous computations required in multiple regression analysis.

The multiple coefficient of determination was presented as a measure of the goodness of fit of the estimated regression equation. It determines the proportion of the variation of y that

can be explained by the estimated regression equation. The adjusted multiple coefficient of determination is a similar measure of goodness of fit that adjusts for the number of independent variables and thus avoids overestimating the impact of adding more independent variables.

An F test and a t test were presented as ways to determine statistically whether the relationship among the variables is significant. The F test is used to determine whether there is a significant overall relationship between the dependent variable and the set of all independent variables. The t test is used to determine whether there is a significant relationship between the dependent variable and an individual independent variable given the other independent variables in the regression model. Correlation among the independent variables, known as multicollinearity, was discussed.

The section on categorical independent variables showed how dummy variables can be used to incorporate categorical data into multiple regression analysis. And, the last section showed how curvilinear relationships can be easily handled using a multiple regression model.

Glossary

Multiple regression analysis Regression analysis involving two or more independent variables.

Multiple regression model The mathematical equation that describes how the dependent variable y is related to the independent variables $x_1, x_2, \ldots, x_p$ and an error term ϵ.

Multiple regression equation The mathematical equation relating the expected value or mean value of the dependent variable to the values of the independent variables; that is, $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p$.

Estimated multiple regression equation The estimate of the multiple regression equation based on sample data and the least squares method; it is $\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \cdots + b_p x_p$.

Least squares method The method used to develop the estimated regression equation. It minimizes the sum of squared residuals (the deviations between the observed values of the dependent variable, y_i, and the estimated values of the dependent variable, $\hat{y}_i$).

Multiple coefficient of determination A measure of the goodness of fit of the estimated multiple regression equation. It can be interpreted as the proportion of the variability in the dependent variable that is explained by the estimated regression equation.

Adjusted multiple coefficient of determination A measure of the goodness of fit of the estimated multiple regression equation that adjusts for the number of independent variables in the model and thus avoids overestimating the impact of adding more independent variables.

Multicollinearity The term used to describe the correlation among the independent variables.

Categorical independent variable An independent variable with categorical data.

Dummy variable A variable used to model the effect of categorical independent variables. A dummy variable may take only the value zero or one.

Key Formulas

Multiple Regression Model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p + \epsilon \qquad (13.1)$$

Multiple Regression Equation

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p \quad (13.2)$$

Estimated Multiple Regression Equation

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \cdots + b_p x_p \quad (13.3)$$

Least Squares Criterion

$$\min \Sigma(y_i - \hat{y}_i)^2 \quad (13.4)$$

Relationship Among SST, SSR, and SSE

$$\text{SST} = \text{SSR} + \text{SSE} \quad (13.7)$$

Multiple Coefficient of Determination

$$R^2 = \frac{\text{SSR}}{\text{SST}} \quad (13.8)$$

Adjusted Multiple Coefficient of Determination

$$R_a^2 = 1 - (1 - R^2)\frac{n-1}{n-p-1} \quad (13.9)$$

Mean Square Regression

$$\text{MSR} = \frac{\text{SSR}}{p} \quad (13.12)$$

Mean Square Error

$$\text{MSE} = \frac{\text{SSE}}{n-p-1} \quad (13.13)$$

***F* Test Statistic**

$$F = \frac{\text{MSR}}{\text{MSE}} \quad (13.14)$$

***t* Test Statistic**

$$t = \frac{b_i}{s_{b_i}} \quad (13.15)$$

Supplementary Exercises

44. The admissions officer for Clearwater College developed the following estimated regression equation relating the final college GPA to the student's SAT mathematics score and high-school GPA.

$$\hat{y} = -1.41 + .0235x_1 + .00486x_2$$

where

x_1 = high-school grade point average
x_2 = SAT mathematics score
y = final college grade point average

a. Interpret the coefficients in this estimated regression equation.
b. Predict the final college GPA for a student who has a high-school average of 84 and a score of 540 on the SAT mathematics test.

45. The personnel director for Electronics Associates developed the following estimated regression equation relating an employee's score on a job satisfaction test to his or her length of service and wage rate.

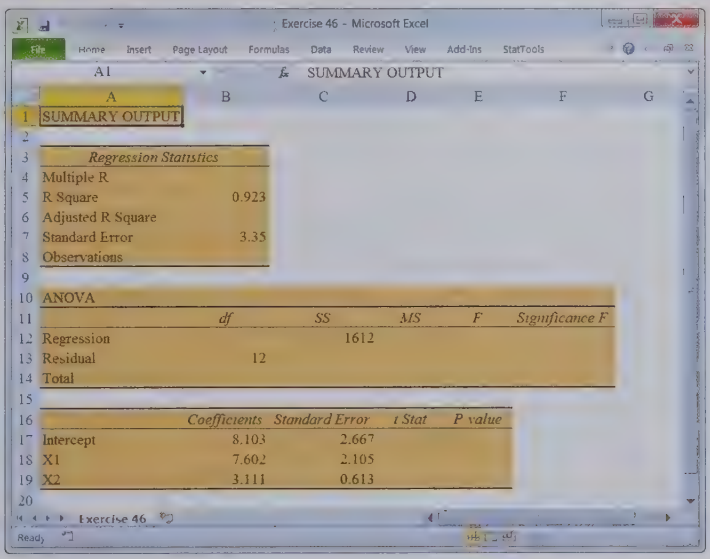
$$\hat{y} = 14.4 - 8.69x_1 + 13.5x_2$$

where

x_1 = length of service (years)
x_2 = wage rate (dollars)
y = job satisfaction test score (higher scores indicate greater job satisfaction)

a. Interpret the coefficients in this estimated regression equation.
b. Predict the job satisfaction test score for an employee who has four years of service and makes $6.50 per hour.

46. A partial computer output from a regression analysis follows.



a. Compute the missing entries in this output.
b. Using $\alpha = .05$, test for overall significance.
c. Use the t test and $\alpha = .05$ to test H_0: $\beta_1 = 0$ and H_0: $\beta_2 = 0$.

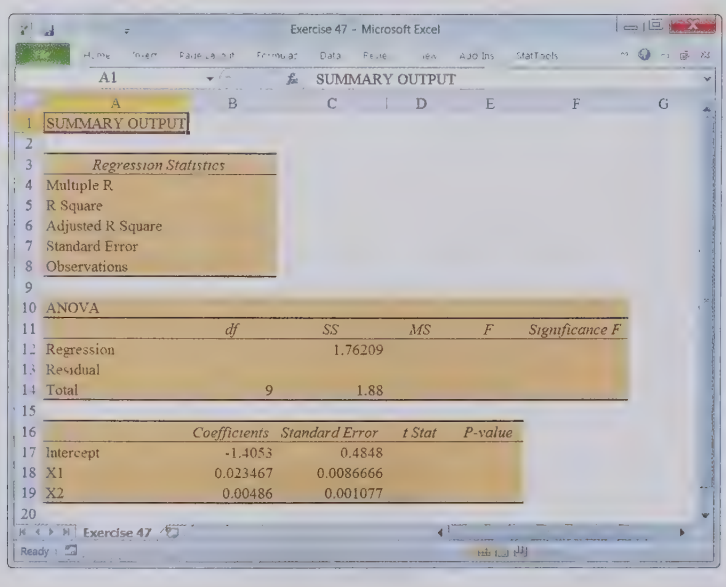
47. Recall that in exercise 44, the admissions officer for Clearwater College developed the following estimated regression equation relating final college GPA to the student's SAT mathematics score and high-school GPA.

$$\hat{y} = -1.41 + .0235x_1 + .00486x_2$$

where

x_1 = high-school grade point average
x_2 = SAT mathematics score
y = final college grade point average

A portion of the regression output follows.



a. Complete the missing entries in this output.
b. Using $\alpha = .05$, test for overall significance.
c. Did the estimated regression equation provide a good fit to the data? Explain.
d. Use the t test and $\alpha = .05$ to test H_0: $\beta_1 = 0$ and H_0: $\beta_2 = 0$.

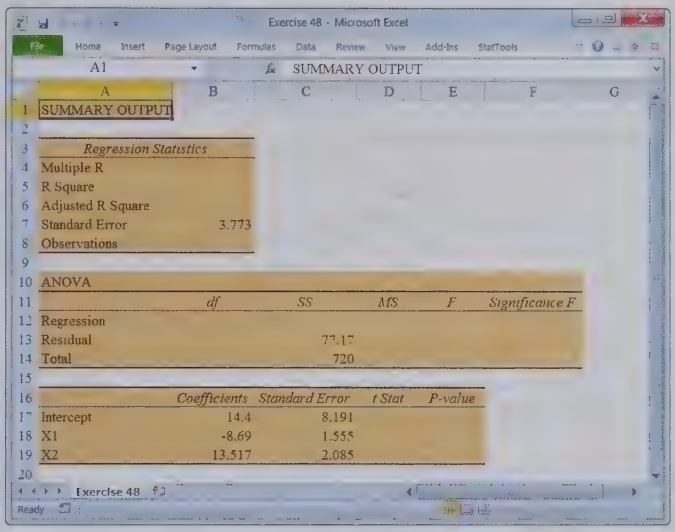
48. Recall that in exercise 45 the personnel director for Electronics Associates developed the following estimated regression equation relating an employee's score on a job satisfaction test to length of service and wage rate.

$$\hat{y} = 14.4 - 8.69x_1 + 13.5x_2$$

where

x_1 = length of service (years)
x_2 = wage rate (dollars)
y = job satisfaction test score (higher scores indicate greater job satisfaction)

A portion of the regression output follows.



a. Complete the missing entries in this output.
b. Using $\alpha = .05$, test for overall significance.
c. Did the estimated regression equation provide a good fit to the data? Explain.
d. Use the t test and $\alpha = .05$ to test H_0: $\beta_1 = 0$ and H_0: $\beta_2 = 0$.

49. The Tire Rack, America's leading online distributor of tires and wheels, conducts extensive testing to provide customers with products that are right for their vehicle, driving style, and driving conditions. In addition, the Tire Rack maintains an independent consumer survey to help drivers help each other by sharing their long-term tire experiences. The following data show survey ratings (1 to 10 scale with 10 the highest rating) for 18 maximum performance summer tires (Tire Rack website, February 3, 2009). The variable Steering rates the tire's steering responsiveness, Tread Wear rates quickness of wear based on the driver's expectations, and Buy Again rates the driver's overall tire satisfaction and desire to purchase the same tire again.

WEB file TireRack

Tire	Steering	Tread Wear	Buy Again
Goodyear Assurance TripleTred	8.9	8.5	8.1
Michelin HydroEdge	8.9	9.0	8.3
Michelin Harmony	8.3	8.8	8.2
Dunlop SP 60	8.2	8.5	7.9
Goodyear Assurance ComforTred	7.9	7.7	7.1
Yokohama Y372	8.4	8.2	8.9
Yokohama Aegis LS4	7.9	7.0	7.1
Kumho Power Star 758	7.9	7.9	8.3
Goodyear Assurance	7.6	5.8	4.5
Hankook H406	7.8	6.8	6.2
Michelin Energy LX4	7.4	5.7	4.8
Michelin MX4	7.0	6.5	5.3
Michelin Symmetry	6.9	5.7	4.2
Kumho 722	7.2	6.6	5.0
Dunlop SP 40 A/S	6.2	4.2	3.4
Bridgestone Insignia SE200	5.7	5.5	3.6
Goodyear Integrity	5.7	5.4	2.9
Dunlop SP20 FE	5.7	5.0	3.3

a. Develop an estimated regression equation that can be used to predict the Buy Again rating given based on the Steering rating. At the .05 level of significance, test for a significant relationship.
b. Did the estimated regression equation developed in part (a) provide a good fit to the data? Explain.
c. Develop an estimated regression equation that can be used to predict the Buy Again rating given the Steering rating and the Tread Wear rating.
d. Is the addition of the Tread Wear independent variable significant? Use $\alpha = .05$.

50. *Consumer Reports* provided extensive testing and ratings for 24 treadmills. An overall score, based primarily on ease of use, ergonomics, exercise range, and quality, was developed for each treadmill tested. In general, a higher overall score indicates better performance. The following data show the price, the quality rating, and overall score for the 24 treadmills (*Consumer Reports*, February 2006).

Brand & Model	Price	Quality	Score
Landice L7	2900	Excellent	86
NordicTrack S3000	3500	Very good	85
SportsArt 3110	2900	Excellent	82
Precor	3500	Excellent	81
True Z4 HRC	2300	Excellent	81
Vision Fitness T9500	2000	Excellent	81
Precor M 9.31	3000	Excellent	79
Vision Fitness T9200	1300	Very good	78
Star Trac TR901	3200	Very good	72
Trimline T350HR	1600	Very good	72
Schwinn 820p	1300	Very good	69
Bowflex 7-Series	1500	Excellent	83
NordicTrack S1900	2600	Very good	83
Horizon Fitness PST8	1600	Very good	82
Horizon Fitness 5.2T	1800	Very good	80
Evo by Smooth Fitness FX30	1700	Very good	75
ProForm 1000S	1600	Very good	75
Horizon Fitness CST4.5	1000	Very good	74
Keys Fitness 320t	1200	Very good	73
Smooth Fitness 7.1HR Pro	1600	Very good	73
NordicTrack C2300	1000	Good	70
Spirit Inspire	1400	Very good	70
ProForm 750	1000	Good	67
Image 19.0 R	600	Good	66

a. Use these data to develop an estimated regression equation that could be used to estimate the overall score given the price.
b. Use $\alpha = .05$ to test for overall significance.
c. To incorporate the effect of quality, a categorical variable with three levels, we used two dummy variables, Quality-E and Quality-VG. Each variable is coded 0 or 1 as follows.

$$\text{Quality-E} = \begin{cases} 1 \text{ if quality rating is excellent} \\ 0 \text{ otherwise} \end{cases}$$

$$\text{Quality-VG} = \begin{cases} 1 \text{ if quality rating is very good} \\ 0 \text{ otherwise} \end{cases}$$

Develop an estimated regression equation that could be used to estimate the overall score given the price and the quality rating.

d. For the estimated regression equation developed in part (c), test for overall significance using $\alpha = .10$.
e. For the estimated regression equation developed in part (c), use the t test to determine the significance of each independent variable. Use $\alpha = .10$.
f. Predict the overall score for a treadmill with a price of $2000 and a good quality rating. How much would the prediction change if the quality rating were very good? Explain.

51. A portion of a data set containing information for 45 mutual funds that are part of the *Morningstar Funds 500* for 2008 follows. The complete data set is available in the file named MutualFunds. The data set includes the following five variables:

Type: The type of fund, labeled DE (Domestic Equity), IE (International Equity), and FI (Fixed Income).

Net Asset Value ($): The closing price per share on December 31, 2007.

5-Year Average Return (%): The average annual return for the fund over the past five years.

Expense Ratio (%): The percentage of assets deducted each fiscal year for fund expenses.

Morningstar Rank: The risk adjusted star rating for each fund; Morningstar ranks go from a low of 1-Star to a high of 5-Stars.

WEB file
MutualFunds

Fund Name	Fund Type	Net Asset Value ($)	Five-Year Average Return (%)	Expense Ratio (%)	Morningstar Rank
Amer Cent Inc & Growth Inv	DE	28.88	12.39	0.67	2-Star
American Century Intl. Disc	IE	14.37	30.53	1.41	3-Star
American Century Tax-Free Bond	FI	10.73	3.34	0.49	4-Star
American Century Ultra	DE	24.94	10.88	0.99	3-Star
Ariel	DE	46.39	11.32	1.03	2-Star
Artisan Intl Val	IE	25.52	24.95	1.23	3-Star
Artisan Small Cap	DE	16.92	15.67	1.18	3-Star
Baron Asset	DE	50.67	16.77	1.31	5-Star
Brandywine	DE	36.58	18.14	1.08	4-Star
⋮	⋮	⋮	⋮	⋮	⋮

a. Develop an estimated regression equation that can be used to predict the five-year average return given fund type. At the .05 level of significance, test for a significant relationship.
b. Did the estimated regression equation developed in part (a) provide a good fit to the data? Explain.
c. Develop the estimated regression equation that can be used to predict the five-year average return given the type of fund, the net asset value, and the expense ratio. At the .05 level of significance, test for a significant relationship. Do you think any variables should be deleted from the estimated regression equation? Explain.
d. Morningstar Rank is a categorical variable. Because the data set contains only funds with four ranks (2-Star through 5-Star), use the following dummy variables: 3StarRank = 1 for a 3-Star fund, 0 otherwise; 4StarRank = 1 for a 4-Star fund, 0 otherwise; and 5StarRank = 1 for a 5-Star fund, 0 otherwise. Develop an estimated regression equation that can be used to predict the five-year average return given the type of fund, the expense ratio, and the Morningstar Rank. Using $\alpha = .05$, remove any independent variables that are not significant.
e. Use the estimated regression equation developed in part (d) to predict the five-year average return for a domestic equity fund with an expense ratio of 1.05% and a 3-Star Morningstar Rank.

52. The U.S. Department of Energy's Fuel Economy Guide provides fuel efficiency data for cars and trucks (U.S. Department of Energy website, February 22, 2008). A portion of the data for 311 compact, midsize, and large cars follows. The column labeled Class identifies the size of the car: Compact, Midsize, or Large. The column labeled Displacement shows the engine's displacement in liters. The column labeled Fuel Type shows whether the car uses premium (P) or regular (R) fuel, and the column labeled Hwy MPG shows the fuel efficiency rating for highway driving in terms of miles per gallon. The complete data set is contained in the file named FuelData.

Car	Class	Displacement	Fuel Type	Hwy MPG
1	Compact	3.1	P	25
2	Compact	3.1	P	25
3	Compact	3	P	25
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
161	Midsize	2.4	R	30
162	Midsize	2	P	29
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
310	Large	3	R	25
311	Large	3	R	25

a. Develop an estimated regression equation that can be used to predict the fuel efficiency for highway driving given the engine's displacement. Test for significance using $\alpha = .05$.
b. Consider the addition of the dummy variables ClassMidsize and ClassLarge. The value of ClassMidsize is 1 if the car is a midsize car and 0 otherwise; the value of ClassLarge is 1 if the car is a large car and 0 otherwise. Thus, for a compact car, the value of ClassMidsize and the value of ClassLarge is 0. Develop the estimated regression equation that can be used to predict the fuel efficiency for highway driving given the engine's displacement and the dummy variables ClassMidsize and ClassLarge.
c. Use $\alpha = .05$ to determine whether the dummy variables added in part (b) are significant.
d. Consider the addition of the dummy variable FuelPremium, where the value of FuelPremium is 1 if the car uses premium fuel and 0 if the car uses regular fuel. Develop the estimated regression equation that can be used to predict the fuel efficiency for highway driving given the engine's displacement, the dummy variables ClassMidsize and ClassLarge, and the dummy variable FuelPremium.
e. For the estimated regression equation developed in part (d), test for overall significance and individual significance using $\alpha = .05$.

Case Problem 1 Consumer Research, Inc.

Consumer Research, Inc., is an independent agency that conducts research on consumer attitudes and behaviors for a variety of firms. In one study, a client asked for an investigation of consumer characteristics that can be used to predict the amount charged by credit card users. Data were collected on annual income, household size, and annual credit card charges for a sample of 50 consumers. The following data appear in the file named Consumer.

WEB file
Consumer

Income ($1000s)	Household Size	Amount Charged ($)	Income ($1000s)	Household Size	Amount Charged ($)
54	3	4016	54	6	5573
30	2	3159	30	1	2583
32	4	5100	48	2	3866
50	5	4742	34	5	3586
31	2	1864	67	4	5037
55	2	4070	50	2	3605
37	1	2731	67	5	5345
40	2	3348	55	6	5370
66	4	4764	52	2	3890
51	3	4110	62	3	4705
25	3	4208	64	2	4157
48	4	4219	22	3	3579
27	1	2477	29	4	3890
33	2	2514	39	2	2972
65	3	4214	35	1	3121
63	4	4965	39	4	4183
42	6	4412	54	3	3730
21	2	2448	23	6	4127
44	1	2995	27	2	2921
37	5	4171	26	7	4603
62	6	5678	61	2	4273
21	3	3623	30	2	3067
55	7	5301	22	4	3074
42	2	3020	46	5	4820
41	7	4828	66	4	5149

Managerial Report

1. Use methods of descriptive statistics to summarize the data. Comment on the findings.
2. Develop estimated regression equations, first using annual income as the independent variable and then using household size as the independent variable. Which variable is the better predictor of annual credit card charges? Discuss your findings.
3. Develop an estimated regression equation with annual income and household size as the independent variables. Discuss your findings.
4. What is the predicted annual credit card charge for a three-person household with an annual income of $40,000?
5. Discuss the need for other independent variables that could be added to the model. What additional variables might be helpful?

Case Problem 2 Alumni Giving

Alumni donations are an important source of revenue for colleges and universities. If administrators could determine the factors that could lead to increases in the percentage of alumni who make a donation, they might be able to implement policies that could lead to increased revenues. Research shows that students who are more satisfied with their contact with teachers are more likely to graduate. As a result, one might suspect that smaller class

sizes and lower student-faculty ratios might lead to a higher percentage of satisfied graduates, which in turn might lead to increases in the percentage of alumni who make a donation. Table 13.8 shows data for 48 national universities (*America's Best Colleges*, Year 2000 Edition). The column labeled Graduation Rate is the percentage of students who initially enrolled at the university and graduated. The column labeled % of Classes Under 20 shows the percentage of classes offered with fewer than 20 students. The column labeled Student-Faculty Ratio is the number of students enrolled divided by the total number of faculty. Finally, the column labeled Alumni Giving Rate is the percentage of alumni who made a donation to the university.

Managerial Report

1. Use methods of descriptive statistics to summarize the data.
2. Develop an estimated regression equation that can be used to predict the alumni giving rate given the number of students who graduate. Discuss your findings.
3. Develop an estimated regression equation that could be used to predict the alumni giving rate using the data provided.
4. What conclusions and recommendations can you derive from your analysis?

Case Problem 3 PGA Tour Statistics

The Professional Golfers Association (PGA) maintains data on performance and earnings for members of the PGA Tour. The top 125 players based on total earnings in PGA Tour events are exempt for the following season. Making the top 125 money list is important because a player who is "exempt" has qualified to be a full-time member of the PGA tour for the following season.

Scoring average is generally considered the most important statistic in terms of success on the PGA Tour. To investigate the effect that variables such as driving distance, driving accuracy, greens in regulation, sand saves, and average putts per round have on average score, year-end performance data for the 125 players who had the highest total earnings in PGA Tour events for 2008 are contained in the file named PGATour (PGA Tour website, 2009). Each row of the data set corresponds to a PGA Tour player, and the data have been sorted based upon total earnings. Descriptions for the variables in the data set follow.

Money: Total earnings in PGA Tour events.

Scoring Average: The average number of strokes per completed round.

DrDist (Driving Distance): The average number of yards per measured drive. On the PGA Tour driving distance is measured on two holes per round. Care is taken to select two holes which face in opposite directions to counteract the effect of wind. Drives are measured to the point at which they come to rest regardless of whether they are in the fairway or not.

DrAccu (Driving Accuracy): The percentage of time a tee shot comes to rest in the fairway (regardless of club). Driving accuracy is measured on every hole, excluding par 3s.

GIR (Greens in Regulation): The percentage of time a player was able to hit the green in regulation. A green is considered hit in regulation if any portion of the ball is touching the putting surface after the GIR stroke has been taken. The GIR stroke is determined by subtracting 2 from par (1st stroke on a par 3, 2nd on a par 4, 3rd on a par 5). In other words, a green is considered hit in regulation if the player has reached the putting surface in par minus two strokes.

TABLE 13.8 DATA FOR 48 NATIONAL UNIVERSITIES

WEB file Alumni

	State	Graduation Rate	% of Classes Under 20	Student-Faculty Ratio	Alumni Giving Rate
Boston College	MA	85	39	13	25
Brandeis University	MA	79	68	8	33
Brown University	RI	93	60	8	40
California Institute of Technology	CA	85	65	3	46
Carnegie Mellon University	PA	75	67	10	28
Case Western Reserve Univ.	OH	72	52	8	31
College of William and Mary	VA	89	45	12	27
Columbia University	NY	90	69	7	31
Cornell University	NY	91	72	13	35
Dartmouth College	NH	94	61	10	53
Duke University	NC	92	68	8	45
Emory University	GA	84	65	7	37
Georgetown University	DC	91	54	10	29
Harvard University	MA	97	73	8	46
Johns Hopkins University	MD	89	64	9	27
Lehigh University	PA	81	55	11	40
Massachusetts Inst. of Technology	MA	92	65	6	44
New York University	NY	72	63	13	13
Northwestern University	IL	90	66	8	30
Pennsylvania State Univ.	PA	80	32	19	21
Princeton University	NJ	95	68	5	67
Rice University	TX	92	62	8	40
Stanford University	CA	92	69	7	34
Tufts University	MA	87	67	9	29
Tulane University	LA	72	56	12	17
U. of California–Berkeley	CA	83	58	17	18
U. of California–Davis	CA	74	32	19	7
U. of California–Irvine	CA	74	42	20	9
U. of California–Los Angeles	CA	78	41	18	13
U. of California–San Diego	CA	80	48	19	8
U. of California–Santa Barbara	CA	70	45	20	12
U. of Chicago	IL	84	65	4	36
U. of Florida	FL	67	31	23	19
U. of Illinois–Urbana Champaign	IL	77	29	15	23
U. of Michigan–Ann Arbor	MI	83	51	15	13
U. of North Carolina–Chapel Hill	NC	82	40	16	26
U. of Notre Dame	IN	94	53	13	49
U. of Pennsylvania	PA	90	65	7	41
U. of Rochester	NY	76	63	10	23
U. of Southern California	CA	70	53	13	22
U. of Texas–Austin	TX	66	39	21	13
U. of Virginia	VA	92	44	13	28
U. of Washington	WA	70	37	12	12
U. of Wisconsin–Madison	WI	73	37	13	13
Vanderbilt University	TN	82	68	9	31
Wake Forest University	NC	82	59	11	38
Washington University–St. Louis	MO	86	73	7	33
Yale University	CT	94	77	7	50

Sand Saves: The percentage of time a player was able to get "up and down" once in a greenside sand bunker (regardless of score). "Up and down" indicates it took the player two shots or less to put the ball in the hole from a greenside sand bunker.

PPR (Putts Per Round): The average number of putts per round.

Scrambling: The percentage of time a player missed the green in regulation but still made par or better.

Managerial Report

1. To predict Scoring Average, develop estimated regression equations, first using DrDist as the independent variable and then using DrAccu as the independent variable. Which variable is the better predictor of Scoring Average? Discuss your findings.
2. Develop an estimated regression equation with GIR as the independent variable. Compare your findings with the results obtained using DrDist and DrAccu.
3. Develop an estimated regression equation with GIR and Sand Saves as the independent variables. Discuss your findings.
4. Develop an estimated regression equation with GIR and PPR as the independent variables. Discuss your findings.
5. Develop an estimated regression equation with GIR and Scrambling as the independent variables. Discuss your findings.
6. Compare the results obtained for the estimated regression equations that use GIR and Sand Saves, GIR and PPR, and GIR and Scrambling as the two independent variables. If you had to select one of these two-independent-variable estimated regression equations to predict Scoring Average, which estimated regression equation would you use? Explain.
7. Develop the estimated regression equation that uses GIR, Sand Saves, and PPR to predict Scoring Average. Compare the results to an estimated regression equation that uses GIR, PPR, and Scrambling as the independent variables.
8. Develop an estimated regression equation that uses GIR, Sand Saves, PPR, and Scrambling to predict Scoring Average. Discuss your results.

Case Problem 4 Predicting Winning Percentage for the NFL

The National Football League (NFL) records a variety of performance data for individuals and teams (NFL website). Some of the year-end performance data for the 2005 season appear in the file named NFLStats. Each row of the data set corresponds to an NFL team, and the teams are ranked by winning percentage. Descriptions for the data follow:

WinPct	Percentage of games won
DefYds/G	Average number of yards per game given up on defense
RushYds/G	Average number of rushing yards per game
PassYds/G	Average number of passing yards per game
FGPct	Percentage of field goals made
TakeInt	Takeaway interceptions; the total number of interceptions made by the team's defense
TakeFum	Takeaway fumbles; the total number of fumbles recovered by the team's defense
GiveInt	Giveaway interceptions; the total number of interceptions thrown by the team's offense
GiveFum	Giveaway fumbles; the total number of fumbles lost by the team's offense

Managerial Report

1. Use methods of descriptive statistics to summarize the data. Comment on the findings.
2. Develop an estimated regression equation that can be used to predict WinPct using DefYds/G, RushYds/G, PassYds/G, and FGPct. Discuss your findings.
3. Starting with the estimated regression equation developed in part (2), delete any independent variables that are not significant and develop a new estimated regression equation that can be used to predict WinPct. Use $\alpha = .05$.
4. Some football analysts believe that turnovers are one of the most important factors in determining a team's success. With Takeaways = TakeInt + TakeFum and Giveaways = GiveInt + GiveFum, let NetDiff = Takeaways − Giveaways. Develop an estimated regression equation that can be used to predict WinPct using NetDiff. Compare your results with the estimated regression equation developed in part (3).
5. Develop an estimated regression equation that can be used to predict WinPct using all the data provided.

Appendix Multiple Regression Analysis Using StatTools

Butler

In this appendix we show how StatTools can be used to perform the regression analysis computations for the Butler Trucking problem. Begin by using the Data Set Manager to create a StatTools data set for these data using the procedure described in the appendix in Chapter 1. The following steps describe how StatTools can be used to provide the regression results.

Step 1. Click the **StatTools** tab on the Ribbon
Step 2. In the **Analyses** group, click **Regression and Classification**
Step 3. Choose the **Regression** option
Step 4. When the StatTools - Regression dialog box appears,
Select **Multiple** in the **Regression Type** box
In the **Variables** section:
Click the **Format** button and select **Unstacked**
In the column labeled **I** select **Miles**
In the column labeled **I** select **Deliveries**
In the column labeled **D** select **Time**
Click **OK**

The regression analysis output will appear in a new worksheet.

In the appendix to Chapter 12 we showed how to compute prediction intervals using StatTools.

The StatTools - Regression dialog box contains a number of more advanced options for developing prediction interval estimates and producing residual plots. The StatTools Help facility provides information on using all of these options.

CHAPTER 14

Statistical Methods for Quality Control

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STATISTICS in PRACTICE

DOW CHEMICAL COMPANY*

FREEPORT, TEXAS

In 1940 the Dow Chemical Company purchased 800 acres of Texas land on the Gulf Coast to build a magnesium production facility. That original site has expanded to cover more than 5000 acres and holds one of the largest petrochemical complexes in the world. Among the products from Dow Texas Operations are magnesium, styrene, plastics, adhesives, solvent, glycol, and chlorine. Some products are made solely for use in other processes, but many end up as essential ingredients in products such as pharmaceuticals, toothpastes, dog food, water hoses, ice chests, milk cartons, garbage bags, shampoos, and furniture.

Dow's Texas Operations produce more than 30% of the world's magnesium, an extremely lightweight metal used in products ranging from tennis racquets to suitcases to "mag" wheels. The Magnesium Department was the first group in Texas Operations to train its technical people and managers in the use of statistical quality control. Some of the earliest successful applications of statistical quality control were in chemical processing.

In one application involving the operation of a drier, samples of the output were taken at periodic intervals; the average value for each sample was computed and recorded on a chart called an $\bar{x}$ chart. Such a chart enabled Dow analysts to monitor trends in the output that might indicate the process was not operating correctly. In one instance, analysts began to observe values for the sample mean that were not indicative of a process operating within its design limits. On further examination of the control chart and the operation itself, the analysts found that the variation could be traced to problems involving one operator. The $\bar{x}$ chart recorded after retraining the operator showed a significant improvement in the process quality.

© PRNewsFoto/The Dow Chemical Company

Statistical quality control has enabled Dow Chemical Company to improve its processing methods and output.

Dow achieves quality improvements everywhere it applies statistical quality control. Documented savings of several hundred thousand dollars per year are realized, and new applications are continually being discovered.

In this chapter we will show how an $\bar{x}$ chart such as the one used by Dow can be developed. Such charts are a part of statistical quality control known as statistical process control. We will also discuss methods of quality control for situations in which a decision to accept or reject a group of items is based on a sample.

*The authors are indebted to Clifford B. Wilson, Magnesium Technical Manager, The Dow Chemical Company, for providing this Statistics in Practice.

ASQ's Vision: "By making quality a global priority, an organizational imperative, and a personal ethic, the American Society for Quality becomes the community for everyone who seeks quality concepts, technology, and tools to improve themselves and their world" (ASQ website).

The American Society for Quality (ASQ) defines quality as "the totality of features and characteristics of a product or service that bears on its ability to satisfy given needs." In other words, quality measures how well a product or service meets customer needs. Organizations recognize that to be competitive in today's global economy, they must strive for a high level of quality. As a result, they place increased emphasis on methods for monitoring and maintaining quality.

Today, the customer-driven focus that is fundamental to high-performing organizations has changed the scope that quality issues encompass, from simply eliminating defects on a production line to developing broad-based corporate quality strategies. Broadening the scope of quality naturally leads to the concept of **total quality (TQ)**.

> Total Quality (TQ) is a people-focused management system that aims at continual increase in customer satisfaction at continually lower real cost. TQ is a total system approach (not a

> separate area or work program) and an integral part of high-level strategy; it works horizontally across function and departments, involves all employees, top to bottom, and extends backward and forward to include the supply chain and the customer chain. TQ stresses learning and adaptation to continual change as keys to organization success.[1]

Regardless of how it is implemented in different organizations, total quality is based on three fundamental principles: a focus on customers and stakeholders; participation and teamwork throughout the organization; and a focus on continuous improvement and learning. In the first section of the chapter we provide a brief introduction to three quality management frameworks: the Malcolm Baldrige Quality Award, ISO 9000 standards, and the Six Sigma philosophy. In the last two sections we introduce two statistical tools that can be used to monitor quality: statistical process control and acceptance sampling.

14.1 Philosophies and Frameworks

After World War II, Dr. W. Edwards Deming became a consultant to Japanese industry; he is credited with being the person who convinced top managers in Japan to use the methods of statistical quality control.

In the early twentieth century, quality control practices were limited to inspecting finished products and removing defective items. But this all changed as the result of the pioneering efforts of a young engineer named Walter A. Shewhart. After completing his doctorate in physics from the University of California in 1917, Dr. Shewhart joined the Western Electric Company, working in the inspection engineering department. In 1924 Dr. Shewhart prepared a memorandum that included a set of principles that are the basis for what is known today as process control. And his memo also contained a diagram that would be recognized as a statistical control chart. Continuing his work in quality at Bell Telephone Laboratories until his retirement in 1956, he brought together the disciplines of statistics, engineering, and economics and in doing so changed the course of industrial history. Dr. Shewhart is recognized as the father of statistical quality control and was the first honorary member of the ASQ.

Two other individuals who have had great influence on quality are Dr. W. Edwards Deming, a student of Dr. Shewhart, and Joseph Juran. These men helped educate the Japanese in quality management shortly after World War II. Although quality is everybody's job, Deming stressed that the focus on quality must be led by managers. He developed a list of 14 points that he believed represent the key responsibilities of managers. For instance, Deming stated that managers must cease dependence on mass inspection; must end the practice of awarding business solely on the basis of price; must seek continual improvement in all production processes and service; must foster a team-oriented environment; and must eliminate goals, slogans, and work standards that prescribe numerical quotas. Perhaps most important, managers must create a work environment in which a commitment to quality and productivity is maintained at all times.

Juran proposed a simple definition of quality: *fitness for use.* Juran's approach to quality focused on three quality processes: quality planning, quality control, and quality improvement. In contrast to Deming's philosophy, which required a major cultural change in the organization, Juran's programs were designed to improve quality by working within the current organizational system. Nonetheless, the two philosophies are similar in that they both focus on the need for top management to be involved and stress the need for continuous improvement, the importance of training, and the use of quality control techniques.

Many other individuals played significant roles in the quality movement, including Philip B. Crosby, A. V. Feigenbaum, Karou Ishikawa, and Genichi Taguchi. More specialized texts dealing exclusively with quality provide details of the contributions of each of

[1] J. R. Evans and W. M. Lindsay, *Managing for Quality and Performance Excellence*, 8th ed. (Cincinnati, OH: South-Western, 2011), p. 11.

these individuals. The contributions of all individuals involved in the quality movement helped define a set of best practices and led to numerous awards and certification programs. The two most significant programs are the U.S. Malcolm Baldrige National Quality Award and the international ISO 9000 certification process. In recent years, use of Six Sigma—a methodology for improving organizational performance based on rigorous data collection and statistical analysis—has also increased.

Malcolm Baldrige National Quality Award

The U.S. Commerce Department's National Institute of Standards and Technology (NIST) manages the Baldrige National Quality Program. More information can be obtained at the NIST website.

The Malcolm Baldrige National Quality Award is given by the president of the United States to organizations that apply and are judged to be outstanding in seven areas: leadership; strategic planning; customer and market focus; measurement, analysis, and knowledge management; human resource focus; process management; and business results. Congress established the award program in 1987 to recognize U.S. organizations for their achievements in quality and performance and to raise awareness about the importance of quality as a competitive edge. The award is named for Malcolm Baldrige, who served as secretary of commerce from 1981 until his death in 1987.

2004 was the final year for the Baldrige Stock Study because of the increase in the number of recipients that are either nonprofit or privately held businesses.

Since the presentation of the first awards in 1988, the Baldrige National Quality Program (BNQP) has grown in stature and impact. Approximately 2 million copies of the criteria have been distributed since 1988, and wide-scale reproduction by organizations and electronic access add to that number significantly. For eight years in a row, a hypothetical stock index, made up of publicly traded U.S. companies that had received the Baldrige Award, outperformed the Standard & Poor's 500. In one year, the "Baldrige Index" outperformed the S&P 500 by 4.4 to 1. Bob Barnett, executive vice president of Motorola, Inc., said, "We applied for the Award, not with the idea of winning, but with the goal of receiving the evaluation of the Baldrige Examiners. That evaluation was comprehensive, professional, and insightful . . . making it perhaps the most cost-effective, value-added business consultation available anywhere in the world today."

ISO 9000

ISO 9000 standards are revised periodically to improve the quality of the standard.

ISO 9000 is a series of five international standards published in 1987 by the International Organization for Standardization (ISO), Geneva, Switzerland. Companies can use the standards to help determine what is needed to maintain an efficient quality conformance system. For example, the standards describe the need for an effective quality system, for ensuring that measuring and testing equipment is calibrated regularly, and for maintaining an adequate record-keeping system. ISO 9000 registration determines whether a company complies with its own quality system. Overall, ISO 9000 registration covers less than 10% of the Baldrige Award criteria.

Six Sigma

In the late 1980s Motorola recognized the need to improve the quality of its products and services; their goal was to achieve a level of quality so good that for every million opportunities no more than 3.4 defects will occur. This level of quality is referred to as the six sigma level of quality, and the methodology created to reach this quality goal is referred to as **Six Sigma**.

An organization may undertake two kinds of Six Sigma projects:

- DMAIC (Define, Measure, Analyze, Improve, and Control) to help redesign existing processes
- DFSS (Design for Six Sigma) to design new products, processes, or services

In helping to redesign existing processes and design new processes, Six Sigma places a heavy emphasis on statistical analysis and careful measurement. Today, Six Sigma is a major tool in helping organizations achieve Baldrige levels of business performance and process quality. Many Baldrige examiners view Six Sigma as the ideal approach for implementing Baldrige improvement programs.

Six Sigma limits and defects per million opportunities In Six Sigma terminology, a *defect* is any mistake or error that is passed on to the customer. The Six Sigma process defines quality performance as defects per million opportunities (dpmo). As we indicated previously, Six Sigma represents a quality level of at most 3.4 dpmo. To illustrate how this quality level is measured, let us consider the situation at KJW Packaging.

KJW operates a production line where boxes of cereal are filled. The filling process has a mean of $\mu = 16.05$ ounces and a standard deviation of $\sigma = .10$ ounces. It is reasonable to assume the filling weights are normally distributed. The distribution of filling weights is shown in Figure 14.1. Suppose management considers 15.45 to 16.65 ounces to be acceptable quality limits for the filling process. Thus, any box of cereal that contains less than 15.45 or more than 16.65 ounces is considered to be a defect. Using Excel, it can be shown that 99.9999998% of the boxes filled will have between $16.05 - 6(.10) = 15.45$ ounces and $16.05 + 6(.10) = 16.65$ ounces. In other words, only .0000002% of the boxes filled will contain less than 15.45 ounces or more than 16.65 ounces. Thus, the likelihood of obtaining a defective box of cereal from the filling process appears to be extremely unlikely, because on average only two boxes in 10 million will be defective.

Using Excel, NORM.S.DIST (6,TRUE)−NORM.S.DIST (−6,TRUE) = .999999998.

Motorola's early work on Six Sigma convinced them that a process mean can shift on average by as much as 1.5 standard deviations. For instance, suppose that the process mean for KJW increases by 1.5 standard deviations or $1.5(.10) = .15$ ounces. With such a shift, the normal distribution of filling weights would now be centered at $\mu = 16.05 + .15 = 16.20$ ounces. With a process mean of $\mu = 16.05$ ounces, the probability of obtaining a box of cereal with more than 16.65 ounces is extremely small. But how does this probability

Using Excel, 1−NORM.S.DIST (4.5,TRUE) = .0000034.

FIGURE 14.1 NORMAL DISTRIBUTION OF CEREAL BOX FILLING WEIGHTS WITH A PROCESS MEAN $\mu = 16.05$

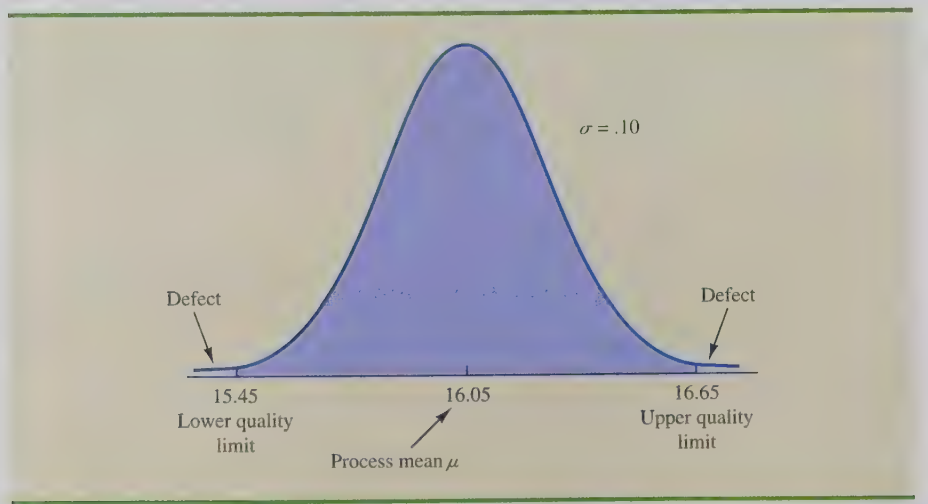
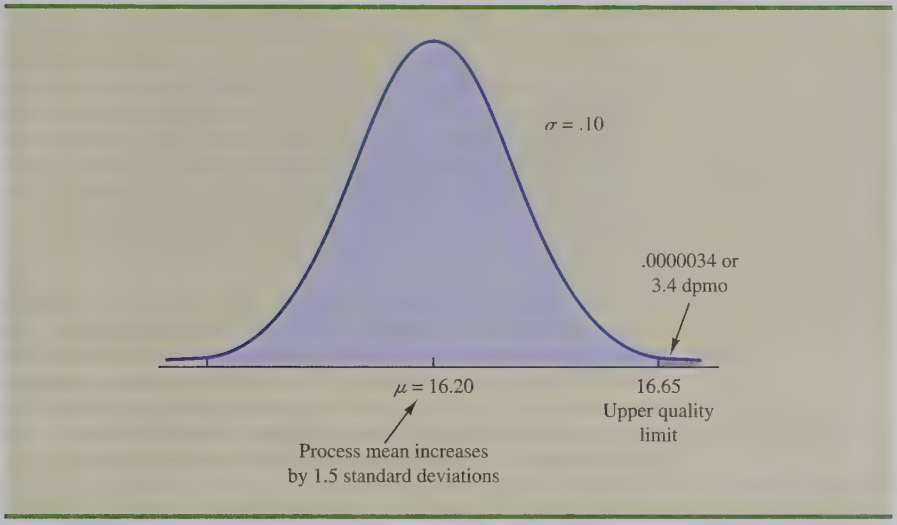


FIGURE 14.2 NORMAL DISTRIBUTION OF CEREAL BOX FILLING WEIGHTS WITH A PROCESS MEAN $\mu = 16.20$



change if the mean of the process shifts up to $\mu = 16.20$ ounces? Figure 14.2 shows that for this case, the upper quality limit of 16.65 is 4.5 standard deviations to the right of the new mean $\mu = 16.20$ ounces. Using this mean and Excel, we find that the probability of obtaining a box with more than 16.65 ounces is .0000034. Thus, if the process mean shifts up by 1.5 standard deviations, approximately 1,000,000(.0000034) = 3.4 boxes of cereal will exceed the upper limit of 16.65 ounces. In Six Sigma terminology, the quality level of the process is said to be 3.4 defects per million opportunities. If management of KJW considers 15.45 to 16.65 ounces to be acceptable quality limits for the filling process, the KJW filling process would be considered a Six Sigma process. Thus, if the process mean stays within 1.5 standard deviations of its target value $\mu = 16.05$ ounces, a maximum of only 3.4 defects per million boxes filled can be expected.

Organizations that want to achieve and maintain a Six Sigma level of quality must emphasize methods for monitoring and maintaining quality. *Quality assurance* refers to the entire system of policies, procedures, and guidelines established by an organization to achieve and maintain quality. Quality assurance consists of two principal functions: quality engineering and quality control. The object of *quality engineering* is to include quality in the design of products and processes and to identify quality problems prior to production. **Quality control** consists of a series of inspections and measurements used to determine whether quality standards are being met. If quality standards are not being met, corrective or preventive action can be taken to achieve and maintain conformance. In the next two sections we present two statistical methods used in quality control. The first method, *statistical process control,* uses graphical displays known as control charts to monitor a process; the goal is to determine whether the process can be continued or whether corrective action should be taken to achieve a desired quality level. The second method, *acceptance sampling,* is used in situations where a decision to accept or reject a group of items must be based on the quality found in a sample.

14.2 Statistical Process Control

In this section we consider quality control procedures for a production process whereby goods are manufactured continuously. On the basis of sampling and inspection of production output, a decision will be made to either continue the production process or adjust it to bring the items or goods being produced up to acceptable quality standards.

Continuous improvement is one of the most important concepts of the total quality management movement. The most important use of a control chart is in improving the process.

Despite high standards of quality in manufacturing and production operations, machine tools will invariably wear out, vibrations will throw machine settings out of adjustment, purchased materials will be defective, and human operators will make mistakes. Any or all of these factors can result in poor quality output. Fortunately, procedures are available for monitoring production output so that poor quality can be detected early and the production process can be adjusted or corrected.

If the variation in the quality of the production output is due to **assignable causes** such as tools wearing out, incorrect machine settings, poor quality raw materials, or operator error, the process should be adjusted or corrected as soon as possible. Alternatively, if the variation is due to what are called **common causes**—that is, randomly occurring variations in materials, temperature, humidity, and so on, which the manufacturer cannot possibly control—the process does not need to be adjusted. The main objective of statistical process control is to determine whether variations in output are due to assignable causes or common causes.

Whenever assignable causes are detected, we conclude that the process is *out of control.* In that case, corrective action will be taken to bring the process back to an acceptable level of quality. However, if the variation in the output of a production process is due only to common causes, we conclude that the process is *in statistical control,* or simply *in control;* in such cases, no changes or adjustments are necessary.

Process control procedures are closely related to hypothesis testing procedures discussed earlier in this text. Control charts provide an ongoing test of the hypothesis that the process is in control.

The statistical procedures for process control are based on the hypothesis testing methodology presented in Chapter 9. The null hypothesis H_0 is formulated in terms of the production process being in control. The alternative hypothesis H_a is formulated in terms of the production process being out of control. Table 14.1 shows that correct decisions to continue an in-control process and adjust an out-of-control process are possible. However, as with other hypothesis testing procedures, both a Type I error (adjusting an in-control process) and a Type II error (allowing an out-of-control process to continue) are also possible.

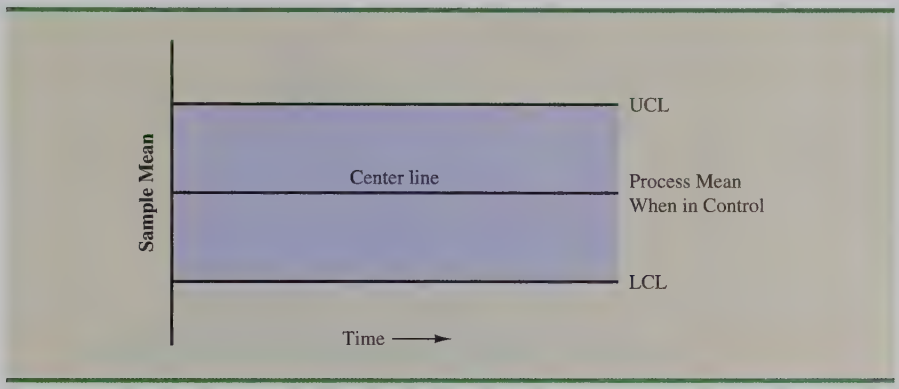
Control Charts

A **control chart** provides a basis for deciding whether the variation in the output is due to common causes (in control) or assignable causes (out of control). Whenever an

TABLE 14.1 THE OUTCOMES OF STATISTICAL PROCESS CONTROL

		State of Production Process	
		H_0 True Process in Control	H_0 False Process Out of Control
Decision	Continue Process	Correct decision	Type II error (allowing an out-of-control process to continue)
	Adjust Process	Type I error (adjusting an in-control process)	Correct decision

FIGURE 14.3 $\bar{x}$ CHART STRUCTURE



out-of-control situation is detected, adjustments or other corrective action will be taken to bring the process back into control.

Control charts based on data that can be measured on a continuous scale are called variables control charts. The $\bar{x}$ chart is a variables control chart.

Control charts can be classified by the type of data they contain. An **$\bar{x}$ chart** is used if the quality of the output of the process is measured in terms of a variable such as length, weight, temperature, and so on. In that case, the decision to continue or to adjust the production process will be based on the mean value found in a sample of the output. To introduce some of the concepts common to all control charts, let us consider some specific features of an $\bar{x}$ chart.

Figure 14.3 shows the general structure of an $\bar{x}$ chart. The center line of the chart corresponds to the mean of the process when the process is *in control.* The vertical line identifies the scale of measurement for the variable of interest. Each time a sample is taken from the production process, a value of the sample mean $\bar{x}$ is computed and a data point showing the value of $\bar{x}$ is plotted on the control chart.

The two lines labeled UCL and LCL are important in determining whether the process is in control or out of control. The lines are called the *upper control limit* and the *lower control limit,* respectively. They are chosen so that when the process is in control, there will be a high probability that the value of $\bar{x}$ will be between the two control limits. Values outside the control limits provide strong statistical evidence that the process is out of control and corrective action should be taken.

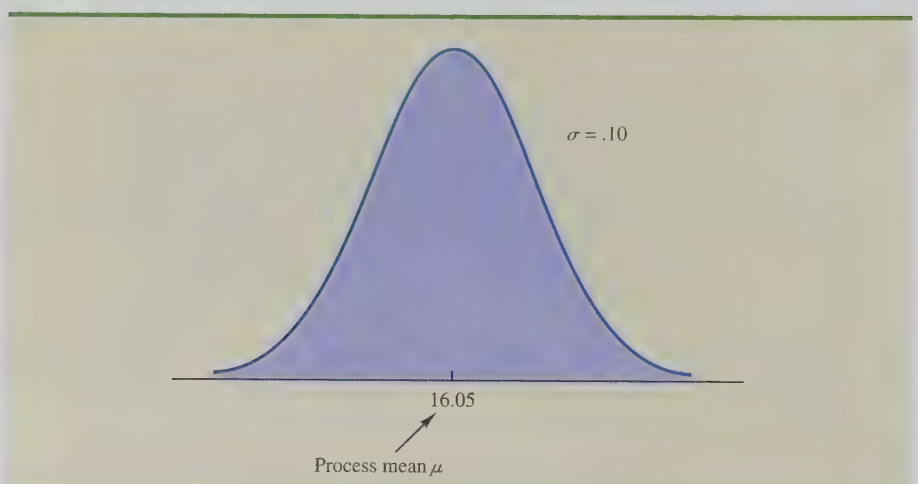
Over time, more and more data points will be added to the control chart. The order of the data points will be from left to right as the process is sampled. In essence, every time a point is plotted on the control chart, we are carrying out a hypothesis test to determine whether the process is in control.

In addition to the $\bar{x}$ chart, other control charts can be used to monitor the range of the measurements in the sample (***R* chart**), the proportion defective in the sample (**p chart**), and the number of defective items in the sample (***np* chart**). In each case, the control chart has a LCL, a center line, and an UCL similar to the $\bar{x}$ chart in Figure 14.3. The major difference among the charts is what the vertical axis measures; for instance, in a p chart the measurement scale denotes the proportion of defective items in the sample instead of the sample mean. In the following discussion, we will illustrate the construction and use of the $\bar{x}$ chart, R chart, p chart, and np chart.

$\bar{x}$ Chart: Process Mean and Standard Deviation Known

To illustrate the construction of an $\bar{x}$ chart, let us reconsider the situation at KJW Packaging. Recall that KJW operates a production line where cartons of cereal are filled. When the process

FIGURE 14.4 NORMAL DISTRIBUTION OF CEREAL CARTON FILLING WEIGHTS



is operating correctly—and hence the system is in control—the mean filling weight is $\mu = 16.05$ ounces, and the process standard deviation is $\sigma = .10$ ounces. In addition, the filling weights are assumed to be normally distributed. This distribution is shown in Figure 14.4.

The sampling distribution of $\bar{x}$, as presented in Chapter 7, can be used to determine the variation that can be expected in $\bar{x}$ values for a process that is in control. Let us first briefly review the properties of the sampling distribution of $\bar{x}$. First, recall that the expected value or mean of $\bar{x}$ is equal to μ, the mean filling weight when the production line is in control. For samples of size n, the equation for the standard deviation of $\bar{x}$, called the standard error of the mean, is

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \tag{14.1}$$

In addition, because the filling weights are normally distributed, the sampling distribution of $\bar{x}$ is normally distributed for any sample size. Thus, the sampling distribution of $\bar{x}$ is a normal distribution with mean μ and standard deviation $\sigma_{\bar{x}}$. This distribution is shown in Figure 14.5.

The sampling distribution of $\bar{x}$ is used to determine what values of $\bar{x}$ are reasonable if the process is in control. The general practice in quality control is to define as reasonable any value of $\bar{x}$ that is within 3 standard deviations, or standard errors, above or below the mean value. Recall from the study of the normal probability distribution that approximately 99.7% of the values of a normally distributed random variable are within ± 3 standard deviations of its mean value. Thus, if a value of $\bar{x}$ is within the interval $\mu - 3\sigma_{\bar{x}}$ to $\mu + 3\sigma_{\bar{x}}$, we will assume that the process is in control. In summary, then, the control limits for an $\bar{x}$ chart are as follows.

CONTROL LIMITS FOR AN $\bar{x}$ CHART: PROCESS MEAN AND STANDARD DEVIATION KNOWN

$$\text{UCL} = \mu + 3\sigma_{\bar{x}} \tag{14.2}$$

$$\text{LCL} = \mu - 3\sigma_{\bar{x}} \tag{14.3}$$

FIGURE 14.5 SAMPLING DISTRIBUTION OF $\bar{x}$ FOR A SAMPLE OF n FILLING WEIGHTS

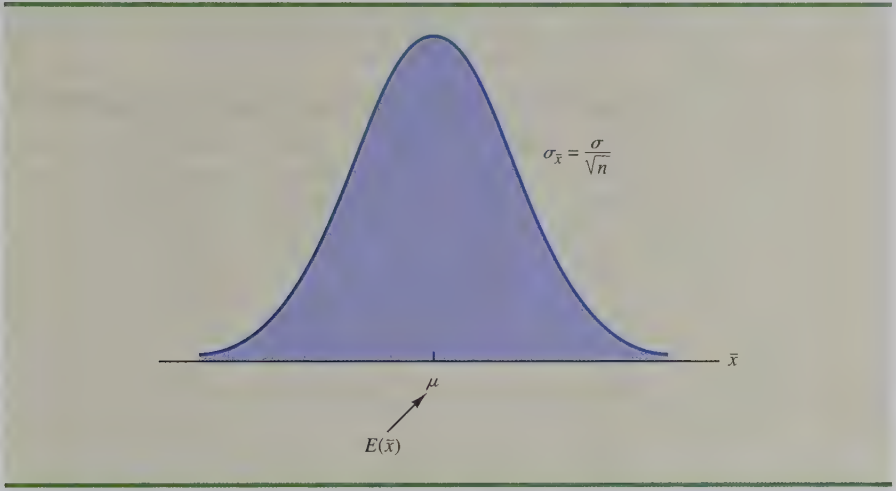
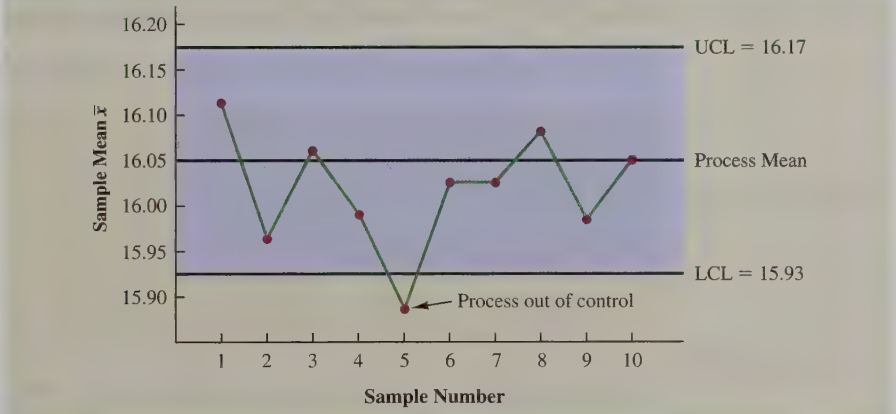


FIGURE 14.6 THE $\bar{x}$ CHART FOR THE CEREAL CARTON FILLING PROCESS



Reconsider the KJW Packaging example with the process distribution of filling weights shown in Figure 14.4 and the sampling distribution of $\bar{x}$ shown in Figure 14.5. Assume that a quality control inspector periodically samples six cartons and uses the sample mean filling weight to determine whether the process is in control or out of control. Using equation (14.1), we find that the standard error of the mean is $\sigma_{\bar{x}} = \sigma/\sqrt{n} = .10/\sqrt{6} = .04$. Thus, with the process mean at 16.05, the control limits are UCL = 16.05 + 3(.04) = 16.17 and LCL = 16.05 − 3(.04) = 15.93. Figure 14.6 is the control chart with the results of 10 samples taken over a 10-hour period. For ease of reading, the sample numbers 1 through 10 are listed below the chart.

Note that the mean for the fifth sample in Figure 14.6 shows there is strong evidence that the process is out of control. The fifth sample mean is below the LCL, indicating that

assignable causes of output variation are present and that underfilling is occurring. As a result, corrective action was taken at this point to bring the process back into control. The fact that the remaining points on the $\bar{x}$ chart are within the upper and lower control limits indicates that the corrective action was successful.

$\bar{x}$ Chart: Process Mean and Standard Deviation Unknown

In the KJW Packaging example, we showed how an $\bar{x}$ chart can be developed when the mean and standard deviation of the process are known. In most situations, the process mean and standard deviation must be estimated by using samples that are selected from the process when it is in control. For instance, KJW might select a random sample of five boxes each morning and five boxes each afternoon for 10 days of in-control operation. For each subgroup, or sample, the mean and standard deviation of the sample are computed. The overall averages of both the sample means and the sample standard deviations are used to construct control charts for both the process mean and the process standard deviation.

It is important to maintain control over both the mean and the variability of a process.

In practice, it is more common to monitor the variability of the process by using the range instead of the standard deviation because the range is easier to compute. The range can be used to provide good estimates of the process standard deviation; thus it can be used to construct upper and lower control limits for the $\bar{x}$ chart with little computational effort. To illustrate, let us consider the problem facing Jensen Computer Supplies, Inc.

Jensen Computer Supplies (JCS) manufactures 3.5-inch-diameter solid-state drives; they just finished adjusting their production process so that it is operating in control. Suppose random samples of five drives were selected during the first hour of operation, five drives were selected during the second hour of operation, and so on, until 20 samples were obtained. Table 14.2 provides the diameter of each drive sampled as well as the mean $\bar{x}_j$ and range R_j for each of the samples.

The estimate of the process mean μ is given by the overall sample mean.

OVERALL SAMPLE MEAN

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_k}{k} \tag{14.4}$$

where

$\bar{x}_j$ = mean of the jth sample $j = 1, 2, \ldots, k$
k = number of samples

For the JCS data in Table 14.2, the overall sample mean is $\bar{\bar{x}} = 3.4995$. This value will be the center line for the $\bar{x}$ chart. The range of each sample, denoted R_j, is simply the difference between the largest and smallest values in each sample. The average range for k samples is computed as follows.

AVERAGE RANGE

$$\bar{R} = \frac{R_1 + R_2 + \cdots + R_k}{k} \tag{14.5}$$

where

R_j = range of the jth sample, $j = 1, 2, \ldots, k$
k = number of samples

TABLE 14.2 DATA FOR THE JENSEN COMPUTER SUPPLIES PROBLEM

Sample Number	Observations					Sample Mean $\bar{x}_j$	Sample Range R_j
1	3.5056	3.5086	3.5144	3.5009	3.5030	3.5065	.0135
2	3.4882	3.5085	3.4884	3.5250	3.5031	3.5026	.0368
3	3.4897	3.4898	3.4995	3.5130	3.4969	3.4978	.0233
4	3.5153	3.5120	3.4989	3.4900	3.4837	3.5000	.0316
5	3.5059	3.5113	3.5011	3.4773	3.4801	3.4951	.0340
6	3.4977	3.4961	3.5050	3.5014	3.5060	3.5012	.0099
7	3.4910	3.4913	3.4976	3.4831	3.5044	3.4935	.0213
8	3.4991	3.4853	3.4830	3.5083	3.5094	3.4970	.0264
9	3.5099	3.5162	3.5228	3.4958	3.5004	3.5090	.0270
10	3.4880	3.5015	3.5094	3.5102	3.5146	3.5047	.0266
11	3.4881	3.4887	3.5141	3.5175	3.4863	3.4989	.0312
12	3.5043	3.4867	3.4946	3.5018	3.4784	3.4932	.0259
13	3.5043	3.4769	3.4944	3.5014	3.4904	3.4935	.0274
14	3.5004	3.5030	3.5082	3.5045	3.5234	3.5079	.0230
15	3.4846	3.4938	3.5065	3.5089	3.5011	3.4990	.0243
16	3.5145	3.4832	3.5188	3.4935	3.4989	3.5018	.0356
17	3.5004	3.5042	3.4954	3.5020	3.4889	3.4982	.0153
18	3.4959	3.4823	3.4964	3.5082	3.4871	3.4940	.0259
19	3.4878	3.4864	3.4960	3.5070	3.4984	3.4951	.0206
20	3.4969	3.5144	3.5053	3.4985	3.4885	3.5007	.0259

WEB file
Jensen

For the JCS data in Table 14.2, the average range is $\bar{R} = .0253$.

In the preceding section we showed that the upper and lower control limits for the $\bar{x}$ chart are

$$\bar{\bar{x}} \pm 3\frac{\sigma}{\sqrt{n}} \tag{14.6}$$

The overall sample mean $\bar{\bar{x}}$ is used to estimate μ and the sample ranges are used to develop an estimate of σ.

Hence, to construct the control limits for the $\bar{x}$ chart, we need to estimate μ and σ, the mean and standard deviation of the process. An estimate of μ is given by $\bar{\bar{x}}$. An estimate of σ can be developed by using the range data.

It can be shown that an estimator of the process standard deviation σ is the average range divided by d_2, a constant that depends on the sample size n. That is,

$$\text{Estimator of } \sigma = \frac{\bar{R}}{d_2} \tag{14.7}$$

The *American Society for Testing and Materials Manual on Presentation of Data and Control Chart Analysis* provides values for d_2 as shown in Table 14.3. For instance, when $n = 5$, $d_2 = 2.326$, and the estimate of σ is the average range divided by 2.326. If we substitute $\bar{R}/d_2$ for σ in expression (14.6), we can write the control limits for the $\bar{x}$ chart as

$$\bar{\bar{x}} \pm 3\frac{\bar{R}/d_2}{\sqrt{n}} = \bar{\bar{x}} \pm \frac{3}{d_2\sqrt{n}}\bar{R} = \bar{\bar{x}} \pm A_2\bar{R} \tag{14.8}$$

TABLE 14.3 FACTORS FOR $\bar{x}$ AND R CONTROL CHARTS

Observations in Sample, n	d_2	A_2	d_3	D_3	D_4
2	1.128	1.880	0.853	0	3.267
3	1.693	1.023	0.888	0	2.574
4	2.059	0.729	0.880	0	2.282
5	2.326	0.577	0.864	0	2.114
6	2.534	0.483	0.848	0	2.004
7	2.704	0.419	0.833	0.076	1.924
8	2.847	0.373	0.820	0.136	1.864
9	2.970	0.337	0.808	0.184	1.816
10	3.078	0.308	0.797	0.223	1.777
11	3.173	0.285	0.787	0.256	1.744
12	3.258	0.266	0.778	0.283	1.717
13	3.336	0.249	0.770	0.307	1.693
14	3.407	0.235	0.763	0.328	1.672
15	3.472	0.223	0.756	0.347	1.653
16	3.532	0.212	0.750	0.363	1.637
17	3.588	0.203	0.744	0.378	1.622
18	3.640	0.194	0.739	0.391	1.608
19	3.689	0.187	0.734	0.403	1.597
20	3.735	0.180	0.729	0.415	1.585
21	3.778	0.173	0.724	0.425	1.575
22	3.819	0.167	0.720	0.434	1.566
23	3.858	0.162	0.716	0.443	1.557
24	3.895	0.157	0.712	0.451	1.548
25	3.931	0.153	0.708	0.459	1.541

Source: Reprinted with permission from Table 27 of ASTM STP 15D, *ASTM Manual on Presentation of Data and Control Chart Analysis*, Copyright ASTM International, 100 Barr Harbor Drive, West Conshohocken, PA 19428.

Note that $A_2 = 3/(d_2\sqrt{n})$ is a constant that depends only on the sample size. Values for A_2 are provided in Table 14.3. For $n = 5$, $A_2 = .577$; thus, the control limits for the $\bar{x}$ chart are

$$3.4995 \pm (.577)(.0253) = 3.4995 \pm .0146$$

Hence, UCL = 3.514 and LCL = 3.485.

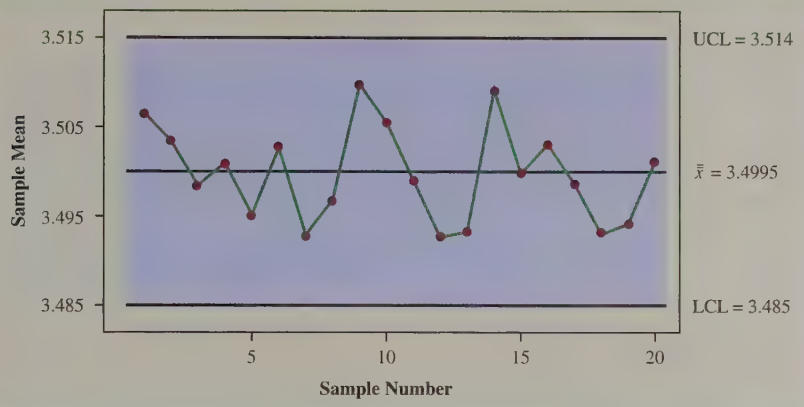
In the chapter appendix, we show how to contract $\bar{x}$ and R charts using StatTools.

Figure 14.7 shows the $\bar{x}$ chart for the Jensen Computer Supplies problem. We used the data in Table 14.2 and StatTools' X/R Charts routine to construct the chart. The center line is shown at the overall sample mean $\bar{\bar{x}} = 3.4995$. The upper control limit (UCL) is 3.514 and the lower control (LCL) is 3.485. The $\bar{x}$ chart shows the 20 sample means plotted over time. Because all 20 sample means are within the control limits, we confirm that the process mean was in control during the sampling period.

R Chart

Let us now consider a range chart (R chart) that can be used to control the variability of a process. To develop the R chart, we need to think of the range of a sample as a random variable with its own mean and standard deviation. The average range $\bar{R}$ provides an estimate

FIGURE 14.7 $\bar{x}$ CHART FOR THE JENSEN COMPUTER SUPPLIES PROBLEM



of the mean of this random variable. Moreover, it can be shown that an estimate of the standard deviation of the range is

$$\hat{\sigma}_R = d_3 \frac{\bar{R}}{d_2} \tag{14.9}$$

where d_2 and d_3 are constants that depend on the sample size; values of d_2 and d_3 are provided in Table 14.3. Thus, the UCL for the R chart is given by

$$\bar{R} + 3\hat{\sigma}_R = \bar{R}\left(1 + 3\frac{d_3}{d_2}\right) \tag{14.10}$$

and the LCL is

$$\bar{R} - 3\hat{\sigma}_R = \bar{R}\left(1 - 3\frac{d_3}{d_2}\right) \tag{14.11}$$

If we let

$$D_4 = 1 + 3\frac{d_3}{d_2} \tag{14.12}$$

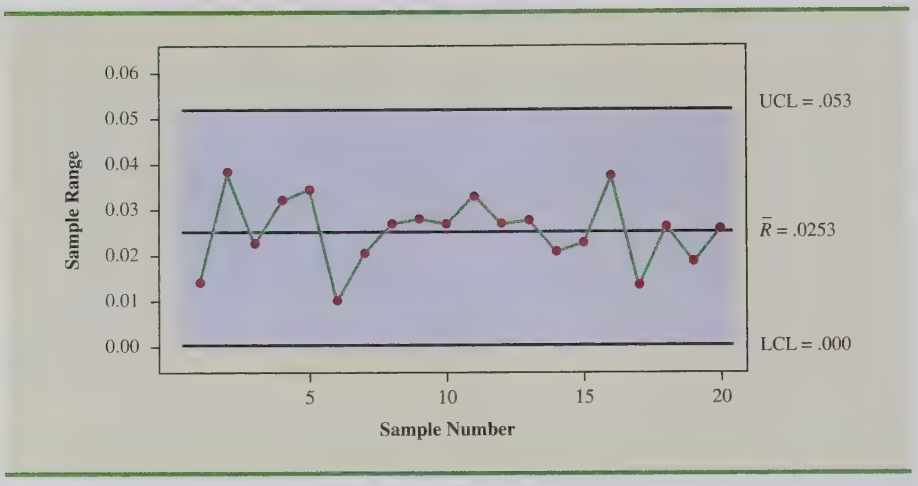
$$D_3 = 1 - 3\frac{d_3}{d_2} \tag{14.13}$$

we can write the control limits for the R chart as

$$\text{UCL} = \bar{R}D_4 \tag{14.14}$$

$$\text{LCL} = \bar{R}D_3 \tag{14.15}$$

FIGURE 14.8 *R* CHART FOR THE JENSEN COMPUTER SUPPLIES PROBLEM



Values for D_3 and D_4 are also provided in Table 14.3. Note that for $n = 5$, $D_3 = 0$ and $D_4 = 2.114$. Thus, with $\bar{R} = .0253$, the control limits are

$$\text{UCL} = .0253(2.114) = .053$$
$$\text{LCL} = .0253(0) = 0$$

If the R chart indicates that the process is out of control, the $\bar{x}$ chart should not be interpreted until the R chart indicates the process variability is in control.

Figure 14.8 shows the *R* chart for the Jensen Computer Supplies problem. We used the data in Table 14.2 and StatTools' X/R Charts routine to construct the chart. The center line is shown at the overall mean of the 20 sample ranges, $\bar{R} = .0253$. The UCL is .053 and the LCL is .000. The *R* chart shows the 20 sample ranges plotted over time. Because all 20 sample ranges are within the control limits, we confirm that the process variability was in control during the sampling period.

p Chart

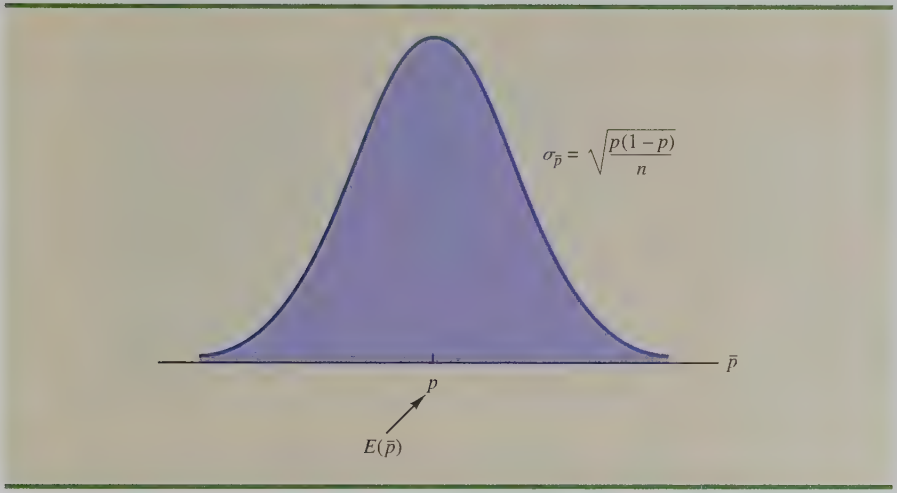
Control charts that are based on data indicating the presence of a defect or a number of defects are called attributes control charts. A p chart is an attributes control chart.

Let us consider the case in which the output quality is measured by either nondefective or defective items. The decision to continue or to adjust the production process will be based on $\bar{p}$, the proportion of defective items found in a sample. The control chart used for proportion-defective data is called a *p* chart.

To illustrate the construction of a *p* chart, consider the use of automated mail-sorting machines in a post office. These automated machines scan the zip codes on letters and divert each letter to its proper carrier route. Even when a machine is operating properly, some letters are diverted to incorrect routes. Assume that when a machine is operating correctly, or in a state of control, 3% of the letters are incorrectly diverted. Thus *p*, the proportion of letters incorrectly diverted when the process is in control, is .03.

The sampling distribution of $\bar{p}$, as presented in Chapter 7, can be used to determine the variation that can be expected in $\bar{p}$ values for a process that is in control. Recall that the expected value or mean of $\bar{p}$ is *p*, the proportion defective when the process is in control. With

FIGURE 14.9 SAMPLING DISTRIBUTION OF $\bar{p}$



samples of size n, the formula for the standard deviation of $\bar{p}$, called the standard error of the proportion, is

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} \tag{14.16}$$

We also learned in Chapter 7 that the sampling distribution of $\bar{p}$ can be approximated by a normal distribution whenever the sample size is large. With $\bar{p}$, the sample size can be considered large whenever the following two conditions are satisfied.

$$np \geq 5$$
$$n(1-p) \geq 5$$

In summary, whenever the sample size is large, the sampling distribution of $\bar{p}$ can be approximated by a normal distribution with mean p and standard deviation $\sigma_{\bar{p}}$. This distribution is shown in Figure 14.9.

To establish control limits for a p chart, we follow the same procedure we used to establish control limits for an chart. That is, the limits for the control chart are set at 3 standard deviations, or standard errors, above and below the proportion defective when the process is in control. Thus, we have the following control limits.

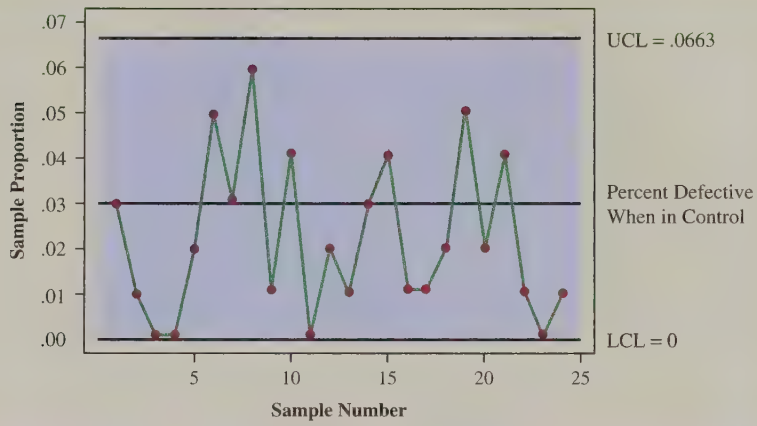
CONTROL LIMITS FOR A p CHART

$$\text{UCL} = p + 3\sigma_{\bar{p}} \tag{14.17}$$
$$\text{LCL} = p - 3\sigma_{\bar{p}} \tag{14.18}$$

With $p = .03$ and samples of size $n = 200$, equation (14.16) shows that the standard error is

$$\sigma_{\bar{p}} = \sqrt{\frac{.03(1-.03)}{200}} = .0121$$

FIGURE 14.10 p CHART FOR THE PROPORTION DEFECTIVE IN A MAIL-SORTING PROCESS



Hence, the control limits are UCL = .03 + 3(.0121) = .0663 and LCL = .03 − 3(.0121) = −.0063. Whenever equation (14.18) provides a negative value for LCL, LCL is set equal to zero in the control chart.

Figure 14.10 is the p chart for the mail-sorting process. The points plotted show the sample proportion defective found in samples of letters taken from the process. All points are within the control limits, providing no evidence to conclude that the sorting process is out of control.

If the proportion of defective items for a process that is in control is not known, that value is first estimated by using sample data. Suppose, for example, that k different samples, each of size n, are selected from a process that is in control. The fraction or proportion of defective items in each sample is then determined. Treating all the data collected as one large sample, we can compute the proportion of defective items for all the data; that value can then be used to estimate p, the proportion of defective items observed when the process is in control. Note that this estimate of p also enables us to estimate the standard error of the proportion; upper and lower control limits can then be established.

np Chart

An np chart is a control chart developed for the number of defective items in a sample. In this case, n is the sample size and p is the probability of observing a defective item when the process is in control. Whenever the sample size is large, that is, when $np \geq 5$ and $n(1 - p) \geq 5$, the distribution of the number of defective items observed in a sample of size n can be approximated by a normal distribution with mean np and standard deviation $\sqrt{np(1 - p)}$. Thus, for the mail-sorting example, with $n = 200$ and $p = .03$, the number of defective items observed in a sample of 200 letters can be approximated by a normal distribution with a mean of 200(.03) = 6 and a standard deviation of $\sqrt{200(.03)(.97)} = 2.4125$.

The control limits for an np chart are set at 3 standard deviations above and below the expected number of defective items observed when the process is in control. Thus, we have the following control limits.

CONTROL LIMITS FOR AN *np* CHART

$$\text{UCL} = np + 3\sqrt{np(1-p)} \quad (14.19)$$

$$\text{LCL} = np - 3\sqrt{np(1-p)} \quad (14.20)$$

For the mail-sorting process example, with $p = .03$ and $n = 200$, the control limits are UCL = 6 + 3(2.4125) = 13.2375 and LCL = 6 − 3(2.4125) = −1.2375. When LCL is negative, LCL is set equal to zero in the control chart. Hence, if the number of letters diverted to incorrect routes is greater than 13, the process is concluded to be out of control.

The information provided by an *np* chart is equivalent to the information provided by the *p* chart; the only difference is that the *np* chart is a plot of the number of defective items observed, whereas the *p* chart is a plot of the proportion of defective items observed. Thus, if we were to conclude that a particular process is out of control on the basis of a *p* chart, the process would also be concluded to be out of control on the basis of an *np* chart.

Interpretation of Control Charts

The location and pattern of points in a control chart enable us to determine, with a small probability of error, whether a process is in statistical control. A primary indication that a process may be out of control is a data point outside the control limits, such as point 5 in Figure 14.6. Finding such a point is statistical evidence that the process is out of control; in such cases, corrective action should be taken as soon as possible.

In addition to points outside the control limits, certain patterns of the points within the control limits can be warning signals of quality control problems. For example, assume that all the data points are within the control limits but that a large number of points are on one side of the center line. This pattern may indicate that an equipment problem, a change in materials, or some other assignable cause of a shift in quality has occurred. Careful investigation of the production process should be undertaken to determine whether quality has changed.

Even if all points are within the upper and lower control limits, a process may not be in control. Trends in the sample data points or unusually long runs above or below the center line may also indicate out-of-control conditions.

Another pattern to watch for in control charts is a gradual shift, or trend, over time. For example, as tools wear out, the dimensions of machined parts will gradually deviate from their designed levels. Gradual changes in temperature or humidity, general equipment deterioration, dirt buildup, or operator fatigue may also result in a trend pattern in control charts. Six or seven points in a row that indicate either an increasing or decreasing trend should be cause for concern, even if the data points are all within the control limits. When such a pattern occurs, the process should be reviewed for possible changes or shifts in quality. Corrective action to bring the process back into control may be necessary.

NOTES AND COMMENTS

1. Because the control limits for the $\bar{x}$ chart depend on the value of the average range, these limits will not have much meaning unless the process variability is in control. In practice, the *R* chart is usually constructed before the $\bar{x}$ chart; if the *R* chart indicates that the process variability is in control, then the $\bar{x}$ chart is constructed. The StatTools X/R Charts option provides the $\bar{x}$ chart and the *R* chart simultaneously. The steps of this procedure are described in the chapter appendix.
2. An *np* chart is used to monitor a process in terms of the number of defects. The Motorola Six Sigma Quality Level sets a goal of producing no more than 3.4 defects per million operations; this goal implies $p = .0000034$.

Exercises

Methods

1. A process that is in control has a mean of $\mu = 12.5$ and a standard deviation of $\sigma = .8$.
   a. Construct the $\bar{x}$ control chart for this process if samples of size 4 are to be used.
   b. Repeat part (a) for samples of size 8 and 16.
   c. What happens to the limits of the control chart as the sample size is increased? Discuss why this is reasonable.
2. Twenty-five samples, each of size 5, were selected from a process that was in control. The sum of all the data collected was 677.5 pounds.
   a. What is an estimate of the process mean (in terms of pounds per unit) when the process is in control?
   b. Develop the $\bar{x}$ control chart for this process if samples of size 5 will be used. Assume that the process standard deviation is .5 when the process is in control, and that the mean of the process is the estimate developed in part (a).
3. Twenty-five samples of 100 items each were inspected when a process was considered to be operating satisfactorily. In the 25 samples, a total of 135 items were found to be defective.
   a. What is an estimate of the proportion defective when the process is in control?
   b. What is the standard error of the proportion if samples of size 100 will be used for statistical process control?
   c. Compute the upper and lower control limits for the control chart.
4. SELF test A process sampled 20 times with a sample of size 8 resulted in $\bar{\bar{x}} = 28.5$ and $\bar{R} = 1.6$. Compute the upper and lower control limits for the $\bar{x}$ and R charts for this process.

Applications

5. Temperature is used to measure the output of a production process. When the process is in control, the mean of the process is $\mu = 128.5$ and the standard deviation is $\sigma = .4$.
   a. Construct the $\bar{x}$ chart for this process if samples of size 6 are to be used.
   b. Is the process in control for a sample providing the following data?

      128.8 128.2 129.1 128.7 128.4 129.2

   c. Is the process in control for a sample providing the following data?

      129.3 128.7 128.6 129.2 129.5 129.0

6. A quality control process monitors the weight per carton of laundry detergent. Control limits are set at UCL = 20.12 ounces and LCL = 19.90 ounces. Samples of size 5 are used for the sampling and inspection process. What are the process mean and process standard deviation for the manufacturing operation?
7. The Goodman Tire and Rubber Company periodically tests its tires for tread wear under simulated road conditions. To study and control the manufacturing process, 20 samples, each containing three radial tires, were chosen from different shifts over several days of operation, with the following results. Assuming that these data were collected when the manufacturing process was believed to be operating in control, develop the R and $\bar{x}$ charts.

Sample	Tread Wear*		
1	31	42	28
2	26	18	35
3	25	30	34
4	17	25	21
5	38	29	35
6	41	42	36
7	21	17	29
8	32	26	28
9	41	34	33
10	29	17	30
11	26	31	40
12	23	19	25
13	17	24	32
14	43	35	17
15	18	25	29
16	30	42	31
17	28	36	32
18	40	29	31
19	18	29	28
20	22	34	26

*Hundredths of an inch

WEB file
Tires

8. Over several weeks of normal, or in-control, operation, 20 samples of 150 packages each of synthetic-gut tennis strings were tested for breaking strength. A total of 141 packages of the 3000 tested failed to conform to the manufacturer's specifications.
   a. What is an estimate of the process proportion defective when the system is in control?
   b. Compute the upper and lower control limits for a p chart.
   c. With the results of part (b), what conclusion should be made about the process if tests on a new sample of 150 packages find 12 defective? Do there appear to be assignable causes in this situation?
   d. Compute the upper and lower control limits for an np chart.
   e. Answer part (c) using the results of part (d).
   f. Which control chart would be preferred in this situation? Explain.

9. An automotive industry supplier produces pistons for several models of automobiles. Twenty samples, each consisting of 200 pistons, were selected when the process was known to be operating correctly. The numbers of defective pistons found in the samples follow.

8	10	6	4	5	7	8	12	8	15
14	10	10	7	5	8	6	10	4	8

   a. What is an estimate of the proportion defective for the piston manufacturing process when it is in control?
   b. Construct the p chart for the manufacturing process, assuming each sample has 200 pistons.
   c. With the results of part (b), what conclusion should be made if a sample of 200 has 20 defective pistons?
   d. Compute the upper and lower control limits for an np chart.
   e. Answer part (c) using the results of part (d).

14.3 Acceptance Sampling

In acceptance sampling, the items of interest can be incoming shipments of raw materials or purchased parts as well as finished goods from final assembly. Suppose we want to decide whether to accept or reject a group of items on the basis of specified quality characteristics. In quality control terminology, the group of items is a **lot**, and **acceptance sampling** is a statistical method that enables us to base the accept-reject decision on the inspection of a sample of items from the lot.

The general steps of acceptance sampling are shown in Figure 14.11. After a lot is received, a sample of items is selected for inspection. The results of the inspection are compared to specified quality characteristics. If the quality characteristics are satisfied, the lot is accepted and sent to production or shipped to customers. If the lot is rejected, managers must decide on its disposition. In some cases, the decision may be to keep the lot and remove the unacceptable or nonconforming items. In other cases, the lot may be returned to the supplier at the supplier's expense; the extra work and cost placed on the supplier can motivate the supplier to provide high-quality lots. Finally, if the rejected lot consists of finished goods, the goods must be scrapped or reworked to meet acceptable quality standards.

FIGURE 14.11 ACCEPTANCE SAMPLING PROCEDURE

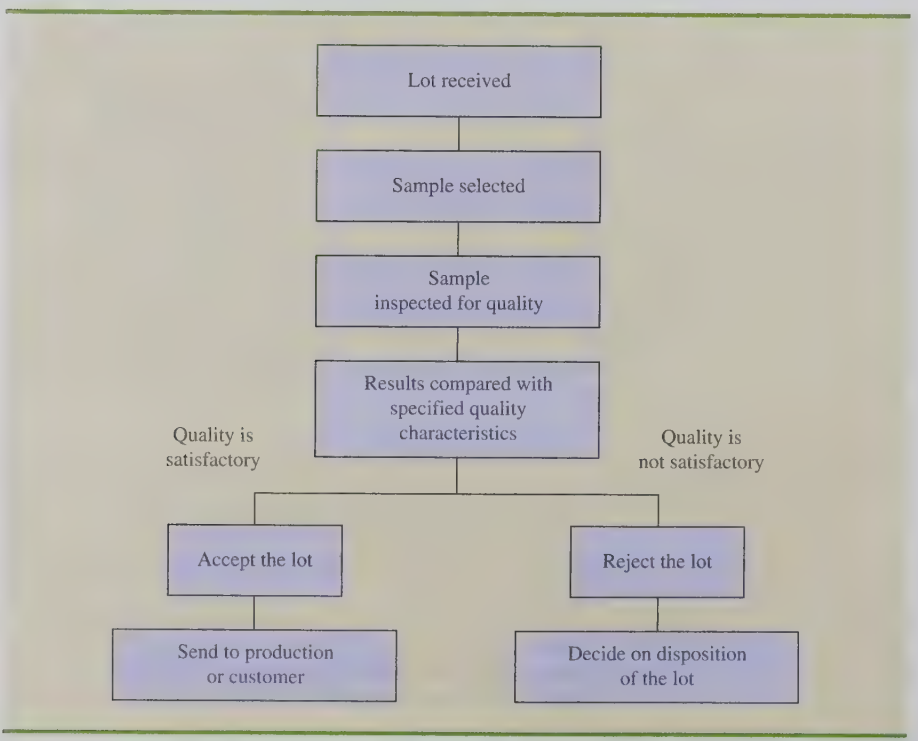


TABLE 14.4 THE OUTCOMES OF ACCEPTANCE SAMPLING

		State of the Lot	
		H_0 True Good-Quality Lot	H_0 False Poor-Quality Lot
Decision	Accept the Lot	Correct decision	Type II error (accepting a poor-quality lot)
	Reject the Lot	Type I error (rejecting a good-quality lot)	Correct decision

Acceptance sampling has the following advantages over 100% inspection:
1. *Usually less expensive*
2. *Less product damage due to less handling and testing*
3. *Fewer inspectors required*
4. *The only approach possible if destructive testing must be used*

The statistical procedure of acceptance sampling is based on the hypothesis testing methodology presented in Chapter 9. The null and alternative hypotheses are stated as follows.

H_0: Good-quality lot

H_a: Poor-quality lot

Table 14.4 shows the results of the hypothesis testing procedure. Note that correct decisions correspond to accepting a good-quality lot and rejecting a poor-quality lot. However, as with other hypothesis testing procedures, we need to be aware of the possibilities of making a Type I error (rejecting a good-quality lot) or a Type II error (accepting a poor-quality lot).

The probability of a Type I error creates a risk for the producer of the lot and is known as the **producer's risk**. For example, a producer's risk of .05 indicates a 5% chance that a good-quality lot will be erroneously rejected. The probability of a Type II error, on the other hand, creates a risk for the consumer of the lot and is known as the **consumer's risk**. For example, a consumer's risk of .10 means a 10% chance that a poor-quality lot will be erroneously accepted and thus used in production or shipped to the customer. Specific values for the producer's risk and the consumer's risk can be controlled by the person designing the acceptance sampling procedure. To illustrate how to assign risk values, let us consider the problem faced by KALI, Inc.

KALI, Inc.: An Example of Acceptance Sampling

KALI, Inc., manufactures home appliances that are marketed under a variety of trade names. However, KALI does not manufacture every component used in its products. Several components are purchased directly from suppliers. For example, one of the components that KALI purchases for use in home air conditioners is an overload protector, a device that turns off the compressor if it overheats. The compressor can be seriously damaged if the overload protector does not function properly, and therefore KALI is concerned about the quality of the overload protectors. One way to ensure quality would be to test every component received through an approach known as 100% inspection. However, to determine proper functioning of an overload protector, the device must be subjected to time-consuming and expensive tests, and KALI cannot justify testing every overload protector it receives.

Instead, KALI uses an acceptance sampling plan to monitor the quality of the overload protectors. The acceptance sampling plan requires that KALI's quality control inspectors select and test a sample of overload protectors from each shipment. If very few defective units are found in the sample, the lot is probably of good quality and should be accepted. However, if a large number of defective units are found in the sample, the lot is probably of poor quality and should be rejected.

An acceptance sampling plan consists of a sample size n and an acceptance criterion c. The **acceptance criterion** is the maximum number of defective items that can be found in the sample and still indicate an acceptable lot. For example, for the KALI problem let us assume that a sample of 15 items will be selected from each incoming shipment or lot. Furthermore, assume that the manager of quality control states that the lot can be accepted only if no defective items are found. In this case, the acceptance sampling plan established by the quality control manager is $n = 15$ and $c = 0$.

This acceptance sampling plan is easy for the quality control inspector to implement. The inspector simply selects a sample of 15 items, performs the tests, and reaches a conclusion based on the following decision rule.

- *Accept the lot* if zero defective items are found.
- *Reject the lot* if one or more defective items are found.

Before implementing this acceptance sampling plan, the quality control manager wants to evaluate the risks or errors possible under the plan. The plan will be implemented only if both the producer's risk (Type I error) and the consumer's risk (Type II error) are controlled at reasonable levels.

Computing the Probability of Accepting a Lot

The key to analyzing both the producer's risk and the consumer's risk is a "what-if" type of analysis. That is, we will assume that a lot has some known percentage of defective items and compute the probability of accepting the lot for a given sampling plan. By varying the assumed percentage of defective items, we can examine the effect of the sampling plan on both types of risks.

Let us begin by assuming that a large shipment of overload protectors has been received and that 5% of the overload protectors in the shipment are defective. For a shipment or lot with 5% of the items defective, what is the probability that the $n = 15$, $c = 0$ sampling plan will lead us to accept the lot? Because each overload protector tested will be either defective or nondefective and because the lot size is large, the number of defective items in a sample of 15 has a *binomial distribution.* The binomial probability function, which was presented in Chapter 5, follows.

BINOMIAL PROBABILITY FUNCTION FOR ACCEPTANCE SAMPLING

$$f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{(n-x)} \qquad (14.21)$$

where

n = the sample size
p = the proportion of defective items in the lot
x = the number of defective items in the sample
$f(x)$ = the probability of x defective items in the sample

For the KALI acceptance sampling plan, $n = 15$; thus, for a lot with 5% defective ($p = .05$), we have

$$f(x) = \frac{15!}{x!(15-x)!} (.05)^x (1-.05)^{(15-x)} \qquad (14.22)$$

TABLE 14.5 PROBABILITY OF ACCEPTING THE LOT FOR THE KALI PROBLEM WITH $n = 15$ AND $c = 0$

Percent Defective in the Lot	Probability of Accepting the Lot
1	.8601
2	.7386
3	.6333
4	.5421
5	.4633
10	.2059
15	.0874
20	.0352
25	.0134

Using equation (14.22), $f(0)$ will provide the probability that zero overload protectors will be defective and the lot will be accepted. In using equation (14.22), recall that $0! = 1$. Thus, the probability computation for $f(0)$ is

$$f(0) = \frac{15!}{0!(15-0)!}(.05)^0(1-.05)^{(15-0)}$$
$$= \frac{15!}{0!(15)!}(.05)^0(.95)^{15} = (.95)^{15} = .4633$$

We now know that the $n = 15$, $c = 0$ sampling plan has a .4633 probability of accepting a lot with 5% defective items. Hence, there must be a corresponding $1 - .4633 = .5367$ probability of rejecting a lot with 5% defective items.

Binomial probabilities can be easily computed using Excel. See Chapter 5.

Excel's BINOM.DIST function can be used to simplify making these binomial probability calculations. Using this function, we can determine that if the lot contains 10% defective items, there is a .2059 probability that the $n = 15$, $c = 0$ sampling plan will indicate an acceptable lot. The probability that the $n = 15$, $c = 0$ sampling plan will lead to the acceptance of lots with 1%, 2%, 3%, . . . defective items is summarized in Table 14.5.

Using the probabilities in Table 14.5, a graph of the probability of accepting the lot versus the percent defective in the lot can be drawn as shown in Figure 14.12. This graph, or curve, is called the **operating characteristic (OC) curve** for the $n = 15$, $c = 0$ acceptance sampling plan.

Perhaps we should consider other sampling plans, ones with different sample sizes n or different acceptance criteria c. First consider the case in which the sample size remains $n = 15$ but the acceptance criterion increases from $c = 0$ to $c = 1$. That is, we will now accept the lot if zero or one defective component is found in the sample. For a lot with 5% defective items ($p = .05$), the binomial probability function in equation (14.21), or Excel's BINOM.DIST function, can be used to compute $f(0) = .4633$ and $f(1) = .3658$. Thus, there is a $.4633 + .3658 = .8291$ probability that the $n = 15$, $c = 1$ plan will lead to the acceptance of a lot with 5% defective items.

Continuing these calculations, we obtain Figure 14.13, which shows the operating characteristic curves for four alternative acceptance sampling plans for the KALI problem. Samples of size 15 and 20 are considered. Note that regardless of the proportion defective in the lot, the $n = 15$, $c = 1$ sampling plan provides the highest probabilities of accepting the lot. The $n = 20$, $c = 0$ sampling plan provides the lowest probabilities of accepting the lot; however, that plan also provides the highest probabilities of rejecting the lot.

FIGURE 14.12 OPERATING CHARACTERISTIC CURVE FOR THE $n = 15$, $c = 0$ ACCEPTANCE SAMPLING PLAN

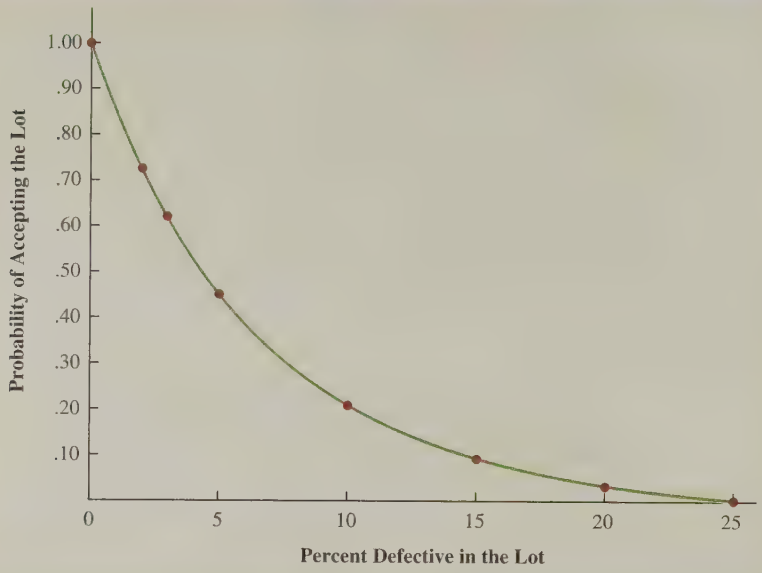
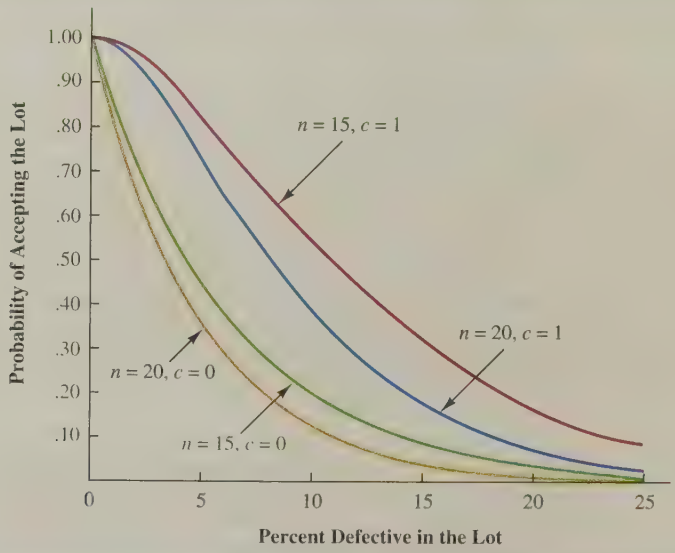


FIGURE 14.13 OPERATING CHARACTERISTIC CURVES FOR FOUR ACCEPTANCE SAMPLING PLANS



Selecting an Acceptance Sampling Plan

Now that we know how to use the binomial distribution to compute the probability of accepting a lot with a given proportion defective, we are ready to select the values of n and c that determine the desired acceptance sampling plan for the application being studied. To develop this plan, managers must specify two values for the fraction defective in the lot. One value, denoted p_0, will be used to control for the producer's risk, and the other value, denoted p_1, will be used to control for the consumer's risk.

We will use the following notation.

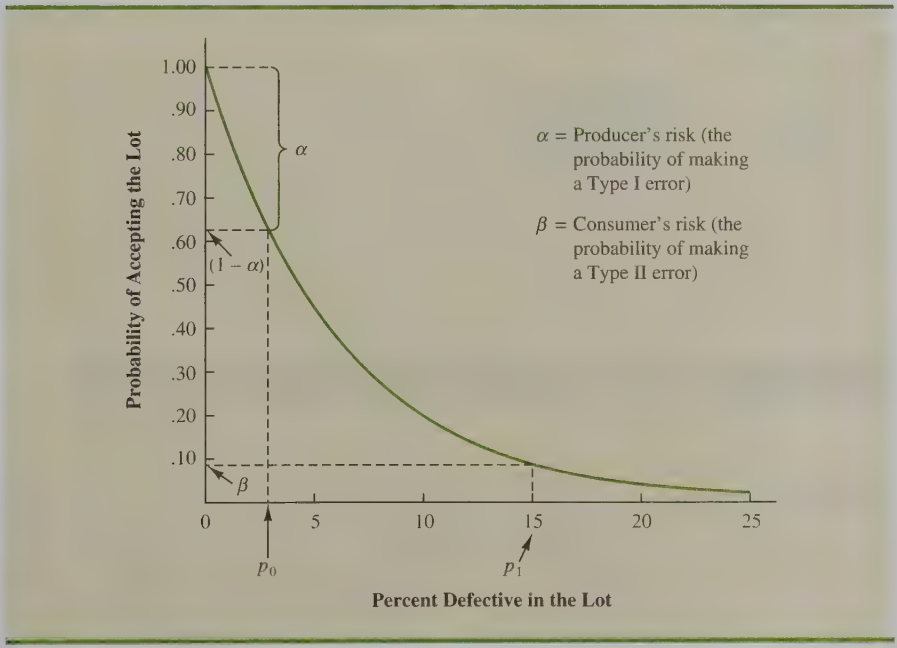
α = the producer's risk; the probability of rejecting a lot with p_0 defective items

β = the consumer's risk; the probability of accepting a lot with p_1 defective items

Suppose that for the KALI problem, the managers specify that $p_0 = .03$ and $p_1 = .15$. From the OC curve for $n = 15$, $c = 0$ in Figure 14.14, we see that $p_0 = .03$ provides a producer's risk of approximately $1 - .63 = .37$, and $p_1 = .15$ provides a consumer's risk of approximately .09. Thus, if the managers are willing to tolerate both a .37 probability of rejecting a lot with 3% defective items (producer's risk) and a .09 probability of accepting a lot with 15% defective items (consumer's risk), the $n = 15$, $c = 0$ acceptance sampling plan would be acceptable.

Suppose, however, that the managers request a producer's risk of $\alpha = .10$ and a consumer's risk of $\beta = .19$. We see that now the $n = 15$, $c = 0$ sampling plan has a better-than-desired consumer's risk but an unacceptably large producer's risk. The fact that

FIGURE 14.14 OPERATING CHARACTERISTIC CURVE FOR $n = 15$, $c = 0$ WITH $p_0 = .03$ AND $p_1 = .15$



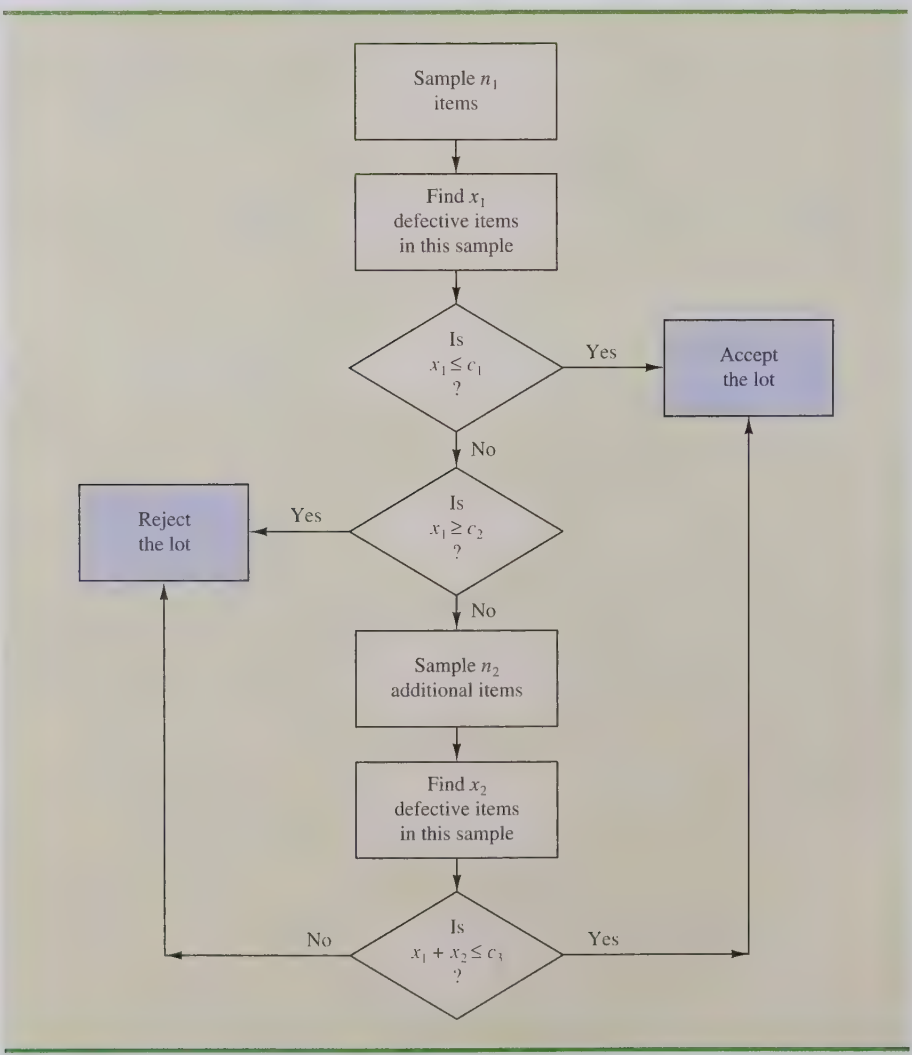
$\alpha = .37$ indicates that 37% of the lots will be erroneously rejected when only 3% of the items in them are defective. The producer's risk is too high, and a different acceptance sampling plan should be considered.

Exercise 13 at the end of this section will ask you to compute the producer's risk and the consumer's risk for the $n = 20$, $c = 1$ sampling plan.

Using $p_0 = .03$, $\alpha = .10$, $p_1 = .15$, and $\beta = .20$, Figure 14.13 shows that the acceptance sampling plan with $n = 20$ and $c = 1$ comes closest to meeting both the producer's and the consumer's risk requirements.

As shown in this section, several computations and several operating characteristic curves may need to be considered to determine the sampling plan with the desired producer's and consumer's risk. Fortunately, tables of sampling plans are published. For example, the American Military Standard Table, MIL-STD-105D, provides information

FIGURE 14.15 A TWO-STAGE ACCEPTANCE SAMPLING PLAN



helpful in designing acceptance sampling plans. More advanced texts on quality control, such as those listed in the bibliography, describe the use of such tables. The advanced texts also discuss the role of sampling costs in determining the optimal sampling plan.

Multiple Sampling Plans

The acceptance sampling procedure we presented for the KALI problem is a *single-sample* plan. It is called a single-sample plan because only one sample or sampling stage is used. After the number of defective components in the sample is determined, a decision must be made to accept or reject the lot. An alternative to the single-sample plan is a **multiple sampling plan**, in which two or more stages of sampling are used. At each stage a decision is made among three possibilities: stop sampling and accept the lot, stop sampling and reject the lot, or continue sampling. Although more complex, multiple sampling plans often result in a smaller total sample size than single-sample plans with the same α and β probabilities.

The logic of a two-stage, or double-sample, plan is shown in Figure 14.15. Initially a sample of n_1 items is selected. If the number of defective components x_1 is less than or equal to c_1, accept the lot. If x_1 is greater than or equal to c_2, reject the lot. If x_1 is between c_1 and c_2 ($c_1 < x_1 < c_2$), select a second sample of n_2 items. Determine the combined, or total, number of defective components from the first sample (x_1) and the second sample (x_2). If $x_1 + x_2 \leq c_3$, accept the lot; otherwise reject the lot. The development of the double-sample plan is more difficult because the sample sizes n_1 and n_2 and the acceptance numbers c_1, c_2, and c_3 must meet both the producer's and consumer's risks desired.

NOTES AND COMMENTS

1. The use of the binomial distribution for acceptance sampling is based on the assumption of large lots. If the lot size is small, the hypergeometric distribution is appropriate.
2. In the MIL-ST-105D sampling tables, p_0 is called the acceptable quality level (AQL). In some sampling tables, p_1 is called the lot tolerance percent defective (LTPD) or the rejectable quality level (RQL). Many of the published sampling plans also use quality indexes such as the indifference quality level (IQL) and the average outgoing quality limit (AOQL). The more advanced texts listed in the bibliography provide a complete discussion of these other indexes.
3. In this section we provided an introduction to *attributes sampling plans.* In these plans each item sampled is classified as nondefective or defective. In *variables sampling plans,* a sample is taken and a measurement of the quality characteristic is taken. For example, for gold jewelry a measurement of quality may be the amount of gold it contains. A simple statistic such as the average amount of gold in the sample jewelry is computed and compared with an allowable value to determine whether to accept or reject the lot.

Exercises

Methods

SELF test

10. For an acceptance sampling plan with $n = 25$ and $c = 0$, find the probability of accepting a lot that has a defect rate of 2%. What is the probability of accepting the lot if the defect rate is 6%?
11. Consider an acceptance sampling plan with $n = 20$ and $c = 0$. Compute the producer's risk for each of the following cases.
    a. The lot has a defect rate of 2%.
    b. The lot has a defect rate of 6%.

12. Repeat exercise 11 for the acceptance sampling plan with $n = 20$ and $c = 1$. What happens to the producer's risk as the acceptance number c is increased? Explain.

Applications

13. Refer to the KALI problem presented in this section. The quality control manager requested a producer's risk of .10 when p_0 was .03 and a consumer's risk of .20 when p_1 was .15. Consider the acceptance sampling plan based on a sample size of 20 and an acceptance number of 1. Answer the following questions.
    a. What is the producer's risk for the $n = 20$, $c = 1$ sampling plan?
    b. What is the consumer's risk for the $n = 20$, $c = 1$ sampling plan?
    c. Does the $n = 20$, $c = 1$ sampling plan satisfy the risks requested by the quality control manager? Discuss.

14. To inspect incoming shipments of raw materials, a manufacturer is considering samples of sizes 10, 15, and 19. Use the binomial probabilities from Table 5 of Appendix B to select a sampling plan that provides a producer's risk of $\alpha = .03$ when p_0 is .05 and a consumer's risk of $\beta = .12$ when p_1 is .30.

15. A domestic manufacturer of watches purchases quartz crystals from a Swiss firm. The crystals are shipped in lots of 1000. The acceptance sampling procedure uses 20 randomly selected crystals.
    a. Construct operating characteristic curves for acceptance numbers of 0, 1, and 2.
    b. If p_0 is .01 and $p_1 = .08$, what are the producer's and consumer's risks for each sampling plan in part (a)?

Summary

In this chapter we discussed how statistical methods can be used to assist in the control of quality. We first presented the $\bar{x}$, R, p, and np control charts as graphical aids in monitoring process quality. Control limits are established for each chart; samples are selected periodically, and the data points plotted on the control chart. Data points outside the control limits indicate that the process is out of control and that corrective action should be taken. Patterns of data points within the control limits can also indicate potential quality control problems and suggest that corrective action may be warranted.

We also considered the technique known as acceptance sampling. With this procedure, a sample is selected and inspected. The number of defective items in the sample provides the basis for accepting or rejecting the lot. The sample size and the acceptance criterion can be adjusted to control both the producer's risk (Type I error) and the consumer's risk (Type II error).

Glossary

Total quality (TQ) A total system approach to improving customer satisfaction and lowering real cost through a strategy of continuous improvement and learning.
Six Sigma A methodology that uses measurement and statistical analysis to achieve a level of quality so good that for every million opportunities no more than 3.4 defects will occur.
Quality control A series of inspections and measurements that determine whether quality standards are being met.
Assignable causes Variations in process outputs that are due to factors such as machine tools wearing out, incorrect machine settings, poor-quality raw materials, operator error,

and so on. Corrective action should be taken when assignable causes of output variation are detected.

Common causes Normal or natural variations in process outputs that are due purely to chance. No corrective action is necessary when output variations are due to common causes.

Control chart A graphical tool used to help determine whether a process is in control or out of control.

$\bar{x}$ chart A control chart used when the quality of the output of a process is measured in terms of the mean value of a variable such as a length, weight, temperature, and so on.

***R* chart** A control chart used when the quality of the output of a process is measured in terms of the range of a variable.

***p* chart** A control chart used when the quality of the output of a process is measured in terms of the proportion defective.

***np* chart** A control chart used to monitor the quality of the output of a process in terms of the number of defective items.

Lot A group of items such as incoming shipments of raw materials or purchased parts as well as finished goods from final assembly.

Acceptance sampling A statistical method in which the number of defective items found in a sample is used to determine whether a lot should be accepted or rejected.

Producer's risk The risk of rejecting a good-quality lot; a Type I error.

Consumer's risk The risk of accepting a poor-quality lot; a Type II error.

Acceptance criterion The maximum number of defective items that can be found in the sample and still indicate an acceptable lot.

Operating characteristic (OC) curve A graph showing the probability of accepting the lot as a function of the percentage defective in the lot. This curve can be used to help determine whether a particular acceptance sampling plan meets both the producer's and the consumer's risk requirements.

Multiple sampling plan A form of acceptance sampling in which more than one sample or stage is used. On the basis of the number of defective items found in a sample, a decision will be made to accept the lot, reject the lot, or continue sampling.

Key Formulas

Standard Error of the Mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \tag{14.1}$$

Control Limits for an $\bar{x}$ Chart: Process Mean and Standard Deviation Known

$$\text{UCL} = \mu + 3\sigma_{\bar{x}} \tag{14.2}$$

$$\text{LCL} = \mu - 3\sigma_{\bar{x}} \tag{14.3}$$

Overall Sample Mean

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_k}{k} \tag{14.4}$$

Average Range

$$\bar{R} = \frac{R_1 + R_2 + \cdots + R_k}{k} \tag{14.5}$$

Control Limits for an x Chart: Process Mean and Standard Deviation Unknown

$$\bar{\bar{x}} \pm A_2\bar{R} \tag{14.8}$$

Control Limits for an R Chart

$$\text{UCL} = \bar{R}D_4 \tag{14.14}$$
$$\text{LCL} = \bar{R}D_3 \tag{14.15}$$

Standard Error of the Proportion

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} \tag{14.16}$$

Control Limits for a p Chart

$$\text{UCL} = p + 3\sigma_{\bar{p}} \tag{14.17}$$
$$\text{LCL} = p - 3\sigma_{\bar{p}} \tag{14.18}$$

Control Limits for an np Chart

$$\text{UCL} = np + 3\sqrt{np(1-p)} \tag{14.19}$$
$$\text{LCL} = np - 3\sqrt{np(1-p)} \tag{14.20}$$

Binomial Probability Function for Acceptance Sampling

$$f(x) = \frac{n!}{x!(n-x)!}p^x(1-p)^{(n-x)} \tag{14.21}$$

Supplementary Exercises

16. Samples of size 5 provided the following 20 sample means for a production process that is believed to be in control.

95.72	95.24	95.18
95.44	95.46	95.32
95.40	95.44	95.08
95.50	95.80	95.22
95.56	95.22	95.04
95.72	94.82	95.46
95.60	95.78	

    a. Based on these data, what is an estimate of the mean when the process is in control?
    b. Assume that the process standard deviation is $\sigma = .50$. Develop the $\bar{x}$ control chart for this production process. Assume that the mean of the process is the estimate developed in part (a).
    c. Do any of the 20 sample means indicate that the process was out of control?

17. Product filling weights are normally distributed with a mean of 350 grams and a standard deviation of 15 grams.
    a. Develop the control limits for the $\bar{x}$ chart for samples of size 10, 20, and 30.
    b. What happens to the control limits as the sample size is increased?
    c. What happens when a Type I error is made?

d. What happens when a Type II error is made?
e. What is the probability of a Type I error for samples of size 10, 20, and 30?
f. What is the advantage of increasing the sample size for control chart purposes? What error probability is reduced as the sample size is increased?

18. Twenty-five samples of size 5 resulted in $\bar{\bar{x}} = 5.42$ and $\bar{R} = 2.0$. Compute control limits for the $\bar{x}$ and R charts, and estimate the standard deviation of the process.

19. The following are quality control data for a manufacturing process at Kensport Chemical Company. The data show the temperature in degrees centigrade at five points in time during a manufacturing cycle. The company is interested in using control charts to monitor the temperature of its manufacturing process. Construct the $\bar{x}$ chart and R chart. What conclusions can be made about the quality of the process?

Sample	$\bar{x}$	R	Sample	$\bar{x}$	R
1	95.72	1.0	11	95.80	.6
2	95.24	.9	12	95.22	.2
3	95.18	.8	13	95.56	1.3
4	95.44	.4	14	95.22	.5
5	95.46	.5	15	95.04	.8
6	95.32	1.1	16	95.72	1.1
7	95.40	.9	17	94.82	.6
8	95.44	.3	18	95.46	.5
9	95.08	.2	19	95.60	.4
10	95.50	.6	20	95.74	.6

20. The following were collected for the Master Blend Coffee production process. The data show the filling weights based on samples of 3-pound cans of coffee. Use these data to construct the $\bar{x}$ and R charts. What conclusions can be made about the quality of the production process?

WEB file
Coffee

	Observations				
Sample	1	2	3	4	5
1	3.05	3.08	3.07	3.11	3.11
2	3.13	3.07	3.05	3.10	3.10
3	3.06	3.04	3.12	3.11	3.10
4	3.09	3.08	3.09	3.09	3.07
5	3.10	3.06	3.06	3.07	3.08
6	3.08	3.10	3.13	3.03	3.06
7	3.06	3.06	3.08	3.10	3.08
8	3.11	3.08	3.07	3.07	3.07
9	3.09	3.09	3.08	3.07	3.09
10	3.06	3.11	3.07	3.09	3.07

21. Consider the following situations. Comment on whether the situation might cause concern about the quality of the process.
a. A p chart has LCL = 0 and UCL = .068. When the process is in control, the proportion defective is .033. Plot the following seven sample results: .035, .062, .055, .049, .058, .066, and .055. Discuss.

b. An $\bar{x}$ chart has LCL = 22.2 and UCL = 24.5. The mean is $\mu = 23.35$ when the process is in control. Plot the following seven sample results: 22.4, 22.6, 22.65, 23.2, 23.4, 23.85, and 24.1. Discuss.

22. Managers of 1200 different retail outlets make twice-a-month restocking orders from a central warehouse. Past experience shows that 4% of the orders result in one or more errors such as wrong item shipped, wrong quantity shipped, and item requested but not shipped. Random samples of 200 orders are selected monthly and checked for accuracy.
    a. Construct a control chart for this situation.
    b. Six months of data show the following numbers of orders with one or more errors: 10, 15, 6, 13, 8, and 17. Plot the data on the control chart. What does your plot indicate about the order process?

23. An $n = 10$, $c = 2$ acceptance sampling plan is being considered; assume that $p_0 = .05$ and $p_1 = .20$.
    a. Compute both producer's and consumer's risk for this acceptance sampling plan.
    b. Would the producer, the consumer, or both be unhappy with the proposed sampling plan?
    c. What change in the sampling plan, if any, would you recommend?

24. An acceptance sampling plan with $n = 15$ and $c = 1$ has been designed with a producer's risk of .075.
    a. Was the value of p_0 .01, .02, .03, .04, or .05? What does this value mean?
    b. What is the consumer's risk associated with this plan if p_1 is .25?

25. A manufacturer produces lots of a canned food product. Let p denote the proportion of the lots that do not meet the product quality specifications. An $n = 25$, $c = 0$ acceptance sampling plan will be used.
    a. Compute points on the operating characteristic curve when $p = .01, .03, .10$, and .20.
    b. Plot the operating characteristic curve.
    c. What is the probability that the acceptance sampling plan will reject a lot containing .01 defective?

Appendix Control Charts Using StatTools

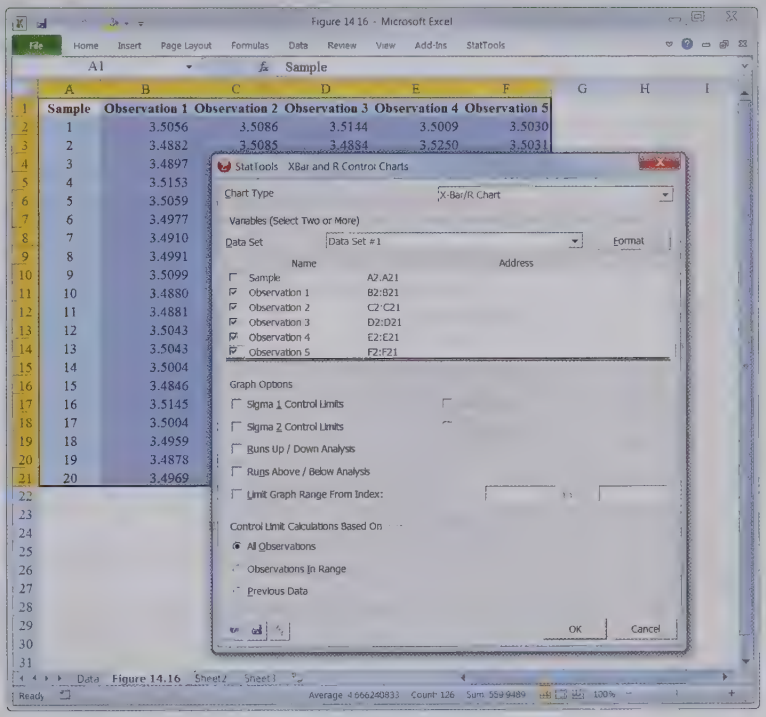
Jensen

In this appendix we show how StatTools can be used to construct an $\bar{x}$ chart and an R chart for the Jensen Computer Supplies data shown in Table 14.2. Figure 14.16 is an Excel worksheet containing the Jensen data with the StatTools - Xbar and R Control Charts dialog box in the foreground. Begin by using the Data Set Manager to create a StatTools data set for these data using the procedure described in the appendix in Chapter 1. The following steps describe how StatTools can be used to provide both control charts.

Step 1. Click the **StatTools** tab on the Ribbon
Step 2. In the **Analyses** group, click **Quality Control**
Step 3. Choose the **X/R Charts** option
Step 4. When the StatTools - Xbar and R Control Charts dialog box appears (Figure 14.16),
Select **X-Bar/R Chart** in the **Chart Type** box
In the **Variables** section, select **Observation 1, Observation 2, Observation 3, Observation 4,** and **Observation 5**
Click **OK**

An $\bar{x}$ chart similar to the one in Figure 14.7 will appear. It will be followed by an R chart similar to the one in Figure 14.8.

FIGURE 14.16 EXCEL DATA WORKSHEET FOR JENSEN COMPUTER SUPPLIES



APPENDIXES

APPENDIX A
References and Bibliography

APPENDIX B
Tables

APPENDIX C
Summation Notation

APPENDIX D
Self-Test Solutions and Answers to Even-Numbered Exercises

APPENDIX E
Microsoft Excel 2010 and Tools for Statistical Analysis

Appendix A: References and Bibliography

General

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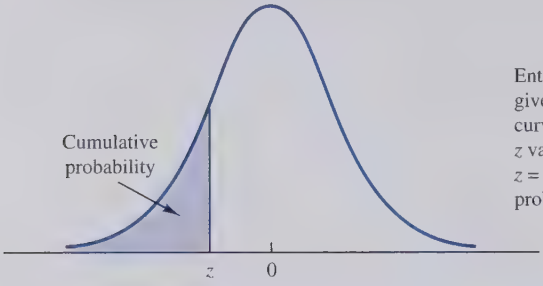
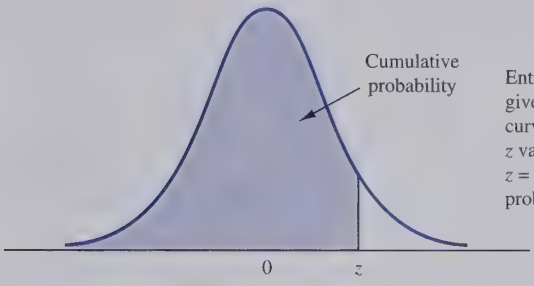
TABLE 1 CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION



Entries in the table give the area under the curve to the left of the z value. For example, for $z = -.85$, the cumulative probability is .1977.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

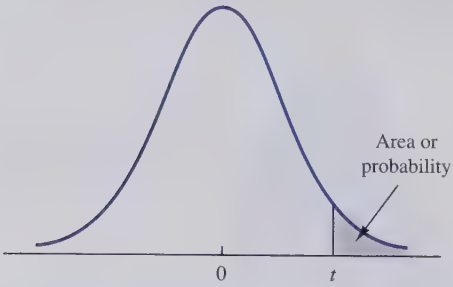
TABLE 1 CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION (*Continued*)



Entries in the table give the area under the curve to the left of the z value. For example, for $z = 1.25$, the cumulative probability is .8944.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990

TABLE 2 t DISTRIBUTION



Entries in the table give t values for an area or probability in the upper tail of the t distribution. For example, with 10 degrees of freedom and a .05 area in the upper tail, $t_{.05} = 1.812$.

Degrees of Freedom	Area in Upper Tail					
	.20	.10	.05	.025	.01	.005
1	1.376	3.078	6.314	12.706	31.821	63.656
2	1.061	1.886	2.920	4.303	6.965	9.925
3	.978	1.638	2.353	3.182	4.541	5.841
4	.941	1.533	2.132	2.776	3.747	4.604
5	.920	1.476	2.015	2.571	3.365	4.032
6	.906	1.440	1.943	2.447	3.143	3.707
7	.896	1.415	1.895	2.365	2.998	3.499
8	.889	1.397	1.860	2.306	2.896	3.355
9	.883	1.383	1.833	2.262	2.821	3.250
10	.879	1.372	1.812	2.228	2.764	3.169
11	.876	1.363	1.796	2.201	2.718	3.106
12	.873	1.356	1.782	2.179	2.681	3.055
13	.870	1.350	1.771	2.160	2.650	3.012
14	.868	1.345	1.761	2.145	2.624	2.977
15	.866	1.341	1.753	2.131	2.602	2.947
16	.865	1.337	1.746	2.120	2.583	2.921
17	.863	1.333	1.740	2.110	2.567	2.898
18	.862	1.330	1.734	2.101	2.552	2.878
19	.861	1.328	1.729	2.093	2.539	2.861
20	.860	1.325	1.725	2.086	2.528	2.845
21	.859	1.323	1.721	2.080	2.518	2.831
22	.858	1.321	1.717	2.074	2.508	2.819
23	.858	1.319	1.714	2.069	2.500	2.807
24	.857	1.318	1.711	2.064	2.492	2.797
25	.856	1.316	1.708	2.060	2.485	2.787
26	.856	1.315	1.706	2.056	2.479	2.779
27	.855	1.314	1.703	2.052	2.473	2.771
28	.855	1.313	1.701	2.048	2.467	2.763
29	.854	1.311	1.699	2.045	2.462	2.756
30	.854	1.310	1.697	2.042	2.457	2.750
31	.853	1.309	1.696	2.040	2.453	2.744
32	.853	1.309	1.694	2.037	2.449	2.738
33	.853	1.308	1.692	2.035	2.445	2.733
34	.852	1.307	1.691	2.032	2.441	2.728

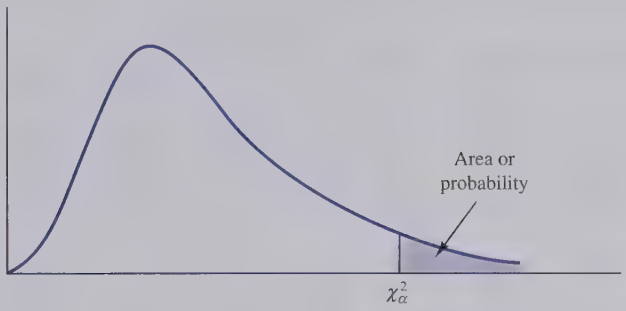
TABLE 2 *t* DISTRIBUTION (*Continued*)

Degrees of Freedom	Area in Upper Tail					
	.20	.10	.05	.025	.01	.005
35	.852	1.306	1.690	2.030	2.438	2.724
36	.852	1.306	1.688	2.028	2.434	2.719
37	.851	1.305	1.687	2.026	2.431	2.715
38	.851	1.304	1.686	2.024	2.429	2.712
39	.851	1.304	1.685	2.023	2.426	2.708
40	.851	1.303	1.684	2.021	2.423	2.704
41	.850	1.303	1.683	2.020	2.421	2.701
42	.850	1.302	1.682	2.018	2.418	2.698
43	.850	1.302	1.681	2.017	2.416	2.695
44	.850	1.301	1.680	2.015	2.414	2.692
45	.850	1.301	1.679	2.014	2.412	2.690
46	.850	1.300	1.679	2.013	2.410	2.687
47	.849	1.300	1.678	2.012	2.408	2.685
48	.849	1.299	1.677	2.011	2.407	2.682
49	.849	1.299	1.677	2.010	2.405	2.680
50	.849	1.299	1.676	2.009	2.403	2.678
51	.849	1.298	1.675	2.008	2.402	2.676
52	.849	1.298	1.675	2.007	2.400	2.674
53	.848	1.298	1.674	2.006	2.399	2.672
54	.848	1.297	1.674	2.005	2.397	2.670
55	.848	1.297	1.673	2.004	2.396	2.668
56	.848	1.297	1.673	2.003	2.395	2.667
57	.848	1.297	1.672	2.002	2.394	2.665
58	.848	1.296	1.672	2.002	2.392	2.663
59	.848	1.296	1.671	2.001	2.391	2.662
60	.848	1.296	1.671	2.000	2.390	2.660
61	.848	1.296	1.670	2.000	2.389	2.659
62	.847	1.295	1.670	1.999	2.388	2.657
63	.847	1.295	1.669	1.998	2.387	2.656
64	.847	1.295	1.669	1.998	2.386	2.655
65	.847	1.295	1.669	1.997	2.385	2.654
66	.847	1.295	1.668	1.997	2.384	2.652
67	.847	1.294	1.668	1.996	2.383	2.651
68	.847	1.294	1.668	1.995	2.382	2.650
69	.847	1.294	1.667	1.995	2.382	2.649
70	.847	1.294	1.667	1.994	2.381	2.648
71	.847	1.294	1.667	1.994	2.380	2.647
72	.847	1.293	1.666	1.993	2.379	2.646
73	.847	1.293	1.666	1.993	2.379	2.645
74	.847	1.293	1.666	1.993	2.378	2.644
75	.846	1.293	1.665	1.992	2.377	2.643
76	.846	1.293	1.665	1.992	2.376	2.642
77	.846	1.293	1.665	1.991	2.376	2.641
78	.846	1.292	1.665	1.991	2.375	2.640
79	.846	1.292	1.664	1.990	2.374	2.639

TABLE 2 *t* DISTRIBUTION (*Continued*)

Degrees of Freedom	Area in Upper Tail					
	.20	.10	.05	.025	.01	.005
80	.846	1.292	1.664	1.990	2.374	2.639
81	.846	1.292	1.664	1.990	2.373	2.638
82	.846	1.292	1.664	1.989	2.373	2.637
83	.846	1.292	1.663	1.989	2.372	2.636
84	.846	1.292	1.663	1.989	2.372	2.636
85	.846	1.292	1.663	1.988	2.371	2.635
86	.846	1.291	1.663	1.988	2.370	2.634
87	.846	1.291	1.663	1.988	2.370	2.634
88	.846	1.291	1.662	1.987	2.369	2.633
89	.846	1.291	1.662	1.987	2.369	2.632
90	.846	1.291	1.662	1.987	2.368	2.632
91	.846	1.291	1.662	1.986	2.368	2.631
92	.846	1.291	1.662	1.986	2.368	2.630
93	.846	1.291	1.661	1.986	2.367	2.630
94	.845	1.291	1.661	1.986	2.367	2.629
95	.845	1.291	1.661	1.985	2.366	2.629
96	.845	1.290	1.661	1.985	2.366	2.628
97	.845	1.290	1.661	1.985	2.365	2.627
98	.845	1.290	1.661	1.984	2.365	2.627
99	.845	1.290	1.660	1.984	2.364	2.626
100	.845	1.290	1.660	1.984	2.364	2.626
∞	.842	1.282	1.645	1.960	2.326	2.576

TABLE 3 CHI-SQUARE DISTRIBUTION



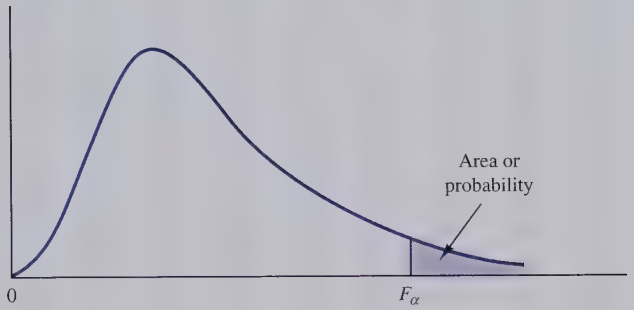
Entries in the table give χ^2_α values, where α is the area or probability in the upper tail of the chi-square distribution. For example, with 10 degrees of freedom and a .01 area in the upper tail, $\chi^2_{.01} = 23.209$.

Degrees of Freedom	Area in Upper Tail									
	.995	.99	.975	.95	.90	.10	.05	.025	.01	.005
1	.000	.000	.001	.004	.016	2.706	3.841	5.024	6.635	7.879
2	.010	.020	.051	.103	.211	4.605	5.991	7.378	9.210	10.597
3	.072	.115	.216	.352	.584	6.251	7.815	9.348	11.345	12.838
4	.207	.297	.484	.711	1.064	7.779	9.488	11.143	13.277	14.860
5	.412	.554	.831	1.145	1.610	9.236	11.070	12.832	15.086	16.750
6	.676	.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.647	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.041	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.558
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.878	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.994
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.335

TABLE 3 CHI-SQUARE DISTRIBUTION (*Continued*)

Degrees of Freedom	Area in Upper Tail									
	.995	.99	.975	.95	.90	.10	.05	.025	.01	.005
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
35	17.192	18.509	20.569	22.465	24.797	46.059	49.802	53.203	57.342	60.275
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
45	24.311	25.901	28.366	30.612	33.350	57.505	61.656	65.410	69.957	73.166
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
55	31.735	33.571	36.398	38.958	42.060	68.796	73.311	77.380	82.292	85.749
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
65	39.383	41.444	44.603	47.450	50.883	79.973	84.821	89.177	94.422	98.105
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
75	47.206	49.475	52.942	56.054	59.795	91.061	96.217	100.839	106.393	110.285
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
85	55.170	57.634	61.389	64.749	68.777	102.079	107.522	112.393	118.236	122.324
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
95	63.250	65.898	69.925	73.520	77.818	113.038	118.752	123.858	129.973	134.247
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.170

TABLE 4 F DISTRIBUTION



Entries in the table give F_α values, where α is the area or probability in the upper tail of the F distribution. For example, with 4 numerator degrees of freedom, 8 denominator degrees of freedom, and a .05 area in the upper tail, $F_{.05} = 3.84$.

Denominator Degrees of Freedom	Area in Upper Tail	Numerator Degrees of Freedom																	
		1	2	3	4	5	6	7	8	9	10	15	20	25	30	40	60	100	1000
1	.10	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	61.22	61.74	62.05	62.26	62.53	62.79	63.01	63.30
	.05	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	245.95	248.02	249.26	250.10	251.14	252.20	253.04	254.19
	.025	647.79	799.48	864.15	899.60	921.83	937.11	948.20	956.64	963.28	968.63	984.87	993.08	998.09	1001.40	1005.60	1009.79	1013.16	1017.76
	.01	4052.18	4999.34	5403.53	5624.26	5763.96	5858.95	5928.33	5980.95	6022.40	6055.93	6156.97	6208.66	6239.86	6260.35	6286.43	6312.97	6333.92	6362.80
2	.10	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.42	9.44	9.45	9.46	9.47	9.47	9.48	9.49
	.05	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.43	19.45	19.46	19.46	19.47	19.48	19.49	19.49
	.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50
	.01	98.50	99.00	99.16	99.25	99.30	99.33	99.36	99.38	99.39	99.40	99.43	99.45	99.46	99.47	99.48	99.48	99.49	99.50
3	.10	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.20	5.18	5.17	5.17	5.16	5.15	5.14	5.13
	.05	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.70	8.66	8.63	8.62	8.59	8.57	8.55	8.53
	.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.25	14.17	14.12	14.08	14.04	13.99	13.96	13.91
	.01	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.34	27.23	26.87	26.69	26.58	26.50	26.41	26.32	26.24	26.14
4	.10	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.87	3.84	3.83	3.82	3.80	3.79	3.78	3.76
	.05	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
	.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.66	8.56	8.50	8.46	8.41	8.36	8.32	8.26
	.01	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.20	14.02	13.91	13.84	13.75	13.65	13.58	13.47
5	.10	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.324	3.21	3.19	3.17	3.16	3.14	3.13	3.11
	.05	6.61.	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.62	4.56	4.52	4.50	4.46	4.43	4.41	4.37
	.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.43	6.33	6.27	6.23	6.18	6.12	6.08	6.02
	.01	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.72	9.55	9.45	9.38	9.29	9.20	9.13	9.03

TABLE 4 *F* DISTRIBUTION (*Continued*)

Denominator Degrees of Freedom	Area in Upper Tail	Numerator Degrees of Freedom																	
		1	2	3	4	5	6	7	8	9	10	15	20	25	30	40	60	100	1000
6	.10	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.87	2.84	2.81	2.80	2.78	2.76	2.75	2.72
	.05	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	3.94	3.87	3.83	3.81	3.77	3.74	3.71	3.67
	025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.27	5.17	5.11	5.07	5.01	4.96	4.92	4.86
	.01	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.56	7.40	7.30	7.23	7.14	7.06	6.99	6.89
7	.10	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.63	2.59	2.57	2.56	2.54	2.51	2.50	2.47
	05	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.51	3.44	3.40	3.38	3.34	3.30	3.27	3.23
	.025	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.57	4.47	4.40	4.36	4.31	4.25	4.21	4.15
	.01	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.31	6.16	6.06	5.99	5.91	5.82	5.75	5.66
8	.10	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.46	2.42	2.40	2.38	2.36	2.34	2.32	2.30
	.05	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.22	3.15	3.11	3.08	3.04	3.01	2.97	2.93
	.025	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.10	4.00	3.94	3.89	3.84	3.78	3.74	3.68
	.01	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.52	5.36	5.26	5.20	5.12	5.03	4.96	4.87
9	.10	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.34	2.30	2.27	2.25	2.23	2.21	2.19	2.16
	.05	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.01	2.94	2.89	2.86	2.83	2.79	2.76	2.71
	.025	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.77	3.67	3.60	3.56	3.51	3.45	3.40	3.34
	.01	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	4.96	4.81	4.71	4.65	4.57	4.48	4.41	4.32
10	.10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.24	2.20	2.17	2.16	2.13	2.11	2.09	2.06
	.05	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.85	2.77	2.73	2.70	2.66	2.62	2.59	2.54
	.025	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.52	3.42	3.35	3.31	3.26	3.20	3.15	3.09
	.01	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.56	4.41	4.31	4.25	4.17	4.08	4.01	3.92
11	.10	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.17	2.12	2.10	2.08	2.05	2.03	2.01	1.98
	.05	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.72	2.65	2.60	2.57	2.53	2.49	2.46	2.41
	.025	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.33	3.23	3.16	3.12	3.06	3.00	2.96	2.89
	.01	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.25	4.10	4.01	3.94	3.86	3.78	3.71	3.61
12	.10	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.10	2.06	2.03	2.01	1.99	1.96	1.94	1.91
	.05	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.62	2.54	2.50	2.47	2.43	2.38	2.35	2.30
	.025	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.18	3.07	3.01	2.96	2.91	2.85	2.80	2.73
	.01	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.01	3.86	3.76	3.70	3.62	3.54	3.47	3.37
13	.10	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.05	2.01	1.98	1.96	1.93	1.90	1.88	1.85
	.05	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.53	2.46	2.41	2.38	2.34	2.30	2.26	2.21
	.025	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.05	2.95	2.88	2.84	2.78	2.72	2.67	2.60
	.01	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.82	3.66	3.57	3.51	3.43	3.34	3.27	3.18
14	.10	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.01	1.96	1.93	1.99	1.89	1.86	1.83	1.80
	.05	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.46	2.39	2.34	2.31	2.27	2.22	2.19	2.14
	.025	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	2.95	2.84	2.78	2.73	2.67	2.61	2.56	2.50
	.01	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.66	3.51	3.41	3.35	3.27	3.18	3.11	3.02
15	.10	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	1.97	1.92	1.89	1.87	1.85	1.82	1.79	1.76
	.05	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.40	2.33	2.28	2.25	2.20	2.16	2.12	2.07
	.025	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.86	2.76	2.69	2.64	2.59	2.52	2.47	2.40
	.01	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.52	3.37	3.28	3.21	3.13	3.05	2.98	2.88

Denominator Degrees of Freedom	Area in Upper Tail	Numerator Degrees of Freedom																	
		1	2	3	4	5	6	7	8	9	10	15	20	25	30	40	60	100	1000
16	.10	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.94	1.89	1.86	1.84	1.81	1.78	1.76	1.72
	.05	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.35	2.28	2.23	2.19	2.15	2.11	2.07	2.02
	.025	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.79	2.68	2.61	2.57	2.51	2.45	2.40	2.32
	.01	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.41	3.26	3.16	3.10	3.02	2.93	2.86	2.76
17	.10	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.91	1.86	1.83	1.81	1.78	1.75	1.73	1.69
	.05	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.31	2.23	2.18	2.15	2.10	2.06	2.02	1.97
	.025	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.72	2.62	2.55	2.50	2.44	2.38	2.33	2.26
	.01	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.31	3.16	3.07	3.00	2.92	2.83	2.76	2.66
18	.10	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.89	1.84	1.80	1.78	1.75	1.72	1.70	1.66
	.05	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.27	2.19	2.14	2.11	2.06	2.02	1.98	1.92
	.025	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.67	2.56	2.49	2.44	2.38	2.32	2.27	2.20
	.01	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.23	3.08	2.98	2.92	2.84	2.75	2.68	2.58
19	.10	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.86	1.81	1.78	1.76	1.73	1.70	1.67	1.64
	.05	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.23	2.16	2.11	2.07	2.03	1.98	1.94	1.88
	.025	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.62	2.51	2.44	2.39	2.33	2.27	2.22	2.14
	.01	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.15	3.00	2.91	2.84	2.76	2.67	2.60	2.50
20	.10	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.84	1.79	1.76	1.74	1.71	1.68	1.65	1.61
	.05	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.20	2.12	2.07	2.04	1.99	1.95	1.91	1.85
	.025	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.57	2.46	2.40	2.35	2.29	2.22	2.17	2.09
	.01	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.09	2.94	2.84	2.78	2.69	2.61	2.54	2.43
21	.10	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.83	1.78	1.74	1.72	1.69	1.66	1.63	1.59
	.05	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.18	2.10	2.05	2.01	1.96	1.92	1.88	1.82
	.025	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.53	2.42	2.36	2.31	2.25	2.18	2.13	2.05
	.01	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.03	2.88	2.79	2.72	2.64	2.55	2.48	2.37
22	.10	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.81	1.76	1.73	1.70	1.67	1.64	1.61	1.57
	.05	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.15	2.07	2.02	1.98	1.94	1.89	1.85	1.79
	.025	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.50	2.39	2.32	2.27	2.21	2.14	2.09	2.01
	.01	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	2.98	2.83	2.73	2.67	2.58	2.50	2.42	2.32
23	.10	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.80	1.74	1.71	1.69	1.66	1.62	1.59	1.55
	.05	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.13	2.05	2.00	1.96	1.91	1.86	1.82	1.76
	.025	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.47	2.36	2.29	2.24	2.18	2.11	2.06	1.98
	.01	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	2.93	2.78	2.69	2.62	2.54	2.45	2.37	2.27
24	.10	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.78	1.73	1.70	1.67	1.64	1.61	1.58	1.54
	.05	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.11	2.03	1.97	1.94	1.89	1.84	1.80	1.74
	.025	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.44	2.33	2.26	2.21	2.15	2.08	2.02	1.94
	.01	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	2.89	2.74	2.64	2.58	2.49	2.40	2.33	2.22

TABLE 4 *F* DISTRIBUTION (*Continued*)

Denominator Degrees of Freedom	Area in Upper Tail	Numerator Degrees of Freedom																	
		1	2	3	4	5	6	7	8	9	10	15	20	25	30	40	60	100	1000
25	.10	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.77	1.72	1.68	1.66	1.63	1.59	1.56	1.52
	.05	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.09	2.01	1.96	1.92	1.87	1.82	1.78	1.72
	.025	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.41	2.30	2.23	2.18	2.12	2.05	2.00	1.91
	.01	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.85	2.70	2.60	2.54	2.45	2.36	2.29	2.18
26	.10	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.76	1.71	1.67	1.65	1.61	1.58	1.55	1.51
	.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.07	1.99	1.94	1.90	1.85	1.80	1.76	1.70
	.025	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.39	2.28	2.21	2.16	2.09	2.03	1.97	1.89
	.01	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.81	2.66	2.57	2.50	2.42	2.33	2.25	2.14
27	.10	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.75	1.70	1.66	1.64	1.60	1.57	1.54	1.50
	.05	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.06	1.97	1.92	1.88	1.84	1.79	1.74	1.68
	.025	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.36	2.25	2.18	2.13	2.07	2.00	1.94	1.86
	.01	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.78	2.63	2.54	2.47	2.38	2.29	2.22	2.11
28	.10	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.74	1.69	1.65	1.63	1.59	1.56	1.53	1.48
	.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.04	1.96	1.91	1.87	1.82	1.77	1.73	1.66
	.025	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.34	2.23	2.16	2.11	2.05	1.98	1.92	1.84
	.01	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.75	2.60	2.51	2.44	2.35	2.26	2.19	2.08
29	.10	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.73	1.68	1.64	1.62	1.58	1.55	1.52	1.47
	.05	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.03	1.94	1.89	1.85	1.81	1.75	1.71	1.65
	.025	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.32	2.21	2.14	2.09	2.03	1.96	1.90	1.82
	.01	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.73	2.57	2.48	2.41	2.33	2.23	2.16	2.05
30	.10	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.72	1.67	1.63	1.61	1.57	1.54	1.51	1.46
	.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.01	1.93	1.88	1.84	1.79	1.74	1.70	1.63
	.025	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.31	2.20	2.12	2.07	2.01	1.94	1.88	1.80
	.01	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.70	2.55	2.45	2.39	2.30	2.21	2.13	2.02
40	.10	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.66	1.61	1.57	1.54	1.51	1.47	1.43	1.38
	.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	1.92	1.84	1.78	1.74	1.69	1.64	1.59	1.52
	.025	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.18	2.07	1.99	1.94	1.88	1.80	1.74	1.65
	.01	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.52	2.37	2.27	2.20	2.11	2.02	1.94	1.82
60	.10	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.60	1.54	1.50	1.48	1.44	1.40	1.36	1.30
	.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.84	1.75	1.69	1.65	1.59	1.53	1.48	1.40
	.025	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.06	1.94	1.87	1.82	1.74	1.67	1.60	1.49
	.01	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.35	2.20	2.10	2.03	1.94	1.84	1.75	1.62
100	.10	2.76	2.36	2.14	2.00	1.91	1.83	1.78	1.73	1.69	1.66	1.56	1.49	1.45	1.42	1.38	1.34	1.29	1.22
	.05	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.77	1.68	1.62	1.57	1.52	1.45	1.39	1.30
	.025	5.18	3.83	3.25	2.92	2.70	2.54	2.42	2.32	2.24	2.18	1.97	1.85	1.77	1.71	1.64	1.56	1.48	1.36
	.01	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50	2.22	2.07	1.97	1.89	1.80	1.69	1.60	1.45
1000	.10	2.71	2.31	2.09	1.95	1.85	1.78	1.72	1.68	1.64	1.61	1.49	1.43	1.38	1.35	1.30	1.25	1.20	1.08
	.05	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89	1.84	1.68	1.58	1.52	1.47	1.41	1.33	1.26	1.11
	.025	5.04	3.70	3.13	2.80	2.58	2.42	2.30	2.20	2.13	2.06	1.85	1.72	1.64	1.58	1.50	1.41	1.32	1.13
	.01	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43	2.34	2.06	1.90	1.79	1.72	1.61	1.50	1.38	1.16

Appendix C: Summation Notation

Summations

Definition

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + \cdots + x_n \tag{C.1}$$

Example for $x_1 = 5, x_2 = 8, x_3 = 14$:

$$\begin{aligned}\sum_{i=1}^{3} x_i &= x_1 + x_2 + x_3 \\ &= 5 + 8 + 14 \\ &= 27\end{aligned}$$

Result 1

For a constant c:

$$\sum_{i=1}^{n} c = \underbrace{(c + c + \cdots + c)}_{n \text{ times}} = nc \tag{C.2}$$

Example for $c = 5, n = 10$:

$$\sum_{i=1}^{10} 5 = 10(5) = 50$$

Example for $c = \bar{x}$:

$$\sum_{i=1}^{n} \bar{x} = n\bar{x}$$

Result 2

$$\begin{aligned}\sum_{i=1}^{n} cx_i &= cx_1 + cx_2 + \cdots + cx_n \\ &= c(x_1 + x_2 + \cdots + x_n) = c\sum_{i=1}^{n} x_i\end{aligned} \tag{C.3}$$

Example for $x_1 = 5, x_2 = 8, x_3 = 14, c = 2$:

$$\sum_{i=1}^{3} 2x_i = 2\sum_{i=1}^{3} x_i = 2(27) = 54$$

Result 3

$$\sum_{i=1}^{n} (ax_i + by_i) = a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} y_i \tag{C.4}$$

Example for $x_1 = 5, x_2 = 8, x_3 = 14, a = 2, y_1 = 7, y_2 = 3, y_3 = 8, b = 4$:

$$\begin{aligned}\sum_{i=1}^{3}(2x_i + 4y_i) &= 2\sum_{i=1}^{3}x_i + 4\sum_{i=1}^{3}y_i \\ &= 2(27) + 4(18) \\ &= 54 + 72 \\ &= 126\end{aligned}$$

Double Summations

Consider the following data involving the variable x_{ij}, where i is the subscript denoting the row position and j is the subscript denoting the column position:

		Column		
		1	**2**	**3**
Row	**1**	$x_{11} = 10$	$x_{12} = 8$	$x_{13} = 6$
	2	$x_{21} = 7$	$x_{22} = 4$	$x_{23} = 12$

Definition

$$\begin{aligned}\sum_{i=1}^{n}\sum_{j=1}^{m}x_{ij} = (x_{11} + x_{12} + \cdots + x_{1m}) + (x_{21} + x_{22} + \cdots + x_{2m}) \\ + (x_{31} + x_{32} + \cdots + x_{3m}) + \cdots + (x_{n1} + x_{n2} + \cdots + x_{nm})\end{aligned} \quad \textbf{(C.5)}$$

Example:

$$\begin{aligned}\sum_{i=1}^{2}\sum_{j=1}^{3}x_{ij} &= x_{11} + x_{12} + x_{13} + x_{21} + x_{22} + x_{23} \\ &= 10 + 8 + 6 + 7 + 4 + 12 \\ &= 47\end{aligned}$$

Definition

$$\sum_{i=1}^{n}x_{ij} = x_{1j} + x_{2j} + \cdots + x_{nj} \quad \textbf{(C.6)}$$

Example:

$$\begin{aligned}\sum_{i=1}^{2}x_{i2} &= x_{12} + x_{22} \\ &= 8 + 4 \\ &= 12\end{aligned}$$

Shorthand Notation

Sometimes when a summation is for all values of the subscript, we use the following shorthand notations:

$$\sum_{i=1}^{n}x_i = \sum x_i \quad \textbf{(C.7)}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m}x_{ij} = \sum\sum x_{ij} \quad \textbf{(C.8)}$$

$$\sum_{i=1}^{n}x_{ij} = \sum_{i}x_{ij} \quad \textbf{(C.9)}$$

Appendix D: Self-Test Solutions and Answers to Even-Numbered Exercises

Chapter 1

2. a. 10
b. 5
c. Categorical variables: Size and Fuel
Quantitiative variables: Cylinders, City MPG, and Highway MPG
d.

Variable	Measurement Scale
Size	Ordinal
Cylinders	Ratio
City MPG	Ratio
Highway MPG	Ratio
Fuel	Nominal

3. a. Average for city driving = 182/10 = 18.2 mpg
b. Average for highway driving = 261/10 = 26.1 mpg
On average, the miles per gallon for highway driving is 7.9 mpg greater than for city driving
c. 3 of 10 or 30% have four-cyclinder engines
d. 6 of 10 or 60% use regular fuel

4. a. 7
b. 5
c. Categorical variables: State, Campus Setting, and NCAA Division
d. Quantitiative variables: Endowment and Applicants Admitted

6. a. Quantitative
b. Categorical
c. Categorical
d. Quantitative
e. Categorical

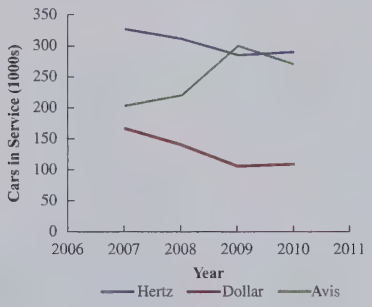
8. a. 1015
b. Categorical
c. Percentages
d. .10(1015) = 101.5; 101 or 102 respondents

10. a. Categorical
b. Percentages
c. 15%
d. Support against

12. a. All visitors to Hawaii
b. Yes
c. First and fourth questions provide quantitative data
Second and third questions provide categorical data

13. a. Federal spending ($ trillions)
b. Quantitative
c. Time series
d. Federal spending has increased over time

14. a. The graph of the time series follows:



b. In 2007 and 2008 Hertz was the clear market share leader. In 2009 and 2010 Hertz and Avis have approximately the same market share. The market share for Dollar appears to be declining.

c. The bar chart for 2010 is shown below.

This chart is based on cross-sectional data.

18. a. 67%
b. 612
c. Categorical

20. a. 43% of managers were bullish or very bullish, and 21% of managers expected health care to be the leading industry over the next 12 months
b. The average 12-month return estimate is 11.2% for the population of investment managers
c. The sample average of 2.5 years is an estimate of how long the population of investment managers think it will take to resume sustainable growth

22. a. The population consists of all customers of the chain stores in Charlotte, North Carolina
b. Some of the ways the grocery store chain could use to collect the data are
- Customers entering or leaving the store could be surveyed

- A survey could be mailed to customers who have a shopper's club card
- Customers could be given a printed survey when they check out
- Customers could be given a coupon that asks them to complete a brief online survey; if they do, they will receive a 5% discount on their next shopping trip

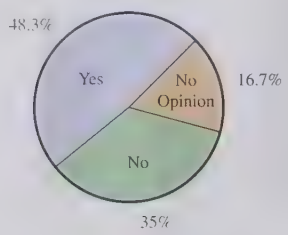
24. a. Correct
b. Incorrect
c. Correct
d. Incorrect
e. Incorrect

Chapter 2

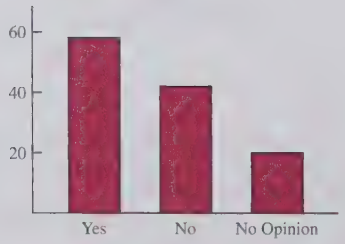
2. a. .20
b. 40
c/d.

Class	Frequency	Percent Frequency
A	44	22
B	36	18
C	80	40
D	40	20
Total	200	100

3. a. $360° \times 58/120 = 174°$
b. $360° \times 42/120 = 126°$
c.



d.



4. a. Categorical
b.

TV Show	Frequency	Percent Frequency
Law & Order	10	20%
CSI	18	36%
Without a Trace	9	18%
Desp Housewives	13	26%
Total:	50	100%

d. *CSI* had the largest viewing audience; *Desperate Housewives* was in second place

6. a.

Network	Frequency	Percent Frequency
ABC	15	30
CBS	17	34
FOX	1	2
NBC	17	34

b. CBS and NBC tied for first; ABC is close with 15

7. a.

Rating	Frequency	Percent Frequency
Excellent	20	40
Very Good	23	46
Good	4	8
Fair	1	2
Poor	2	4
Total	50	100

Management should be very pleased; 86% of the ratings are very good or excellent.

b. Review explanations from the three with Fair or Poor ratings to identify reasons for the low ratings

8. a.

Position	Frequency	Relative Frequency
P	17	.309
H	4	.073
1	5	.091
2	4	.073
3	2	.036
S	5	.091
L	6	.109
C	5	.091
R	7	.127
Totals	55	1.000

b. Pitcher
c. 3rd base
d. Right field
e. Infielders 16 to outfielders 18

10. a/b.

Rating	Frequency	Percent Frequency
Excellent	20	2
Good	101	10
Fair	528	52
Bad	244	24
Terrible	122	12
Total	1015	100

c.

d. 36% a bad or a terrible job
12% a good or excellent job

e. 50% a bad or a terrible job
4% a good or excellent job
More pessimism in Spain

12.

Class	Cumulative Frequency	Cumulative Relative Frequency
≤19	10	.20
≤29	24	.48
≤39	41	.82
≤49	48	.96
≤59	50	1.00

14. b/c.

Class	Frequency	Percent Frequency
6.0–7.9	4	20
8.0–9.9	2	10
10.0–11.9	8	40
12.0–13.9	3	15
14.0–15.9	3	15
Totals	20	100

15. a/b.

Waiting Time	Frequency	Relative Frequency
0–4	4	.20
5–9	8	.40
10–14	5	.25
15–19	2	.10
20–24	1	.05
Totals	20	1.00

c/d.

Waiting Time	Cumulative Frequency	Cumulative Relative Frequency
≤4	4	.20
≤9	12	.60
≤14	17	.85
≤19	19	.95
≤24	20	1.00

e. 12/20 = .60

16. a.

Salary	Frequency
150–159	1
160–169	3
170–179	7
180–189	5
190–199	1
200–209	2
210–219	1
Total	20

b.

Salary	Percent Frequency
150–159	5
160–169	15
170–179	35
180–189	25
190–199	5
200–209	10
210–219	5
Total	100

c.

Salary	Cumulative Percent Frequency
Less than or equal to 159	5
Less than or equal to 169	20
Less than or equal to 179	55
Less than or equal to 189	80
Less than or equal to 199	85
Less than or equal to 209	95
Less than or equal to 219	100
Total	100

e. There is skewness to the right

f. 15%

18. a. Lowest $180; highest $2050

b.

Spending	Frequency	Percent Frequency
$0–249	3	12
250–499	6	24
500–749	5	20
750–999	5	20
1000–1249	3	12
1250–1499	1	4
1500–1749	0	0
1750–1999	1	4
2000–2249	1	4
Total	25	100

c. The distribution shows a positive skewness

d. Majority (64%) of consumers spend between $250 and $1000; the middle value is about $750; and two high spenders are above $1750

20. a.

Off-Course Income ($1000s)	Frequency	Percent Frequency
0–4,999	30	60
5,000–9,999	9	18
10,000–14,999	4	8
15,000–19,999	0	0
20,000–24,999	3	6
25,000–29,999	2	4
30,000–34,999	0	0
35,000–39,999	0	0
40,000–44,999	1	2
45,000–49,999	0	0
Over 50,000	1	2
Total	50	100

c. Off-course income is skewed to the right; only Tiger Woods earns over $50 million

d. The majority (60%) earn less that $5 million; 78% earn less than $10 million; five golfers (10%) earn between $20 million and $30 million; only Tiger Woods and Phil Mickelson earn more than $40 million

22.

5	7 8
6	4 5 8
7	0 2 2 5 5 6 8
8	0 2 3 5

23. Leaf unit = .1

6	3
7	5 5 7
8	1 3 4 8
9	3 6
10	0 4 5
11	3

24. Leaf unit = 10

11	6
12	0 2
13	0 6 7
14	2 2 7
15	5
16	0 2 8
17	0 2 3

25.

9	8 9
10	2 4 6 6
11	4 5 7 8 8 9
12	2 4 5 7
13	1 2
14	4
15	1

26. Median Pay

6	6 7 7
7	2 4 6 7 7 8 9
8	0 0 1 3 7
9	9
10	0 6
11	0
12	1

Top Pay

10	0 6 9
11	1 6 9
12	2 5 6
13	0 5 8 8
14	0 6
15	2 5 7
16	
17	
18	
19	
20	
21	4
22	1

28. a.

2	14
2	67
3	011123
3	5677
4	003333344
4	6679
5	00022
5	5679
6	14
6	6
7	2

b. 40–44 with 9

c. 43 with 5

d. 10%; relatively small participation in the race

29. a.

		y = 1	y = 2	Total
x	A	5	0	5
	B	11	2	13
	C	2	10	12
	Total	18	12	30

b.

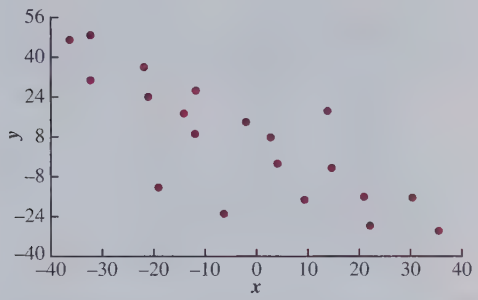
		y = 1	y = 2	Total
x	A	100.0	0.0	100.0
	B	84.6	15.4	100.0
	C	16.7	83.3	100.0

c.

		y = 1	y = 2
x	A	27.8	0.0
	B	61.1	16.7
	C	11.1	83.3
	Total	100.0	100.0

d. *A* values are always in $y = 1$
B values are most often in $y = 1$
C values are most often in $y = 2$

30. a.



b. A negative relationship between x and y; y decreases as x increases

32. a.

	Household Income ($1000s)					
Education Level	Under 25	25.0–49.9	50.0–74.9	75.0–99.9	100 or more	Total
Not H.S. Graduate	32.10	18.71	9.13	5.26	2.20	13.51
H.S. Graduate	37.52	37.05	33.04	25.73	16.00	29.97
Some College	21.42	28.44	30.74	31.71	24.43	27.21
Bachelor's Degree	6.75	11.33	18.72	25.19	32.26	18.70
Beyond Bach. Deg.	2.21	4.48	8.37	12.11	25.11	10.61
Total	100.00	100.00	100.00	100.00	100.00	100.00

13.51% of the heads of households did not graduate from high school

b. 25.11%, 53.54%

c. Positive relationship between income and education level

34. a.

	5-Year Average Return						
Fund Type	0–9.99	10–19.99	20–29.99	30–39.99	40–49.99	50–59.99	Total
DE	1	25	1	0	0	0	27
FI	9	1	0	0	0	0	10
IE	0	2	3	2	0	1	8
Total	10	28	4	2	0	1	45

b.

5-Year Average Return	Frequency
0–9.99	10
10–19.99	28
20–29.99	4
30–39.99	2
40–49.99	0
50–59.99	1
Total	45

c.

Fund Type	Frequency
DE	27
FI	10
IE	8
Total	45

d. The margin of the crosstabulation shows these frequency distributions

e. Higher returns—International Equity funds
Lower returns—Fixed Income funds

36. b. Higher 5-year returns are associated with higher net asset values.

38. a.

	Highway MPG						
Displacement	15–19	20–24	25–29	30–34	35–39	40–44	Total
1–2	0	0	0	0	7	7	14
2–3	0	0	14	41	9	1	65
3–4	0	1	34	4	0	0	39
4–5	0	10	4	0	0	0	14
5–6	0	3	2	0	0	0	5
6–7	8	4	0	0	0	0	12
Total	8	18	54	45	16	8	149

b. Higher fuel efficiencies are associated with smaller displacement engines
Lower fuel efficiencies are associated with larger displacement engines

d. Lower fuel efficiencies are associated with larger displacement engines

e. Scatter diagram

40. a.

Division	Frequency	Percent
Buick	10	5
Cadillac	10	5
Chevrolet	122	61
GMC	24	12
Hummer	2	1
Pontiac	18	9
Saab	2	1
Saturn	12	6
Total	200	100

b. Chevrolet, 61%

c. Hummer and Saab, both only 1%
Maintain Chevrolet and GMC

42. a.

SAT Score	Frequency
800–999	1
1000–1199	3
1200–1399	6
1400–1599	10
1600–1799	7
1800–1999	2
2000–2199	1
Total	30

b. Nearly symmetrical

c. 33% of the scores fall between 1400 and 1599
A score below 800 or above 2200 is unusual
The average is near or slightly above 1500

44. a.

Population	Frequency	Percent Frequency
0.0–2.4	17	34
2.5–4.9	12	24
5.0–7.4	9	18
7.5–9.9	4	8
10.0–12.4	3	6
12.5–14.9	1	2
15.0–17.4	1	2
17.5–19.9	1	2
20.0–22.4	0	0
22.5–24.9	1	2
25.0–27.4	0	0
27.5–29.9	0	0
30.0–32.4	0	0
32.5–34.9	0	0
35.0–37.4	1	2
Total	50	100

c. High positive skewness

d. 17 (34%) with population less than 2.5 million
29 (58%) with population less than 5 million
8 (16%) with population greater than 10 million
Largest 35.9 million (California)
Smallest .5 million (Wyoming)

46. a. High Temperatures

```
1 |
2 |
3 | 0
4 | 1 2 2 5
5 | 2 4 5
6 | 0 0 0 1 2 2 5 6 8
7 | 0 7
8 | 4
```

b. Low Temperatures

```
1 | 1
2 | 1 2 6 7 9
3 | 1 5 6 8 9
4 | 0 3 3 6 7
5 | 0 0 4
6 | 5
7 |
8 |
```

c. The most frequent range for high is in 60s (9 of 20) with only one low temperature above 54
High temperatures range mostly from 41 to 68, while low temperatures range mostly from 21 to 47
Low was 11; high was 84

d.

High Temp	Frequency	Low Temp	Frequency
10–19	0	10–19	1
20–29	0	20–29	5
30–39	1	30–39	5
40–49	4	40–49	5
50–59	3	50–59	3
60–69	9	60–69	1
70–79	2	70–79	0
80–89	1	80–89	0
Total	20	Total	20

48. a.

Level of Support	Percent Frequency
Strongly favor	30.10
Favor more than oppose	34.83
Oppose more than favor	21.13
Strongly oppose	13.94
Total	100.00

Overall favor higher tax = 30.10% + 34.83%
= 64.93%

b. 20.2, 19.5, 20.6, 20.7, 19.0
Roughly 20% per country

c. The crosstabulation with column percentages:

	Country				
Support	Great Britain	Italy	Spain	Germany	United States
Strongly favor	31.00	31.96	45.99	19.98	20.98
Favor more than oppose	34.04	39.04	32.01	36.99	32.06
Oppose more than favor	23.00	17.99	13.98	24.03	26.96
Strongly oppose	11.96	11.01	8.03	18.99	20.00
Total	100.00	100.00	100.00	100.00	100.00

Considering the percentage of respondents who favor the higher tax by either saying "strongly favor" or "favor more than oppose," 65.04%, 71.00%, 78.00%, 56.97%, and 53.04% for the five countries; all show more than 50% support, but all European countries show more support for the tax than the United States; Italy and Spain show the highest level of support.

50. a. Row totals: 247; 54; 82; 121
Column totals: 149; 317; 17; 7; 14

b.

Year	Freq.	Fuel	Freq.
1973 or before	247	Elect.	149
1974–79	54	Nat. Gas	317
1980–86	82	Oil	17
1987–91	121	Propane	7
Total	504	Other	14
		Total	504

c. Crosstabulation of column percentages

Year Constructed	Fuel Type				
	Elect.	Nat. Gas	Oil	Propane	Other
1973 or before	26.9	57.7	70.5	71.4	50.0
1974–1979	16.1	8.2	11.8	28.6	0.0
1980–1986	24.8	12.0	5.9	0.0	42.9
1987–1991	32.2	22.1	11.8	0.0	7.1
Total	100.0	100.0	100.0	100.0	100.0

d. Crosstabulation of row percentages

Year Constructed	Fuel Type					
	Elect.	Nat. Gas	Oil	Propane	Other	Total
1973 or before	16.2	74.1	4.9	2.0	2.8	100.0
1974–1979	44.5	48.1	3.7	3.7	0.0	100.0
1980–1986	45.1	46.4	1.2	0.0	7.3	100.0
1987–1991	39.7	57.8	1.7	0.0	0.8	100.0

52. a. Crosstabulation of market value and profit

Market Value ($1000s)	Profit ($1000s)				
	0–300	300–600	600–900	900–1200	Total
0–8000	24	4			28
8000–16,000	4	4	2	1	11
16,000–24,000		2	1		3
24,000–32,000		1	2	2	5
32,000–40,000		2	1		3
Total	28	13	6	3	50

b. Crosstabulation of row percentages

Market Value ($1000s)	Profit ($1000s)				
	0–300	300–600	600–900	900–1200	Total
0–8000	85.71	14.29	0.00	0.00	100
8000–16,000	36.36	36.36	18.18	9.09	100
16,000–24,000	0.00	66.67	33.33	0.00	100
24,000–32,000	0.00	20.00	40.00	40.00	100
32,000–40,000	0.00	66.67	33.33	0.00	100

c. There appears to be a positive relationship between Profit and Market Value. As profit goes up, Market Value goes up.

54. b. A positive relationship is demonstrated between market value and stockholders' equity

Chapter 3

2. 16, 16.5

3. Arrange data in order: 15, 20, 25, 25, 27, 28, 30, 34

$i = \frac{20}{100}(8) = 1.6$; round up to position 2

20th percentile = 20

$i = \frac{25}{100}(8) = 2$; use positions 2 and 3

$$25\text{th percentile} = \frac{20 + 25}{2} = 22.5$$

$i = \frac{65}{100}(8) = 5.2$; round up to position 6

65th percentile = 28

$i = \frac{75}{100}(8) = 6$; use positions 6 and 7

$$75\text{th percentile} = \frac{28 + 30}{2} = 29$$

4. 59.73, 57, 53

6. a. 18.42

b. 6.32

c. 34.3%

d. Reductions of only .65 shots and .9% made shots per game
Yes, agree but not dramatically

8. a. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{3200}{20} = 160$

Order the data from low 100 to high 360

Median: $i = \left(\frac{50}{100}\right)20 = 10$; use 10th and 11th positions

$$\text{Median} = \left(\frac{130 + 140}{2}\right) = 135$$

Mode = 120 (occur 3 times)

b. $i = \left(\frac{25}{100}\right)20 = 5$; use 5th and 6th positions

$$Q_1 = \left(\frac{115 + 115}{2}\right) = 115$$

$i = \left(\frac{75}{100}\right)20 = 15$; use 15th and 16th positions

$$Q_3 = \left(\frac{180 + 195}{2}\right) = 187.5$$

c. $i = \left(\frac{90}{100}\right)20 = 18$; use 18th and 19th positions

$$90\text{th percentile} = \left(\frac{235 + 255}{2}\right) = 245$$

90% of the tax returns cost $245 or less

10. a. .4%, 3.5%

b. 2.3%, 2.5%, 2.7%

c. 2.0%, 2.8%

d. optimistic

12. Disney: 3321, 255.5, 253, 169, 325
Pixar: 3231, 538.5, 505, 363, 631
Pixar films generate approximately twice as much box office revenue per film

14. 16, 4

15. Range = 34 − 15 = 19
Arrange data in order: 15, 20, 25, 25, 27, 28, 30, 34

$i = \frac{25}{100}(8) = 2$; $Q_1 = \frac{20 + 25}{2} = 22.5$

$i = \frac{75}{100}(8) = 6$; $Q_3 = \frac{28 + 30}{2} = 29$

$\text{IQR} = Q_3 - Q_1 = 29 - 22.5 = 6.5$

$\bar{x} = \dfrac{\Sigma x_i}{n} = \dfrac{204}{8} = 25.5$

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
27	1.5	2.25
25	−.5	.25
20	−5.5	30.25
15	−10.5	110.25
30	4.5	20.25
34	8.5	72.25
28	2.5	6.25
25	−.5	.25
		242.00

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n - 1} = \frac{242}{8 - 1} = 34.57$$

$$s = \sqrt{34.57} = 5.88$$

16. a. Range = 190 − 168 = 22

b. $\bar{x} = \dfrac{\Sigma x_i}{n} = \dfrac{1068}{6} = 178$

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n - 1} = \frac{4^2 + (-10)^2 + 6^2 + 12^2 + (-8)^2 + (-4)^2}{6 - 1} = \frac{376}{5} = 75.2$$

c. $s = \sqrt{75.2} = 8.67$

d. $\dfrac{s}{\bar{x}}(100) = \dfrac{8.67}{178}(100\%) = 4.87\%$

18. a. 38, 97, 9.85

b. Eastern shows more variation

20. *Dawson:* range = 2, s = .67
Clark: range = 8, s = 2.58

22. a. 1285, 433
Freshmen spend more

b. 1720, 352

c. 404, 131.5

d. 367.04, 96.96

e. Freshmen have more variability

24. *Quarter-milers:* s = .0564, Coef. of Var. = 5.8%
Milers: s = .1295, Coef. of Var. = 2.9%

26. .20, 1.50, 0, −.50, −2.20

27. Chebyshev's theorem: *at least* $(1 - 1/z^2)$

a. $z = \dfrac{40 - 30}{5} = 2$; $1 - \dfrac{1}{(2)^2} = .75$

b. $z = \dfrac{45 - 30}{5} = 3$; $1 - \dfrac{1}{(3)^2} = .89$

c. $z = \dfrac{38 - 30}{5} = 1.6$; $1 - \dfrac{1}{(1.6)^2} = .61$

d. $z = \dfrac{42 - 30}{5} = 2.4$; $1 - \dfrac{1}{(2.4)^2} = .83$

e. $z = \dfrac{48 - 30}{5} = 3.6$; $1 - \dfrac{1}{(3.6)^2} = .92$

28. a. 95%

b. Almost all

c. 68%

29. a. z = 2 standard deviations

$1 - \dfrac{1}{z^2} = 1 - \dfrac{1}{2^2} = \dfrac{3}{4}$; at least 75%

b. z = 2.5 standard deviations

$1 - \dfrac{1}{z^2} = 1 - \dfrac{1}{2.5^2} = .84$; at least 84%

c. z = 2 standard deviations
Empirical rule: 95%

30. a. 68%

b. 81.5%

c. 2.5%

32. a. −.67

b. 1.50

c. Neither an outlier

d. Yes; z = 8.25

34. a. 76.5, 7

b. 16%, 2.5%

c. 12.2, 7.89; no

36. 15, 22.5, 26, 29, 34

38. Arrange data in order: 5, 6, 8, 10, 10, 12, 15, 16, 18

$i = \dfrac{25}{100}(9) = 2.25$; round up to position 3

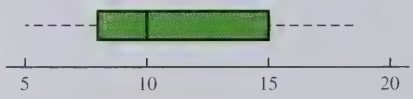
$Q_1 = 8$

Median (5th position) = 10

$i = \dfrac{75}{100}(9) = 6.75$; round up to position 7

$Q_3 = 15$

5-number summary: 5, 8, 10, 15, 18



40. a. Subway with 29,612 locations

b. Order data smallest to largest
Median (middle value) = 5889

c. Smallest = 1397; Largest = 29,612

$i = \dfrac{25}{100}(13) = 3.25$ Round to 4th position

$Q_1 = 4516$

$i = \dfrac{75}{100}(13) = 9.75$ Round to 10th position

$Q_3 = 10{,}238$

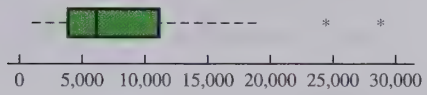
5-number summary: 1397 4516 5889 10,238 29,612

d. IQR = $Q_3 - Q_1$ = 10,238 − 4516 = 5722
Lower: Q_1 − 1.5 (IQR) = 4516 − 1.5(5722) = −4067

Upper: $Q_3 + 1.5\,(\text{IQR}) = 10{,}238 + 1.5(5722) = 18{,}821$

Subway and McDonald's are outliers

e.



41. a. Arrange data in order low to high

$i = \frac{25}{100}(21) = 5.25$; round up to 6th position

$Q_1 = 1872$

Median (11th position) = 4019

$i = \frac{75}{100}(21) = 15.75$; round up to 16th position

$Q_3 = 8305$

5-number summary: 608, 1872, 4019, 8305, 14,138

b. IQR $= Q_3 - Q_1 = 8305 - 1872 = 6433$

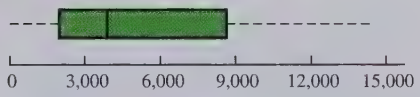
Lower limit: $1872 - 1.5(6433) = -7777.5$

Upper limit: $8305 + 1.5(6433) = 17{,}955$

c. No; data are within limits

d. 41,138 > 27,604; 41,138 would be an outlier; data value would be reviewed and corrected

e.



42. a. 73.5

b. 68, 71.5, 73.5, 74.5, 77

c. Limits: 67 and 79; no outliers

d. 66, 68, 71, 73, 75; 60.5 and 80.5
63, 65, 66, 67.6, 69; 61.25 and 71.25
75, 77, 78.5, 79.5, 81; 73.25 and 83.25
No outliers for any of the services

e. Verizon is highest rated
Sprint is lowest rated

44. a. 18.2, 15.35

b. 11.7, 23.5

c. 3.4, 11.7, 15.35, 23.5, 41.3

d. Yes; Alger Small Cap 41.3

45. b. There appears to be a negative linear relationship between x and y

c.

x_i	y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})(y_i - \bar{y})$
4	50	−4	4	−16
6	50	−2	4	−8
11	40	3	−6	−18
3	60	−5	14	−70
16	30	8	−16	−128
40	230	0	0	−240

$\bar{x} = 8$; $\bar{y} = 46$

$$s_{xy} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{n-1} = \frac{-240}{4} = -60$$

The sample covariance indicates a negative linear association between x and y

d. $r_{xy} = \frac{s_{xy}}{s_x s_y} = \frac{-60}{(5.43)(11.40)} = -.969$

The sample correlation coefficient of −.969 is indicative of a strong negative linear relationship

46. b. There appears to be a positive linear relationship between x and y

c. $s_{xy} = 26.5$

d. $r_{xy} = .693$

48. −.91; negative relationship

50. b. .910

c. Strong positive linear relationship; no

52. a. 3.69

b. 3.175

53. a.

f_i	M_i	$f_i M_i$
4	5	20
7	10	70
9	15	135
5	20	100
25		325

$$\bar{x} = \frac{\Sigma f_i M_i}{n} = \frac{325}{25} = 13$$

b.

f_i	M_i	$(M_i - \bar{x})$	$(M_i - \bar{x})^2$	$f_i(M_i - \bar{x})^2$
4	5	−8	64	256
7	10	−3	9	63
9	15	2	4	36
5	20	7	49	245
25				600

$$s^2 = \frac{\Sigma f_i (M_i - \bar{x})^2}{n-1} = \frac{600}{25-1} = 25$$

$$s = \sqrt{25} = 5$$

54. a.

Grade x_i	Weight w_i
4 (A)	9
3 (B)	15
2 (C)	33
1 (D)	3
0 (F)	0
	60 credit hours

$$\bar{x} = \frac{\Sigma w_i x_i}{\Sigma w_i} = \frac{9(4) + 15(3) + 33(2) + 3(1)}{9 + 15 + 33 + 3} = \frac{150}{60} = 2.5$$

b. Yes

56. 3.8, 3.7

58. a. 1800, 1351

b. 387, 1710

c. 7280, 1323
d. 3,675,303, 1917
e. High positive skewness
f. Using a box plot: 4135 and 7450 are outliers

60. a. 2.3, 1.85
b. 1.90, 1.38
c. Altria Group 5%
d. −.51, below mean
e. 1.02, above mean
f. No

62. a. $670
b. $456
c. $z = 3$; yes
d. Save time and prevent a penalty cost

64. a. 215.9
b. 55%
c. 175.0, 628.3
d. 48.8, 175.0, 215.9, 628.3, 2325.0
e. Yes, any price over 1308.25
f. 482.1; prefer median

66. a. 364 rooms
b. $457
c. −.293; slight negative correlation
Higher cost per night tends to be associated with smaller hotels

68. a. .268, low or weak positive correlation
b. Very poor predictor; spring training is practice and does not count toward standings or playoffs

70. a. 60.68
b. $s^2 = 31.23$; $s = 5.59$

Chapter 4

2. $\binom{6}{3} = \frac{6!}{3!3!} = \frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{(3\cdot2\cdot1)(3\cdot2\cdot1)} = 20$

ABC	ACE	BCD	BEF
ABD	ACF	BCE	CDE
ABE	ADE	BCF	CDF
ABF	ADF	BDE	CEF
ACD	AEF	BDF	DEF

4. b. (H,H,H), (H,H,T), (H,T,H), (H,T,T), (T,H,H), (T,H,T), (T,T,H), (T,T,T)
c. 1/8

6. $P(E_1) = .40$, $P(E_2) = .26$, $P(E_3) = .34$
The relative frequency method was used

8. a. 4: Commission Positive—Council Approves
Commission Positive—Council Disapproves
Commission Negative—Council Approves
Commission Negative—Council Disapproves

9. $\binom{50}{4} = \frac{50!}{4!46!} = \frac{50\cdot49\cdot48\cdot47}{4\cdot3\cdot2\cdot1} = 230{,}300$

10. a. Using the table, $P(\text{Debt}) = .94$
b. Five of the 8 institutions, $P(\text{over } 60\%) = 5/8 = .625$
c. Two of the 8 institutions, $P(\text{more than } \$30{,}000) = 2/8 = .25$
d. $P(\text{No debt}) = 1 - P(\text{debt}) = 1 - .72 = .28$
e. A weighted average with 72% having average debt of $32,980 and 28% having no debt

$$\text{Average debt per graduate} = \frac{.72(\$32{,}980) + .28(\$0)}{.72 + .28} = \$23{,}746$$

12. a. 3,478,761
b. 1/3,478,761
c. 1/146,107,962

14. a. 1/4
b. 1/2
c. 3/4

15. a. S = {ace of clubs, ace of diamonds, ace of hearts, ace of spades}
b. S = {2 of clubs, 3 of clubs, . . . , 10 of clubs, J of clubs, Q of clubs, K of clubs, A of clubs}
c. There are 12; jack, queen, or king in each of the four suits
d. For (a): 4/52 = 1/13 = .08
For (b): 13/52 = 1/4 = .25
For (c): 12/52 = .23

16. a. 36
c. 1/6
d. 5/18
e. No; $P(\text{odd}) = P(\text{even}) = \frac{1}{2}$
f. Classical

17. a. (4, 6), (4, 7), (4, 8)
b. .05 + .10 + .15 = .30
c. (2, 8), (3, 8), (4, 8)
d. .05 + .05 + .15 = .25
e. .15

18. a. .0222
b. .8226
c. .1048

20. a. .108
b. .096
c. .434

22. a. .40, .40, .60
b. .80, yes
c. $A^c = \{E_3, E_4, E_5\}$; $C^c = \{E_1, E_4\}$;
$P(A^c) = .60$; $P(C^c) = .40$
d. (E_1, E_2, E_5); .60
e. .80

23. a. $P(A) = P(E_1) + P(E_4) + P(E_6)$
$= .05 + .25 + .10 = .40$
$P(B) = P(E_2) + P(E_4) + P(E_7)$
$= .20 + .25 + .05 = .50$
$P(C) = P(E_2) + P(E_3) + P(E_5) + P(E_7)$
$= .20 + .20 + .15 + .05 = .60$
b. $A \cup B = \{E_1, E_2, E_4, E_6, E_7\}$;
$P(A \cup B) = P(E_1) + P(E_2) + P(E_4) + P(E_6) + P(E_7)$
$= .05 + .20 + .25 + .10 + .05$
$= .65$
c. $A \cap B = \{E_4\}$; $P(A \cap B) = P(E_4) = .25$
d. Yes, they are mutually exclusive

e. $B^c = \{E_1, E_3, E_5, E_6\}$;
$P(B^c) = P(E_1) + P(E_3) + P(E_5) + P(E_6)$
$= .05 + .20 + .15 + .10$
$= .50$

24. a. .05
b. .70

26. a. .64
b. .48
c. .36
d. .76

28. Let B = rented a car for business reasons
P = rented a car for personal reasons
a. $P(B \cup P) = P(B) + P(P) - P(B \cap P)$
$= .540 + .458 - .300$
$= .698$
b. $P(\text{Neither}) = 1 - .698 = .302$

30. a. $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.40}{.60} = .6667$

b. $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{.40}{.50} = .80$

c. No, because $P(A \mid B) \neq P(A)$

32. a.

	Car	Light Truck	Total
U.S.	.1330	.2939	.4269
Non-U.S.	.3478	.2253	.5731
Total	.4808	.5192	1.0000

b. .4269, .5731 Non-U.S. higher
.4808, .5192 Light Truck slightly higher
c. .3115, .6885 Light Truck higher
d. .6909, .3931 Car higher
e. .5661, U.S. higher for Light Trucks

33. a.

	Reason for Applying			
	Quality	Cost/ Convenience	Other	Total
Full-time	.218	.204	.039	.461
Part-time	.208	.307	.024	.539
Total	.426	.511	.063	1.000

b. A student is most likely to cite cost or convenience as the first reason (probability = .511); school quality is the reason cited by the second largest number of students (probability = .426)
c. $P(\text{quality} \mid \text{full-time}) = .218/.461 = .473$
d. $P(\text{quality} \mid \text{part-time}) = .208/.539 = .386$
e. For independence, we must have $P(A)P(B) = P(A \cap B)$; from the table
$P(A \cap B) = .218$, $P(A) = .461$, $P(B) = .426$
$P(A)P(B) = (.461)(.426) = .196$
Because $P(A)P(B) \neq P(A \cap B)$, the events are not independent

34. a.

	On Time	Late	Total
Southwest	.3336	.0664	.40
US Airways	.2629	.0871	.35
JetBlue	.1753	.0747	.25
Total	.7718	.2282	1.00

b. Southwest (.40)
c. .7718
d. US Airways (.3817); Southwest (.2910)

36. a. .7921
b. .9879
c. .0121
d. .3364, .8236, .1764
Don't foul Jerry Stackhouse

38. a.

	Met Proficiency Standards		
Grade	Yes	No	Total
3	.11196	.05663	0.16858
4	.08271	.08205	0.16476
5	.08517	.07922	0.16439
6	.08588	.07777	0.16365
7	.09671	.07031	0.16702
8	.09618	.07542	0.17159
Total	.55861	.44139	1.00000

b. The column marginal probabilities are .55861 and .44139; the row marginal probabilities are .16858, .16476, and so on
c. .6651; .5020
d. .2004; .1481

39. a. Yes, because $P(A_1 \cap A_2) = 0$
b. $P(A_1 \cap B) = P(A_1)P(B \mid A_1) = .40(.20) = .08$
$P(A_2 \cap B) = P(A_2)P(B \mid A_2) = .60(.05) = .03$
c. $P(B) = P(A_1 \cap B) + P(A_2 \cap B) = .08 + .03 = .11$
d. $P(A_1 \mid B) = \dfrac{.08}{.11} = .7273$

$P(A_2 \mid B) = \dfrac{.03}{.11} = .2727$

40. a. .10, .20, .09
b. .51
c. .26, .51, .23

42. M = missed payment
D_1 = customer defaults
D_2 = customer does not default
$P(D_1) = .05, P(D_2) = .95, P(M \mid D_2) = .2, P(M \mid D_1) = 1$

a. $P(D_1 \mid M) = \dfrac{P(D_1)P(M \mid D_1)}{P(D_1)P(M \mid D_1) + P(D_2)P(M \mid D_2)}$

$= \dfrac{(.05)(1)}{(.05)(1) + (.95)(.2)} = \dfrac{.05}{.24} = .21$

b. Yes, the probability of default is greater than .20

44. a. .47, .53, .50, .45
b. .4963
c. .4463
d. 47%, 53%

46. a. .60
b. .26
c. .40
d. .74

48. a. .2675
b. .3376
c. .0892
d. .5159
e. .0255

50. a. .76
b. .24

52. b. .2022
c. .4618
d. .4005

54. a. .49
b. .44
c. .54
d. No
e. Yes

56. a. .25
b. .125
c. .0125
d. .10
e. No

58. a.

	Young Adult	Older Adult	Total
Blogger	.0432	.0368	.08
Nonblogger	.2208	.6992	.92
Total	.2640	.7360	1.00

b. .2640
c. .0432
d. .1636

60. a. .40
b. .67

Chapter 5

1. a. Head, Head (H, H)
Head, Tail (H, T)
Tail, Head (T, H)
Tail, Tail (T, T)
b. x = number of heads on two coin tosses
c.

Outcome	Values of x
(H, H)	2
(H, T)	1
(T, H)	1
(T, T)	0

d. Discrete; 0, 1, and 2

2. a. x = time in minutes to assemble product
b. Any positive value: $x > 0$
c. Continuous

3. Let Y = position is offered
N = position is not offered
a. $S = \{(Y, Y, Y), (Y, Y, N), (Y, N, Y), (Y, N, N), (N, Y, Y), (N, Y, N), (N, N, Y), (N, N, N)\}$
b. Let N = number of offers made; N is a discrete random variable
c.

Experimental Outcome	(Y, Y, Y)	(Y, Y, N)	(Y, N, Y)	(Y, N, N)	(N, Y, Y)	(N, Y, N)	(N, N, Y)	(N, N, N)
Value of N	3	2	2	1	2	1	1	0

4. $x = 0, 1, 2, \ldots, 9$

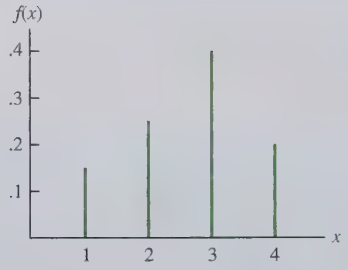
6. a. 0, 1, 2, . . . , 20; discrete
b. 0, 1, 2, . . . ; discrete
c. 0, 1, 2, . . . , 50; discrete
d. $0 \le x \le 8$; continuous
e. $x > 0$; continuous

7. a. $f(x) \ge 0$ for all values of x
$\Sigma f(x) = 1$; therefore, it is a valid probability distribution
b. Probability $x = 30$ is $f(30) = .25$
c. Probability $x \le 25$ is $f(20) + f(25) = .20 + .15 = .35$
d. Probability $x > 30$ is $f(35) = .40$

8. a.

x	$f(x)$
1	3/20 = .15
2	5/20 = .25
3	8/20 = .40
4	4/20 = .20
	Total 1.00

b.



c. $f(x) \ge 0$ for $x = 1, 2, 3, 4$
$\Sigma f(x) = 1$

10. a.

x	1	2	3	4	5
$f(x)$	.05	.09	.03	.42	.41

b.

x	1	2	3	4	5
$f(x)$	.04	.10	.12	.46	.28

c. .83
d. .28
e. Senior executives are more satisfied

12. a. Yes
b. .15
c. .10

14. **a.** .05
b. .70
c. .40

16. **a.**

y	$f(y)$	$yf(y)$
2	.20	.4
4	.30	1.2
7	.40	2.8
8	.10	.8
Totals	1.00	5.2

$E(y) = \mu = 5.2$

b.

y	$y - \mu$	$(y - \mu)^2$	$f(y)$	$(y - \mu)^2 f(y)$
2	−3.20	10.24	.20	2.048
4	−1.20	1.44	.30	.432
7	1.80	3.24	.40	1.296
8	2.80	7.84	.10	.784
			Total	4.560

$\text{Var}(y) = 4.56$
$\sigma = \sqrt{4.56} = 2.14$

18. **a/b.**

x	$f(x)$	$xf(x)$	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2 f(x)$
0	0.04	0.00	−1.84	3.39	0.12
1	0.34	0.34	−0.84	0.71	0.24
2	0.41	0.82	0.16	0.02	0.01
3	0.18	0.53	1.16	1.34	0.24
4	0.04	0.15	2.16	4.66	0.17
Total	1.00	1.84			0.79
		↑ $E(x)$			↑ $\text{Var}(x)$

c/d.

y	$f(y)$	$yf(y)$	$y - \mu$	$(y - \mu)^2$	$y - \mu^2 f(y)$
0	0.00	0.00	−2.93	8.58	0.01
1	0.03	0.03	−1.93	3.72	0.12
2	0.23	0.45	−0.93	0.86	0.20
3	0.52	1.55	0.07	0.01	0.00
4	0.22	0.90	1.07	1.15	0.26
Total	1.00	2.93			0.59
		↑ $E(y)$			↑ $\text{Var}(y)$

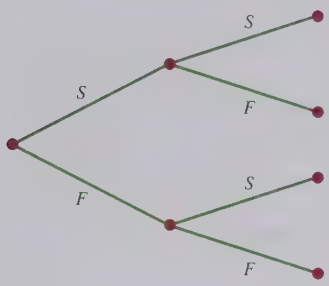
e. The number of bedrooms in owner-occupied houses is greater than in renter-occupied houses; the expected number of bedrooms is 2.93 − 1.84 = 1.09 greater; and the variability in the number of bedrooms is less for the owner-occupied houses

20. **a.** 430
b. −90; concern is to protect against the expense of a large loss

22. **a.** 445
b. $1250 loss

24. **a.** Medium: 145; large: 140
b. Medium: 2725; large: 12,400

25. **a.**



b. $f(1) = \binom{2}{1}(.4)^1(.6)^1 = \frac{2!}{1!1!}(.4)(.6) = .48$

c. $f(0) = \binom{2}{0}(.4)^0(.6)^2 = \frac{2!}{0!2!}(1)(.36) = .36$

d. $f(2) = \binom{2}{2}(.4)^2(.6)^0 = \frac{2!}{2!0!}(.16)(.1) = .16$

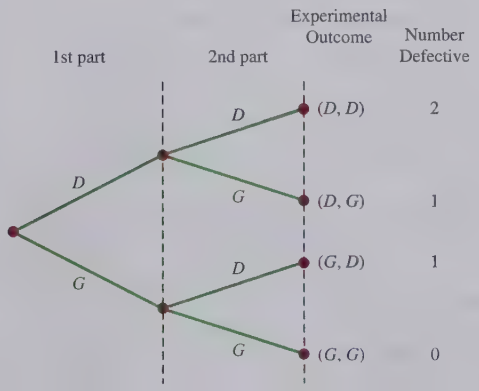
e. $P(x \geq 1) = f(1) + f(2) = .48 + .16 = .64$

f. $E(x) = np = 2(.4) = .8$
$\text{Var}(x) = np(1 - p) = 2(.4)(.6) = .48$
$\sigma = \sqrt{.48} = .6928$

26. **a.** .3487
b. .1937
c. .9298
d. .6513
e. 1
f. .9, .95

28. **a.** .2789
b. .4181
c. .0733

30. **a.** Probability of a defective part being produced must be .03 for each part selected; parts must be selected independently
b. Let D = defective
G = not defective



c. Two outcomes result in exactly one defect

d. $P(\text{no defects}) = (.97)(.97) = .9409$
$P(1 \text{ defect}) = 2(.03)(.97) = .0582$
$P(2 \text{ defects}) = (.03)(.03) = .0009$

32. a. .90
b. .99
c. .999
d. Yes

34. a. .2262
b. .8355

36. a. .1897
b. .9757
c. $f(12) = .0008$; yes
d. 5

38. a. $f(x) = \dfrac{3^x e^{-3}}{x!}$
b. .2241
c. .1494
d. .8008

39. a. $f(x) = \dfrac{2^x e^{-2}}{x!}$
b. $\mu = 6$ for 3 time periods
c. $f(x) = \dfrac{6^x e^{-6}}{x!}$
d. $f(2) = \dfrac{2^2 e^{-2}}{2!} = \dfrac{4(.1353)}{2} = .2706$
e. $f(6) = \dfrac{6^6 e^{-6}}{6!} = .1606$
f. $f(5) = \dfrac{4^5 e^{-4}}{5!} = .1563$

40. a. .1952
b. .1048
c. .0183
d. .0907

42. a. $f(0) = \dfrac{7^0 e^{-7}}{0!} = e^{-7} = .0009$
b. probability $= 1 - [f(0) + f(1)]$
$f(1) = \dfrac{7^1 e^{-7}}{1!} = 7e^{-7} = .0064$
probability $= 1 - [.0009 + .0064] = .9927$
c. $\mu = 3.5$
$f(0) = \dfrac{3.5^0 e^{-3.5}}{0!} = e^{-3.5} = .0302$
probability $= 1 - f(0) = 1 - .0302 = .9698$
d.
probability $= 1 - [f(0) + f(1) + f(2) + f(3) + f(4)]$
$= 1 - [.0009 + .0064 + .0223 + .0521 + .0912]$
$= .8271$

44. a. $\mu = 1.25$
b. .2865
c. .3581
d. .3554

46. a. $f(1) = \dfrac{\binom{3}{1}\binom{10-3}{4-1}}{\binom{10}{4}} = \dfrac{\left(\frac{3!}{1!2!}\right)\left(\frac{7!}{3!4!}\right)}{\frac{10!}{4!6!}}$
$= \dfrac{(3)(35)}{210} = .50$
b. $f(2) = \dfrac{\binom{3}{2}\binom{10-3}{2-2}}{\binom{10}{2}} = \dfrac{(3)(1)}{45} = .067$
c. $f(0) = \dfrac{\binom{3}{0}\binom{10-3}{2-0}}{\binom{10}{2}} = \dfrac{(1)(21)}{45} = .4667$
d. $f(2) = \dfrac{\binom{3}{2}\binom{10-3}{4-2}}{\binom{10}{4}} = \dfrac{(3)(21)}{210} = .30$
e. $x = 4$ is *greater than* $r = 3$; thus, $f(4) = 0$

48. a. .5250
b. .8167

50. $N = 60, n = 10$
a. $r = 20, x = 0$
$f(0) = \dfrac{\binom{20}{0}\binom{40}{10}}{\binom{60}{10}} = \dfrac{(1)\left(\frac{40!}{10!30!}\right)}{\frac{60!}{10!50!}}$
$= \left(\dfrac{40!}{10!30!}\right)\left(\dfrac{10!50!}{60!}\right)$
$= \dfrac{40 \cdot 39 \cdot 38 \cdot 37 \cdot 36 \cdot 35 \cdot 34 \cdot 33 \cdot 32 \cdot 31}{60 \cdot 59 \cdot 58 \cdot 57 \cdot 56 \cdot 55 \cdot 54 \cdot 53 \cdot 52 \cdot 51}$
$= .0112$
b. $r = 20, x = 1$
$f(1) = \dfrac{\binom{20}{1}\binom{40}{9}}{\binom{60}{10}} = 20\left(\dfrac{40!}{9!31!}\right)\left(\dfrac{10!50!}{60!}\right)$
$= .0725$
c. $1 - f(0) - f(1) = 1 - .0112 - .0725 = .9163$
d. Same as the probability one will be from Hawaii; .0725

52. a. .2917
b. .0083
c. .5250, .1750; 1 bank
d. .7083
e. .90, .49, .70

54. a.

x	1	2	3	4	5
$f(x)$	.24	.21	.10	.21	.24

b. 3.00, 2.34
c. Bonds: $E(x) = 1.36$, $\text{Var}(x) = .23$
Stocks: $E(x) = 4$, $\text{Var}(x) = 1$

56. a. .0596
b. .3585

c. 100
d. 95, 9.75

58. a. .9510
b. .0480
c. .0490

60. a. 240
b. 12.96
c. 12.96

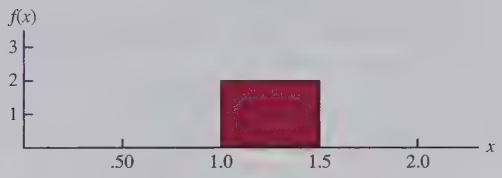
62. .1912

64. a. .2240
b. .5767

66. a. .4667
b. .4667
c. .0667

Chapter 6

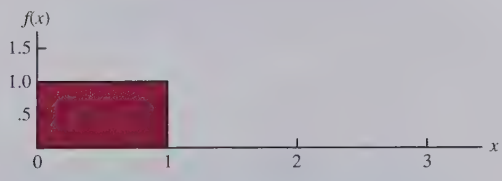
1. a.



b. $P(x = 1.25) = 0$; the probability of any single point is zero because the area under the curve above any single point is zero
c. $P(1.0 \le x \le 1.25) = 2(.25) = .50$
d. $P(1.20 < x < 1.5) = 2(.30) = .60$

2. b. .50
c. .60
d. 15
e. 8.33

4. a.



b. $P(.25 < x < .75) = 1(.50) = .50$
c. $P(x \le .30) = 1(.30) = .30$
d. $P(x > .60) = 1(.40) = .40$

6. a. .125
b. .50
c. .25

10. a. .9332
b. .8413
c. .0919
d. .4938

12. a. .2967
b. .4418
c. .3300
d. .5910
e. .8849
f. .2389

13. a. $P(-1.98 \le z \le .49) = P(z \le .49) - P(z < -1.98) = .6879 - .0239 = .6640$
b. $P(.52 \le z \le 1.22) = P(z \le 1.22) - P(z < .52) = .8888 - .6985 = .1903$
c. $P(-1.75 \le z \le -1.04) = P(z \le -1.04) - P(z < -1.75) = .1492 - .0401 = .1091$

14. a. $z = 1.96$
b. $z = 1.96$
c. $z = .61$
d. $z = 1.12$
e. $z = .44$
f. $z = .44$

15. a. The z value corresponding to a cumulative probability of .2119 is $z = -.80$
b. Compute .9030/2 = .4515; the cumulative probability of .5000 + .4515 = .9515 corresponds to $z = 1.66$
c. Compute .2052/2 = .1026; z corresponds to a cumulative probability of .5000 + .1026 = .6026, so $z = .26$
d. The z value corresponding to a cumulative probability of .9948 is $z = 2.56$
e. The area to the left of z is 1 − .6915 = .3085, so $z = -.50$

16. a. $z = 2.33$
b. $z = 1.96$
c. $z = 1.645$
d. $z = 1.28$

18. $\mu = 30$ and $\sigma = 8.2$
a. At $x = 40$, $z = \dfrac{40 - 30}{8.2} = 1.22$
$P(z \le 1.22) = .8888$
$P(x \ge 40) = 1.000 - .8888 = .1112$
b. At $x = 20$, $z = \dfrac{20 - 30}{8.2} = -1.22$
$P(z \le -1.22) = .1112$
$P(x \le 20) = .1112$
c. A z value of 1.28 cuts off an area of approximately 10% in the upper tail
$x = 30 + 8.2(1.28)$
$= 40.50$
A stock price of $40.50 or higher will put a company in the top 10%

20. a. .6553
b. 13.05 hours
c. .9838

22. a. .6553
b. 13.05 hours
c. .9838

24. a. 200, 26.04
b. .2206

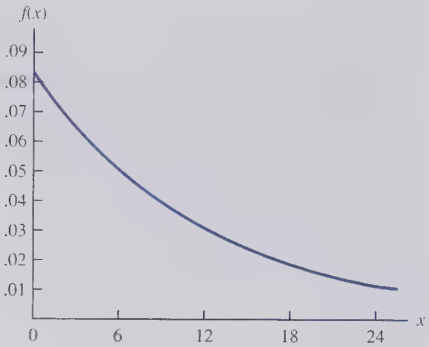
c. .1251
d. 242.84 million

26. a. .5276
b. .3935
c. .4724
d. .1341

27. a. $P(x \leq x_0) = 1 - e^{-x_0/3}$
b. $P(x \leq 2) = 1 - e^{-2/3} = 1 - .5134 = .4866$
c. $P(x \geq 3) = 1 - P(x \leq 3) = 1 - (1 - e^{-3/3})$
$= e^{-1} = .3679$
d. $P(x \leq 5) = 1 - e^{-5/3} = 1 - .1889 = .8111$
e. $P(2 \leq x \leq 5) = P(x \leq 5) - P(x \leq 2)$
$= .8111 - .4866 = .3245$

28. a. .5624
b. .1915
c. .2461
d. .2259

29. a.



b. $P(x \leq 12) = 1 - e^{-12/12} = 1 - .3679 = .6321$
c. $P(x \leq 6) = 1 - e^{-6/12} = 1 - .6065 = .3935$
d. $P(x \geq 30) = 1 - P(x < 30)$
$= 1 - (1 - e^{-30/12})$
$= .0821$

30. a. .3935
b. .2386
c. .1353

32. a. $f(x) = 5.5e^{-5.5x}$
b. .2528
c. .6002

34. a. $3780 or less
b. 19.22%
c. $8167.50

36. a. 3229
b. .2244
c. $12,382 or more

38. a. .0228
b. $50

40. a. 38.3%
b. 3.59% better, 96.41% worse
c. 38.21%

42. $\mu = 19.23$ ounces

44. a. 1/7 minute
b. $7e^{-7x}$
c. .0009
d. .2466

46. a. 2 minutes
b. .2212
c. .3935
d. .0821

Chapter 7

1. a. AB, AC, AD, AE, BC, BD, BE, CD, CE, DE
b. With 10 samples, each has a 1/10 probability
c. B and D because the two smallest random numbers are .0476 and .0957

2. Elements 2, 3, 5, and 10

3. The simple random sample consists of New York, Detroit, Oakland, Boston, and Kansas City

4. Step 1. Generate a random number for each stock
Step 2. Sort with respect to random numbers and select the first three stocks

6. a. finite
b. infinite
c. infinite
d. finite
e. infinite

7. a. $\bar{x} = \dfrac{\Sigma x_i}{n} = \dfrac{54}{6} = 9$

b. $s = \sqrt{\dfrac{\Sigma(x_i - \bar{x})^2}{n - 1}}$

$\Sigma(x_i - \bar{x})^2 = (-4)^2 + (-1)^2 + 1^2 + (-2)^2 + 1^2 + 5^2$
$= 48$

$s = \sqrt{\dfrac{48}{6 - 1}} = 3.1$

8. a. .50
b. .3667

9. a. $\bar{x} = \dfrac{\Sigma x_i}{n} = \dfrac{465}{5} = 93$

b.

	x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
	94	+1	1
	100	+7	49
	85	−8	64
	94	+1	1
	92	−1	1
Totals	465	0	116

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n - 1}} = \sqrt{\frac{116}{4}} = 5.39$$

10. a. .45
b. .15
c. .45

12. a. .10
b. 20
c. .72

15. a. The sampling distribution is normal with:

$$E(\bar{x}) = \mu = 200$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{50}{\sqrt{100}} = 5$$

For +5, $(\bar{x} - \mu) = 5$,

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{5}{5} = 1$$

Area = .8413 − .1587 = .6826

b. For ±10, $(\bar{x} - \mu) = 10$,

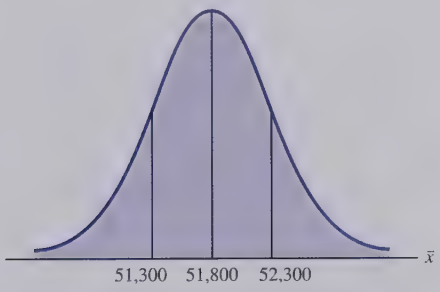
$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{10}{5} = 2$$

Area = .9772 − .0228 = .9544

16. 3.54, 2.50, 2.04, 1.77
$\sigma_{\bar{x}}$ decreases as n increases

18. a. Normal with $E(\bar{x}) = 51{,}800$ and $\sigma_{\bar{x}} = 516.40$
b. $\sigma_{\bar{x}}$ decreases to 365.15
c. $\sigma_{\bar{x}}$ decreases as n increases

19. a.



$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4000}{\sqrt{60}} = 516.40$$

$$\text{At } \bar{x} = 52{,}300,\ z = \frac{52{,}300 - 51{,}800}{516.40} = .97$$

$$P(\bar{x} \le 52{,}300) = P(z \le .97) = .8340$$

$$\text{At } \bar{x} = 51{,}300,\ z = \frac{51{,}300 - 51{,}800}{516.40} = -.97$$

$$P(\bar{x} < 51{,}300) = P(z < -.97) = .1660$$

$$P(51{,}300 \le \bar{x} \le 52{,}300) = .8340 - .1660 = .6680$$

Using Excel:
NORM.DIST(52300,51800,516.40,TRUE)-
NORM.DIST(51300,51800,516.40,TRUE) = .6671

b.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4000}{\sqrt{120}} = 365.15$$

$$\text{At } \bar{x} = 52{,}300,\ z = \frac{52{,}300 - 51{,}800}{365.15} = 1.37$$

$$P(\bar{x} \le 52{,}300) = P(z \le 1.37) = .9147$$

$$\text{At } \bar{x} = 51{,}300,\ z = \frac{51{,}300 - 51{,}800}{365.15} = -1.37$$

$$P(\bar{x} < 51{,}300) = P(z < -1.37) = .0853$$

$$P(51{,}300 \le \bar{x} \le 52{,}300) = .9147 - .0853 = .8294$$

Using Excel:
NORM.DIST(52300,51800,365.15,TRUE)-
NORM.DIST(51300,51800,365.15,TRUE) = .8291

20. a. Normal with $E(\bar{x}) = 17.5$ and $\sigma_{\bar{x}} = .57$
b. .9198
c. .6212

22. a. Using table: .4246, .5284, .6922, .9586
b. Higher probability with a larger sample size

24. a. Normal with $E(\bar{x}) = 95$ and $\sigma_{\bar{x}} = 2.56$
b. Using table: .7580; using NORM.DIST: .7595
c. Using table: .8502; using NORM.DIST: .8494
d. Part (c) because of the larger sample size

26. a. $n/N = .01$; no
b. 1.29, 1.30; little difference
c. Using table: .8764

28. a. $E(\bar{p}) = .40$

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(.40)(.60)}{200}} = .0346$$

Within ±.03 means $.37 \le \bar{p} \le .43$

Using table: $z = \dfrac{\bar{p} - p}{\sigma_{\bar{p}}} = \dfrac{.03}{.0346} = .87$

$$P(.37 \le \bar{p} \le .43) = P(-.87 \le z \le .87)$$
$$= .8078 - .1922$$
$$= .6156$$

Using Excel:
NORM.DIST(.43,.40,.0346,TRUE)-
NORM.DIST(.37,.40,.0346,TRUE) = .6141

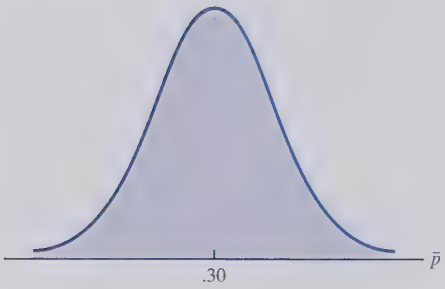
b. Using table: $z = \dfrac{\bar{p} - p}{\sigma_{\bar{p}}} = \dfrac{.05}{.0346} = 1.44$

$$P(.35 \le \bar{p} \le .45) = P(-1.44 \le z \le 1.44)$$
$$= .9251 - .0749$$
$$= .8502$$

Using Excel:
NORM.DIST(.45,.40,.0346,TRUE)-
NORM.DIST(.35,.40,.0346,TRUE) = .8516

30. a. Using table: .6156; using NORM.DIST: .6175
b. Using table: .7814; using NORM.DIST: .7830
c. Using table: .9488; using NORM.DIST: .9490
d. Using table: .9942; using NORM.DIST: .9942
e. Higher probability with larger n

31. a.



$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.30(.70)}{100}} = .0458$$

The normal distribution is appropriate because $np = 100(.30) = 30$ and $n(1-p) = 100(.70) = 70$ are both greater than 5

b. $P(.20 \le \bar{p} \le .40) = ?$

$$z = \frac{.40 - .30}{.0458} = 2.18$$

$$P(.20 \le \bar{p} \le .40) = P(-2.18 \le z \le 2.18)$$
$$= .9854 - .0146$$
$$= .9708$$

Using Excel:

NORM.DIST(.40,.30,.0458,TRUE)-
NORM.DIST(.20,.30,.0458,TRUE) = .9710

c. $P(.25 \le \bar{p} \le .35) = ?$

$$z = \frac{.35 - .30}{.0458} = 1.09$$

$$P(.25 \le \bar{p} \le .35) = P(-1.09 \le z \le 1.09)$$
$$= .8621 - .1379$$
$$= .7242$$

Using Excel:

NORM.DIST(.35,.30,.0458,TRUE)-
NORM.DIST(.25,.30,.0458,TRUE) = .7250

32. a. Normal with $E(\bar{p}) = .66$ and $\sigma_{\bar{p}} = .0273$
b. Using table: .8584; using NORM.DIST: .8571
c. Using table: .9606; using NORM.DIST: .9608
d. Yes, standard error is smaller for youth population
e. Using table: .9616; using NORM.DIST: .9618

34. a. Normal with $E(\bar{p}) = .56$ and $\sigma_{\bar{p}} = .0248$
b. Using table: .5820; using NORM.DIST: .5800
c. Using table: .8926; using NORM.DIST: .8932

36. a. Normal with $E(\bar{p}) = .76$ and $\sigma_{\bar{p}} = .0214$
b. Using table: .8384; using NORM.DIST: .8390
c. Using table: .9452; using NORM.DIST: .9455

38. Thoroughbreds, The Marker, Officers Club, SeaBlue, Crickets

40. a. Normal with $E(\bar{x}) = 115.50$ and $\sigma_{\bar{x}} = 5.53$
b. Using table: .9298; using NORM.DIST: .9292
c. Using table: $z = -2.80$, .0026;
using NORM.DIST: .0025

42. a. 955
b. .50
c. Using table: $z = \pm 1.05$, .7062;
using NORM.DIST: .7050
d. .8230
using NORM.DIST: .8234

44. a. 625
b. .7888

46. a. Normal with $E(\bar{p}) = .28$ and $\sigma_{\bar{p}} = .0290$
b. Using table: $z = \pm 1.38$, .8324;
using NORM.DIST: .8322
c. Using table: $z = \pm .69$, .5098;
using NORM.DIST: .5096

48. a. Using table: $z = \pm 1.59$, .8882; using NORM.DIST: .8900
b. Using table: $z = +1.99$, .0233; using NORM.DIST: .0232

50. a. 48
b. Normal, $E(\bar{p}) = .25$, $\sigma_{\bar{p}} = .0625$
c. .2119

Chapter 8

2. Use $\bar{x} \pm z_{\alpha/2}(\sigma/\sqrt{n})$
a. $32 \pm 1.645(6/\sqrt{50})$
32 ± 1.4; 30.6 to 33.4
b. $32 \pm 1.96(6/\sqrt{50})$
32 ± 1.66; 30.34 to 33.66
c. $32 \pm 2.576(6/\sqrt{50})$
32 ± 2.19; 29.81 to 34.19

4. 54

5. a. $1.96\sigma/\sqrt{n} = 1.96(5/\sqrt{49}) = 1.40$
b. 24.80 ± 1.40; 23.40 to 26.20

6. 8.1 to 8.9

8. a. Population is at least approximately normal
b. 3.1
c. 4.1

10. a. \$113,638 to \$124,672
b. \$112,581 to \$125,729
c. \$110,515 to \$127,795
d. Width increases as confidence level increases

12. a. 2.179
b. −1.676
c. 2.457
d. −1.708 and 1.708
e. −2.014 and 2.014

13. a. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{80}{8} = 10$

b. $s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{84}{7}} = 3.46$

c. $t_{.025}\left(\frac{s}{\sqrt{n}}\right) = 2.365\left(\frac{3.46}{\sqrt{8}}\right) = 2.9$

d. $\bar{x} \pm t_{.025}\left(\frac{s}{\sqrt{n}}\right)$

10 ± 2.9 (7.1 to 12.9)

14. a. 21.5 to 23.5
b. 21.3 to 23.7
c. 20.9 to 24.1
d. A larger margin of error and a wider interval

15. $\bar{x} \pm t_{\alpha/2}(s/\sqrt{n})$

90% confidence: $df = 64$ and $t_{.05} = 1.669$

$19.5 \pm 1.669\left(\frac{5.2}{\sqrt{65}}\right)$

19.5 ± 1.08 (18.42 to 20.58)

95% confidence: $df = 64$ and $t_{.025} = 1.998$

$19.5 \pm 1.998\left(\frac{5.2}{\sqrt{65}}\right)$

19.5 ± 1.29 (18.21 to 20.79)

16. a. 1.69
b. 47.31 to 50.69
c. Fewer hours and higher cost for United

18. a. 22
b. 3.8014
c. 18.20 to 25.80
d. Larger n next time

20. $\bar{x} = 22$; 21.48 to 22.52

22. a. \$9269.52 to \$12,540.48
b. 1523
c. 4,748,714; 434 million

24. a. Planning value of $\sigma = \frac{\text{Range}}{4} = \frac{36}{4} = 9$

b. $n = \frac{z_{.025}^2\sigma^2}{E^2} = \frac{(1.96)^2(9)^2}{(3)^2} = 34.57$; use $n = 35$

c. $n = \frac{(1.96)^2(9)^2}{(2)^2} = 77.79$; use $n = 78$

25. a. Use $n = \frac{z_{\alpha/2}^2\sigma^2}{E^2}$

$n = \frac{(1.96)^2(6.84)^2}{(1.5)^2} = 79.88$; use $n = 80$

b. $n = \frac{(1.645)^2(6.84)^2}{(2)^2} = 31.65$; use $n = 32$

26. a. 18
b. 35
c. 97

28. a. 328
b. 465
c. 803
d. n gets larger; no to 99% confidence

30. 81

31. a. $\bar{p} = \frac{100}{400} = .25$

b. $\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\frac{.25(.75)}{400}} = .0217$

c. $\bar{p} \pm z_{.025}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

$.25 \pm 1.96(.0217)$

$.25 \pm .0424$; .2076 to .2924

32. a. .6733 to .7267
b. .6682 to .7318

34. 1068

35. a. $\bar{p} = \frac{1760}{2000} = .88$

b. Margin of error

$z_{.05}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 1.645\sqrt{\frac{.88(1-.88)}{2000}} = .0120$

c. Confidence interval:
$.88 \pm .0120$
or .868 to .892

d. Margin of error

$z_{.025}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 1.96\sqrt{\frac{.88(1-.88)}{2000}} = .0142$

95% Confidence interval
$.88 \pm .0142$ or .8658 to .8942

36. a. .23
b. .1716 to .2884

38. a. .1790
b. .0738; .5682 to .7158
c. 354

39. a. $n = \frac{1.96^2p^*(1-p^*)}{E^2}$

$n = \frac{1.96^2(.156)(1-.156)}{(.03)^2} = 562$

b. $n = \frac{2.576^2(.156)(1-.156)}{(.03)^2} = 970.77$; use $n = 971$

40. .0346; .4854 to .5546

42. a. .0442
b. 601, 1068, 2401, 9604

44. a. 4.00
b. 29.77 to 37.77

46. a. 122
b. \$1751 to \$1995
c. \$172.316 billion
d. Less than \$1873

48. a. 14 minutes
b. 13.38 to 14.62
c. 32 per day
d. Staff reduction

50. 37

52. 176

54. a. .5420
b. .0508
c. .4912 to .5928

56. a. .8273
b. .7957 to .8589

58. a. 1267
b. 1509

60. a. .3101
b. .2898 to .3304
c. 8219; no, this sample size is unnecessarily large

Chapter 9

2. a. H_0: $\mu \leq 14$
H_a: $\mu > 14$
b. No evidence that the new plan increases sales
c. The research hypothesis $\mu > 14$ is supported; the new plan increases sales

4. a. H_0: $\mu \geq 220$
H_a: $\mu < 220$

5. a. Rejecting H_0: $\mu \leq 56.2$ when it is true
b. Accepting H_0: $\mu \leq 56.2$ when it is false

6. a. H_0: $\mu \leq 1$
H_a: $\mu > 1$
b. Claiming $\mu > 1$ when it is not true
c. Claiming $\mu \leq 1$ when it is not true

8. a. H_0: $\mu \geq 220$
H_a: $\mu < 220$
b. Claiming $\mu < 220$ when it is not true
c. Claiming $\mu \geq 220$ when it is not true

10. a. $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{26.4 - 25}{6/\sqrt{40}} = 1.48$
b. Using normal table with $z = 1.48$: p-value = 1.0000 − .9306 = .0694
Using Excel: p-value = 1 − NORM.S.DIST(1.48,TRUE) = .0694
c. p-value > .01, do not reject H_0
d. Reject H_0 if $z \geq 2.33$
1.48 < 2.33, do not reject H_0

11. a. $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{14.15 - 15}{3/\sqrt{50}} = -2.00$
b. p-value = 2(.0228) = .0456
c. p-value ≤ .05, reject H_0
d. Reject H_0 if $z \leq -1.96$ or $z \geq 1.96$
$-2.00 \leq -1.96$, reject H_0

12. a. .1056; do not reject H_0
b. .0062; reject H_0
c. ≈0; reject H_0
d. .7967; do not reject H_0

14. a. .3844; do not reject H_0
b. .0074; reject H_0
c. .0836; do not reject H_0

15. a. H_0: $\mu \geq 1056$
H_a: $\mu < 1056$
b. $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{910 - 1056}{1600/\sqrt{400}} = -1.83$
p-value = .0336
c. p-value ≤ .05, reject H_0; the mean refund of "last-minute" filers is less than $1056
d. Reject H_0 if $z \leq -1.645$
$-1.83 \leq -1.645$; reject H_0

16. a. H_0: $\mu \leq 3173$
H_a: $\mu > 3173$
b. .0207
c. Reject H_0; the mean credit card balance has increased

18. a. H_0: $\mu = 4.1$
H_a: $\mu \neq 4.1$
b. −2.21, .0272
c. Reject H_0

20. a. H_0: $\mu \geq 32.79$
H_a: $\mu < 32.79$
b. −2.73
c. .0032
d. Reject H_0; mean monthly bill in southern state is less

22. a. H_0: $\mu = 8$
H_a: $\mu \neq 8$
b. .1706
c. Do not reject H_0
d. 7.83 to 8.97; yes

24. a. $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{17 - 18}{4.5/\sqrt{48}} = -1.54$
b. Degrees of freedom = $n - 1 = 47$
Area in lower tail is between .05 and .10
p-value (two-tail) is between .10 and .20
Using Excel: p-value = 2*T.DIST(−1.54,47,TRUE) = .1303
c. p-value > .05; do not reject H_0
d. With $df = 47$, $t_{.025} = 2.012$
Reject H_0 if $t \leq -2.012$ or $t \geq 2.012$
$t = -1.54$; do not reject H_0

26. a. Between .02 and .05; using Excel: p-value = 2*[1−T.DIST(2.10,64,TRUE)] = .0397; reject H_0
b. Between .01 and .02; using Excel: p-value = 2*T.DIST(−2.57,64,TRUE) = .0125; reject H_0
c. Between .10 and .20; using Excel: p-value = 2*[1−T.DIST(1.54,64,TRUE)] = .1285; do not reject H_0

27. a. H_0: $\mu \geq 238$
H_a: $\mu < 238$
b. $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{231 - 238}{80/\sqrt{100}} = -.88$
Degrees of freedom = $n - 1 = 99$
p-value is between .10 and .20
Using Excel: p-value = T.DIST(−.88,99,TRUE) = .1905
c. p-value > .05; do not reject H_0
Cannot conclude mean weekly benefit in Virginia is less than the national mean

d. $df = 99, t_{.05} = -1.66$
Reject H_0 if $t \leq -1.66$
$-.88 > -1.66$; do not reject H_0

28. a. $H_0: \mu \geq 9$
$H_a: \mu < 9$
b. Between .005 and .01
Using Excel: p-value = T.DIST(−2.50,84,TRUE)
= .0072
c. Reject H_0

30. a. $H_0: \mu = 600$
$H_a: \mu \neq 600$
b. Between .20 and .40
Using Excel: p-value = 2*[1−T.DIST(1.17,39,TRUE)]
= .2491
c. Do not reject H_0
d. A larger sample size

32. a. $H_0: \mu = 10{,}192$
$H_a: \mu \neq 10{,}192$
b. Between .02 and .05
Using Excel: p-value = T.DIST(−2.23,49,TRUE)
= .0304
c. Reject H_0

34. a. $H_0: \mu = 2$
$H_a: \mu \neq 2$
b. 2.2
c. .52
d. Between .20 and .40
Using Excel: p-value = 2*[1−T.DIST(1.22,9,TRUE)]
= .2535
e. Do not reject H_0

36. a. $$z = \frac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1 - p_0)}{n}}} = \frac{.68 - .75}{\sqrt{\dfrac{.75(1 - .75)}{300}}} = -2.80$$
p-value = .0026
p-value ≤ .05; reject H_0

b. $$z = \frac{.72 - .75}{\sqrt{\dfrac{.75(1 - .75)}{300}}} = -1.20$$
p-value = .1151
p-value > .05; do not reject H_0

c. $$z = \frac{.70 - .75}{\sqrt{\dfrac{.75(1 - .75)}{300}}} = -2.00$$
p-value = .0228
p-value ≤ .05; reject H_0

d. $$z = \frac{.77 - .75}{\sqrt{\dfrac{.75(1 - .75)}{300}}} = .80$$
p-value = .7881
p-value > .05; do not reject H_0

38. a. $H_0: p = .64$
$H_a: p \neq .64$
b. $\bar{p} = 52/100 = .52$
$$z = \frac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1 - p_0)}{n}}} = \frac{.52 - .64}{\sqrt{\dfrac{.64(1 - .64)}{100}}} = -2.50$$
Area = .4938
p-value = 2(.0062) = .0124
c. p-value ≤ .05; reject H_0
Proportion differs from the reported .64
d. Yes, because $\bar{p} = .52$ indicates that fewer believe the supermarket brand is as good as the name brand

40. a. .35
b. $H_0: p \geq .46$
$H_a: p < .46$
p-value = .0436
c. Proportion providing gifts has decreased

42. a. $\bar{p} = .15$
b. .0718 to .2218
c. Houston proportion is different

44. a. $H_0: p \leq .51$
$H_a: p > .51$
b. p-value = 1−NORM.S.DIST(2.80,TRUE) = .0026
c. reject H_0

46. a. $H_0: \mu = 16$
$H_a: \mu \neq 16$
b. .0286; reject H_0
Readjust line
c. .2186; do not reject H_0
Continue operation
d. $z = 2.19$; reject H_0
$z = -1.23$; do not reject H_0
Yes, same conclusion

48. a. $H_0: \mu \leq 119{,}155$
$H_a: \mu > 119{,}155$
b. .0047
c. Reject H_0

50. $t = -.93$
p-value between .20 and .40
Using Excel: p-value = 2*T.DIST(−1.05,41,TRUE)
= .2999
Do not reject H_0

52. $t = 2.26$
p-value between .01 and .025
Using Excel: p-value = 1−T.DIST(2.26,31,TRUE)
= .0155
Reject H_0

54. a. $H_0: p \leq .80$
$H_a: p > .80$
p-value = .0099
Over 80% feel body scanners will improve security
b. $H_0: p \leq .75$
$H_a: p > .75$
p-value = .0537
Cannot conclude that over 75% approve using

56. a. H_0: $p \leq .80$
H_a: $p > .80$
b. .84
c. .0418; reject H_0; more than 80% are satisfied with service

58. H_0: $p \geq .90$
H_a: $p < .90$
p-value = .0808
Do not reject H_0

Chapter 10

1. a. $\bar{x}_1 - \bar{x}_2 = 13.6 - 11.6 = 2$
b. $z_{\alpha/2} = z_{.05} = 1.645$

$$\bar{x}_1 - \bar{x}_2 \pm 1.645\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$2 \pm 1.645\sqrt{\frac{(2.2)^2}{50} + \frac{(3)^2}{35}}$$

$2 \pm .98$ (1.02 to 2.98)

c. $z_{\alpha/2} = z_{.05} = 1.96$

$$2 \pm 1.96\sqrt{\frac{(2.2)^2}{50} + \frac{(3)^2}{35}}$$

2 ± 1.17 (.83 to 3.17)

2. a. $$z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(25.2 - 22.8) - 0}{\sqrt{\frac{(5.2)^2}{40} + \frac{(6)^2}{50}}} = 2.03$$

b. p-value = 1.0000 − .9788 = .0212
c. p-value ≤ .05; reject H_0

4. a. $\bar{x}_1 - \bar{x}_2 = 85.36 - 81.40 = 3.96$
b.

$$z_{.025}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$1.96\sqrt{\frac{(4.55)^2}{37} + \frac{(3.97)^2}{44}} = 1.88$$

c. 3.96 ± 1.88 (2.08 to 5.84)

6. p-value = .0351
Reject H_0; mean price in Atlanta is lower than in Houston

8. a. $z = 2.74$, p-value = .0031
Conclude that customer service has improved for RiteAid
b. $z = .39$, p-value = .3483
Difference is not statistically significant
c. $z = 1.83$, p-value = .0336
Conclude customer service has improved
d. Increase must be 1.80 or more for the difference to be statistically significant
e. Not statistically significant

9. a. $\bar{x}_1 - \bar{x}_2 = 22.5 - 20.1 = 2.4$
b.

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2}$$

$$= \frac{\left(\frac{2.5^2}{20} + \frac{4.8^2}{30}\right)^2}{\frac{1}{19}\left(\frac{2.5^2}{20}\right)^2 + \frac{1}{29}\left(\frac{4.8^2}{30}\right)^2} = 45.8$$

c. $df = 45$, $t_{.025} = 2.014$

$$t_{.025}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 2.014\sqrt{\frac{2.5^2}{20} + \frac{4.8^2}{30}} = 2.1$$

d. 2.4 ± 2.1 (.3 to 4.5)

10. a. $$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(13.6 - 10.1) - 0}{\sqrt{\frac{5.2^2}{35} + \frac{8.5^2}{40}}} = 2.18$$

b.

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2}$$

$$= \frac{\left(\frac{5.2^2}{35} + \frac{8.5^2}{40}\right)^2}{\frac{1}{34}\left(\frac{5.2^2}{35}\right)^2 + \frac{1}{39}\left(\frac{8.5^2}{40}\right)^2} = 65.7$$

Use $df = 65$
c. $df = 65$, area in tail is between .01 and .025; two-tailed p-value is between .02 and .05
Exact p-value = .0329
d. p-value ≤ .05; reject H_0

12. a. $\bar{x}_1 - \bar{x}_2 = 22.5 - 18.6 = 3.9$ miles
b.

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2}$$

$$= \frac{\left(\frac{8.4^2}{50} + \frac{7.4^2}{40}\right)^2}{\frac{1}{49}\left(\frac{8.4^2}{50}\right)^2 + \frac{1}{39}\left(\frac{7.4^2}{40}\right)^2} = 87.1$$

Use $df = 87$, $t_{.025} = 1.988$

$$3.9 \pm 1.988\sqrt{\frac{8.4^2}{50} + \frac{7.4^2}{40}}$$

3.9 ± 3.3 (.6 to 7.2)

14. a. H_0: $\mu_1 - \mu_2 \geq 0$
H_a: $\mu_1 - \mu_2 < 0$
b. −2.41
c. Using t table, p-value is between .005 and .01
Exact p-value = .009
d. Reject H_0; conclude salaries of staff nurses are lower in Tampa

16. a. H_0: $\mu_1 - \mu_2 \leq 0$
H_a: $\mu_1 - \mu_2 > 0$
b. 3 points higher for midpriced family sedans
c. Using t table, p-value is greater than .20
Exact p-value is .2671
d. p-value > .05; we cannot conclude that the mean road-test score for midpriced family sedans is significantly greater

18. a. H_0: $\mu_1 - \mu_2 \geq 120$
H_a: $\mu_1 - \mu_2 < 120$
b. -2.10
Using t table, p-value is between .01 and .025
Exact p-value = .0195
c. 32 to 118
d. Larger sample size

19. a. 1, 2, 0, 0, 2
b. $\bar{d} = \Sigma d_i / n = 5/5 = 1$
c. $s_d = \sqrt{\dfrac{\Sigma(d_i - \bar{d})^2}{n-1}} = \sqrt{\dfrac{4}{5-1}} = 1$
d. $t = \dfrac{\bar{d} - \mu}{s_d/\sqrt{n}} = \dfrac{1-0}{1/\sqrt{5}} = 2.24$
$df = n - 1 = 4$
Using t table, p-value is between .025 and .05
Exact p-value = .0443
p-value $\leq$ .05; reject H_0

20. a. 3, -1, 3, 5, 3, 0, 1
b. 2
c. 2.08
d. 2
e. .07 to 3.93

21. H_0: $\mu_d \leq 0$
H_a: $\mu_d > 0$
$\bar{d} = .625$
$s_d = 1.30$
$t = \dfrac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \dfrac{.625 - 0}{1.30/\sqrt{8}} = 1.36$
$df = n - 1 = 7$
Using t table, p-value is between .10 and .20
Exact p-value = .1080
p-value $>$ .05; do not reject H_0

22. \$.10 to \$.32

24. $t = 1.32$
Using t table, p-value is greater than .10
Exact p-value = .1142
Do not reject H_0

26. a. H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$
$t = -1.42$, p-value = .1718
Conclude there is no significant difference between first and fourth round scores
b. $\bar{d} = -1.05$, first round scores lower
c. 1.28, yes

28. a. $\bar{\bar{x}} = (156 + 142 + 134)/3 = 144$

$$\text{SSTR} = \sum_{j=1}^{k} n_j(\bar{x}_j - \bar{\bar{x}})^2$$
$$= 6(156 - 144)^2 + 6(142 - 144)^2 + 6(134 - 144)^2$$
$$= 1488$$

b. $\text{MSTR} = \dfrac{\text{SSTR}}{k-1} = \dfrac{1488}{2} = 744$

c. $s_1^2 = 164.4$, $s_2^2 = 131.2$, $s_3^2 = 110.4$

$$\text{SSE} = \sum_{j=1}^{k} (n_j - 1)s_j^2$$
$$= 5(164.4) + 5(131.2) + 5(110.4)$$
$$= 2030$$

d. $\text{MSE} = \dfrac{\text{SSE}}{n_T - k} = \dfrac{2030}{18 - 3} = 135.3$

e.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Treatments	1488	2	744	5.50	.0162
Error	2030	15	135.3		
Total	3518	17			

f. $F = \dfrac{\text{MSTR}}{\text{MSE}} = \dfrac{744}{135.3} = 5.50$
From the F table (2 numerator degrees of freedom and 15 denominator), p-value is between .01 and .025
Using Excel, the p-value corresponding to $F = 5.50$ is .0162
Because p-value $\leq \alpha = .05$, we reject the hypothesis that the means for the three treatments are equal

30. a. H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$
H_a: Not all the population means are equal
b. Yes; p-value = .0000

32. a.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Treatments	1200	2	600	43.99	.0000
Error	600	44	13.64		
Total	1800	46			

Significant relationship

34. a.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Treatments	4560	2	2280	9.87	.0006
Error	6240	27	231.11		
Total	10800	29			

b. Significant relationship

36. No significant relationship: p-value = .2104

38. p-value = .0931; at the .05 level of significance we cannot reject the null hypothesis that the mean drying times for the four paints are equal

40. a. 412
b. 267.79
c. 412 ± 267.79

42. a. $H_0: \mu_1 - \mu_2 \leq 0$
$H_a: \mu_1 - \mu_2 > 0$

b. $t = .60$, $df = 57$
Using t table, p-value is greater than .20
Exact p-value = .2754
Do not reject H_0

44. a. $\bar{d} = 2.45$
b. \$.30 to \$4.60
c. 8% decrease
d. \$23.93

46. Significant relationship; p-value = .0061

48. Significant difference; p-value = .0002

50. Significant relationship; p-value = .0340

Chapter 11

1. a. $\bar{p}_1 - \bar{p}_2 = .48 - .36 = .12$

b. $$\bar{p}_1 - \bar{p}_2 \pm z_{.05}\sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \frac{\bar{p}_2(1-\bar{p}_2)}{n_2}}$$

$$.12 \pm 1.645\sqrt{\frac{.48(1-.48)}{400} + \frac{.36(1-.36)}{300}}$$

$.12 \pm .0614$ (.0586 to .1814)

c. $$.12 \pm 1.96\sqrt{\frac{.48(1-.48)}{400} + \frac{.36(1-.36)}{300}}$$

$.12 \pm .0731$ (.0469 to .1931)

2. a. $$\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2} = \frac{200(.22) + 300(.16)}{200 + 300} = .1840$$

$$z = \frac{\bar{p}_1 - \bar{p}_2}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{.22 - .16}{\sqrt{.1840(1-.1840)\left(\frac{1}{200} + \frac{1}{300}\right)}} = 1.70$$

p-value = 1.0000 − .9554 = .0446

b. p-value ≤ .05; reject H_0

3. a. Professional Golfers: $\bar{p}_1 = 688/1075 = .64$
Amateur Golfers: $\bar{p}_2 = 696/1200 = .58$
Professional golfers have the better putting accuracy

b. $\bar{p}_1 - \bar{p}_2 = .64 - .58 = .06$
Professional golfers make 6% more 6-foot putts than the very best amateur golfers

c. $$\bar{p}_1 - \bar{p}_2 \pm z_{.025}\sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \frac{\bar{p}_2(1-\bar{p}_2)}{n_2}}$$

$$.64 - .58 \pm 1.96\sqrt{\frac{.64(1-.64)}{1075} + \frac{.58(1-.58)}{1200}}$$

$.06 \pm .04$ (.02 to .10)

The confidence interval shows that professional golfers make from 2% to 10% more 6-foot putts than the best amateur golfers

4. a. $H_0: p_w \leq p_m$
$H_a: p_w > p_m$
b. $\bar{p}_w = .3699$
c. $\bar{p}_m = .3400$
d. p-value = .1093
Do not reject H_0

6. a. $H_0: p_1 - p_2 = 0$
$H_a: p_1 - p_2 \neq 0$
b. .28
c. .26
d. .3078, do not reject

8. a. $H_0: p_1 - p_2 = 0$
$H_a: p_1 - p_2 \neq 0$
b. .13
c. p-value = .0404

10. a. $H_0: p_1 - p_2 = 0$
$H_a: p_1 - p_2 \neq 0$
b. .27, .17
c. p-value ≈ 0
Difference is significant

11. a. Expected frequencies: $e_1 = 200(.40) = 80$
$e_2 = 200(.40) = 80$
$e_3 = 200(.20) = 40$
Actual frequencies: $f_1 = 60, f_2 = 120, f_3 = 20$

$$\chi^2 = \frac{(60-80)^2}{80} + \frac{(120-80)^2}{80} + \frac{(20-40)^2}{40}$$

$$= \frac{400}{80} + \frac{1600}{80} + \frac{400}{40}$$

$$= 5 + 20 + 10 = 35$$

Degrees of freedom: $k - 1 = 2$
$\chi^2 = 35$ shows p-value is less than .005
p-value ≤ .01; reject H_0

b. Reject H_0 if $\chi^2 \geq 9.210$
$\chi^2 = 35$; reject H_0

12. $\chi^2 = 15.33$, $df = 3$
p-value less than .005
Reject H_0

13. $H_0: p_{ABC} = .29, p_{CBS} = .28, p_{NBC} = .25, p_{IND} = .18$
H_a: The proportions are not
$p_{ABC} = .29, p_{CBS} = .28, p_{NBC} = .25, p_{IND} = .18$
Expected frequencies: 300(.29) = 87, 300(.28) = 84
300(.25) = 75, 300(.18) = 54
$e_1 = 87, e_2 = 84, e_3 = 75, e_4 = 54$
Actual frequencies: $f_1 = 95, f_2 = 70, f_3 = 89, f_4 = 46$

$$\chi^2 = \frac{(95-87)^2}{87} + \frac{(70-84)^2}{84} + \frac{(89-75)^2}{75} + \frac{(46-54)^2}{54} = 6.87$$

Degrees of freedom: $k - 1 = 3$
$\chi^2 = 6.87$, p-value between .05 and .10
Do not reject H_0

14. $\chi^2 = 29.51$, $df = 5$
p-value is less than .005
Reject H_0

16. a. $\chi^2 = 12.21$, $df = 3$
p-value is between .005 and .01
Conclude difference for 2003
b. 21%, 30%, 15%, 34%
Increased use of debit card
c. 51%

18. $\chi^2 = 16.31$, $df = 3$
p-value less than .005
Reject H_0

19. H_0: The column variable is independent of the row variable
H_a: The column variable is not independent of the row variable

Expected frequencies:

	A	B	C
P	28.5	39.9	45.6
Q	21.5	30.1	34.4

$$\chi^2 = \frac{(20 - 28.5)^2}{28.5} + \frac{(44 - 39.9)^2}{39.9} + \frac{(50 - 45.6)^2}{45.6} + \frac{(30 - 21.5)^2}{21.5} + \frac{(26 - 30.1)^2}{30.1} + \frac{(30 - 34.4)^2}{34.4} = 7.86$$

Degrees of freedom: $(2 - 1)(3 - 1) = 2$
$\chi^2 = 7.86$, p-value between .01 and .025
Reject H_0

20. $\chi^2 = 19.77$, $df = 4$
p-value less than .005
Reject H_0

21. H_0: Type of ticket purchased is independent of the type of flight
H_a: Type of ticket purchased is not independent of the type of flight

Expected frequencies:

$e_{11} = 35.59$ $e_{12} = 15.41$
$e_{21} = 150.73$ $e_{22} = 65.27$
$e_{31} = 455.68$ $e_{32} = 197.32$

Ticket	Flight	Observed Frequency (f_i)	Expected Frequency (e_i)	$(f_i - e_i)^2/e_i$
First	Domestic	29	35.59	1.22
First	International	22	15.41	2.82
Business	Domestic	95	150.73	20.61
Business	International	121	65.27	47.59
Full-fare	Domestic	518	455.68	8.52
Full-fare	International	135	197.32	19.68
Totals		920		$\chi^2 = 100.43$

Degrees of freedom: $(3 - 1)(2 - 1) = 2$
$\chi^2 = 100.43$, p-value is less than .005
Reject H_0

22. a. $\chi^2 = 7.95$, $df = 3$
p-value is between .025 and .05
Reject H_0
b. 18 to 24 use most

24. a. $\chi^2 = 8.47$; p-value is between .025 and .05
Reject H_0; intent to purchase again is not independent of the automobile
b. Accord 77, Camry 71, Taurus 62, Impala 57
c. Impala and Taurus below, Accord and Camry above; Accord and Camry have greater owner satisfaction, which may help future market share

26. a. 6448
b. $\chi^2 = 424.6674$
p-value ≈ 0
Reject H_0
c. Italy

28. $\chi^2 = 3.01$, $df = 2$
p-value is greater than .10
Do not reject H_0; 63.3%

30. a. p-value ≈ 0; reject H_0
b. .0468 to .1332

32. a. .35, .47
b. .0163 to .2237
c. Yes

34. $\chi^2 = 8.04$, $df = 3$
p-value between .025 and .05
Reject H_0

36. $\chi^2 = 4.64$, $df = 2$
p-value between .05 and .10
Do not reject H_0

38. $\chi^2 = 42.53$, $df = 4$
p-value is less than .005
Reject H_0

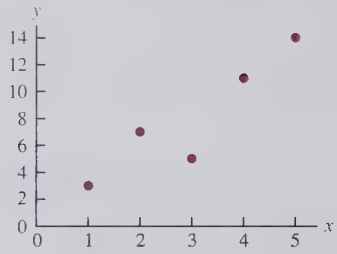
40. $\chi^2 = 23.37$, $df = 3$
p-value is less than .005
Reject H_0

42. a. 71%, 22%, slower preferred
b. $\chi^2 = 2.99$, $df = 2$
p-value greater than .10
Do not reject H_0; cannot conclude men and women differ in preference

44. $\chi^2 = 7.75$, $df = 3$
p-value is between .05 and .10
Do not reject H_0

Chapter 12

1. a.



b. There appears to be a positive linear relationship between x and y

c. Many different straight lines can be drawn to provide a linear approximation of the relationship between x and y; in part (d) we will determine the equation of a straight line that "best" represents the relationship according to the least squares criterion

d. Summations needed to compute the slope and y-intercept:

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{15}{5} = 3, \quad \bar{y} = \frac{\Sigma y_i}{n} = \frac{40}{5} = 8,$$

$$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 26, \quad \Sigma(x_i - \bar{x})^2 = 10$$

$$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{26}{10} = 2.6$$

$$b_0 = \bar{y} - b_1\bar{x} = 8 - (2.6)(3) = 0.2$$

$$\hat{y} = 0.2 + 2.6x$$

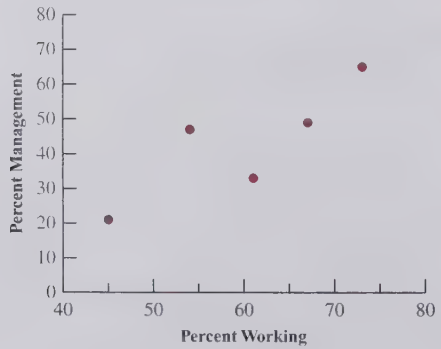
e. $\hat{y} = .2 + 2.6x = .2 + 2.6(4) = 10.6$

2. b. There appears to be a negative linear relationship between x and y

d. $\hat{y} = 68 - 3x$

e. 38

4. a.



b. There appears to be a positive linear relationship between the percentage of women working in the five companies (x) and the percentage of management jobs held by women in that company (y)

c. Many different straight lines can be drawn to provide a linear approximation of the relationship between x and y; in part (d) we will determine the equation of a straight line that "best" represents the relationship according to the least squares criterion.

d. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{300}{5} = 60 \quad \bar{y} = \frac{\Sigma y_i}{n} = \frac{215}{5} = 43$

$$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 624 \quad \Sigma(x_i - \bar{x})^2 = 480$$

$$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{624}{480} = 1.3$$

$$b_0 = \bar{y} - b_1\bar{x} = 43 - 1.3(60) = -35$$

$$\hat{y} = -35 + 1.3x$$

e. $\hat{y} = -35 + 1.3x = -35 + 1.3(60) = 43\%$

6. c. $\hat{y} = 8.9412 - .02633x$

e. 6.3 or approximately \$6300

8. c. $\hat{y} = 359.2668 - 5.2772x$

d. \$254

10. c. $\hat{y} = -6{,}745.44 + 149.29x$

d. 4003 or \$4,003,000

12. c. $\hat{y} = -8129.4439 + 22.4443x$

d. \$8704

14. c. $\hat{y} = 37.1217 + .51758x$

d. 73

15. a. $\hat{y}_i = .2 + 2.6x_i$ and $\bar{y} = 8$

x_i	y_i	$\hat{y}_i$	$y_i - \hat{y}_i$	$(y_i - \hat{y}_i)^2$	$y_i - \bar{y}$	$(y_i - \bar{y})^2$
1	3	2.8	.2	.04	−5	25
2	7	5.4	1.6	2.56	−1	1
3	5	8.0	−3.0	9.00	−3	9
4	11	10.6	.4	.16	3	9
5	14	13.2	.8	.64	6	36
				SSE = 12.40		SST = 80

SSR = SST − SSE = 80 − 12.4 = 67.6

b. $r^2 = \frac{\text{SSR}}{\text{SST}} = \frac{67.6}{80} = .845$

The least squares line provided a good fit; 84.5% of the variability in y has been explained by the least squares line

c. $r_{xy} = \sqrt{.845} = +.9192$

16. a. SSE = 230, SST = 1850, SSR = 1620

b. $r^2 = .876$

c. $r_{xy} = -.936$

18. a. The estimated regression equation and the mean for the dependent variable:

$\hat{y} = 1790.5 + 581.1x, \quad \bar{y} = 3650$

The sum of squares due to error and the total sum of squares:

$\text{SSE} = \Sigma(y_i - \hat{y}_i)^2 = 85{,}135.14$

$\text{SST} = \Sigma(y_i - \bar{y})^2 = 335{,}000$

Thus, SSR = SST − SSE

= 335,000 − 85,135.14 = 249,864.86

b. $r^2 = \dfrac{\text{SSR}}{\text{SST}} = \dfrac{249{,}864.86}{335{,}000} = .746$

The least squares line accounted for 74.6% of the total sum of squares

c. $r_{xy} = \sqrt{.746} = +.8637$

20. a. $\hat{y} = 12.0169 + .0127x$

b. $r^2 = .4503$

c. 53

22. a. .77

b. Yes

c. $r_{xy} = +.88$, strong

23. a. $s^2 = \text{MSE} = \dfrac{\text{SSE}}{n-2} = \dfrac{12.4}{3} = 4.133$

b. $s = \sqrt{\text{MSE}} = \sqrt{4.133} = 2.033$

c. $\Sigma(x_i - \bar{x})^2 = 10$

$$s_{b_1} = \frac{s}{\sqrt{\Sigma(x_i - \bar{x})^2}} = \frac{2.033}{\sqrt{10}} = .643$$

d. $t = \dfrac{b_1 - \beta_1}{s_{b_1}} = \dfrac{2.6 - 0}{.643} = 4.044$

From the t table (3 degrees of freedom), area in tail is between .01 and .025

p-value is between .02 and .05

Using Excel, the p-value corresponding to $t = 4.04$ is .0272

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$

e. $\text{MSR} = \dfrac{\text{SSR}}{1} = 67.6$

$$F = \frac{\text{MSR}}{\text{MSE}} = \frac{67.6}{4.133} = 16.36$$

From the F table (1 numerator degree of freedom and 3 denominator), p-value is between .025 and .05

Using Excel, the p-value corresponding to $F = 16.36$ is .0272

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Regression	67.6	1	67.6	16.36	.0272
Error	12.4	3	4.133		
Total	80	4			

24. a. 76.6667

b. 8.7560

c. .6526

d. Significant; p-value = .0193

e. Significant; p-value = .0193

26. a. $s^2 = \text{MSE} = \dfrac{\text{SSE}}{n-2} = \dfrac{85{,}135.14}{4} = 21{,}283.79$

$s = \sqrt{\text{MSE}} = \sqrt{21{,}283.79} = 145.89$

$\Sigma(x_i - \bar{x})^2 = .74$

$$s_{b_1} = \frac{s}{\sqrt{\Sigma(x_i - \bar{x})^2}} = \frac{145.89}{\sqrt{.74}} = 169.59$$

$$t = \frac{b_1 - \beta_1}{s_{b_1}} = \frac{581.08 - 0}{169.59} = 3.43$$

From the t table (4 degrees of freedom), area in tail is between .01 and .025

p-value is between .02 and .05

Using Excel, the p-value corresponding to $t = 3.43$ is .0266

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$

b. $\text{MSR} = \dfrac{\text{SSR}}{1} = \dfrac{249{,}864.86}{1} = 249{,}864.86$

$$F = \frac{\text{MSR}}{\text{MSE}} = \frac{249{,}864.86}{21{,}283.79} = 11.74$$

From the F table (1 numerator degree of freedom and 4 denominator), p-value is between .025 and .05

Using Excel, the p-value corresponding to $F = 11.74$ is .0266

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$

c.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Regression	249,864.86	1	249,864.86	11.74	.0266
Error	85,135.14	4	21,283.79		
Total	335,000	5			

28. They are related; p-value = .000

30. Significant; p-value = .0042

32. a. $s = 2.033$

$\bar{x} = 3$, $\Sigma(x_i - \bar{x})^2 = 10$

$$s_{\hat{y}^*} = s\sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}}$$

$$= 2.033\sqrt{\frac{1}{5} + \frac{(4-3)^2}{10}} = 1.11$$

b. $\hat{y}^* = .2 + 2.6x^* = .2 + 2.6(4) = 10.6$

$\hat{y}^* \pm t_{\alpha/2} s_{\hat{y}^*}$

$10.6 \pm 3.182(1.11)$

10.6 ± 3.53, or 7.07 to 14.13

c. $s_{\text{pred}} = s\sqrt{1 + \dfrac{1}{n} + \dfrac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}}$

$$= 2.033\sqrt{1 + \frac{1}{5} + \frac{(4-3)^2}{10}} = 2.32$$

d. $\hat{y}^* \pm t_{\alpha/2} s_{\text{pred}}$

$10.6 \pm 3.182(2.32)$

10.6 ± 7.38, or 3.22 to 17.98

34. Confidence interval: 8.65 to 21.15
Prediction interval: −4.50 to 41.30

35. a. $s = 145.89$, $\bar{x} = 3.2$, $\Sigma(x_i - \bar{x})^2 = .74$

$\hat{y}^* = 1790.5 + 581.1x^* = 1790.5 + 581.1(3)$
$= 3533.8$

$$s_{\hat{y}^*} = s\sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}}$$

$$= 145.89\sqrt{\frac{1}{6} + \frac{(3 - 3.2)^2}{.74}} = 68.54$$

$\hat{y}^* \pm t_{\alpha/2}s_{\hat{y}}$
$3533.8 \pm 2.776(68.54)$
3533.8 ± 190.27, or \$3343.53 to \$3724.07

b. $$s_{\text{pred}} = s\sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}}$$

$$= 145.89\sqrt{1 + \frac{1}{6} + \frac{(3 - 3.2)^2}{.74}} = 161.19$$

$\hat{y}^* \pm t_{\alpha/2}s_{\text{pred}}$
$3533.8 \pm 2.776(161.19)$
3533.8 ± 447.46, or \$3086.34 to \$3981.26

36. a. \$201
b. 167.25 to 234.65
c. 108.75 to 293.15

38. a. \$5046.67
b. \$3815.10 to \$6278.24
c. Not out of line

40. a. 9
b. $\hat{y} = 20.0 + 7.21x$
c. $t = 5.29$
From the t table (7 degrees of freedom), area in tail is less than .005
p-value is less than .01
Actual p-value = .000
Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$
d. SSE = SST − SSR = 51,984.1 − 41,587.3 = 10,396.8
MSE = 10,396.8/7 = 1485.3

$$F = \frac{\text{MSR}}{\text{MSE}} = \frac{41{,}587.3}{1485.3} = 28.00$$

From the F table (1 degree of freedom numerator and 7 denominator), p-value is less than .01
Actual p-value = .001
Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$
e. $\hat{y} = 20.0 + 7.21x = 20.0 + 7.21(50)$
$= 380.5$ or \$380,500

42. a. $\hat{y} = 80.0 + 50.0x$
b. $F = 83.17$; significant (p-value = .000)
c. $t = 9.12$; significant (p-value = .000)
d. \$680,000

44. b. Yes
c. $\hat{y} = 2044.38 - 28.35$ Weight
d. Significant; p-value = .000
e. $r^2 = .774$; a good fit

45. a. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{70}{5} = 14$, $\bar{y} = \frac{\Sigma y_i}{n} = \frac{76}{5} = 15.2$,
$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 200$, $\Sigma(x_i - \bar{x})^2 = 126$

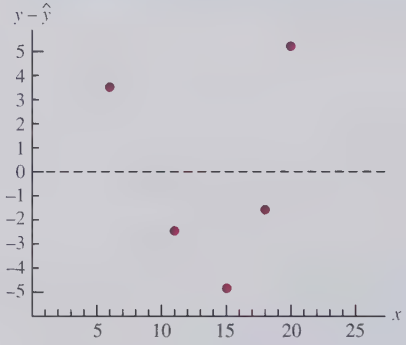
$$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{200}{126} = 1.5873$$

$b_0 = \bar{y} - b_1\bar{x} = 15.2 - (1.5873)(14) = -7.0222$
$\hat{y} = -7.02 + 1.59x$

b.

x_i	y_i	$\hat{y}_i$	$y_i - \hat{y}_i$
6	6	2.52	3.48
11	8	10.47	−2.47
15	12	16.83	−4.83
18	20	21.60	−1.60
20	30	24.78	5.22

c.



With only five observations, it is difficult to determine whether the assumptions are satisfied; however, the plot does suggest curvature in the residuals, which would indicate that the error term assumptions are not satisfied; the scatter diagram for these data also indicates that the underlying relationship between x and y may be curvilinear

46. a. $\hat{y} = 2.32 + .64x$
b. No; the variance appears to increase for larger values of x

47. a. Let x = advertising expenditures and y = revenue
$\hat{y} = 29.4 + 1.55x$
b. SST = 1002, SSE = 310.28, SSR = 691.72

$$\text{MSR} = \frac{\text{SSR}}{1} = 691.72$$

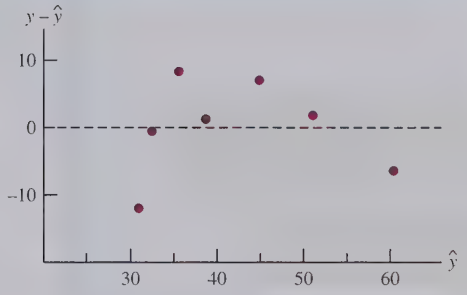
$$\text{MSE} = \frac{\text{SSE}}{n - 2} = \frac{310.28}{5} = 62.0554$$

$$F = \frac{\text{MSR}}{\text{MSE}} = \frac{691.72}{62.0554} = 11.15$$

From the F table (1 numerator degree of freedom and 5 denominator), p-value is between .01 and .025
Using Excel, p-value = .0206
Because p-value $\leq \alpha = .05$, we conclude that the two variables are related

c.

x_i	y_i	$\hat{y}_i = 29.40 + 1.55x_i$	$y_i - \hat{y}_i$
1	19	30.95	−11.95
2	32	32.50	−.50
4	44	35.60	8.40
6	40	38.70	1.30
10	52	44.90	7.10
14	53	51.10	1.90
20	54	60.40	−6.40



d. The residual plot leads us to question the assumption of a linear relationship between x and y; even though the relationship is significant at the $\alpha = .05$ level, it would be extremely dangerous to extrapolate beyond the range of the data

48. b. Yes

50. a. $\hat{y} = 9.26 + .711x$
b. Significant; p-value = .0013
c. $r^2 = .744$; good fit
d. $13.53

52. b. GR(%) = 25.4229 + .2845 RR(%)
c. Significant; p-value = .000
d. No; $r^2 = .4492$
e. Yes
f. Yes

54. a. $\hat{y} = 2.1739 - .1478x$
b. Significant relationship; p-value = .0281
c. Good fit; $r^2 = .739$
d. 12.294 to 17.271

56. a. $\hat{y} = 220 + 131.6667x$
b. Significant; p-value = .000
c. $r^2 = .873$; very good fit
d. $559.50 to $933.90

58. a. Market beta = .95
b. Significant; p-value = .029
c. $r^2 = .470$; not a good fit
d. Xerox has a higher risk

60. b. There appears to be a positive linear relationship between the two variables
c. $\hat{y} = 9.3742 + 1.2875$ Top Five (%)
d. Significant; p-value = .0002
e. $r^2 = .7413$; good fit
f. $r_{xy} = .86$

Chapter 13

2. a. The estimated regression equation is
$\hat{y} = 45.0594 + 1.9436x_1$
An estimate of y when $x_1 = 45$ is
$\hat{y} = 45.0594 + 1.9436(45) = 132.52$
b. The estimated regression equation is
$\hat{y} = 85.2171 + 4.3215x_2$
An estimate of y when $x_2 = 15$ is
$\hat{y} = 85.2171 + 4.3215(15) = 150.04$
c. The estimated regression equation is
$\hat{y} = -18.3683 + 2.0102x_1 + 4.7378x_2$
An estimate of y when $x_1 = 45$ and $x_2 = 15$ is
$\hat{y} = -18.3683 + 2.0102(45) + 4.7378(15) = 143.16$

4. a. $255,000

5. a. A portion of the Excel output is shown in Figure D13.5a
b. A portion of the Excel output is shown in Figure D13.5b
c. It is 1.6039 in part (a) and 2.2902 in part (b); in part (a) the coefficient is an estimate of the change in revenue due to a one-unit change in television advertising expenditures; in part (b) it represents an estimate of the change in revenue due to a one-unit change in television advertising expenditures when the amount of newspaper advertising is held constant
d. Revenue = 83.2301 + 2.2902(3.5) + 1.3010(1.8)
= 93.59 or $93,590

6. a. $\hat{y} = -20.0228 + 33.7264$ Recruiter Score
b. $\hat{y} = 0.8790 + 14.5907$ Recruiter Score
$+ 1.3687$ Tuition & Fees ($1000s)
c. No
d. 119.5550 or approximately $120,000

8. a. $\hat{y} = 31054.26 - 1328.722$ Reliability
b. $\hat{y} = 21,312.9147 + 136.6908$ Road-Test Score
$- 1446.3449$ Reliability
c. 26,462.7991 or approximately $26,463

10. a. $\hat{y} = -1.2207 + 3.9576$ FG%
b. .04
c. $\hat{y} = -1.2346 + 4.8166$ FG% $- 2.5895$ Opp 3 Pt%
$+ .0344$ Opp TO
e. .6372

12. a. $R^2 = \dfrac{\text{SSR}}{\text{SST}} = \dfrac{14,052.2}{15,182.9} = .926$

b. $R_a^2 = 1 - (1 - R^2)\dfrac{n-1}{n-p-1}$
$= 1 - (1 - .926)\dfrac{10-1}{10-2-1} = .905$

c. Yes; after adjusting for the number of independent variables in the model, we see that 90.5% of the variability in y has been accounted for

14. a. .75
b. .68

FIGURE D13.5a

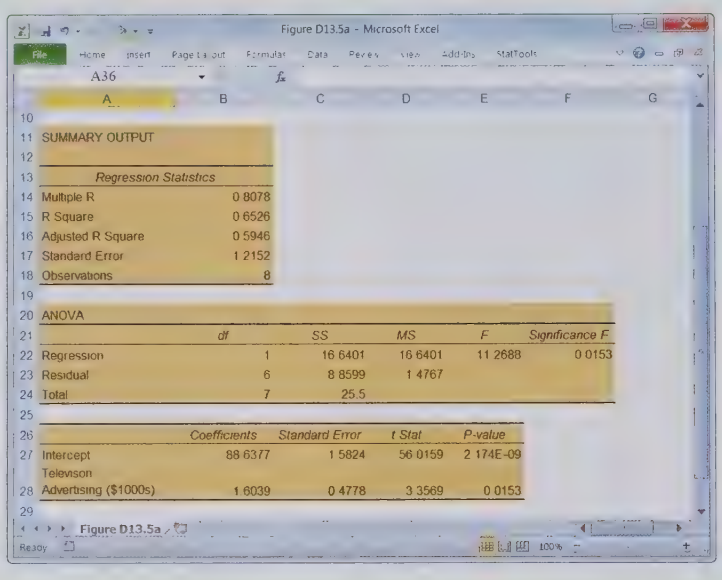


Figure D13.5a - Microsoft Excel

	A	B	C	D	E	F	G
10							
11	SUMMARY OUTPUT						
12							
13	*Regression Statistics*						
14	Multiple R	0 8078					
15	R Square	0 6526					
16	Adjusted R Square	0 5946					
17	Standard Error	1 2152					
18	Observations	8					
19							
20	ANOVA						
21		*df*	*SS*	*MS*	*F*	*Significance F*	
22	Regression	1	16 6401	16 6401	11 2688	0 0153	
23	Residual	6	8 8599	1 4767			
24	Total	7	25.5				
25							
26		*Coefficients*	*Standard Error*	*t Stat*	*P-value*		
27	Intercept	88 6377	1 5824	56 0159	2 174E-09		
28	Television Advertising ($1000s)	1 6039	0 4778	3 3569	0 0153		
29							

Figure D13.5a

Ready

FIGURE D13.5b

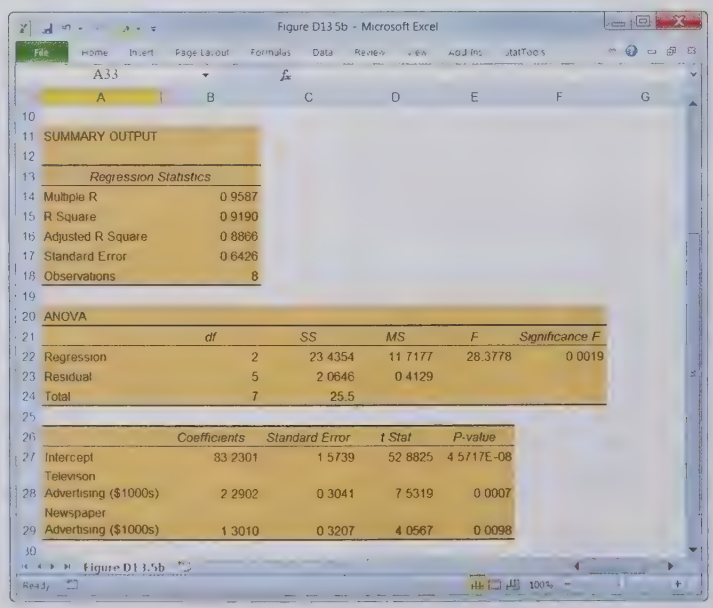


Figure D13.5b - Microsoft Excel

	A	B	C	D	E	F	G
10							
11	SUMMARY OUTPUT						
12							
13	*Regression Statistics*						
14	Multiple R	0 9587					
15	R Square	0 9190					
16	Adjusted R Square	0 8866					
17	Standard Error	0 6426					
18	Observations	8					
19							
20	ANOVA						
21		*df*	*SS*	*MS*	*F*	*Significance F*	
22	Regression	2	23 4354	11 7177	28 3778	0 0019	
23	Residual	5	2 0646	0 4129			
24	Total	7	25.5				
25							
26		*Coefficients*	*Standard Error*	*t Stat*	*P-value*		
27	Intercept	83 2301	1 5739	52 8825	4 5717E-08		
28	Television Advertising ($1000s)	2 2902	0 3041	7 5319	0 0007		
29	Newspaper Advertising ($1000s)	1 3010	0 3207	4 0567	0 0098		
30							

Figure D13.5b

Ready

15. a. $R^2 = \dfrac{\text{SSR}}{\text{SST}} = \dfrac{23.435}{25.5} = .919$

$$R_a^2 = 1 - (1 - R^2)\frac{n-1}{n-p-1}$$

$$= 1 - (1 - .919)\frac{8-1}{8-2-1} = .887$$

b. Multiple regression analysis is preferred because both R^2 and R_a^2 show an increased percentage of the variability of y explained when both independent variables are used

16. a. $r^2 = .6133$; not too bad a fit using only one independent variable

b. $R^2 = .793$ and $R_a^2 = .768$
The multiple linear regression equation will provide a better estimate of the starting salary and bonus of the graduates

18. a. $R^2 = .5638$, $R_a^2 = .5114$

b. The fit is not very good; but it does explain over 50% of the variability in y

19. a. $\text{MSR} = \dfrac{\text{SSR}}{p} = \dfrac{6216.375}{2} = 3108.188$

$$\text{MSE} = \frac{\text{SSE}}{n-p-1} = \frac{507.75}{10-2-1} = 72.536$$

b. $F = \dfrac{\text{MSR}}{\text{MSE}} = \dfrac{3108.188}{72.536} = 42.85$

p-value (2 degrees of freedom numerator and 7 denominator) = .0001

Because the p-value $\leq \alpha = .05$, the overall model is significant

c. $t = \dfrac{b_1}{s_{b_1}} = \dfrac{.5906}{.0813} = 7.26$

p-value (7 degrees of freedom) = .0002

Because the p-value $\leq \alpha = .05$, β_1 is significant

d. $t = \dfrac{b_2}{s_{b_2}} = \dfrac{.4980}{.0567} = 8.78$

p-value (7 degrees of freedom) = .0001

Because the p-value $\leq \alpha = .05$, β_2 is significant

20. a. Significant; p-value = .0001

b. Significant; p-value = .0000

c. Significant; p-value = .0016

22. a. SSE = 4000, MSE = 571.43, MSR = 6000

b. Significant; p-value = .0078

23. a. $F = 28.38$

p-value (2 degrees of freedom numerator and 1 denominator) = .0019

Because the p-value $\leq \alpha = .01$, reject H_0

b. $t = 7.53$

p-value = .0007

Because p-value $\leq \alpha = .05$, β_1 is significant and x_1 should not be dropped from the model

c. $t = 4.06$

$t_{.025} = 2.571$

With $t > t_{.025} = 2.571$, β_2 is significant and x_2 should not be dropped from the model

24. a. $\hat{y} = -.6820 + .0498$ Revenue $+ .0147$ %Wins

b. Significant; p-value = .001

c. Revenue is significant; p-value = .0007
%Wins is significant; p-value = .0063

26. a. Significant; p-value = .0000

b. All of the independent variables are significant

28. a. 143.15

b. Using StatTools, the 95% prediction interval is 111.16 to 175.16

29. a. See Excel output in Figure D13.5b.

$\hat{y} = 83.2 + 2.29(3.5) + 1.30(1.8) = 93.555$
or \$93,555

b. Using StatTools, 91.774 to 95.401, or \$91,774 to \$95,401

30. a. 49

b. 44.815 to 52.589

32. a. $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

where $x_2 = \begin{cases} 0 \text{ if level 1} \\ 1 \text{ if level 2} \end{cases}$

b. $E(y) = \beta_0 + \beta_1 x_1 + \beta_2(0) = \beta_0 + \beta_1 x_1$

c. $E(y) = \beta_0 + \beta_1 x_1 + \beta_2(1) = \beta_0 + \beta_1 x_1 + \beta_2$

d. $\beta_2 = E(y \mid \text{level 2}) - E(y \mid \text{level 1})$
β_1 is the change in $E(y)$ for a one-unit change in x_1 holding x_2 constant

34. a. \$15,300

b. $\hat{y} = 10.1 - 4.2(2) + 6.8(8) + 15.3(0) = 56.1$
Sales prediction: \$56,100

c. $\hat{y} = 10.1 - 4.2(1) + 6.8(3) + 15.3(1) = 41.6$
Sales prediction: \$41,600

36. a. $\hat{y} = 1.8602 + 0.2914$ Months $+ 1.1024$ Type $- 0.6091$ Person

b. Significant; p-value = .0021 $< \alpha = .05$

c. Person is not significant

38. a. $\hat{y} = -91.7595 + 1.0767$ Age $+ .2518$ Pressure $+ 8.7399$ Smoker

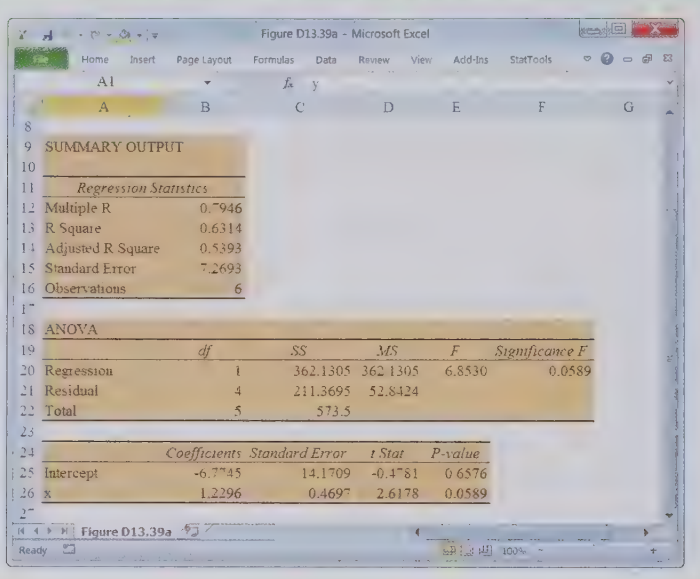
b. Significant; p-value = .0102 $< \alpha = .05$

c. 95% prediction interval is 21.35 to 47.18 or a probability of .2135 to .4718; quit smoking and begin some type of treatment to reduce his blood pressure

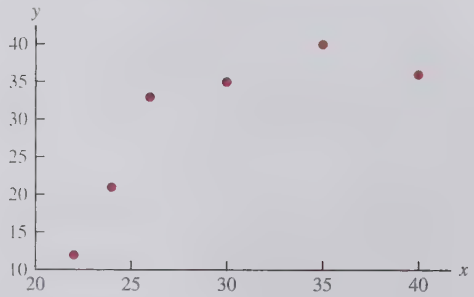
39. a. A portion of the Excel output is shown in Figure D13.39a

b. The p-value corresponding to $F = 6.8530$ is .0589 $> \alpha = .05$; therefore, the relationship is not significant

FIGURE D13.39a



c.



The scatter diagram suggests that a curvilinear relationship may be appropriate

d. A portion of the Excel output is shown in Figure D13.39d

e. The p-value corresponding to $F = 25.6778$ is $.013 < \alpha = .05$; therefore, the relationship is significant

f. $\hat{y} = -168.8848 + 12.1870(25) - .1770(25)^2 = 25.165$

40. a. $\hat{y} = 9.3152 + .4242x$; p-value $= .1171$ indicates that the relationship between x and y is not significant

b. $\hat{y} = -8.1014 + 2.4127x - .0480x^2$
$R^2_a = .9324$; good fit

c. 20.953

42. b. No, the relationship appears to be curvilinear

c. $\hat{y} = 2.90 - .185x + .00351x^2$

44. b. 3.19

46. a. $\hat{y} = 8.103 + 7.602\,x_1 + 3.111x_2$

b. Significant; p-value $= .000$

c. β_1 is significant; p-value $= .0036$
β_2 is significant; p-value $= .0003$

48. a. $\hat{y} = 14.4 - 8.69\,x_1 + 13.517x_2$

b. Significant; p-value $= .0031$

c. Good fit

d. β_1 is significant; p-value $= .0025$
β_2 is significant; p-value $= .0013$

50. a. $\hat{y} = 67.6762 + .0046$ Price

b. Significant; p-value $= .0004$

c. $\hat{y} = 65.6597 + .0023$ Price $+ 10.2097$ Quality-E $+ 5.9246$ Quality-VG

d. Significant; p-value $= .0002$

e. All significant

f. 70; 6

52. a. $\hat{y} = 35.3950 - 2.8821$ Displacement
p-value $= .0000 < \alpha = .05$; significant relationship

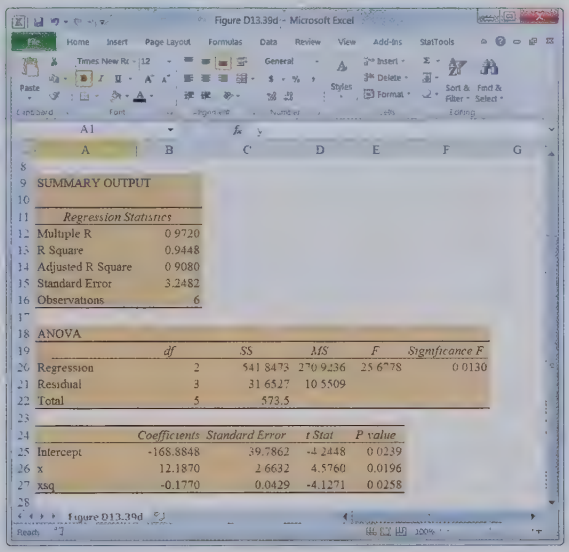
b. $\hat{y} = 35.2289 + 3.4563$ ClassMidsize $+ 1.7078$ ClassLarge $- 1.9347$ Displacement

c. ClassMidsize: p-value $= .000 < \alpha = .05$; significant
ClassLarge: p-value $= .000 < \alpha = .05$

d. $\hat{y} = 30.9548 + 2.9832$ ClassMidsize $+ 1.5088$ ClassLarge $- 1.8533$ Displacement $- 1.4306$ FuelPremium

e. p-value corresponding to $F = 317.1329$ is $.0000 < \alpha = .05$; there is a significant overall relationship
Because the p-values for each independent variable are also $< \alpha = .05$, each of the independent variables is significant

FIGURE D13.39d



Chapter 14

2. **a.** 5.42
b. UCL = 6.09, LCL = 4.75

4. *R chart:*
$UCL = \bar{R}D_4 = 1.6(1.864) = 2.98$
$LCL = \bar{R}D_3 = 1.6(.136) = .22$
$\bar{x}$ chart:
$UCL = \bar{\bar{x}} + A_2\bar{R} = 28.5 + .373(1.6) = 29.10$
$LCL = \bar{\bar{x}} - A_2\bar{R} = 28.5 - .373(1.6) = 27.90$

6. 20.01, .082

8. **a.** .0470
b. UCL = .0989, LCL = −0.0049 (use LCL = 0)
c. $\bar{p} = .08$; in control
d. UCL = 14.826, LCL = −0.726 (use LCL = 0)
Process is out of control if more than 14 defective
e. In control with 12 defective
f. *np* chart

10. $f(x) = \dfrac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$

When $p = .02$, the probability of accepting the lot is

$$f(0) = \frac{25!}{0!(25-0)!}(.02)^0(1-.02)^{25} = .6035$$

When $p = .06$, the probability of accepting the lot is

$$f(0) = \frac{25!}{0!(25-0)!}(.06)^0(1-.06)^{25} = .2129$$

12. $p_0 = .02$; producer's risk = .0599
$p_0 = .06$; producer's risk = .3396
Producer's risk decreases as the acceptance number c is increased

14. $n = 20, c = 3$

16. **a.** 95.4
b. UCL = 96.07, LCL = 94.73
c. No

18.

	R Chart	$\bar{x}$ Chart
UCL	4.23	6.57
LCL	0	4.27

Estimate of standard deviation = .86

20.

	R Chart	$\bar{x}$ Chart
UCL	.1121	3.112
LCL	0	3.051

22. **a.** UCL = .0817, LCL = −.0017 (use LCL = 0)

24. **a.** .03
b. $\beta = .0802$

Chapter 15

1. The following table shows the calculations for parts (a), (b), and (c):

Week	Time Series Value	Forecast	Forecast Error	Absolute Value of Forecast Error	Squared Forecast Error	Percentage Error	Absolute Value of Percentage Error
1	18						
2	13	18	−5	5	25	−38.46	38.46
3	16	13	3	3	9	18.75	18.75
4	11	16	−5	5	25	−45.45	45.45
5	17	11	6	6	36	35.29	35.29
6	14	17	−3	3	9	−21.43	21.43
			Totals	22	104	−51.30	159.38

a. $\text{MAE} = \frac{22}{5} = 4.4$

b. $\text{MSE} = \frac{104}{5} = 20.8$

c. $\text{MAPE} = \frac{159.38}{5} = 31.88$

d. Forecast for week 7 is 14

2. The following table shows the calculations for parts (a), (b), and (c):

Week	Time Series Value	Forecast	Forecast Error	Absolute Value of Forecast Error	Squared Forecast Error	Percentage Error	Absolute Value of Percentage Error
1	18						
2	13	18.00	−5.00	5.00	25.00	−38.46	38.46
3	16	15.50	0.50	0.50	0.25	3.13	3.13
4	11	15.67	−4.67	4.67	21.81	−42.45	42.45
5	17	14.50	2.50	2.50	6.25	14.71	14.71
6	14	15.00	−1.00	1.00	1.00	−7.14	7.14
			Totals	13.67	54.31	−70.21	105.86

a. $\text{MAE} = \frac{13.67}{5} = 2.73$

b. $\text{MSE} = \frac{54.31}{5} = 10.86$

c. $\text{MAPE} = \frac{105.89}{5} = 21.18$

d. Forecast for week 7 is

$$\frac{18 + 13 + 16 + 11 + 17 + 14}{6} = 14.83$$

4. a. $\text{MSE} = \frac{363}{6} = 60.5$

Forecast for month 8 is 15

b. $\text{MSE} = \frac{216.72}{6} = 36.12$

Forecast for month 8 is 18

c. The average of all the previous values is better because MSE is smaller

5. a. The data appear to follow a horizontal pattern

b. Three-week moving average

Week	Time Series Value	Forecast	Forecast Error	Squared Forecast Error
1	18			
2	13			
3	16			
4	11	15.67	−4.67	21.78
5	17	13.33	3.67	13.44
6	14	14.67	−0.67	0.44
			Total	35.67

$$\text{MSE} = \frac{35.67}{3} = 11.89$$

$$\text{The forecast for week 7} = \frac{(11 + 17 + 14)}{3} = 14$$

c. Smoothing constant = .2

Week	Time Series Value	Forecast	Forecast Error	Squared Forecast Error
1	18			
2	13	18.00	−5.00	25.00
3	16	17.00	−1.00	1.00
4	11	16.80	−5.80	33.64
5	17	15.64	1.36	1.85
6	14	15.91	−1.91	3.66
			Total	65.15

$$\text{MSE} = \frac{65.15}{5} = 13.03$$

The forecast for week 7 is .2(14) + (1 − .2)15.91 = 15.53

d. The three-week moving average provides a better forecast since it has a smaller MSE

e. Smoothing constant = .4

Week	Time Series Value	Forecast	Forecast Error	Squared Forecast Error
1	18			
2	13	18.00	−5.00	25.00
3	16	16.00	0.00	0.00
4	11	16.00	−5.00	25.00
5	17	14.00	3.00	9.00
6	14	15.20	−1.20	1.44
			Total	60.44

$$\text{MSE} = \frac{60.44}{5} = 12.09$$

The exponential smoothing forecast using $\alpha = .4$ provides a better forecast than the exponential smoothing forecast using $\alpha = .2$ since it has a smaller MSE

6. a. The data appear to follow a horizontal pattern

b. $\text{MSE} = \frac{110}{4} = 27.5$

The forecast for week 8 is 19

c. $\text{MSE} = \frac{252.87}{6} = 42.15$

The forecast for week 7 is 19.12

d. The three-week moving average provides a better forecast since it has a smaller MSE

e. MSE = 39.79

The exponential smoothing forecast using $\alpha = .4$ provides a better forecast than the exponential smoothing forecast using $\alpha = .2$ since it has a smaller MSE

8. a.

Week	4	5	6	7	8	9	10	11	12
Forecast	19.33	21.33	19.83	17.83	18.33	18.33	20.33	20.33	17.83

b. MSE = 11.49

Prefer the unweighted moving average here; it has a smaller MSE

c. You could always find a weighted moving average at least as good as the unweighted one; actually the unweighted moving average is a special case of the weighted ones where the weights are equal

10. b. The more recent data receive the greater weight or importance in determining the forecast; the moving averages method weights the last n data values equally in determining the forecast

12. a. The data appear to follow a horizontal pattern

b. MSE(3-Month) = .12
MSE(4-Month) = .14
Use 3-Month moving averages

c. 9.63

13. a. The data appear to follow a horizontal pattern

b.

Month	Time-Series Value	3-Month Moving Average Forecast	$(\text{Error})^2$	$\alpha = .2$ Forecast	$(\text{Error})^2$
1	240				
2	350			240.00	12100.00
3	230			262.00	1024.00
4	260	273.33	177.69	255.60	19.36
5	280	280.00	0.00	256.48	553.19
6	320	256.67	4010.69	261.18	3459.79
7	220	286.67	4444.89	272.95	2803.70
8	310	273.33	1344.69	262.36	2269.57
9	240	283.33	1877.49	271.89	1016.97
10	310	256.67	2844.09	265.51	1979.36
11	240	286.67	2178.09	274.41	1184.05
12	230	263.33	1110.89	267.53	1408.50
		Totals	17,988.52		27,818.49

MSE(3-Month) = 17,988.52/9 = 1998.72

MSE(α = .2) = 27,818.49/11 = 2528.95

Based on the preceding MSE values, the 3-Month moving averages appear better; however, exponential smoothing was penalized by including month 2, which was difficult for any method to forecast; using only the errors for months 4 to 12, the MSE for exponential smoothing is

$$\text{MSE}(\alpha = .2) = 14{,}694.49/9 = 1632.72$$

Thus, exponential smoothing was better considering months 4 to 12

c. Using exponential smoothing,

$$F_{13} = \alpha Y_{12} + (1 - \alpha)F_{12}$$
$$= .20(230) + .80(267.53) = 260$$

14. a. The data appear to follow a horizontal pattern

b. Values for months 2–12 are as follows:

105.00 114.00 115.80 112.56 105.79 110.05
120.54 126.38 118.46 106.92 104.85

MSE = 510.29

c. Values for months 2–12 are as follows:

105.00 120.00 120.00 112.50 101.25 110.63
127.81 133.91 116.95 98.48 99.24

$$\text{MSE} = 540.55$$

Conclusion: A smoothing constant of .3 is better than a smoothing constant of .5 since the MSE is less for 0.3

16. a. The time series plot indicates a possible linear trend in the data; this could be due to decreasing viewer interest in watching the Masters, but closer inspection of the data indicates that the two highest ratings correspond to years 1997 and 2001, years in which Tiger Woods won the tournament; the pattern observed may be simply due to the effect Tiger Woods has on ratings and not necessarily on any long-term decrease in viewer interest

b. The methods discussed in this section are only applicable for a time series that has a horizontal pattern, so if there is a really a long-term linear trend in the data, the methods discussed in this section are not appropriate

c. The time series plot for the data for years 2002–2008 exhibits a horizontal pattern; it seems reasonable to conclude that the extreme values observed in 1997 and 2001 are more attributable to viewer interest in the performance of Tiger Woods; basing the forecast on years 2002–2008 does seem reasonable, but, because of the injury that Tiger Woods experienced in the 2008 reason, if he is able to play in the 2009 Masters, then the rating for 2009 may be significantly higher than suggested by the data for years 2002–2008

17. a. The time series plot shows a linear trend

b. $\bar{t} = \dfrac{\sum_{t=1}^{n} t}{n} = \dfrac{15}{5} = 3 \qquad \bar{Y} = \dfrac{\sum_{t=1}^{n} Y_t}{n} = \dfrac{55}{5} = 11$

$$\Sigma(t - \bar{t})(Y_t - \bar{Y}) = 21 \qquad \Sigma(t - \bar{t})^2 = 10$$

$$b_1 = \frac{\sum_{t=1}^{n}(t - \bar{t})(Y_t - \bar{Y})}{\sum_{t=1}^{n}(t - \bar{t})^2} = \frac{21}{10} = 2.1$$

$$b_0 = \bar{Y} - b_1\bar{t} = 11 - (2.1)(3) = 4.7$$

$$T_t = 4.7 + 2.1t$$

c. $T_6 = 4.7 + 2.1(6) = 17.3$

18. a. The time series plot exhibits a curvilinear trend

b. Using Excel's chart tools, the quadratic trend equation is $T_t = 1.1429 + 5.3571t - .5714t^2$

c. Forecast for $t = 8$ is $T_8 = 1.1429 + 5.3571(8) - .5714(8)^2 = 7.4$

d. $T_8 = 4.7 + 2.1(8) = 21.5$

The forecast obtained using the linear trend equation provides a much different forecast because it has not picked up the change that has occurred in the time series in periods 7 and 8; each time a new time series value becomes available we need to develop a new time series plot and make a new decision as to what the underlying pattern of the time series is

20. a. The time series plot exhibits a curvilinear trend

b. $T_t = 107.857 - 28.9881t + 2.65476t^2$

c. 45.86

21. a. The time series plot shows a linear trend

b. $\bar{t} = \dfrac{\sum_{t=1}^{n} t}{n} = \dfrac{45}{9} = 5 \qquad \bar{Y} = \dfrac{\sum_{t=1}^{n} Y_t}{n} = \dfrac{108}{9} = 12$

$$\Sigma(t - \bar{t})(Y_t - \bar{Y}) = 87.4 \qquad \Sigma(t - \bar{t})^2 = 60$$

$$b_1 = \frac{\sum_{t=1}^{n}(t - \bar{t})(Y_t - \bar{Y})}{\sum_{t=1}^{n}(t - t)^2} = \frac{87.4}{60} = 1.4567$$

$$b_0 = \bar{Y} - b_1\bar{t} = 12 - (1.4567)(5) = 4.7165$$

$$T_t = 4.7165 + 1.4567t$$

c. $T_{10} = 4.7165 + 1.4567(10) = 19.28$

22. a. The time series plot shows a downward linear trend

b. $T_t = 13.8 - .7t$

c. 8.2

d. If SCF can continue to decrease the percentage of funds spent on administrative and fund-raising by .7% per year, the forecast of expenses for 2018 is 4.70%

24. a. The time series plot shows a linear trend

b. $T_t = 7.5623 - .07541t$

c. 6.7328

d. Given the uncertainty in global market conditions, making a prediction for December using only time is not recommended

26. a. A linear trend is not appropriate

b. $T_t = 5.702 + 2.889t - 1618t^2$

c. 17.90

28. a. The time series plot shows a horizontal pattern, but there is a seasonal pattern in the data; for instance, in each year the lowest value occurs in quarter 2 and the highest value occurs in quarter 4

b. The estimated multiple regression equation is

```
Value = 77.0 - 10.0 Qtr1 - 30.0
        Qtr2 - 20.0 Qtr3
```

c. The quarterly forecasts for next year are as follows:

Quarter 1 forecast = 77.0 − 10.0(1) − 30.0(0) − 20.0(0) = 67

Quarter 2 forecast = 77.0 − 10.0(0) − 30.0(1) − 20.0(0) = 47

Quarter 3 forecast = 77.0 − 10.0(0) − 30.0(0) − 20.0(1) = 57
Quarter 4 forecast = 77.0 − 10.0(0) − 30.0(0) − 20.0(0) = 77

30. a. There appears to be a seasonal pattern in the data and perhaps a moderate upward linear trend

b. The estimated multiple regression equation is

```
Value = 2492 - 712 Qtr1 - 1512
        Qtr2 + 327 Qtr3
```

c. The quarterly forecasts for next year are as follows:

Quarter 1 forecast is 1780
Quarter 2 forecast is 980
Quarter 3 forecast is 2819
Quarter 4 forecast is 2492

d. The estimated multiple regression equation is

```
Value = 2307 - 642 Qtr1 - 1465
        Qtr2 + 350 Qtr3 + 23.1 t
```

The quarterly forecasts for next year are as follows:

Quarter 1 forecast is 2058
Quarter 2 forecast is 1258
Quarter 3 forecast is 3096
Quarter 4 forecast is 2769

32. a. The time series plot shows both a linear trend and seasonal effects

b. The estimated multiple regression equation is

```
Revenue = 70.0 + 10.0 Qtr1 + 105
          Qtr2 + 245 Qtr3
```

Quarter 1 forecast is 80
Quarter 2 forecast is 175
Quarter 3 forecast is 315
Quarter 4 forecast is 70

c. The estimated multiple regression equation is

```
Revenue = -70.1 + 45.0 Qtr1 + 128
          Qtr2 + 257 Qtr3 + 11.7 Period
```

Quarter 1 forecast = is 221
Quarter 2 forecast = is 315
Quarter 3 forecast = is 456
Quarter 4 forecast = is 211

34. a. The time series plot shows seasonal and linear trend effects

b. *Note*: Jan = 1 if January, 0 otherwise; Feb = 1 if February, 0 otherwise; and so on

The estimated multiple regression equation is

```
Expense = 175 - 18.4 Jan - 3.72 Feb +
          12.7 Mar + 45.7 Apr + 57.1
          May + 135 Jun + 181 Jul + 105
          Aug + 47.6 Sep + 50.6 Oct +
          35.3 Nov + 1.96 Period
```

c. *Note*: The next time period in the time series is Period = 37 (January of Year 4); the forecasts for January–December are 229; 246; 264; 299; 312; 392; 440; 366; 311; 316; 302; 269

35. a. The time series plot indicates a linear trend and a seasonal pattern

b.

Year	Quarter	Time Series Value	Four-Quarter Moving Average	Centered Moving Average
1	1	4		
	2	2		
			3.50	
	3	3		3.750
			4.00	
	4	5		4.125
			4.25	
2	1	6		4.500
			4.75	
	2	3		5.000
			5.25	
	3	5		5.375
			5.50	
	4	7		5.875
			6.25	
3	1	7		6.375
			6.50	
	2	6		6.625
			6.75	
	3	6		
	4	8		

c.

Year	Quarter	Time Series Value	Centered Moving Average	Seasonal-Irregular Component
1	1	4		
	2	2		
	3	3	3.750	0.800
	4	5	4.125	1.212
2	1	6	4.500	1.333
	2	3	5.000	0.600
	3	5	5.375	0.930
	4	7	5.875	1.191
3	1	7	6.375	1.098
	2	6	6.625	0.906
	3	6		
	4	8		

Quarter	Seasonal-Irregular Values		Seasonal Index	Adjusted Seasonal Index
1	1.333	1.098	1.216	1.205
2	0.600	0.906	0.752	0.746
3	0.800	0.930	0.865	0.857
4	1.212	1.191	1.201	1.191
		Total	4.036	

$$\text{Adjustment for seasonal index} = \frac{4.000}{4.036} = 0.991$$

36. a.

Year	Quarter	Deseasonalized Value
1	1	3.320
	2	2.681
	3	3.501
	4	4.198
2	1	4.979
	2	4.021
	3	5.834
	4	5.877
3	1	5.809
	2	8.043
	3	7.001
	4	6.717

b. Let Period = 1 denote the time series value in Year 1—Quarter 1; Period = 2 denote the time series value in Year 1—Quarter 2; and so on; treating Period as the independent variable and the Deseasonlized Values as the values of the dependent variable, the estimated regression equation is

```
Deseasonalized Value = 2.42 + 0.422
                       Period
```

c. The quarterly deseasonalized trend forecasts for Year 4 (Periods 13, 14, 15, and 16) are as follows:

Forecast for quarter 1 is 7.906
Forecast for quarter 2 is 8.328
Forecast for quarter 3 is 8.750
Forecast for quarter 4 is 9.172

d. Adjusting the quarterly deseasonalized trend forecasts provides the following quarterly estimates:

Forecast for quarter 1 is 9.527
Forecast for quarter 2 is 6.213
Forecast for quarter 3 is 7.499
Forecast for quarter 4 is 10.924

38. a. The time series plot shows a linear trend and seasonal effects

b. 0.71 0.78 0.83 0.97 1.02 1.30 1.50 1.23 0.98 0.99 0.93 0.79

c.

Month	Deaseasonalized Expense
1	239.44
2	230.77
3	246.99
4	237.11
5	235.29
6	242.31
7	240.00
8	235.77
9	244.90
10	242.42
11	247.31
12	246.84
13	253.52
14	262.82
15	259.04
16	252.58
17	259.80
18	253.85
19	266.67
20	272.36
21	265.31
22	272.73
23	274.19
24	278.48
25	274.65
26	269.23
27	277.11
28	288.66
29	284.31
30	300.00
31	280.00
32	268.29
33	295.92
34	297.98
35	301.08
36	316.46

d. Let Period = 1 denote the time series value in January—Year 1; Period = 2 denote the time series value in February—Year 2; and so on; treating Period as the independent variable and the Deseasonlized Values as the values of the dependent variable the estimated regression equation is

```
Deseasonalized Expense = 228 + 1.96
                         Period
```

e.

Month	Monthly Forecast
January	213.37
February	235.93
March	252.69
April	297.21
May	314.53
June	403.42
July	486.42
August	386.52
September	309.88
October	314.98
November	297.71
December	254.44

40. a. The time series plot indicates a seasonal effect; power consumption is lowest in the time period 12–4 A.M., steadily increases to the highest value in the 12–4 P.M. time period, and then decreases again. There may also be some linear trend in the data

b.

Time Period	Adjusted Seasonal Index
12–4 A.M.	0.3256
4–8 A.M.	0.4476
8–12 noon	1.3622
12–4 P.M.	1.6959
4–8 P.M.	1.4578
8–12 midnight	0.7109

c. The following estimated regression equation shows the results of fitting a linear trend equation to the deseasonalized time series:

```
Deseasonalized Power = 63108 + 1854 t
```

Deaseasonalized Power (t = 19) = 63,108 + 1854(19) = 98,334

Forecast for 12–4 P.M. = 1.6959(98,334) = 166,764.63 or approximately 166,765 kWh

Deaseasonalized Power (t = 20) = 63,108 + 1854(20) = 100,188

Forecast for 4–8 P.M. = 1.4578(100,188) = 146,054.07 or approximately 146,054 kWh

Thus, the forecast of power consumption from noon to 8 P.M. is 166,765 + 146,054 = 312,819 kWh

42. a. The time series plot indicates a horizontal pattern

b. MSE(α = .2) = 1.40
MSE(α = .3) = 1.27
MSE(α = .4) = 1.23

A smoothing constant of α = .4 provides the best forecast because it has a smaller MSE

c. 31.00

44. a. There appears to be an increasing trend in the data

b. *Note*: t = 1 corresponds to 2001, t = 2 corresponds to 2002, and so on. The estimated regression equation is

```
Balance($) = 1984 + 146 t
```

The forecast for 2009 (t = 9) is Balance($) = 1984 + 146(9) = $3298

c. *Note*: t = 1 corresponds to 2001, t = 2 corresponds to 2002, and so on. The estimated regression equation is

```
Balance($) = 2924 - 419 t + 62.7 tsq
```

The forecast for 2009 (t = 9) is Balance($) = 2924 − 419(9) + 62.7(9)^2 = $4232

d. The quadratic trend equation provides the best forecast accuracy for the historical data

e. Linear trend equation

46. a. The forecast for July is 236.97
Forecast for August, using forecast for July as the actual sales in July, is 236.97
Exponential smoothing provides the same forecast for every period in the future; this is why it is not usually recommended for long-term forecasting

b. The linear trend equation is

$$T_t = 149.72 + 18.451t$$

Forecast for July is 278.88
Forecast for August is 297.33

c. The proposed settlement is not fair since it does not account for the upward trend in sales; based upon trend projection, the settlement should be based on forecasted lost sales of $278,880 in July and $297,330 in August

48. a. The time series plot shows a linear trend

b. $T_t = -5 + 15t$
The slope of 15 indicates that the average increase in sales is 15 pianos per year

c. 85, 100

50. a.

Quarter	Adjusted Seasonal Index
1	1.2717
2	0.6120
3	0.4978
4	1.6185

Note: Adjustment for seasonal index = $\frac{4}{3.8985}$ = 1.0260

b. The largest effect is in quarter 4; this seems reasonable since retail sales are generally higher during October, November, and December

52. a. Yes, a linear trend pattern appears to be present

b. The estimated regression equation is

```
Number Sold = 22.9 + 15.5 Year
```

c. Forecast in year 8 is or approximately 147 units

54. b. The centered moving average values smooth out the time series by removing seasonal effects and some of the random variability; the centered moving average time series shows the trend in the data

c.

Quarter	Adjusted Seasonal Index
1	0.899
2	1.362
3	1.118
4	0.621

d. Hudson Marine experiences the largest seasonal increase in quarter 2; since this quarter occurs prior to the peak summer boating season, this result seems reasonable, but the largest seasonal effect is the seasonal decrease in quarter 4; this is also reasonable because of decreased boating in the fall and winter

Appendix E: Microsoft Excel 2010 and Tools for Statistical Analysis

Microsoft Excel 2010, part of the Microsoft Office 2010 system, is a spreadsheet program that can be used to organize and analyze data, perform complex calculations, and create a wide variety of graphical displays. We assume that readers are familiar with basic Excel operations such as selecting cells, entering formulas, copying, and so on. But we do not assume readers are familiar with Excel 2010 or the use of Excel for statistical analysis.

The purpose of this appendix is twofold. First, we provide an overview of Excel 2010 and discuss the basic operations needed to work with Excel 2010 workbooks and worksheets. Second, we provide an overview of the tools that are available for conducting statistical analysis with Excel. These include Excel functions and formulas which allow users to conduct their own analyses and add-ins that provide more comprehensive analysis tools.

Excel's Data Analysis add-in, included with the basic Excel system, is a valuable tool for conducting statistical analysis. In the last section of this appendix we provide instruction for installing the Data Analysis add-in. Other add-ins have been developed by outside suppliers to supplement the basic statistical capabilities provided by Excel. In the last section we also discuss StatTools, a commercially available add-in developed by Palisade Corporation.

Overview of Microsoft Excel 2010

A workbook is a file containing one or more worksheets.

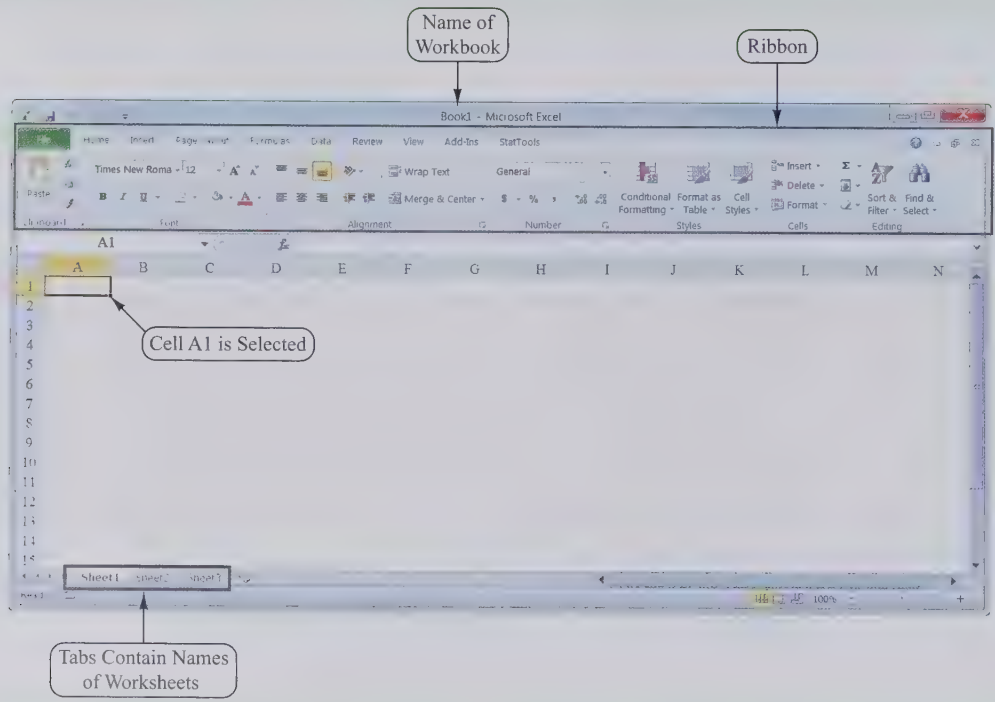
When using Excel for statistical analysis, data is displayed in workbooks, each of which contains a series of worksheets that typically include the original data as well as any resulting analysis, including charts. Figure 1 shows the layout of a blank workbook created each time Excel is opened. The workbook is named Book1, and consists of three worksheets named Sheet1, Sheet2, and Sheet3. Excel highlights the worksheet currently displayed (Sheet1) by setting the name on the worksheet tab in bold. To select a different worksheet simply click on the corresponding tab. Note that cell A1 is initially selected.

The wide bar located across the top of the workbook is referred to as the Ribbon. Tabs, located at the top of the Ribbon, provide quick access to groups of related commands. There are nine tabs shown on the workbook in Figure 1: File; Home; Insert; Page Layout; Formulas; Data; Review; View; and Add-Ins. Each tab contains a series of groups of related commands. Note that the Home tab is selected when Excel is opened. Figure 2 displays the groups available when the Home tab is selected. Under the Home tab there are seven groups: Clipboard; Font; Alignment; Number; Styles; Cells; and Editing. Commands are arranged within each group. For example, to change selected text to boldface, click the Home tab and click the Bold button in the Font group.

Figure 3 illustrates the location of the Quick Access Toolbar and the Formula Bar. The Quick Access Toolbar allows you to quickly access workbook options. To add or remove features on the Quick Access Toolbar click the Customize Quick Access Toolbar button on the Quick Access Toolbar.

The Formula Bar (see Figure 3) contains a Name box, the Insert Function button f_x, and a Formula box. In Figure 3, "A1" appears in the name box because cell A1 is selected. You can select any other cell in the worksheet by using the mouse to move the cursor to another cell and clicking or by typing the new cell location in the Name box. The Formula box is used to display the formula in the currently selected cell. For instance, if you had entered =A1+A2 into cell A3, whenever you select cell A3 the formula =A1+A2 will be shown

FIGURE 1 BLANK WORKBOOK CREATED WHEN EXCEL IS OPENED



in the Formula box. This feature makes it very easy to see and edit a formula in a particular cell. The Insert Function button allows you to quickly access all of the functions available in Excel. Later we show how to find and use a particular function.

Basic Workbook Operations

Figure 4 illustrates the worksheet options that can be performed after right-clicking on a worksheet tab. For instance, to change the name of the current worksheet from "Sheet1" to "Data," right-click the worksheet tab named "Sheet1" and select the Rename option. The current worksheet name (Sheet1) will be highlighted. Then, simply type the new name (Data) and press the Enter key to rename the worksheet.

Suppose that you wanted to create a copy of "Sheet 1." After right-clicking the tab named "Sheet1," select the Move or Copy option. When the Move or Copy dialog box appears, select Create a Copy and click OK. The name of the copied worksheet will appear as "Sheet1 (2)." You can then rename it, if desired.

To add a worksheet to the workbook, right-click any worksheet tab and select the Insert option; when the Insert dialog box appears, select Worksheet and click OK. An additional blank worksheet titled "Sheet 4" will appear in the workbook. You can also insert a new worksheet by clicking the Insert Worksheet tab button that appears to the right of the last worksheet tab displayed. Worksheets can be deleted by right-clicking the worksheet tab and choosing Delete. After clicking Delete a window will appear warning you that any data appearing in the worksheet will be lost. Click Delete to confirm that you do want to delete

FIGURE 2 PORTION OF THE HOME TAB

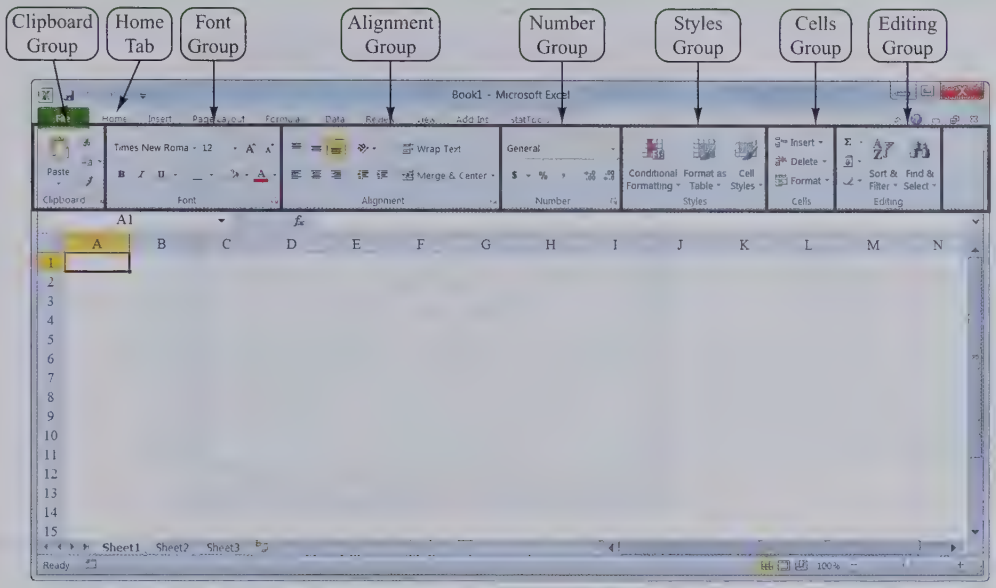


FIGURE 3 EXCEL 2010 QUICK ACCESS TOOLBAR AND FORMULA BAR

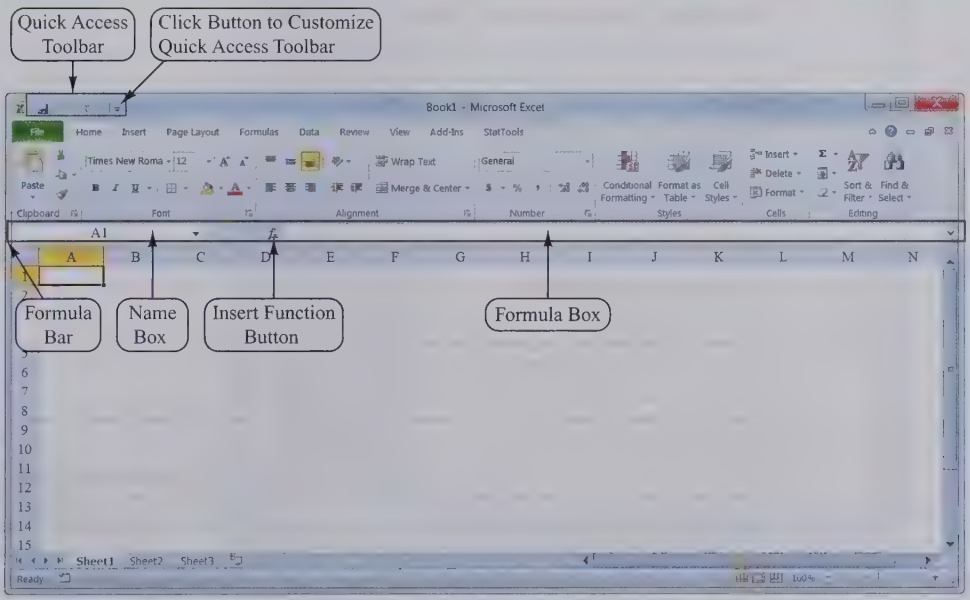
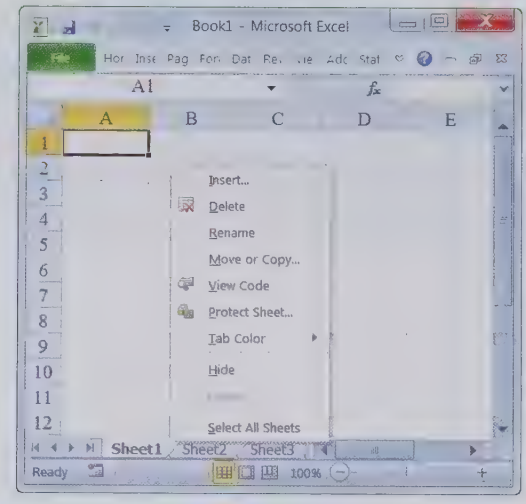


FIGURE 4 WORKSHEET OPTIONS OBTAINED AFTER RIGHT-CLICKING ON A WORKSHEET TAB



the worksheet. Worksheets can also be moved to other workbooks or a different position in the current workbook by using the Move or Copy option.

Creating, Saving, and Opening Files

Data can be entered into an Excel worksheet by manually entering the data into the worksheet or by opening another workbook that already contains the data. As an illustration of manually entering, saving, and opening a file we will use the example from Chapter 2 involving data for a sample of 50 soft drink purchases. The original data are shown in Table 1.

Suppose you just opened Excel and want to work with this data. A blank workbook containing three worksheets will be displayed. The soft drink data can now be entered by simply typing it into one of the worksheets. If Excel is currently running and no blank workbook is displayed, you can create a blank workbook using the following steps:

Step 1: Click the **File** tab
Step 2: Click **New** in the list of options
Step 3: When the Available Templates dialog box appears, Double-click **Blank Workbook**

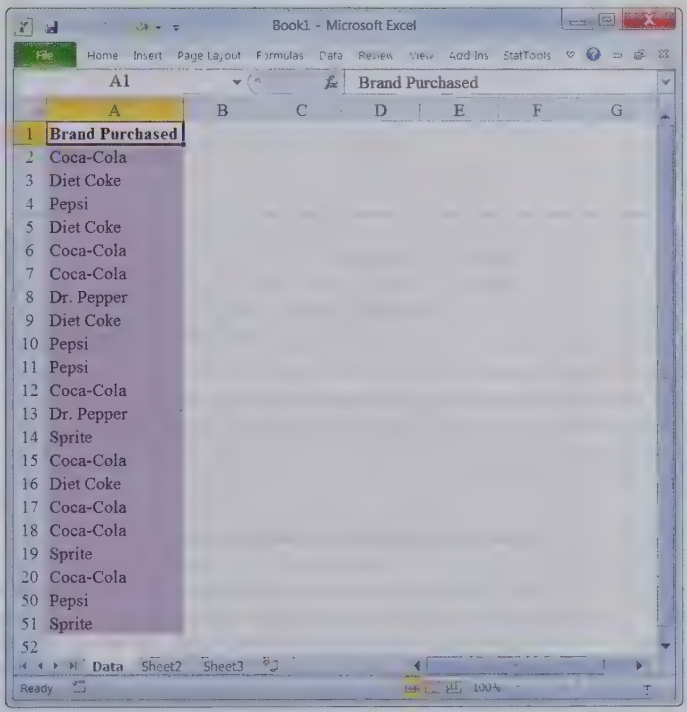
A new workbook containing three worksheets labeled Sheet1, Sheet2, and Sheet3 will appear.

Suppose we want to enter the data for the sample of 50 soft drink purchases into Sheet1 of the new workbook. First, we enter the label "Brand Purchased" into cell A1; then, we enter the data for the 50 soft drink purchases into cells A2:A51. As a reminder that this worksheet contains the data, we will change the name of the worksheet from "Sheet1" to "Data" using the procedure described previously. Figure 5 shows the data worksheet that we just developed.

TABLE 1 DATA FROM A SAMPLE OF 50 SOFT DRINK PURCHASES

Coca-Cola	Sprite	Pepsi
Diet Coke	Coca-Cola	Coca-Cola
Pepsi	Diet Coke	Coca-Cola
Diet Coke	Coca-Cola	Coca-Cola
Coca-Cola	Diet Coke	Pepsi
Coca-Cola	Coca-Cola	Dr. Pepper
Dr. Pepper	Sprite	Coca-Cola
Diet Coke	Pepsi	Diet Coke
Pepsi	Coca-Cola	Pepsi
Pepsi	Coca-Cola	Pepsi
Coca-Cola	Coca-Cola	Pepsi
Dr. Pepper	Pepsi	Pepsi
Sprite	Coca-Cola	Coca-Cola
Coca-Cola	Sprite	Dr. Pepper
Diet Coke	Dr. Pepper	Pepsi
Coca-Cola	Pepsi	Sprite
Coca-Cola	Diet Coke	

FIGURE 5 WORKSHEET CONTAINING THE SOFT DRINK DATA



Note: Rows 21–49 are hidden.

Before doing any analysis with these data, we recommend that you first save the file; this will prevent you from having to reenter the data in case something happens that causes Excel to close. To save the file as an Excel 2010 workbook using the filename SoftDrink we perform the following steps:

Step 1: Click the **File** tab
Step 2: Click **Save** in the list of options
Step 3: When the Save As dialog box appears,
Select the location where you want to save the file
Type the filename **SoftDrink** in the **File name** box
Click **Save**

Keyboard shortcut: To save the file, press CTRL+S

Excel's Save command is designed to save the file as an Excel 2010 workbook. As you work with the file to do statistical analysis you should follow the practice of periodically saving the file so you will not lose any statistical analysis you may have performed. Simply click the File tab and select Save in the list of options.

Sometimes you may want to create a copy of an existing file. For instance, suppose you would like to save the soft drink data and any resulting statistical analysis in a new file named "SoftDrink Analysis." The following steps show how to create a copy of the SoftDrink workbook and analysis with the new filename, "SoftDrink Analysis."

Step 1: Click the **File** tab
Step 2: Click **Save As**
Step 3: When the Save As dialog box appears,
Select the location where you want to save the file
Type the filename **SoftDrink Analysis** in the **File name** box
Click **Save**

Once the workbook has been saved, you can continue to work with the data to perform whatever type of statistical analysis is appropriate. When you are finished working with the file simply click the close window button located at the top right-hand corner of the Ribbon. To access the SoftDrink Analysis file at another point in time you can open the file by performing the following steps:

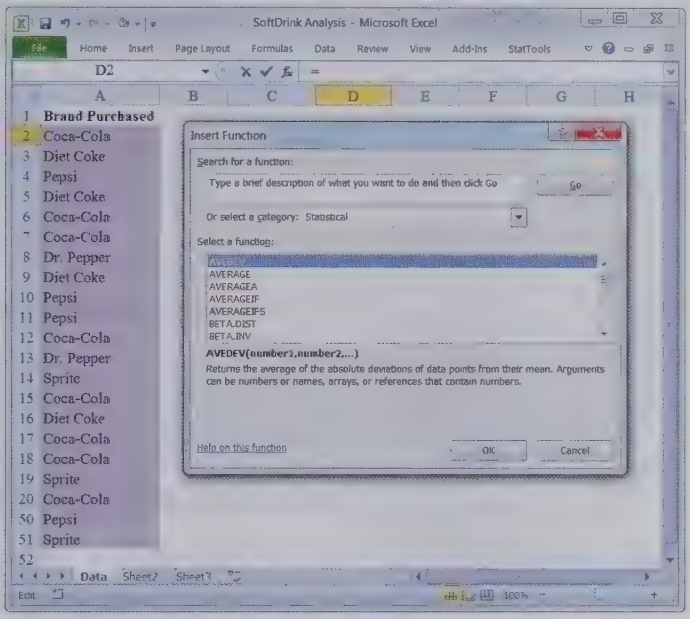
Step 1: Click the **File** tab
Step 2: Click **Open**
Step 3: When the Open dialog box appears,
Select the location where you previously saved the file
Enter the filename **SoftDrink Analysis** in the **File name** box
Click **Open**

The procedures we showed for saving or opening a workbook begin by clicking File tab to access the Save and Open commands. Once you have used Excel for a while you will probably find it more convenient to add these commands to the Quick Access Toolbar.

Using Excel Functions

Excel 2010 provides a wealth of functions for data management and statistical analysis. If we know what function is needed, and how to use it, we can simply enter the function into the appropriate worksheet cell. However, if we are not sure what functions are available to accomplish a task or are not sure how to use a particular function, Excel can provide assistance. Many new functions for statistical analysis have been added with Excel 2010. To illustrate we will use the SoftDrink Analysis workbook created in the previous subsection.

FIGURE 6 INSERT FUNCTION DIALOG BOX



Note: Rows 21–49 are hidden.

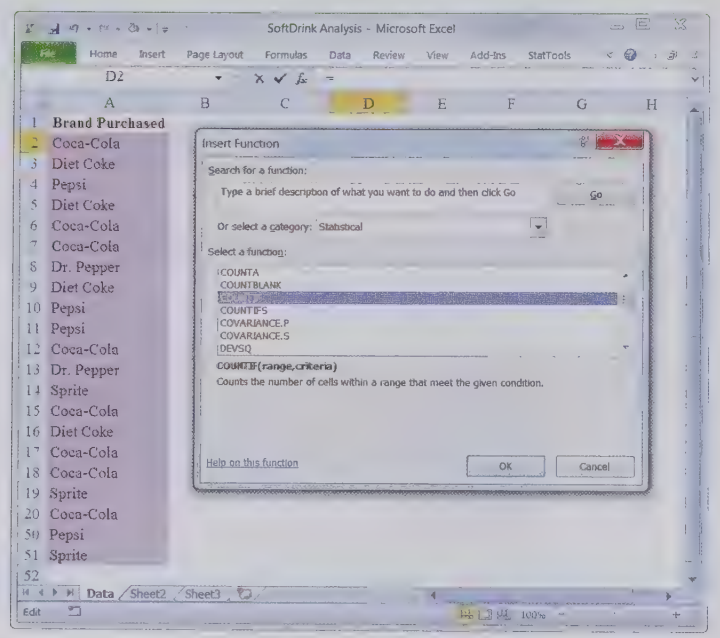
Finding the Right Excel Function

To identify the functions available in Excel, select the cell where you want to insert the function; we have selected cell D2. Click the **Formulas** tab on the Ribbon and then click the **Insert Function** button in the **Function Library** group. Alternatively, click the f_x button on the formula bar. Either approach provides the **Insert Function** dialog box shown in Figure 6.

The **Search for a function** box at the top of the Insert Function dialog box enables us to type a brief description of what we want to do. After doing so and clicking **Go**, Excel will search for and display, in the **Select a function** box, the functions that may accomplish our task. In many situations, however, we may want to browse through an entire category of functions to see what is available. For this task, the **Or select a category** box is helpful. It contains a drop-down list of several categories of functions provided by Excel. Figure 6 shows that we selected the **Statistical** category. As a result, Excel's statistical functions appear in alphabetic order in the Select a function box. We see the AVEDEV function listed first, followed by the AVERAGE function, and so on.

The AVEDEV function is highlighted in Figure 6, indicating it is the function currently selected. The proper syntax for the function and a brief description of the function appear below the Select a function box. We can scroll through the list in the Select a function box to display the syntax and a brief description for each of the statistical functions that are available. For instance, scrolling down farther, we select the COUNTIF function as shown in Figure 7. Note that COUNTIF is now highlighted, and that immediately below the Select a function box we see **COUNTIF(range,criteria)**, which indicates that the COUNTIF function contains two inputs, range and criteria. In addition, we see that the description of the COUNTIF function is "Counts the number of cells within a range that meet the given condition."

FIGURE 7 DESCRIPTION OF THE COUNTIF FUNCTION IN THE INSERT FUNCTION DIALOG BOX



Note: Rows 21–49 are hidden.

If the function selected (highlighted) is the one we want to use, we click **OK**; the **Function Arguments** dialog box then appears. The Function Arguments dialog box for the COUNTIF function is shown in Figure 8. This dialog box assists in creating the appropriate arguments for the function selected. When finished entering the arguments, we click OK; Excel then inserts the function into a worksheet cell.

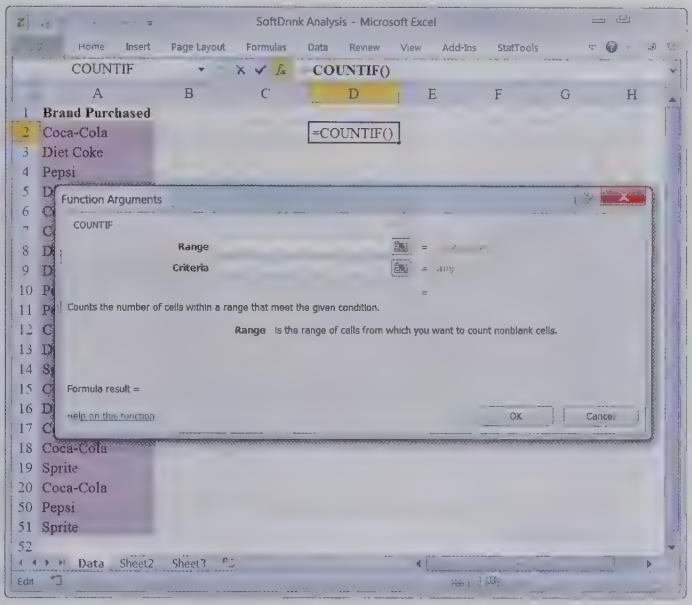
Inserting a Function into a Worksheet Cell

We will now show how to use the Insert Function and Function Arguments dialog boxes to select a function, develop its arguments, and insert the function into a worksheet cell.

Suppose we want to construct a frequency distribution for the soft drink purchase data in Table 1. Figure 9 displays an Excel worksheet containing the soft drink data and labels for the frequency distribution we would like to construct. We see that the frequency of Coca-Cola purchases will go into cell D2, the frequency of Diet Coke purchases will go into cell D3, and so on. Suppose we want to use the COUNTIF function to compute the frequencies and would like some assistance from Excel.

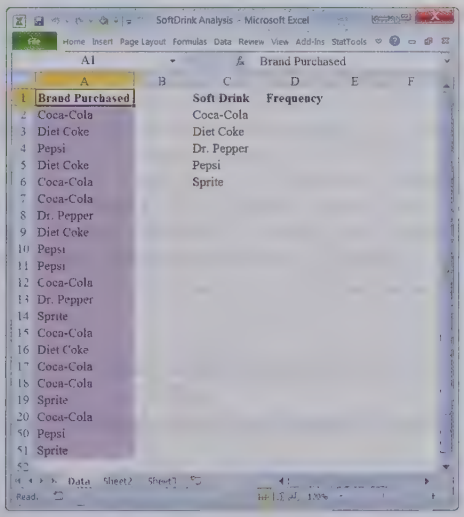
Step 1. Select cell D2
Step 2. Click f_x on the formula bar
Step 3. When the Insert Function dialog box appears,
Select **Statistical** in the **Or select a category** box
Select **COUNTIF** in the **Select a function** box
Click **OK**

FIGURE 8 FUNCTION ARGUMENTS DIALOG BOX FOR THE COUNTIF FUNCTION



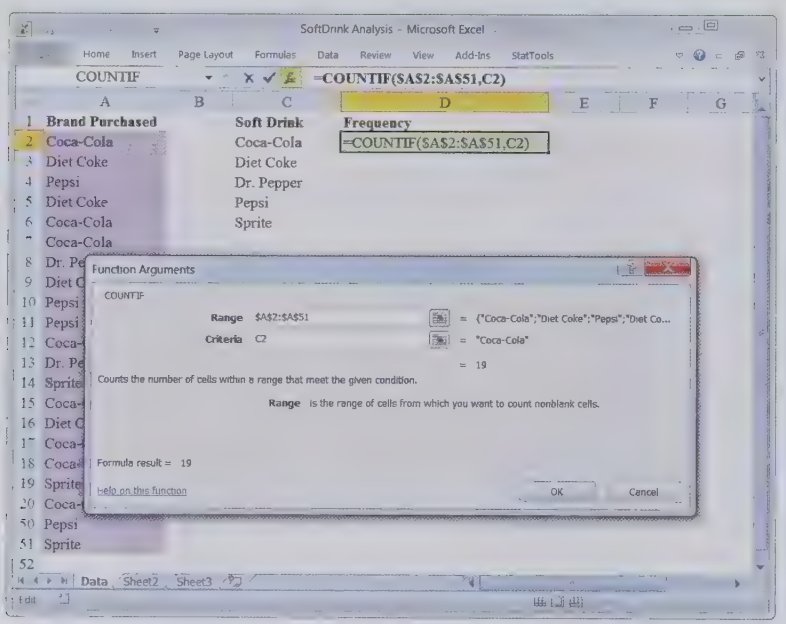
Note: Rows 21–49 are hidden.

FIGURE 9 EXCEL WORKSHEET WITH SOFT DRINK DATA AND LABELS FOR THE FREQUENCY DISTRIBUTION WE WOULD LIKE TO CONSTRUCT



Note: Rows 21–49 are hidden.

FIGURE 10 COMPLETED FUNCTION ARGUMENTS DIALOG BOX FOR THE COUNTIF FUNCTION



Note: Rows 21–49 are hidden.

Step 4. When the **Function Arguments** dialog box appears (see Figure 10),
Enter $A$2:$A$51 in the **Range** box
Enter C2 in the **Criteria** box (At this point, the value of the function will appear on the next-to-last line of the dialog box. Its value is 19.)
Click **OK**

Step 5. Copy cell D2 to cells D3:D6

The worksheet then appears as in Figure 11. The formula worksheet is in the background; the value worksheet appears in the foreground. The formula worksheet shows that the COUNTIF function was inserted into cells D2:D6. The value worksheet shows the proper class frequencies as computed.

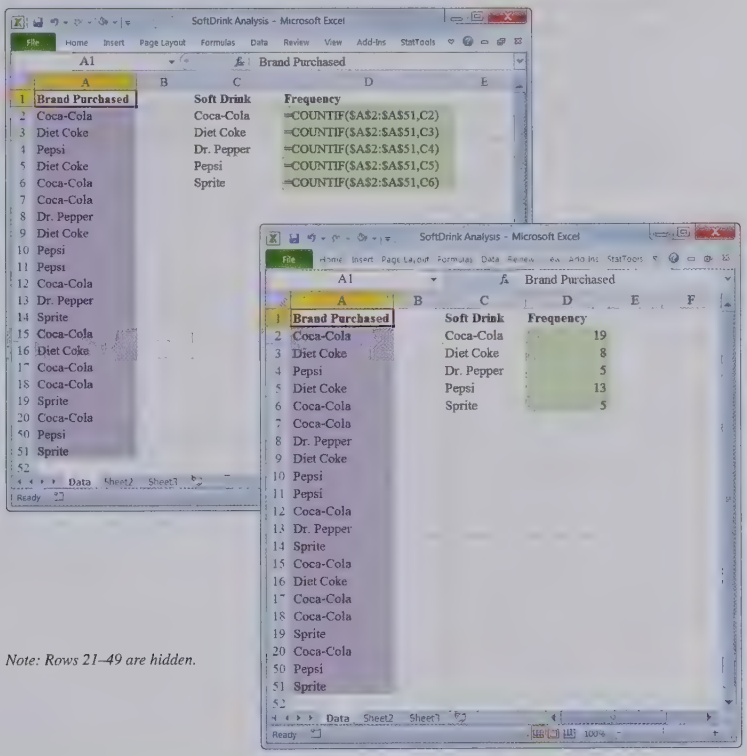
We illustrated the use of Excel's capability to provide assistance in using the COUNTIF function. The procedure is similar for all Excel functions. This capability is especially helpful if you do not know what function to use or forget the proper name and/or inputs required for a function.

Using Excel Add-Ins

Excel's Data Analysis Add-In

Excel's Data Analysis add-in, included with the basic Excel package, is a valuable tool for conducting statistical analysis. Before you can use the Data Analysis add-in it must be installed. To see if the Data Analysis add-in has already been installed, click the Data tab on the Ribbon. In the Analysis group you should see the Data Analysis command. If you do

FIGURE 11 EXCEL WORKSHEET SHOWING THE USE OF EXCEL'S COUNTIF FUNCTION TO CONSTRUCT A FREQUENCY DISTRIBUTION



Note: Rows 21–49 are hidden.

not have an Analysis group and/or the Data Analysis command does not appear in the Analysis group, you will need to install the Data Analysis add-in. The steps needed to install the Data Analysis add-in are as follows:

Step 1. Click the **File** tab
Step 2. Click **Options**
Step 3. When the Excel Options dialog box appears,
Select **Add-Ins** from the list of options (on the pane on the left)
In the **Manage** box, select **Excel Add-Ins**
Click **Go**
Step 4. When the Add-Ins dialog box appears,
Select **Analysis ToolPak**
Click **OK**

Outside Vendor Add-Ins

One of the leading companies in the development of Excel add-ins for statistical analysis is Palisade Corporation. In this text we use StatTools, an Excel add-in developed by Palisade.

StatTools provides a powerful statistics toolset that enables users to perform statistical analysis in the familiar Microsoft Office environment.

In the appendix to Chapter 1 we describe how to download and install the StatTools add-in and provide a brief introduction to using the software. In several appendices throughout the text we show how StatTools can be used when no corresponding basic Excel procedure is available or when additional statistical capabilities would be useful.

Typically the add-ins offered with textbooks are designed primarily for classroom use. StatTools, however, was developed for commercial applications. As a result, students who learn how to use StatTools will be able to continue using StatTools throughout their professional career.

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Note: Chapter 15 can be found with the Online Content for this book. Index entries found in this chapter are denoted by chapter number **15,** hyphen, and page number.

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Preface

The purpose of *Modern Business Statistics with Microsoft® Office Excel®* is to provide students, primarily in the fields of business administration and economics, with a sound conceptual introduction to the field of statistics and its many applications. The text is applications-oriented and has been written with the needs of the nonmathematician in mind.

The solutions manual furnishes assistance by identifying learning objectives and providing detailed solutions for all exercises in the text.

Note: The solutions to the case problems are included in a separate manual.

Acknowledgements

We would like to provide special recognition to Catherine J. Williams for her efforts in preparing the solutions manual.

David R. Anderson
Dennis J. Sweeney
Thomas A. Williams

Chapter 1
Data and Statistics

Learning Objectives

1. Obtain an appreciation for the breadth of statistical applications in business and economics.
2. Understand the meaning of the terms elements, variables, and observations as they are used in statistics.
3. Obtain an understanding of the difference between categorical, quantitative, crossectional and time series data.
4. Learn about the sources of data for statistical analysis both internal and external to the firm.
5. Be aware of how errors can arise in data.
6. Know the meaning of descriptive statistics and statistical inference.
7. Be able to distinguish between a population and a sample.
8. Understand the role a sample plays in making statistical inferences about the population.
9. Know the meaning of the term data mining.
10. Be aware of ethical guidelines for statistical practice.

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Solutions:

1. Statistics can be referred to as numerical facts. In a broader sense, statistics is the field of study dealing with the collection, analysis, presentation and interpretation of data.

2. a. The ten elements are the ten cars

   b. 5 variables: Size, Cylinders, City MPG, Highway MPG, and Fuel

   c. Categorical variables: Size and Fuel

      Quantitative variables: Cylinders, City MPG, and Highway MPG

   d.

Variable	Measurement Scale
Size	Ordinal
Cylinders	Ratio
City MPG	Ratio
Highway MPG	Ratio
Fuel	Nominal

3. a. Average mpg for city driving = 182/10 = 18.2 mpg

   b. Average mpg for highway driving = 261/10 = 26.1 mpg

      On average, the miles per gallon for highway driving is 26.1 – 18.2 = 7.9 mpg greater compared to city driving.

   c. 3 of 10 or 30% have four cylinder engines

   d. 6 of 10 or 60% use regular fuel

4. a. The seven elements are the seven schools shown

   b. 5 variables: State, Campus Setting, Endowment, Applicants Admitted, and NCAA Division

   c. Categorical variables: State, Campus Setting, and NCAA Division

      Quantitative variables: Endowment and Applicants Admitted

5. a. Average endowment = 74.6/7 = $10.657 billion

   b. Average percentage admitted = 111/7 = 15.86%

   c. 3 of 7 or 42.9% have NCAA Division III varsity teams

   d. 3 of 7 or 42.9% have a City: Midsize campus setting

6. a. Quantitative

   b. Categorical

   c. Categorical

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d. Quantitative

e. Categorical

7. a. Although the data are recorded as numbers, the numbers are codes for the ratings of Fair (1), Average (2), Good (3) and Excellent (4). Thus the variables are categorical with each data value corresponding to a rating category for the variable.

b. The data may also be ranked in order of the quality. A higher number indicates a higher rating on a scale from Fair (1) to Excellent (4). Since the data can be ranked or ordered, the scale of measurement is ordinal.

8. a. 1015

b. Categorical

c. Percentages

d. $.10(1015) = 101.5$

101 or 102 respondents said the Federal Bank is doing a good job.

9. a. Categorical

b. 30 of 71; 42.3%

10. a. Categorical

b. Percentages

c. 44 of 1080 respondents or approximately 4% strongly agree with allowing drivers of motor vehicles to talk on a hand-held cell phone while driving.

d. 165 of the 1080 respondents or 15% of said they somewhat disagree and 741 or 69% said they strongly disagree. Thus, there does not appear to be general support for allowing drivers of motor vehicles to talk on a hand-held cell phone while driving.

11. a. Quantitative; ratio

b. Categorical; nominal

c. Categorical; ordinal

d. Quantitative; ratio

e. Categorical; ordinal. The response to this question was recorded as a numerical value from 1 to 10. While the data are numerical, they are not quantitative. The numerical values from 1 to 10 represent categories that *order* the overall rating somewhere between unacceptable and truly exceptional. The data may be ordered by response category with a higher number category indicating a higher overall rating.

While we prefer the categorical; ordinal answer above, at times statisticians may *make the assumption* that the numerical responses are equal-interval measures on a quantitative scale from 1 to 10. When this assumption is made, the data may be considered quantitative with an interval scale of measurement. In this case, additional statistical computations such as the average overall rating become helpful in summarizing the data.

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12. a. The population is all visitors coming to the state of Hawaii.

    b. Since airline flights carry the vast majority of visitors to the state, the use of questionnaires for passengers during incoming flights is a good way to reach this population. The questionnaire actually appears on the back of a mandatory plants and animals declaration form that passengers must complete during the incoming flight. A large percentage of passengers complete the visitor information questionnaire.

    c. Questions 1 and 4 provide quantitative data indicating the number of visits and the number of days in Hawaii. Questions 2 and 3 provide categorical data indicating the categories of reason for the trip and where the visitor plans to stay.

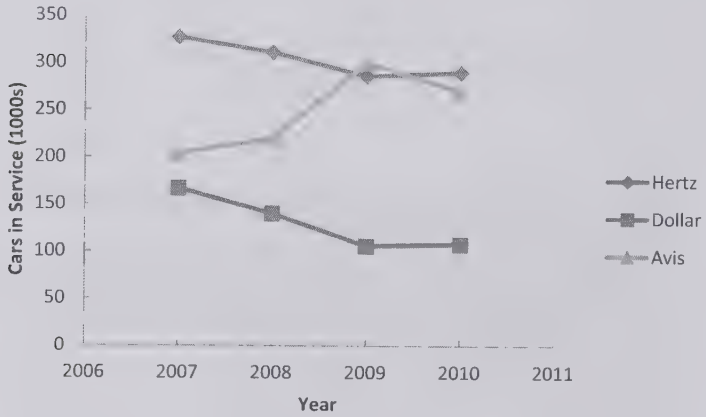
13. a. Federal spending measured in trillions of dollars

    b. Quantitative

    c. Time series

    d. Federal spending has increased over time

14. a. The graph of the time series follows:



    b. In 2007 and 2008 Hertz was the clear market share leader. In 2009 and 2010 Hertz and Avis have approximately the same market share. The market share for Dollar appears to be declining.

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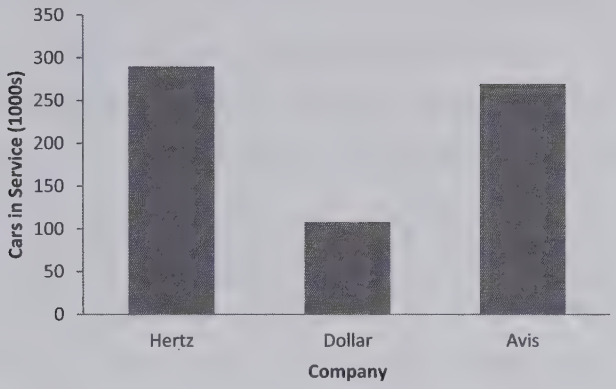
c. The bar chart for 2010 is shown below.



This chart is based on cross-sectional data.

15. a. Quantitative – number of new drugs approved

b. Time series

c. July; 1100

d. 2.9%; Yes, because most recreational boating takes place during the summer months.

e. The bar graph follows the shape of a bell curve.

16. The answer to this exercise depends upon the time series of the average price per gallon of conventional regular gasoline since April 2009. Contact the website *www.eia.doe.gov* to obtain the more recent time series data. In the first quarter of 2011, the average price per gallon was once again increasing. A continuation of the usual summer peak in gasoline prices was anticipated.

17. Internal data on salaries of other employees can be obtained from the personnel department. External data might be obtained from the Department of Labor or industry associations.

18. a. 684/1021; or approximately 67%

b. 612

c. Categorical

19. a. All subscribers of Business Week in North America at the time the survey was conducted.

b. Quantitative

c. Categorical (yes or no)

d. Crossectional - all the data relate to the same time.

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e. Using the sample results, we could infer or estimate 59% of the population of subscribers have an annual income of $75,000 or more and 50% of the population of subscribers have an American Express credit card.

20. a. 43% of managers were bullish or very bullish.

21% of managers expected health care to be the leading industry over the next 12 months.

b. We estimate the average 12-month return estimate for the population of investment managers to be 11.2%.

c. We estimate the average over the population of investment managers to be 2.5 years.

21. a. The two populations are the population of women whose mothers took the drug DES during pregnancy and the population of women whose mothers did not take the drug DES during pregnancy.

b. It was a survey.

c. 63 / 3.980 = 15.8 women out of each 1000 developed tissue abnormalities.

d. The article reported "twice" as many abnormalities in the women whose mothers had taken DES during pregnancy. Thus, a rough estimate would be 15.8/2 = 7.9 abnormalities per 1000 women whose mothers had *not* taken DES during pregnancy.

e. In many situations, disease occurrences are rare and affect only a small portion of the population. Large samples are needed to collect data on a reasonable number of cases where the disease exists.

22. a. The population consists of all customers of the chain's stores in Charlotte, North Carolina.

b. Some of the ways that could be used to collect the data are as follows:

- Customers entering or leaving the store could be surveyed
- A survey could be mailed to customers who have a shopper's club card for the stores
- Customers could be given a printed survey when they check out
- Customers could be given a coupon that asks them to complete a brief on-line survey; if they do, they will receive a 5% discount on their next shopping trip.

23. a. This finding is applicable to the population of all American adults.

b. This finding is applicable to the population of American adults that own a cellphone and/or a tablet computer.

c. They conducted a sample survey. It would be way too costly to survey all American adults or all American adults who own cellphones and/or tablet computers. As we will see later in the text, very good results can be obtained using a sample survey.

d. These results should be quite interesting to restaurant owners. It suggests that it would be worthwhile for them to have a website and to consider advertising through an internet search company, such as Google.

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24. a. This is a statistically correct descriptive statistic for the sample.

    b. An incorrect generalization since the data was not collected for the entire population.

    c. An acceptable statistical inference based on the use of the word "estimate."

    d. While this statement is true for the sample, it is not a justifiable conclusion for the entire population.

    e. This statement is not statistically supportable. While it is true for the particular sample observed, it is entirely possible and even very likely that at least some students will be outside the 65 to 90 range of grades.

25. a. There are five variables: Exchange, Ticker Symbol, Market Cap, Price/Earnings Ratio and Gross Profit Margin.

    b. Categorical variables: Exchange and Ticker Symbol

    Quantitative variables: Market Cap, Price/Earnings Ratio, Gross Profit Margin

    c. Exchange variable:

Exchange	Frequency	Percent Frequency
AMEX	5	(5/25) 20%
NYSE	3	(3/25) 12%
OTC	17	(17/25) 68%
	25	

    d. Gross Profit Margin variable:

Gross Profit Margin	Frequency
0.0 – 14.9	2
15.0 – 29.9	6
30.0 – 44.9	8
45.0 – 59.9	6
60.0 – 74.9	3

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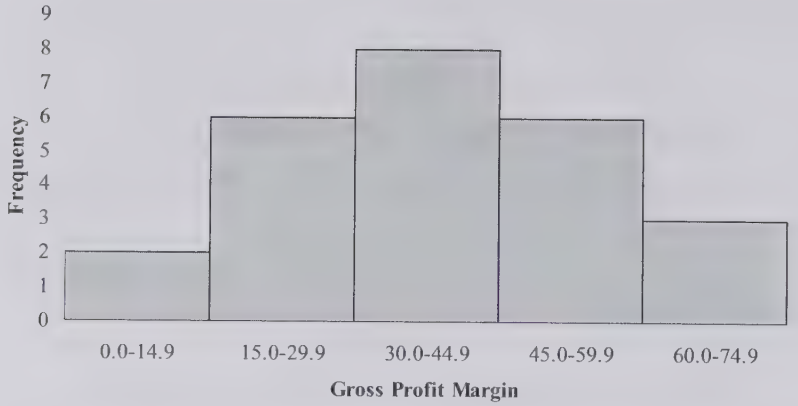

e. Sum the Price/Earnings Ratio data for all 25 companies.

Sum = 505.4

Average Price/Earnings Ratio = Sum/25 = 505.4/25 = 20.2

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Chapter 2
Descriptive Statistics: Tabular and Graphical Presentations

Learning Objectives

1. Learn how to construct and interpret summarization procedures for qualitative data such as: frequency and relative frequency distributions, bar graphs and pie charts.

2. Learn how to construct and interpret tabular summarization procedures for quantitative data such as: frequency and relative frequency distributions, cumulative frequency and cumulative relative frequency distributions.

3. Learn how to construct a dot plot, a histogram, and an ogive as graphical summaries of quantitative data.

4. Learn how the shape of a data distribution is revealed by a histogram. Learn how to recognize when a data distribution is negatively skewed, symmetric, and positively skewed.

5. Be able to use and interpret the exploratory data analysis technique of a stem-and-leaf display.

6. Learn how to construct and interpret cross tabulations and scatter diagrams of bivariate data.

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Solutions:

1.

Class	Frequency	Relative Frequency
A	60	60/120 = 0.50
B	24	24/120 = 0.20
C	36	36/120 = 0.30
	120	1.00

2. a. 1 - (.22 + .18 + .40) = .20

   b. .20(200) = 40

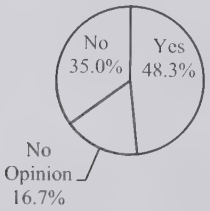
   c/d.

Class	Frequency	Percent Frequency
A	.22(200) = 44	22
B	.18(200) = 36	18
C	.40(200) = 80	40
D	.20(200) = 40	20
Total	200	100

3. a. 360° x 58/120 = 174°

   b. 360° x 42/120 = 126°

   c.



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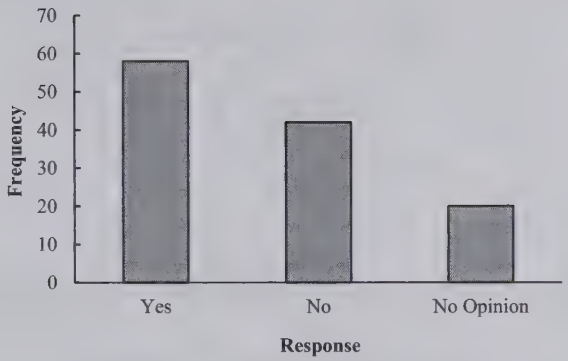
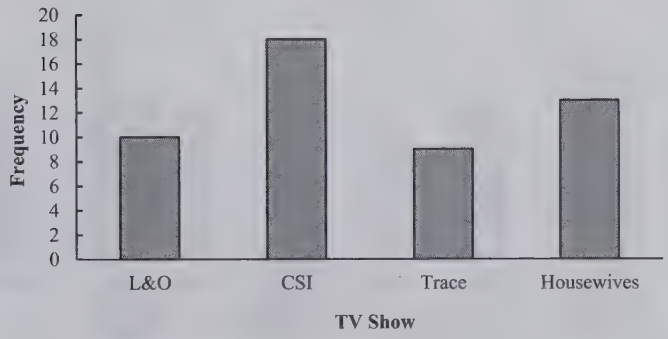
d.



4. a. Categorical

b.

TV Show	Frequency	Percent Frequency
Law & Order	10	20%
CSI	18	36%
Without a Trace	9	18%
Desperate Housewives	13	26%
Total:	50	100%



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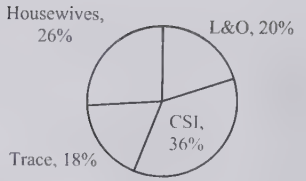
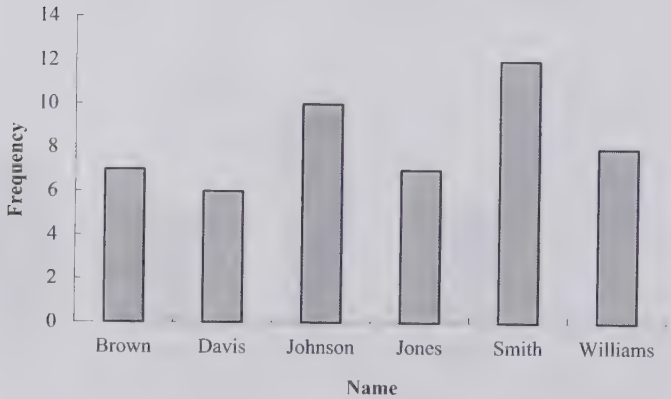


d. CSI had the largest viewing audience. Desperate Housewives was in second place.

5. a.

Name	Frequency	Relative Frequency	Percent Frequency
Brown	7	.14	14%
Davis	6	.12	12%
Johnson	10	.20	20%
Jones	7	.14	14%
Smith	12	.24	24%
Williams	8	.16	16%
	50	1.00	

b.



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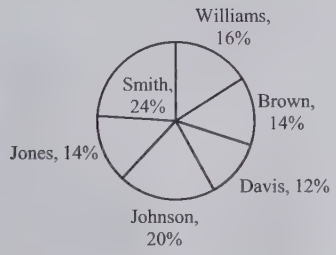
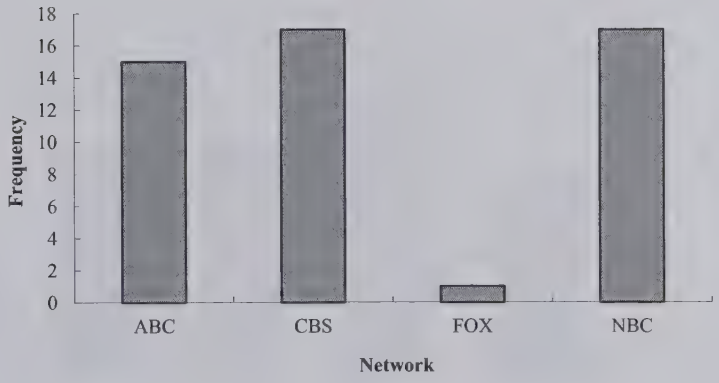
c. Brown .14 x 360 = 50.4°
Davis .12 x 360 = 43.2°
Johnson .20 x 360 = 72.0°
Jones .14 x 360 = 50.4°
Smith .24 x 360 = 86.4°
Williams .16 x 360 = 57.6°



d. Most common: Smith, Johnson and Williams

6. a.

Network	Frequency	Percent Frequency
ABC	15	30%
CBS	17	34%
FOX	1	2%
NBC	17	34%
Total	50	100%



b. CBS and NBC are tied, each with 17 of the top rated television shows. ABC is a close third with 15. The fact that the three networks are so close is surprising. FOX, the newest television network, does not have the history to compete with the other three networks in term of the top rated shows in television history.

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7. a.

Rating	Frequency	Percent Frequency
Excellent	20	40
Very Good	23	46
Good	4	8
Fair	1	2
Poor	2	4
	50	100

Management should be very pleased with the survey results. 40% + 46% = 86% of the ratings are very good to excellent. 94% of the ratings are good or better. This does not look to be a Delta flight where significant changes are needed to improve the overall customer satisfaction ratings.

b. While the overall ratings look fine, note that one customer (2%) rated the overall experience with the flight as Fair and two customers (4%) rated the overall experience with the flight as Poor. It might be insightful for the manager to review explanations from these customers as to how the flight failed to meet expectations. Perhaps, it was an experience with other passengers that Delta could do little to correct or perhaps it was an isolated incident that Delta could take steps to correct in the future.

8. a.

Position	Frequency	Relative Frequency
Pitcher	17	0.309
Catcher	4	0.073
1st Base	5	0.091
2nd Base	4	0.073
3rd Base	2	0.036
Shortstop	5	0.091
Left Field	6	0.109
Center Field	5	0.091
Right Field	7	0.127
	55	1.000

b. Pitchers (Almost 31%)

c. 3rd Base (3 - 4%)

d. Right Field (Almost 13%)

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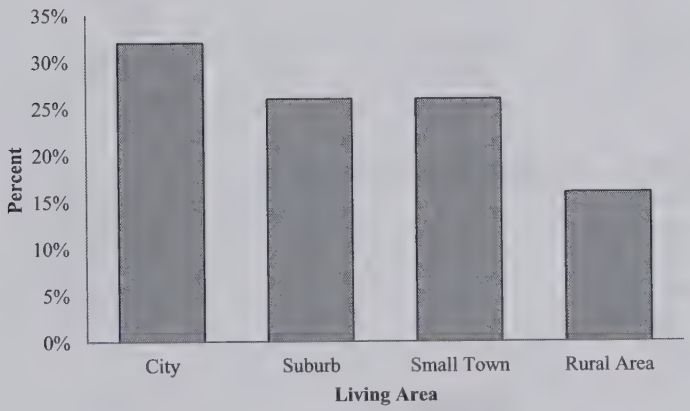
e. Infielders (16 or 29.1%) to Outfielders (18 or 32.7%)

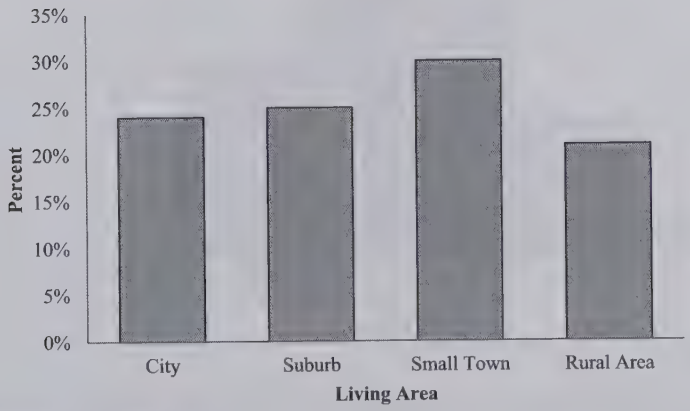
9. a.

Living Area	Live Now	Ideal Community
City	32%	24%
Suburb	26%	25%
Small Town	26%	30%
Rural Area	16%	21%
Total	100%	100%

b. Where do you live now?



What do you consider the ideal community?



c. Most adults are now living in a city (32%).

d. Most adults consider the ideal community a small town (30%).

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e. Percent changes by living area: City -8%, Suburb -1%, Small Town +4%, and Rural Area +5%.

Suburb living is steady, but the trend would be that living in the city would decline while living in small towns and rural areas would increase.

10. a.

Rating	Frequency
Excellent	20
Good	101
Fair	528
Bad	244
Terrible	122
Total	1015

b.

Rating	Percent Frequency
Excellent	2
Good	10
Fair	52
Bad	24
Terrible	12
Total	100

c.

d. 24% + 12% = 36% of adults in the United Sates think the Federal Bank is doing a bad or a terrible job in handling the credit problems. Only 10% + 2% = 12% think the Federal Bank is doing a good or excellent job.

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e. 40% + 10% = 50% of adults in Spain think the European Central Bank is doing a bad or terrible job in handling the credit problems. Only 4% of adults in Spain think the European Central Bank is doing a good or excellent job.

Both countries show pessimism and relatively low confidence in how the banks are handling the credit problems in the financial markets. But in comparing the two countries, adults in Spain show more concern and more pessimism about the bank's ability compared to adults in the United States.

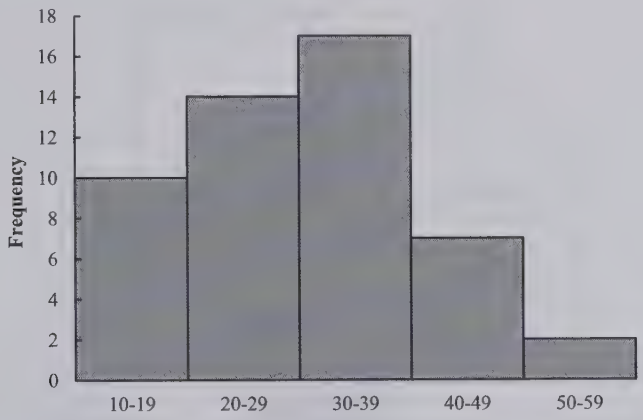
11.

Class	Frequency	Relative Frequency	Percent Frequency
12-14	2	0.050	5.0
15-17	8	0.200	20.0
18-20	11	0.275	27.5
21-23	10	0.250	25.0
24-26	9	0.225	22.5
Total	40	1.000	100.0

12.

Class	Cumulative Frequency	Cumulative Relative Frequency
less than or equal to 19	10	.20
less than or equal to 29	24	.48
less than or equal to 39	41	.82
less than or equal to 49	48	.96
less than or equal to 59	50	1.00

13.



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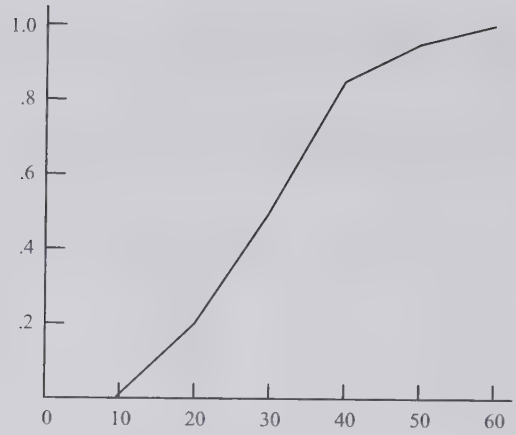

14. a.

b/c.

Class	Frequency	Percent Frequency
6.0 - 7.9	4	20
8.0 - 9.9	2	10
10.0 - 11.9	8	40
12.0 - 13.9	3	15
14.0 - 15.9	3	15
	20	100

15. a/b.

Waiting Time	Frequency	Relative Frequency
0 - 4	4	0.20
5 - 9	8	0.40
10 - 14	5	0.25
15 - 19	2	0.10
20 - 24	1	0.05
Totals	20	1.00

c/d.

Waiting Time	Cumulative Frequency	Cumulative Relative Frequency
Less than or equal to 4	4	0.20
Less than or equal to 9	12	0.60
Less than or equal to 14	17	0.85
Less than or equal to 19	19	0.95
Less than or equal to 24	20	1.00

e. 12/20 = 0.60

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16. a.

Salary	Frequency
150-159	1
160-169	3
170-179	7
180-189	5
190-199	1
200-209	2
210-219	1
Total	20

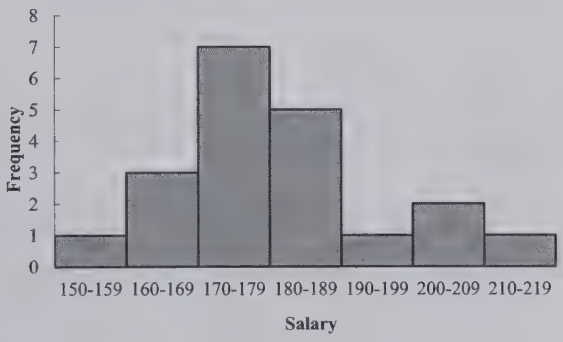
b.

Salary	Percent Frequency
150-159	5
160-169	15
170-179	35
180-189	25
190-199	5
200-209	10
210-219	5
Total	100

c.

Salary	Cumulative Percent Frequency
Less than or equal to 159	5
Less than or equal to 169	20
Less than or equal to 179	55
Less than or equal to 189	80
Less than or equal to 199	85
Less than or equal to 209	95
Less than or equal to 219	100
Total	100

d.



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e. There is skewness to the right.

f. (3/20)(100) = 15%

17. a. The highest price stock is for IBM with a price of $109 per share.

The lowest price stock is for Alcoa with a price of $11 per share.

b. A class size of 10 results in 10 classes.

Price per Share	Frequency
$10-19	5
$20-29	10
$30-39	3
$40-49	2
$50-59	6
$60-69	2
$70-79	1
$80-89	0
$90-99	0
$100-109	1

c

The general shape of the distribution is skewed to the right. Half of the companies (15) have a price per share less than $30. A mid-priced stock appears to be in the $30 to $49 range, while the most frequently priced stock is in the $20 to $29 range.

Five stocks are less than $20 per share (Alcoa, Bank of America, General Electric, Intel and Pfizer). Four stocks are $60 or more per share (3M, Chevron, ExxonMobil and IBM).

d. A variety of comparisons are possible depending upon when the study is done.

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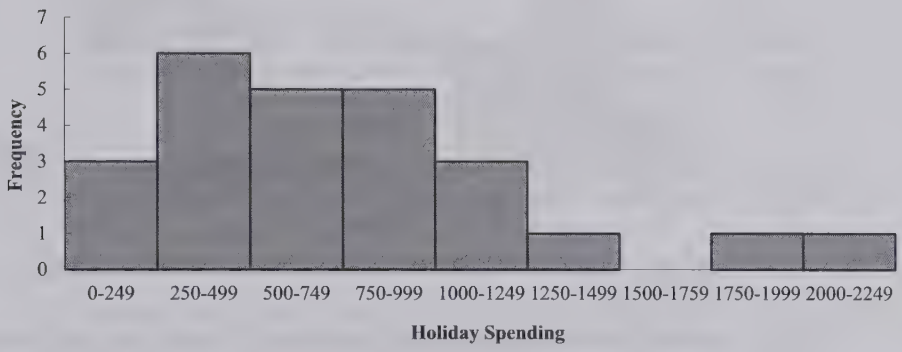
18. a. The lowest holiday spending is $180; the highest $2050.

b.

Spending	Frequency	Percent
0-249	3	12
250-499	6	24
500-749	5	20
750-999	5	20
1000-1249	3	12
1250-1499	1	4
1500-1759	0	0
1750-1999	1	4
2000-2249	1	4
Total	25	100

c. The distribution shows a positive skewness.



d. The holiday spending ranges from $0 to less than $2250. The majority of the spending is between $250 and $1000 with 16 of the 25 customers, 64%, in this range. The middle or average spending is around $750 per customer. The distribution has a positive skewness with two consumers above $1750. One consumer is above $2000.

19. a/b/c/d.

Revenue	Frequency	Relative Frequency	Cumulative Frequency	Cumulative Relative Frequency
0-49	6	.12	6	.12
50-99	29	.58	35	.70
100-149	11	.22	46	.92
150-199	0	.00	46	.92
200-249	1	.02	47	.94
250-299	1	.02	48	.96
300-349	0	.00	48	.96
350-399	0	.00	48	.96
400-449	2	.04	50	1.00
Total	50	1.00		

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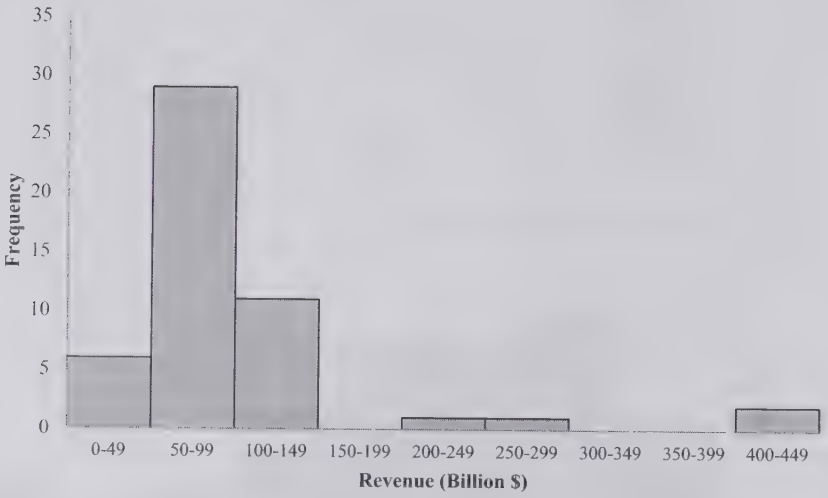
e. The majority of the large corporations (40) have revenues in the $50 billion to $149 billion range. Only 4 corporations have revenues of over $200 billion and only 2 corporations have revenues over $400 billion. .70, or 70%, of the corporations have revenues under $100 billion. .30, or 30%, of the corporations have revenues of $100 billion or more.

f.



The histogram shows the distribution is skewed to the right with four corporations in the $200 to $449 billion range.

g. Exxon-Mobil is America's largest corporation with an annual revenue of $443 billion. Walmart is the second largest corporation with an annual revenue of $406 billion. All other corporations have annual revenues less than $300 billion. Most (92%) have annual revenues less than $150 billion.

20. a.

Off-Course Income ($1000s)	Frequency	Percent Frequency
0-4,999	30	60
5,000-9,999	9	18
10,000-14,999	4	8
15,000-19,999	0	0
20,000-24,999	3	6
25,000-29,999	2	4
30,000-34,999	0	0
35000-39,999	0	0
40,000-44,999	1	2
45,000-49,999	0	0
Over 50,000	1	2
Total	50	100

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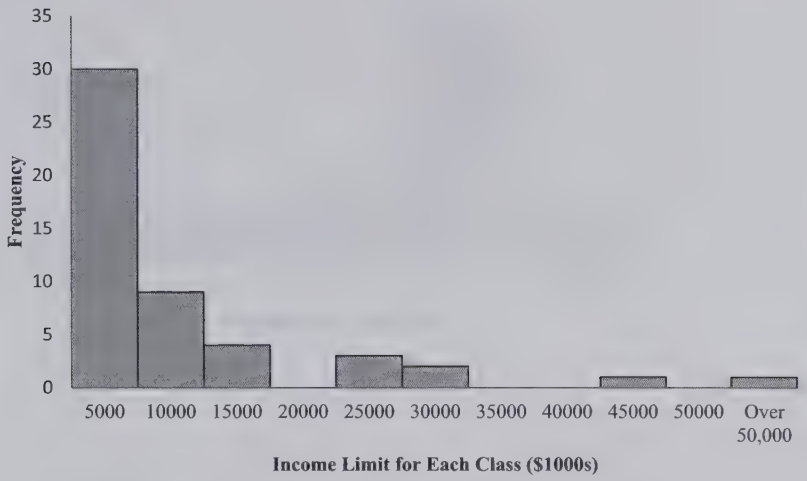
b. Histogram of Off-Course Income



Note: The first class is labeled 5000 and provides the golfers who had an off-course income in the range 0 to 4999 or *less than* 5000. These were the golfers with less than $5 million in off-course income.

c. Off-course income is skewed to the right. Only Tiger Woods earns over $50 million.

d. Considering the top 50 golfers, the majority (60%) earn less than $5 million in off-course income per year. 60% + 18% = 78% earn less than $10 million. Five golfers (10%) earn between $20 million and $30 million. Tiger Woods with $99.8 million and Phil Mickelson with $40.2 million in off-course income are clearly the leaders in this income category.

21. a/b.

Computer Usage (Hours)	Frequency	Relative Frequency
0.0 - 2.9	5	0.10
3.0 - 5.9	28	0.56
6.0 - 8.9	8	0.16
9.0 - 11.9	6	0.12
12.0 - 14.9	3	0.06
Total	50	1.00

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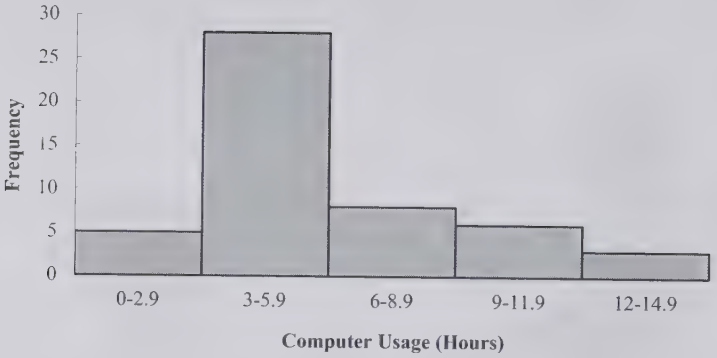
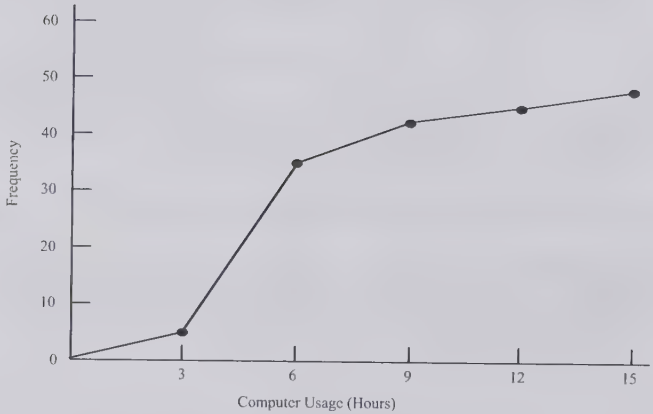
c.



d.



e. The majority of the computer users are in the 3 to 6 hour range. Usage is somewhat skewed toward the right with 3 users in the 12 to 14.9 hour range.

22.

5	7 8
6	4 5 8
7	0 2 2 5 5 6 8
8	0 2 3 5

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23. Leaf Unit = .1

6	3
7	5 5 7
8	1 3 4 8
9	3 6
10	0 4 5
11	3

24. Leaf Unit = 10

11	6
12	0 2
13	0 6 7
14	2 2 7
15	5
16	0 2 8
17	0 2 3

25.

9	8 9
10	2 4 6 6
11	4 5 7 8 8 9
12	2 4 5 7
13	1 2
14	4
15	1

26. Median Pay

6	6 7 7
7	2 4 6 7 7 8 9
8	0 0 1 3 7
9	9
10	0 6
11	0
12	1

The median pay for these careers is generally in the $70 and $80 thousands. Only four careers have a median pay of $100 thousand or more. The highest median pay is $121 thousand for a finance director.

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Top Pay

10	0 6 9
11	1 6 9
12	2 5 6
13	0 5 8 8
14	0 6
15	2 5 7
16	
17	
18	
19	
20	
21	4
22	1

The most frequent top pay is in the $130 thousand range. However, the top pay is rather evenly distributed between $100 and $160 thousand. Two unusually high top pay values occur at $214 thousand for a finance director and $221 thousand for an investment banker. Also, note that the top pay has more variability than the median pay.

27. a.

7	5 9
8	3 6
9	5 6 8
10	0 4 4
11	1 5
12	
13	7
14	5 5

b. Observations such as the following can be made using the stem-and-leaf display.

- The daily rate varies from $75 to $145
- Typical mid-priced daily rates are $95 to $115 with the average daily rate around $100.
- A daily rate in excess of $115 should be considered relatively high. High daily rates of $137 and $145 were found at three ski resorts.

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28. a.

2	1 4
2	6 7
3	0 1 1 1 2 3
3	5 6 7 7
4	0 0 3 3 3 3 3 4 4
4	6 6 7 9
5	0 0 0 2 2
5	5 6 7 9
6	1 4
6	6
7	2

b. Most frequent age group: 40-44 with 9 runners

c. 43 was the most frequent age with 5 runners

d. 4/40 = 10% of the runners were "20-something." With only 10% of the registrants "20-something," the article pointed out that surprisingly few registrants were in this age group. One suggested reason was that "20-somethings" don't have the time to train for a 13.1 mile race. For "20-somethings," college, starting careers, and starting families may take priority over training for long distance races.

29. a.

		y		
		1	2	Total
	A	5	0	5
x	B	11	2	13
	C	2	10	12
	Total	18	12	30

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b.

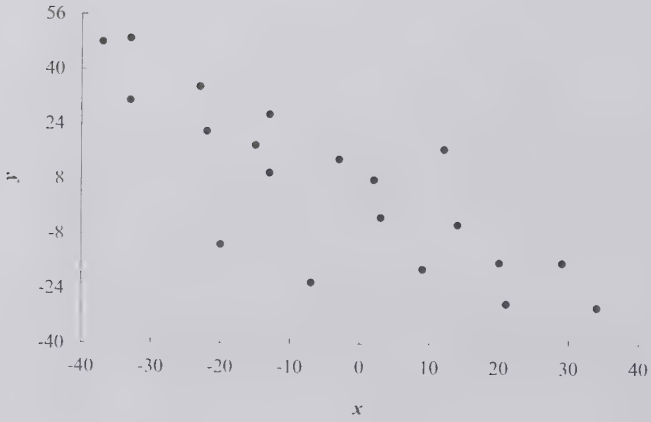
		y = 1	y = 2	Total
x	A	100.0	0.0	100.0
	B	84.6	15.4	100.0
	C	16.7	83.3	100.0

c.

		y = 1	y = 2
x	A	27.8	0.0
	B	61.1	16.7
	C	11.1	83.3
	Total	100.0	100.0

d. Category A values for x are always associated with category 1 values for y. Category B values for x are usually associated with category 1 values for y. Category C values for x are usually associated with category 2 values for y.

30. a.



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b. There is a negative relationship between x and y; y decreases as x increases.

31. a. Row Percentages:

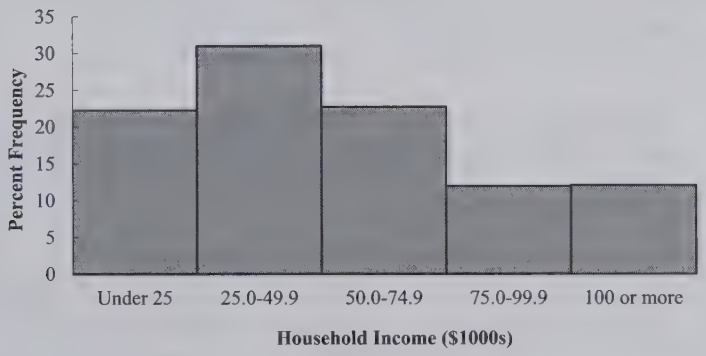
Education Level	Household Income ($1000s)					Total
	Under 25	25.0-49.9	50.0-74.9	75.0-99.9	100 or More	
Not H.S. Graduate	42.23	34.73	13.94	5.41	3.68	100.00
H.S. Graduate	22.25	31.00	22.75	11.93	12.07	100.00
Some College	13.99	26.20	23.31	16.20	20.30	100.00
Bachelor's Degree	6.42	15.19	20.66	18.72	39.02	100.00
Beyond Bach. Deg.	3.71	10.60	16.29	15.87	53.54	100.00
Total	17.77	25.08	20.64	13.90	22.62	100.00

There are six percent frequency distributions in this table with row percentages. The first five give the percent frequency distribution of income for each educational level. The total row provides an overall percent frequency distribution for household income.

The second row, labeled H.S. Graduate, is the percent frequency distribution for households headed by high school graduates. The fourth row, labeled Bachelor's Degree, is the percent frequency distribution for households headed by bachelor's degree recipients.

b. The percentage of households headed by high school graduates earning $75,000 or more is 11.93% + 12.07 = 24.00%. The percent of households headed by bachelor's degree recipients earning $75,000 or more is 18.72% + 39.02% = 57.74%.

c. The percent frequency histogram for high school graduates.



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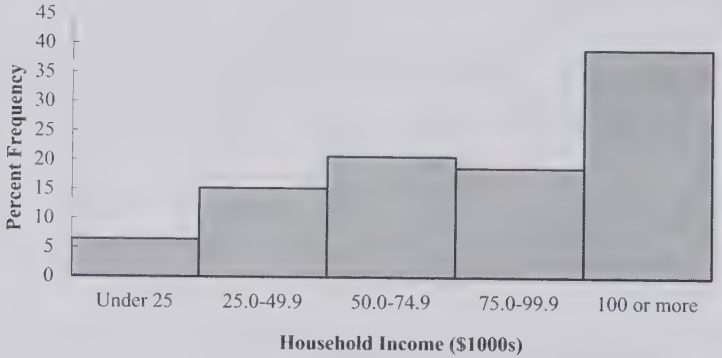
The percent frequency distribution for college graduates with a bachelor's degree.



The histograms show that households headed by a college graduate with a bachelor's degree earn more than households headed by a high school graduate. Yes, there is a positive relationship between education level and income.

32. a. Column Percentages:

	Household Income ($1000s)					
Education Level	Under 25	25.0-49.9	50.0-74.9	75.0-99.9	100 or More	Total
Not H.S. Graduate	32.10	18.71	9.13	5.26	2.20	13.51
H.S. Graduate	37.52	37.05	33.04	25.73	16.00	29.97
Some College	21.42	28.44	30.74	31.71	24.43	27.21
Bachelor's Degree	6.75	11.33	18.72	25.19	32.26	18.70
Beyond Bach. Deg.	2.21	4.48	8.37	12.11	25.11	10.61
Total	100.00	100.00	100.00	100.00	100.00	100.00

There are six percent frequency distributions in this table of column percentages. The first five columns give the percent frequency distributions for each income level. The percent frequency distribution in the "Total" column gives the overall percent frequency distributions for educational level. From that percent frequency distribution we see that 13.51% of the heads of households did not graduate from high school.

b. The column percentages show that 25.11% of households earning $100,000 or more were headed by persons having schooling beyond a bachelor's degree. The row percentages show that 53.54% of the households headed by persons with schooling beyond a bachelor's degree earned $100,000 or more. These percentages are different because they came from different percent frequency distributions and provide different kinds of information.

c. Compare the "under 25" percent frequency distributions to the "Total" percent frequency distributions. We see that for this low income level the percentage with lower levels of education is higher than for the overall population and the percentage with higher levels of education is lower than for the overall population.

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Compare the "100 or more" percent frequency distribution to "Total" percent frequency distribution. We see that for this high income level the percentage with lower levels of education is lower than for the overall population and the percentage with higher levels of education is higher than for the overall population.

From the comparisons it is clear that there is a positive relationship between household incomes and the education level of the head of the household.

33. a. The crosstabulation of condition of the greens by gender is below.

	Green Condition		
Gender	Too Fast	Fine	Total
Male	35	65	100
Female	40	60	100
Total	75	125	200

The female golfers have the highest percentage saying the greens are too fast: 40/100 = 40%. Male golfers have 35/100 = 35% saying the greens are too fast.

b. Among low handicap golfers, 1/10 = 10% of the women think the greens are too fast and 10/50 = 20% of the men think the greens are too fast. So, for the low handicappers, the men show a higher percentage who think the greens are too fast.

c. Among the higher handicap golfers, 39/51 = 43% of the woman think the greens are too fast and 25/50 = 50% of the men think the greens are too fast. So, for the higher handicap golfers, the men show a higher percentage who think the greens are too fast.

d. This is an example of Simpson's Paradox. At each handicap level a smaller percentage of the women think the greens are too fast. But, when the crosstabulations are aggregated, the result is reversed and we find a higher percentage of women who think the greens are too fast.

The hidden variable explaining the reversal is handicap level. Fewer people with low handicaps think the greens are too fast, and there are more men with low handicaps than women.

34. a.

	5 Year Average Return						
Fund Type	0-9.99	10-19.99	20-29.99	30-39.99	40-49.99	50-59.99	Total
DE	1	25	1	0	0	0	27
FI	9	1	0	0	0	0	10
IE	0	2	3	2	0	1	8
Total	10	28	4	2	0	1	45

b.

5 Year Average Return	Frequency
0-9.99	10
10-19.99	28
20-29.99	4
30-39.99	2
40-49.99	0
50-59.99	1
Total	45

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c.

Fund Type	Frequency
DE	27
FI	10
IE	8
Total	45

d. The right margin shows the frequency distribution for the fund type variable and the bottom margin shows the frequency distribution for the 5 year average return variable.

e. Higher returns are associated with International Equity funds and lower returns are associated with Fixed Income funds.

35. a.

	Expense Ratio (%)						
Fund Type	0-0.24	0.25-0.49	0.50-0.74	0.75-0.99	1.00-1.24	1.25-1.49	Total
DE	1	1	3	5	10	7	27
FI	2	4	3	0	0	1	10
IE	0	0	1	2	4	1	8
Total	3	5	7	7	14	9	45

b.

Expense Ratio (%)	Frequency	Percent
0-0.24	3	6.7
0.25-0.49	5	11.1
0.50-0.74	7	15.6
0.75-0.99	7	15.6
1.00-1.24	14	31.0
1.25-1.49	9	20.0
Total	45	100

c. Higher expense ratios are associated with Domestic Equity funds and lower expense ratios are associated with fixed income funds.

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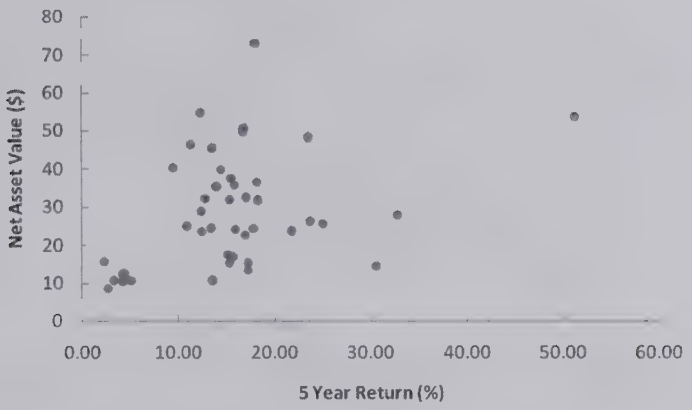
36. a. The scatter diagram is shown below:



b. There is some indication that higher 5-year returns are associated with higher net asset values.

37. a.

	Highway MPG						
Size	15-19	20-24	25-29	30-34	35-39	40-44	Total
Compact	3	4	17	22	5	5	56
Midsize	3	4	30	20	9	3	69
Large	2	10	7	3	2	0	24
Total	8	18	54	45	16	8	149

b. For these data, the larger cars had the lower fuel efficiencies.

c.

	City MPG							
Drive	10-14	15-19	20-24	25-29	30-34	35-39	40-44	Total
A	7	18	3	0	0	0	0	28
F	0	17	49	19	2	0	3	90
R	10	20	0	1	0	0	0	31
Total	17	55	52	20	2	0	3	149

d. Higher fuel efficiencies are associated with front wheel drive cars. Rear wheel drive cars had somewhat lower fuel efficiencies than four wheel drive cars.

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e.

	City MPG							
Drive	10-14	15-19	20-24	25-29	30-34	35-39	40-44	Total
P	17	24	12	3	0	0	0	56
R	0	31	40	17	2	0	3	93
Total	17	55	52	20	2	0	3	149

f. Higher fuel efficiencies are associated with cars that use regular fuel.

38. a.

	Highway MPG						
Displacement	15-19	20-24	25-29	30-34	35-39	40-44	Total
1-2	0	0	0	0	7	7	14
2-3	0	0	14	41	9	1	65
3-4	0	1	34	4	0	0	39
4-5	0	10	4	0	0	0	14
5-6	0	3	2	0	0	0	5
6-7	8	4	0	0	0	0	12
Total	8	18	54	45	16	8	149

b. Higher fuel efficiencies are associated with smaller displacement engines and lower fuel efficiencies are associated with larger displacement engines.

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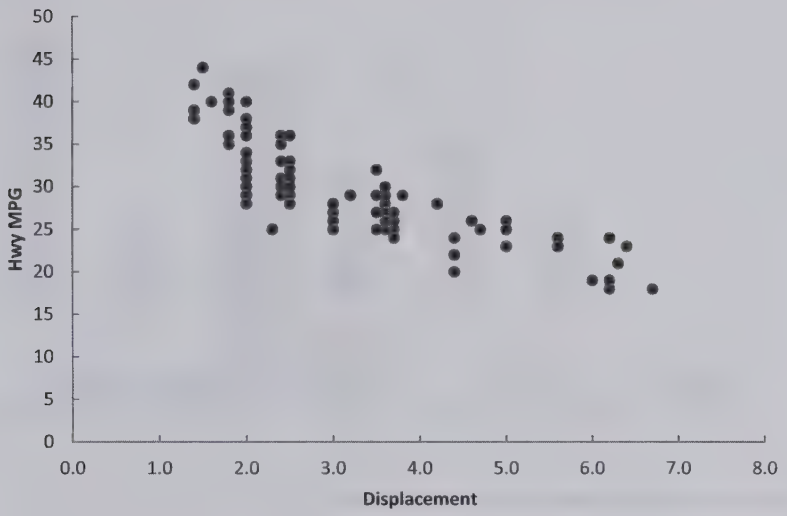
c. The scatter diagram is shown below:



d. The scatter diagram shows that lower fuel efficiencies are associated with larger displacement engines.

e. It is easier to see the relationship between the two variables using the scatter diagram.

39. a.

Major	Frequency	Percent Frequency
Arts/Humanities	7	10.9
Business Administration	13	20.3
Engineering	11	17.2
Professional	6	9.4
Social Science	5	7.8
Other	22	34.4
Total	64	100.0

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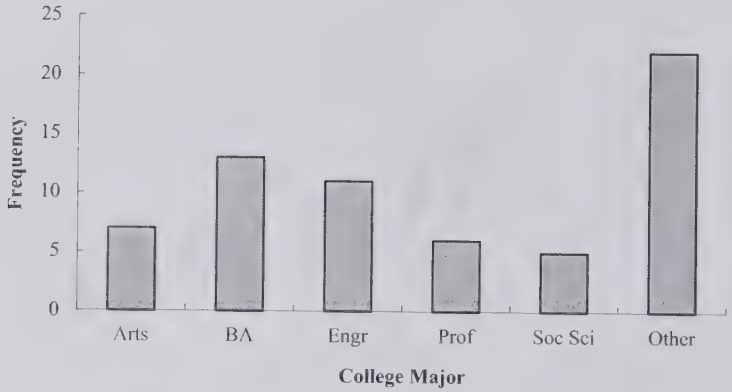
b.



c. 34.4% select another major. So 100% - 34.4% = 65.6% select one of the five most popular majors.

d. Business Administration is the most popular major selected by incoming freshmen, 20.3%

40. a. Frequency distribution and percent frequency distribution of sales by division.

Division	Frequency	Percent
Buick	10	5
Cadillac	10	5
Chevrolet	122	61
GMC	24	12
Hummer	2	1
Pontiac	18	9
Saab	2	1
Saturn	12	6
Total	200	100

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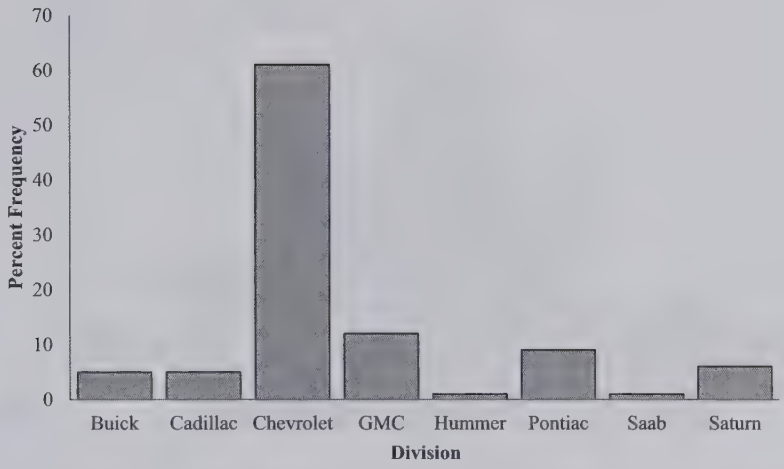
b.



c. Chevrolet is General Motors leading division with 61% of the vehicles sold. This is considered General Motors most important division.

d. Based on the percentages shown, the Hummer division at 1% and Saab division at 1% would be good candidates for General Motors to consider discontinuing. Chevrolet at 61% and GMC at 12% account for 73% of the total vehicles sold. General Motors would be almost certain to maintain these two divisions.

Pontiac remains a solid contributor with 9% of vehicles sold. At the time it was doubtful than General Motors would be able to maintain all three of the other divisions. Some elimination or merging of divisions was anticipated for Saturn 6%, Buick 5%, and Cadillac 5%.

41. a.

Yield%	Frequency	Percent Frequency
0.0-0.9	4	13.3
1.0-1.9	2	6.7
2.0-2.9	6	20.0
3.0-3.9	10	33.3
4.0-4.9	3	10.0
5.0-5.9	2	6.7
6.0-6.9	2	6.7
7.0-7.9	0	0.0
8.0-8.9	0	0.0
9.0-9.9	1	3.3
Total	30	100.0

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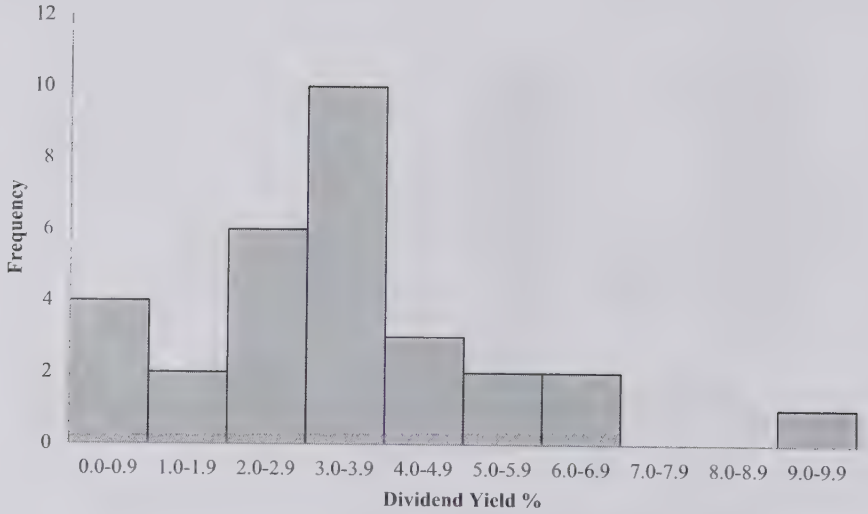
b.



c. The distribution is skewed to the right.

d. Dividend yield ranges from 0% to over 9%. The most frequent range is 3.0% to 3.9%. Average dividend yields looks to be between 3% and 4%. Over 50% of the companies (16) pay from 2.0 % to 3.9%. Five companies (AT&T, DuPont, General Electric, Merck, and Verizon) pay 5.0% or more. Four companies (Bank of America, Cisco Systems, Hewlett-Packard, and J.P. Morgan Chase) pay less than 1%.

e. General Electric had an unusually high dividend yield of 9.2%. 500 shares at $14 per share is an investment of 500($14) = $7,000. A 9.2% dividend yield provides .092(7,000) = $644 of dividend income per year.

42. a.

Class	Frequency
800-999	1
1000-1199	3
1200-1399	6
1400-1599	10
1600-1799	7
1800-1999	2
2000-2199	1
Total	30

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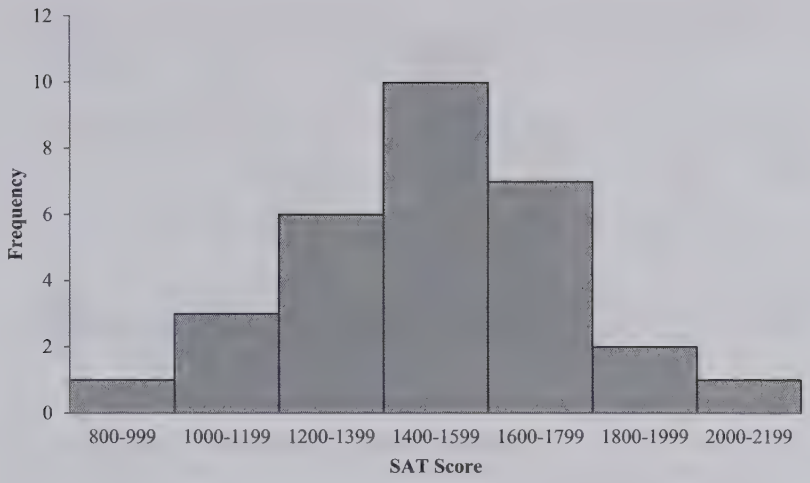

b. The distribution if nearly symmetrical. It could be approximated by a bell-shaped curve.

c. 10 of 30 or 33% of the scores are between 1400 and 1599. The average SAT score looks to be a little over 1500. Scores below 800 or above 2200 are unusual.

43. a.

State	Frequency
Arizona	2
California	11
Florida	15
Georgia	2
Louisiana	8
Michigan	2
Minnesota	1
Texas	2
Total	43

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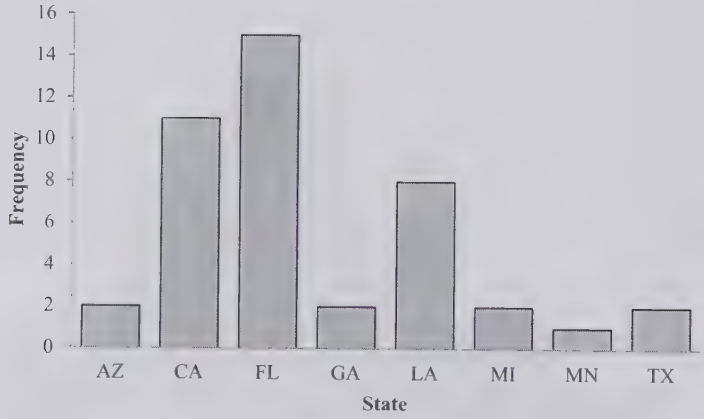
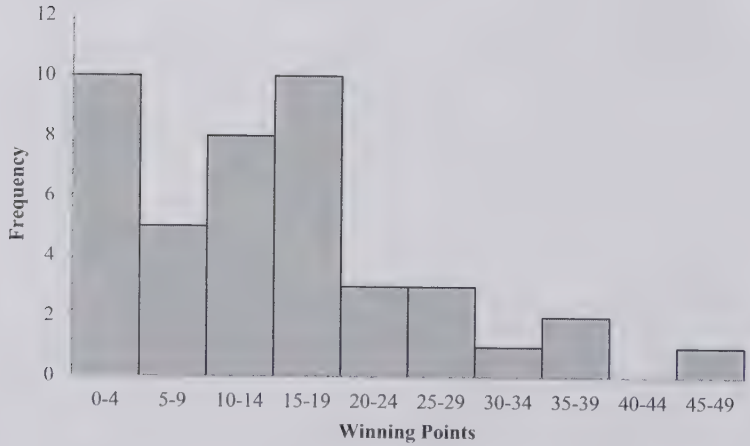


b. A total of 15 + 11 = 26, or 26/43(100) = 60% of the Super Bowls were played in Florida or California. Only 3 Super Bowls, or 3/43(100) = 7%, have been played in the cold-weather states of Michigan and Minnesota.

c.

0	1 3 3 3 3 3 4 4 4 4
0	5 7 7 7 9
1	0 0 0 1 2 2 3 4
1	5 6 7 7 7 7 8 9 9 9
2	1 2 3
2	5 7 7
3	2
3	5 6
4	
4	5



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d. The most frequent winning points have been 0 to 4 points and 15 to 19 points. Both occurred in 10 Super Bowls. There were 10 close games with a margin of victory less than 5 points, 10/43(100) = 23% of the Super Bowls. There have also been 10 games, 23%, with a margin of victory more than 20 points.

e. The closest games was the 25th Super Bowl with a 1 point margin. It was played in Florida. The largest margin of victory occurred one year earlier in the 24th Super Bowl. It had a 45 point margin and was played in Louisiana. More detailed information not available from the text information.

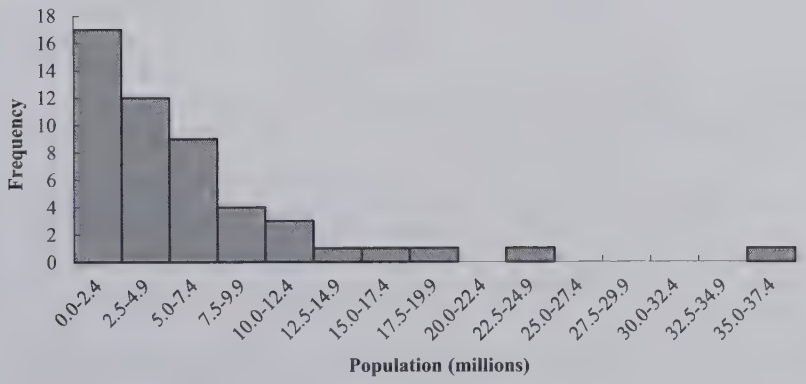
25th Super Bowl: 1991 New York Giants 20 Buffalo Bills 19, Tampa Stadium, Tampa, FL
24th Super Bowl: 1990 San Francisco 49ers 55 Denver Broncos 10, Superdome, New Orleans, LA

Note: The data set Super Bowl contains a list of the teams and the final scores of the 43 Super Bowls. This data set can be used in Chapter 2 and Chapter 3 to provide interesting data summaries about the points scored by the winning team and the points scored by the losing team in the Super Bowl. For example, using the median scores, the median Super Bowl score was 28 to 13.

44. a.

Population	Frequency	Percent Frequency
0.0-2.4	17	34
2.5-4.9	12	24
5.0-7.4	9	18
7.5-9.9	4	8
10.0-12.4	3	6
12.5-14.9	1	2
15.0-17.4	1	2
17.5-19.9	1	2
20.0-22.4	0	0
22.5-24.9	1	2
25.0-27.4	0	0
27.5-29.9	0	0
30.0-32.4	0	0
32.5-34.9	0	0
35.0-37.4	1	2
Total	50	100

b.



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c. High positive skewness.

d. 17 states (34%) have a population less than 2.5 million. Over half of the states have population less than 5 million (29 states – 58%). Only eight states have a population greater than 10 million (California, Florida, Illinois, Michigan, New York, Ohio, Pennsylvania and Texas). The largest state is California (35.9 million) and the smallest state is Wyoming (500 thousand).

45. a.

1	7 7 8
2	1
3	4
4	
5	
6	
7	2 7
8	6
9	
10	
11	6
12	7

b. Smallest roughly \$3 billion or less; medium \$7-\$8 billion; largest \$11-\$12 billion.

c. CVS (\$12,700) and Walgreens (\$11,660)

46. a& b.

High Temperature		Low Temperature	
1		1	1
2		2	1 2 6 7 9
3	0	3	1 5 6 8 9
4	1 2 2 5	4	0 3 3 6 7
5	2 4 5	5	0 0 4
6	0 0 0 1 2 2 5 6 8	6	5
7	0 7	7	
8	4	8	

c. The most frequent range for temperature was in the 60s (9 of 20). Only one low temperature was above 54. High temperatures were mostly 41 to 68, while low temperatures were mostly 21 to 47.

Low was 11; High was 84.

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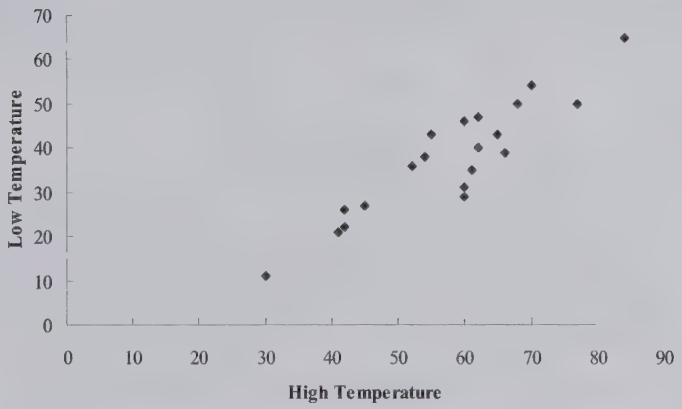
d.

High Temp	Frequency	Low Temp	Frequency
10-19	0	10-19	1
20-29	0	20-29	5
30-39	1	30-39	5
40-49	4	40-49	5
50-59	3	50-59	3
60-69	9	60-69	1
70-79	2	70-79	0
80-89	1	80-89	0
Total	20	Total	20

47. a.



b. There is a positive relationship between high temperature and low temperature for these cities. As one goes up so does the other.

48. a.

Level of Support	Percent Frequency
Strongly favor	1617/5372 = 30.10
Favor more than oppose	1871/5372 = 34.83
Oppose more than favor	1135/5372 = 21.13
Strongly oppose	749/5372 = 13.94
Total	100.00

The results show support for a higher tax. Note that 30.10% + 34.83% = 64.93% of the respondents said they strongly favor or favor more than oppose a higher tax on higher carbon emission cars.

b.

Country	Percent Frequency
Great Britain	1087/5372 = 20.2
Italy	1045/5372 = 19.5
Spain	1109/5372 = 20.6
Germany	1111/5372 = 20.7
United States	1020/5372 = 19.0
Total	100.0

The poll had an approximately equal representation of the five countries with roughly 20% of the poll respondents coming from each country.

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c. Converting the entries in the crosstabulation into column percentages provides the following results:

	Country				
Support	Great Britain	Italy	Spain	Germany	United States
Strongly favor	31.00	31.96	45.99	19.98	20.98
Favor more than oppose	34.04	39.04	32.01	36.99	32.06
Oppose more than favor	23.00	17.99	13.98	24.03	26.96
Strongly oppose	11.96	11.01	8.03	18.99	20.00
Total	100.00	100.00	100.00	100.00	100.00

Considering the percentage of respondents who favor the higher tax by either saying "strongly favor" or "favor more than oppose", we have the following favorable support for the higher tax in each country.

Great Britain	31.00 + 34.04 = 65.04%
Italy	31.96 + 39.04 = 71.00%
Spain	45.99 + 32.01 = 78.00%
Germany	19.98 + 36.99 = 56.97%
United States	20.98 + 32.06 = 53.04%

More that 50% of the respondents favor the higher tax for the higher carbon emission cars in all five countries. But the support for the higher tax is greater in the European countries. Spain and Italy have the greatest support for the higher tax with 78% and 71% respectively. Germany is close in views to the United States with 56.97% expressing favor for the higher tax. United States shows the lowest level of support for the higher tax with 53.04%. Note that United States ranks first in terms of the response "strongly oppose" the higher tax with 20% of the respondents providing this opinion.

49. a. The batting averages for the junior and senior years for each player are as follows:

Junior year:

Allison Fealey	15/40 = .375
Emily Janson	70/200 = .350

Senior year:

Allison Fealey	75/250 = .300
Emily Janson	35/120 = .292

Because Allison Fealey had the higher batting average in both her junior year and senior year, Allison Fealey should receive the scholarship offer.

b. The combined or aggregated two-year crosstabulation is as follows:

	Combined 2-Year Batting	
Outcome	**A. Fealey**	**E. Jansen**
Hit	90	105
No Hit	200	215
Total At Bats	290	320

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Based on this crosstabulation, the batting average for each player is as follows:

Combined Junior/Senior Years

Allison Fealey	90/290 = .310
Emily Janson	105/320 = .328

Because Emily Janson has the higher batting average over the combined junior and senior years, Emily Janson should receive the scholarship offer.

c. The recommendations in parts (a) and (b) are not consistent. This is an example of Simpson's Paradox. It shows that in interpreting the results based upon separate or un-aggregated crosstabulations, the conclusion can be reversed when the crosstabulations are grouped or aggregated. When Simpson's Paradox is present, the decision maker will have to decide whether the un-aggregated or the aggregated form of the crosstabulation is the most helpful in identifying the desired conclusion. Note: The authors prefer the recommendation to offer the scholarship to Emily Janson because it is based upon the aggregated performance for both players over a larger number of at-bats. But this is a judgment or personal preference decision. Others may prefer the conclusion based on using the un-aggregated approach in part (a).

50. a.

	Fuel Type					
Year Constructed	Elec	Nat. Gas	Oil	Propane	Other	Total
1973 or before	40	183	12	5	7	247
1974-1979	24	26	2	2	0	54
1980-1986	37	38	1	0	6	82
1987-1991	48	70	2	0	1	121
Total	149	317	17	7	14	504

b.

Year Constructed	Frequency	Fuel Type	Frequency
1973 or before	247	Electricity	149
1974-1979	54	Nat. Gas	317
1980-1986	82	Oil	17
1987-1991	121	Propane	7
Total	504	Other	14
		Total	504

c. Crosstabulation of Column Percentages

	Fuel Type				
Year Constructed	Elec	Nat. Gas	Oil	Propane	Other
1973 or before	26.9	57.7	70.5	71.4	50.0
1974-1979	16.1	8.2	11.8	28.6	0.0
1980-1986	24.8	12.0	5.9	0.0	42.9
1987-1991	32.2	22.1	11.8	0.0	7.1
Total	100.0	100.0	100.0	100.0	100.0

d. Crosstabulation of row percentages.

	Fuel Type					
Year Constructed	Elec	Nat. Gas	Oil	Propane	Other	Total
1973 or before	16.2	74.1	4.9	2.0	2.8	100.0
1974-1979	44.5	48.1	3.7	3.7	0.0	100.0
1980-1986	45.1	46.4	1.2	0.0	7.3	100.0
1987-1991	39.7	57.8	1.7	0.0	0.8	100.0

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e. Observations from the column percentages crosstabulation

For those buildings using electricity, the percentage has not changed greatly over the years. For the buildings using natural gas, the majority were constructed in 1973 or before; the second largest percentage was constructed in 1987-1991. Most of the buildings using oil were constructed in 1973 or before. All of the buildings using propane are older.

Observations from the row percentages crosstabulation

Most of the buildings in the CG&E service area use electricity or natural gas. In the period 1973 or before most used natural gas. From 1974-1986, it is fairly evenly divided between electricity and natural gas. Since 1987 almost all new buildings are using electricity or natural gas with natural gas being the clear leader.

51. a. Crosstabulation for stockholder's equity and profit.

	Profits ($000)						
Stockholders' Equity ($000)	0-200	200-400	400-600	600-800	800-1000	1000-1200	Total
0-1200	10	1				1	12
1200-2400	4	10			2		16
2400-3600	4	3	3	1	1	1	13
3600-4800					1	2	3
4800-6000		2	3	1			6
Total	18	16	6	2	4	4	50

b. Crosstabulation of Row Percentages.

	Profits ($000)						
Stockholders' Equity ($1000s)	0-200	200-400	400-600	600-800	800-1000	1000-1200	Total
0-1200	83.33	8.33	0.00	0.00	0.00	8.33	100
1200-2400	25.00	62.50	0.00	0.00	12.50	0.00	100
2400-3600	30.77	23.08	23.08	7.69	7.69	7.69	100
3600-4800		0.00	0.00	0.00	33.33	66.67	100
4800-6000	0.00	33.33	50.00	16.67	0.00	0.00	100

c. Stockholder's equity and profit seem to be related. As profit goes up, stockholder's equity goes up. The relationship, however, is not very strong.

52. a. Crosstabulation of market value and profit.

	Profit ($1000s)				
Market Value ($1000s)	0-300	300-600	600-900	900-1200	Total
0-8000	23	4			27
8000-16000	4	4	2	2	12
16000-24000		2	1	1	4
24000-32000		1	2	1	4
32000-40000		2	1		3
Total	27	13	6	4	50

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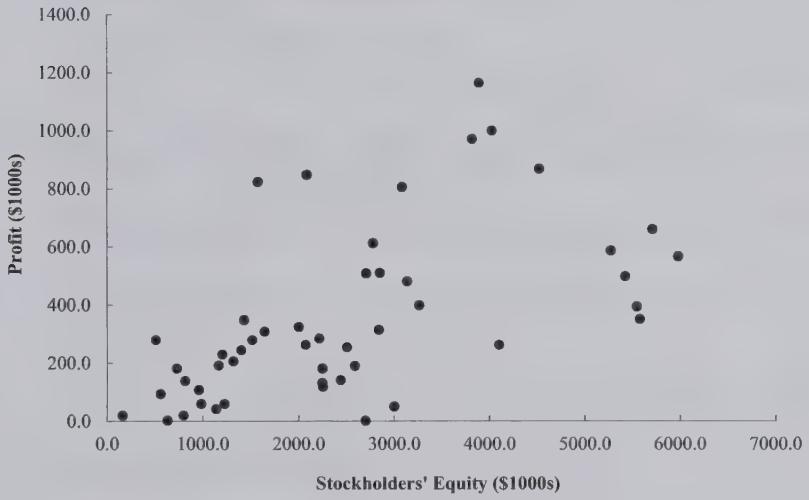
b. Crosstabulation of Row Percentages.

	Profit ($1000s)				
Market Value ($1000s)	0-300	300-600	600-900	900-1200	Total
0-8000	85.19	14.81	0.00	0.00	100
8000-16000	33.33	33.33	16.67	16.67	100
16000-24000	0.00	50.00	25.00	25.00	100
24000-32000	0.00	25.00	50.00	25.00	100
32000-40000	0.00	66.67	33.33	0.00	100

c. There appears to be a positive relationship between Profit and Market Value. As profit goes up, Market Value goes up.

53. a. Scatter diagram of Profit vs. Stockholders' Equity.



b. Profit and Stockholders' Equity appear to be positively related.

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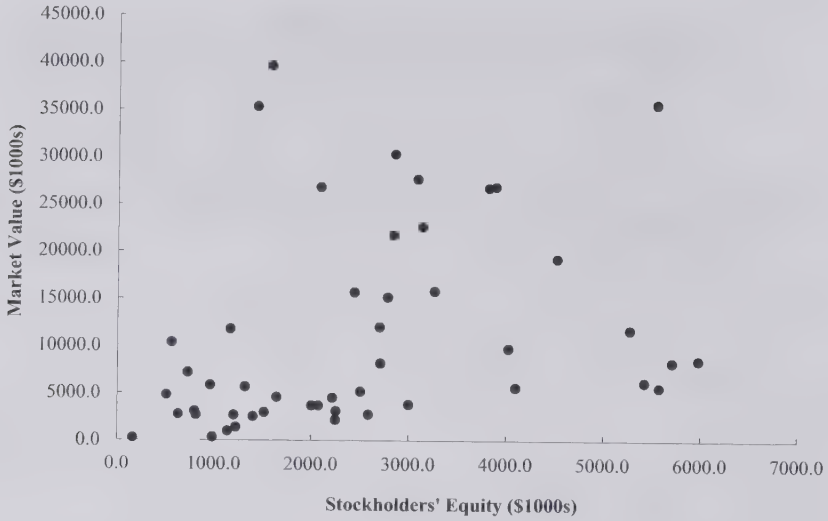
54. a. Scatter diagram of Market Value and Stockholders' Equity.



b. There is a positive relationship between Market Value and Stockholders' Equity.

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Chapter 3
Descriptive Statistics: Numerical Measures

Learning Objectives

1. Understand the purpose of measures of location.
2. Be able to compute the mean, median, mode, quartiles, and various percentiles.
3. Understand the purpose of measures of variability.
4. Be able to compute the range, interquartile range, variance, standard deviation, and coefficient of variation.
5. Understand skewness as a measure of the shape of a data distribution. Learn how to recognize when a data distribution is negatively skewed, roughly symmetric, and positively skewed.
6. Understand how z scores are computed and how they are used as a measure of relative location of a data value.
7. Know how Chebyshev's theorem and the empirical rule can be used to determine the percentage of the data within a specified number of standard deviations from the mean.
8. Learn how to construct a 5-number summary and a box plot.
9. Be able to compute and interpret covariance and correlation as measures of association between two variables.
10. Be able to compute a weighted mean.

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Solutions:

1. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{75}{5} = 15$

   10, 12, 16, 17, 20

   Median = 16 (middle value)

2. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{96}{6} = 16$

   10, 12, 16, 17, 20, 21

   Median $= \frac{16+17}{2} = 16.5$

3. 15, 20, 25, 25, 27, 28, 30, 34

   $i = \frac{20}{100}(8) = 1.6$ 2nd position = 20

   $i = \frac{25}{100}(8) = 2$ $\frac{20+25}{2} = 22.5$

   $i = \frac{65}{100}(8) = 5.2$ 6th position = 28

   $i = \frac{75}{100}(8) = 6$ $\frac{28+30}{2} = 29$

4. Mean $= \frac{\Sigma x_i}{n} = \frac{657}{11} = 59.73$

   Median = 57 6th item

   Mode = 53 It appears 3 times

5. a. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{3181}{20} = \$159$

   b. Median 10th \$160 Los Angeles
   11th \$162 Seattle

   Median $= \frac{160+162}{2} = \$161$

   c. Mode = \$167 San Francisco and New Orleans

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d. $i = \left(\frac{25}{100}\right)20 = 5$

5th $134
6th $139

$Q_1 = \frac{134+139}{2} = \$136.50$

e. $i = \left(\frac{75}{100}\right)20 = 15$

15th $167
16th $173

$Q_3 = \frac{167+173}{2} = \$170$

6. a. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{350}{19} = 18.42$

b. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{120}{19} = 6.32$

c. $\frac{120}{350}(100) = 34.3\%$ of 3-point shots were made from the 20 feet, 9 inch line during the 19 games.

d. Moving the 3-point line back to 20 feet, 9 inches has reduced the number of 3-point shots taken per game from 19.07 to 18.42, or 19.07 – 18.42 = .65 shots per game. The percentage of 3-points made per game has been reduced from 35.2% to 34.3%, or only .9%. The move has reduced both the number of shots taken per game and the percentage of shots made per game, but the differences are small. The data support the Associated Press Sports conclusion that the move has not changed the game dramatically.

The 2008-09 sample data shows 120 3-point baskets in the 19 games. Thus, the mean number of points scored from the 3-point line is 120(3)/19 = 18.95 points per game. With the previous 3-point line at 19 feet, 9 inches, 19.07 shots per game and a 35.2% success rate indicate that the mean number of points scored from the 3-point line was 19.07(.352)(3) = 20.14 points per game. There is only a mean of 20.14 – 18.95 = 1.19 points per game less being scored from the 20 feet, 9 inch 3-point line.

7. a. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{148}{10} = 14.8$

b. Order the data from low 6.7 to high 36.6

Median $i = \left(\frac{50}{100}\right)10 = 5$ Use 5^{th} and 6^{th} positions.

$\text{Median} = \frac{10.1+16.1}{2} = 13.1$

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c. Mode = 7.2 (occurs 2 times)

d. $i = \left(\frac{25}{100}\right)10 = 2.5$ Use 3rd position. $Q_1 = 7.2$

$i = \left(\frac{75}{100}\right)10 = 7.5$ Use 8th position. $Q_3 = 17.2$

e. $\Sigma x_i = \$148$ billion

The percentage of total endowments held by these 2.3% of colleges and universities is (148/413)(100) = 35.8%.

f. A decline of 23% would be a decline of .23(148) = $34 billion for these 10 colleges and universities. With this decline, administrators might consider budget cutting strategies such as

- Hiring freezes for faculty and staff
- Delaying or eliminating construction projects
- Raising tuition
- Increasing enrollments

8. a. $\bar{x} = \frac{\sum x_i}{n} = \frac{3200}{20} = 160$

Order the data from low 100 to high 360

Median $i = \left(\frac{50}{100}\right)20 = 10$ Use 10th and 11th positions

Median $= \left(\frac{130+140}{2}\right) = 135$

Mode = 120 (occurs 3 times)

b. $i = \left(\frac{25}{100}\right)20 = 5$ Use 5th and 6th positions

$Q_1 = \left(\frac{115+115}{2}\right) = 115$

$i = \left(\frac{75}{100}\right)20 = 15$ Use 15th and 16th positions

$Q_3 = \left(\frac{180+195}{2}\right) = 187.5$

c. $i = \left(\frac{90}{100}\right)20 = 18$ Use 18th and 19th positions

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$$90^{\text{th}} \text{ percentile} = \left(\frac{235+255}{2}\right) = 245$$

90% of the tax returns cost $245 or less. 10% of the tax returns cost $245 or more.

9. a. Ordered data: 112.8 140.2 169.9 177.5 181.3 202.5 230.0 315.5 470.2

With $n = 9$, the median is the 5th position.

The median sales price of existing homes is $181.3 thousand.

b. Ordered data: 149.5 175.0 195.8 215.5 225.3 275.9 350.2 525.0

With $n = 8$, the median is the average of the 4th and 5th positions.

$$\text{The median sales price of new homes} = \frac{215.5+225.3}{2} = \$220.4 \text{ thousand.}$$

c. New homes have the higher median sale price by $220.4 – 181.3 = $39.1 thousand

d. Existing homes: $\frac{181.3-208.4}{208.4} = \frac{-27.1}{208.4} = -.130$ or a 13.0% decrease in the median sales price.

New homes: $\frac{220.4-249.0}{249.0} = \frac{-28.6}{249.0} = -.115$ or an 11.5% decrease in the median sales price.

Existing homes had the larger one-year percentage decrease in the median sales price. However, new homes have had the larger one-year decrease in the median sales price; a median sales price decrease of $28.6 thousand for new homes and a median sales price decrease of $27.1 thousand for existing homes.

10. a. Minimum = .4%; Maximum = 3.5%

b. $\Sigma x_i = 69$

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{69}{30} = 2.3\%$$

Median is average of 15th and 16th items.

Both are 2.5%, so the median is 2.5%.

The mode is 2.7%; forecast by 4 economists.

c. For Q_1,

$i = \left(\frac{25}{100}\right)30 = 7.5$; round up and use the 8th item

$$Q_1 = 2.0\%$$

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For Q_3,

$i = \left(\frac{75}{100}\right)30 = 22.5$; round up and use the 23rd item

$Q_3 = 2.8\%$

d. Generally, the 2% to 3% growth should be considered optimistic.

11. Using the mean we get $\bar{x}_{\text{city}} = 15.58$, $\bar{x}_{\text{highway}} = 18.92$

For the samples we see that the mean mileage is better on the highway than in the city.

City

13.2 14.4 15.2 15.3 15.3 15.3 15.9 16 16.1 16.2 16.2 16.7 16.8

↑

Median

Mode: 15.3

Highway

17.2 17.4 18.3 18.5 18.6 18.6 18.7 19.0 19.2 19.4 19.4 20.6 21.1

↑

Median

Mode: 18.6, 19.4

The median and modal mileages are also better on the highway than in the city.

12. Disney

Total Revenue: \$3,321 million (13 movies)

$\bar{x} = \frac{\Sigma x_i}{n} = \frac{3321}{13} = \$255.5$

104 110 136 169 249 250 253 273 304 325 346 354 448

Median 7th position

Median = \$253

Q_1: $i = .25(13)$ 3.25 4th position

$Q_1 = \$169$

Q_3: $i = .75(13) = 9.75$ 10th position

$Q_3 = \$325$

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Pixar

Total Revenue: \$3,231 million (6 movies)

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{3231}{6} = \$538.5$$

362 363 485 525 631 865

Median (3rd and 4th positions)

$$\text{Median} = \frac{485 + 525}{2} = \$505$$

Q_1: $i = .25(6) = 1.5$ 2nd position

$Q_1 = \$363$

Q_3: $i = .75(6) = 4.5$ 5th position

$Q_3 = \$631$

The total box office revenues for the two companies over the 10 year period are approximately the same: Disney \$3321 million; Pixar \$3231 million. But Disney generated its revenue with 13 films while Pixar generated its revenue with only 6 films.

	Disney	Pixar
Mean	\$225.5	\$538.5
Median	\$253	\$505

The first quartiles show 75% of Disney films do better than \$169 million while 75% of Pixar films do better than \$363 million. The third quartiles show 25% of Disney films do better than \$325 million while 25% of Pixar films do better than \$631. In all of these comparisons, Pixar films are about twice as successful as Disney films when it comes to box office revenue. In buying Pixar, Disney looks to acquire Pixar's ability to make higher revenue films.

13. Range 20 - 10 = 10

10, 12, 16, 17, 20

$$i = \frac{25}{100}(5) = 1.25$$

Q_1 (2nd position) = 12

$$i = \frac{75}{100}(5) = 3.75$$

Q_3 (4th position) = 17

IQR = Q_3 - Q_1 = 17 - 12 = 5

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14. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{75}{5} = 15$

$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1} = \frac{64}{4} = 16$

$s = \sqrt{16} = 4$

15. 15, 20, 25, 25, 27, 28, 30, 34 Range = 34 - 15 = 19

$i = \frac{25}{100}(8) = 2$ $Q_1 = \frac{20+25}{2} = 22.5$

$i = \frac{75}{100}(8) = 6$ $Q_3 = \frac{28+30}{2} = 29$

IQR = Q_3 - Q_1 = 29 - 22.5 = 6.5

$\bar{x} = \frac{\Sigma x_i}{n} = \frac{204}{8} = 25.5$

$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1} = \frac{242}{7} = 34.57$

$s = \sqrt{34.57} = 5.88$

16. a. Range = 190 - 168 = 22

b. $\Sigma(x_i - \bar{x})^2 = 376$

$s^2 = \frac{376}{5} = 75.2$

c. $s = \sqrt{75.2} = 8.67$

d. Coefficient of Variation $= \left(\frac{8.67}{178}\right)100\% = 4.87\%$

17. a. With DVD $\bar{x} = \frac{\Sigma x_i}{n} = \frac{2050}{5} = 410$

Without DVD $\bar{x} = \frac{\Sigma x_i}{n} = \frac{1550}{5} = 310$

With DVD \$410 - \$310 = \$100 more expensive

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b. With DVD Range = 500 - 300 = 200

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1} = \frac{22000}{4} = 5500$$

$$s = \sqrt{5500} = 74.2$$

Without DVD Range = 360 - 290 = 70

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1} = \frac{3200}{4} = 800$$

$$s = \sqrt{800} = 28.3$$

Models with DVD players have the greater variation in prices. The price range is \$300 to \$500.

Models without a DVD player have less variation in prices. The price range is \$290 to \$360.

18. a. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{266}{7} = \$38$ per day

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1} = \frac{582}{6} = 97$$

$$s = \sqrt{97} = \$9.85$$

b. The mean car-rental rate per day is \$38 for both Eastern and Western cities. However, Eastern cities show a greater variation in rates per day. This greater variation is most likely due to the inclusion of the most expensive city (New York) in the Eastern city sample.

19. a. Range = 60 - 28 = 32

IQR = Q_3 - Q_1 = 55 - 45 = 10

b. $\bar{x} = \frac{435}{9} = 48.33$

$$\Sigma(x_i - \bar{x})^2 = 742$$

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1} = \frac{742}{8} = 92.75$$

$$s = \sqrt{92.75} = 9.63$$

c. The average air quality is about the same. But, the variability is greater in Anaheim.

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20. Dawson Supply: Range = 11 - 9 = 2

$$s = \sqrt{\frac{4.1}{9}} = 0.67$$

J.C. Clark: Range = 15 - 7 = 8

$$s = \sqrt{\frac{60.1}{9}} = 2.58$$

21. a. Cities:

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{198}{6} = \$33$$

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1} = \frac{72}{6-1} = 14.40$$

$$s = \sqrt{14.40} = 3.79$$

Retirement Areas:

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{192}{6} = \$32$$

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1} = \frac{18}{6-1} = 3.60$$

$$s = \sqrt{3.60} = 1.90$$

b. Mean cost of the market basket is roughly the same with the retirement areas sample mean $1 less. However, there is more variation in the cost in cities than in retirement areas.

22. a. Freshmen $\bar{x} = \frac{\Sigma x_i}{n} = \frac{32125}{25} = 1285$

Seniors $\bar{x} = \frac{\Sigma x_i}{n} = \frac{8660}{20} = \$433$

Freshmen spend almost three times as much on back-to-school items as seniors.

b. Freshmen Range = 2094 – 374 = 1720

Seniors Range = 632 280 = 352

c. Freshmen

$$i = \left(\frac{25}{100}\right)25 = 6.25 \qquad Q_1 = 1079 \quad (7^{\text{th}} \text{ item})$$

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$$i = \left(\frac{75}{100}\right)25 = 18.75 \quad Q_3 = 1475 \ (19^{th} \text{ item})$$

IQR = Q_3 - Q_1 = 1479 – 1075 = 404

Seniors

$$i = \left(\frac{25}{100}\right)20 = 5 \qquad Q_1 = \frac{368+373}{2} = 370.5$$

$$i = \left(\frac{75}{100}\right)20 = 15 \qquad Q_1 = \frac{489+515}{2} = 502$$

IQR = Q_3 - Q_1 = 502 – 370.5 = 131.5

d. $s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$

Freshmen $s = \sqrt{\frac{3233186}{24}} = 367.04$

Seniors $s = \sqrt{\frac{178610}{19}} = 96.96$

e. All measures of variability show freshmen have more variation in back-to-school expenditures.

23. a. For 2005

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{608}{8} = 76$$

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{30}{7}} = 2.07$$

For 2006

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{608}{8} = 76$$

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{194}{7}} = 5.26$$

b. The mean score is 76 for both years, but there is an increase in the standard deviation for the scores in 2006. The golfer is not as consistent in 2006 and shows a sizeable increase in the variation with golf scores ranging from 71 to 85. The increase in variation might be explained by the golfer trying to change or modify the golf swing. In general, a loss of consistency and an increase in the standard deviation could be viewed as a poorer performance in 2006. The optimism in 2006 is that three of the eight scores were better than any score reported for 2005. If the golfer can work for consistency, eliminate the high score rounds, and reduce the standard deviation, golf scores should show improvement.

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24. Quarter milers

$s = 0.0564$

Coefficient of Variation = $(s/\bar{x})100\% = (0.0564/0.966)100\% = 5.8\%$

Milers

$s = 0.1295$

Coefficient of Variation = $(s/\bar{x})100\% = (0.1295/4.534)100\% = 2.9\%$

Yes; the coefficient of variation shows that as a percentage of the mean the quarter milers' times show more variability.

25. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{75}{5} = 15$

$s^2 = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{64}{4}} = 4$

10 $z = \frac{10-15}{4} = -1.25$

20 $z = \frac{20-15}{4} = +1.25$

12 $z = \frac{12-15}{4} = -.75$

17 $z = \frac{17-15}{4} = +.50$

16 $z = \frac{16-15}{4} = +.25$

26. $z = \frac{520-500}{100} = +.20$

$z = \frac{650-500}{100} = +1.50$

$z = \frac{500-500}{100} = 0.00$

$z = \frac{450-500}{100} = -.50$

$z = \frac{280-500}{100} = -2.20$

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27. a. $z = \dfrac{40-30}{5} = 2$ $\quad 1 - \dfrac{1}{2^2} = .75$ At least 75%

b. $z = \dfrac{45-30}{5} = 3$ $\quad 1 - \dfrac{1}{3^2} = .89$ At least 89%

c. $z = \dfrac{38-30}{5} = 1.6$ $\quad 1 - \dfrac{1}{1.6^2} = .61$ At least 61%

d. $z = \dfrac{42-30}{5} = 2.4$ $\quad 1 - \dfrac{1}{2.4^2} = .83$ At least 83%

e. $z = \dfrac{48-30}{5} = 3.6$ $\quad 1 - \dfrac{1}{3.6^2} = .92$ At least 92%

28. a. Approximately 95%

b. Almost all

c. Approximately 68%

29. a. This is from 2 standard deviations below the mean to 2 standard deviations above the mean.

With $z = 2$, Chebyshev's theorem gives:

$$1 - \frac{1}{z^2} = 1 - \frac{1}{2^2} = 1 - \frac{1}{4} = \frac{3}{4}$$

Therefore, at least 75% of adults sleep between 4.5 and 9.3 hours per day.

b. This is from 2.5 standard deviations below the mean to 2.5 standard deviations above the mean.

With $z = 2.5$, Chebyshev's theorem gives:

$$1 - \frac{1}{z^2} = 1 - \frac{1}{2.5^2} = 1 - \frac{1}{6.25} = .84$$

Therefore, at least 84% of adults sleep between 3.9 and 9.9 hours per day.

c. With $z = 2$, the empirical rule suggests that 95% of adults sleep between 4.5and 9.3 hours per day. The percentage obtained using the empirical rule is greater than the percentage obtained using Chebyshev's theorem.

30. a. \$2.71 is one standard deviation below the mean and \$2.91 is one standard deviation above the mean. The empirical rule says that approximately 68% of gasoline sales are in the price range.

b. Part (a) shows that approximately 68% of the gasoline sales are between \$2.71 and \$2.91. Since the bell-shaped distribution is symmetric, approximately half of 68%, or 34%, of the gasoline sales should be between \$2.71 and the mean price of \$2.81. \$3.01 is two standard deviations above the mean price of \$2.81. The empirical rule says that approximately 95% of the gasoline sales should be within two standard deviations of the mean. Thus, approximately half of 95%, or 47.5%, of the gasoline sales should be between the mean price of \$2.81 and \$3.01. The percentage of gasoline sales between \$2.71 and \$3.01 should be approximately 34% + 47.5% = 81.5%.

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c. $3.01 is two standard deviations above the mean and the empirical rule says that approximately 95% of the gasoline sales should be within two standard deviations of the mean. Thus, 1 - 95% = 5% of the gasoline sales should be more than two standard deviations from the mean. Since the bell-shaped distribution is symmetric, we expected half of 5%, or 2.5%, would be more than $3.01.

31. a. 615 is one standard deviation above the mean. Approximately 68% of the scores are between 415 and 615 with half of 68%, or 34%, of the scores between the mean of 515 and 615. Also, since the distribution is symmetric, 50% of the scores are above the mean of 515. With 50% of the scores above 515 and with 34% of the scores between 515 and 615, 50% - 34% = 16% of the scores are above 615.

b. 715 is two standard deviations above the mean. Approximately 95% of the scores are between 315 and 715 with half of 95%, or 47.5%, of the scores between the mean of 515 and 715. Also, since the distribution is symmetric, 50% of the scores are above the mean of 515. With 50% of the scores above 515 and with 47.5% of the scores between 515 and 715, 50%- 47.5% = 2.5% of the scores are above 715.

c. Approximately 68% of the scores are between 415 and 615 with half of 68%, or 34%, of the scores between 415 and the mean of 515.

d. Approximately 95% of the scores are between 315 and 715 with half of 95%, or 47.5%, of the scores between 315 and the mean of 515. Approximately 68% of the scores are between 415 and 615 with half of 68%, or 34%, of the scores between the mean of 515 and 615. Thus, 47.5% + 34% = 81.5% of the scores are between 315 and 615.

32. a. $z = \dfrac{x-\mu}{\sigma} = \dfrac{2300-3100}{1200} = -.67$

b. $z = \dfrac{x-\mu}{\sigma} = \dfrac{4900-3100}{1200} = 1.50$

c. $2300 is .67 standard deviations below the mean. $4900 is 1.50 standard deviations above the mean. Neither is an outlier.

d. $z = \dfrac{x-\mu}{\sigma} = \dfrac{13000-3100}{1200} = 8.25$

$13,000 is 8.25 standard deviations above the mean. This cost is an outlier.

33. a. $\bar{x} = \dfrac{\Sigma x_i}{n} = \dfrac{64}{7} = 9.14$ days

Median: with $n = 7$, use 4th position

2, 3, 8, 8, 12, 13, 18

Median = 8 days

Mode: 8 days (occurred twice)

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b. Range = Largest value – Smallest value
= 18 – 2 = 16

$$s=\sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n-1}}$$

$$\begin{aligned}\Sigma(x_i-\bar{x})^2 &= (13-9.14)^2+(12-9.14)^2+(8-9.14)^2+(3-9.14)^2\\ &\quad +(8-9.14)^2+(2-9.14)^2+(18-9.14)^2\\ &= 192.86\end{aligned}$$

$$s=\sqrt{\frac{192.86}{6}}=5.67$$

c. $$z=\frac{x-\bar{x}}{s}=\frac{18-9.14}{5.67}=1.56$$

The 18 days required to restore service after hurricane Wilma is not an outlier.

d. Yes, FP&L should consider ways to improve its emergency repair procedures. The mean, median and mode show repairs requiring an average of 8 to 9 days can be expected if similar hurricanes are encountered in the future. The 18 days required to restore service after hurricane Wilma should not be considered unusual if FP&L continues to use its current emergency repair procedures. With the number of customers affected running into the millions, plans to shorten the number of days to restore service should be undertaken by the company.

34. a. $$\bar{x}=\frac{\Sigma x_i}{n}=\frac{765}{10}=76.5$$

$$s=\sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n-1}}=\sqrt{\frac{442.5}{10-1}}=7$$

b. $$z=\frac{x-\bar{x}}{s}=\frac{84-76.5}{7}=1.07$$

Approximately one standard deviation above the mean. Approximately 68% of the scores are within one standard deviation. Thus, half of (100-68), or 16%, of the games should have a winning score of 84 or more points.

$$z=\frac{x-\bar{x}}{s}=\frac{90-76.5}{7}=1.93$$

Approximately two standard deviations above the mean. Approximately 95% of the scores are within two standard deviations. Thus, half of (100-95), or 2.5%, of the games should have a winning score of more than 90 points.

c. $$\bar{x}=\frac{\Sigma x_i}{n}=\frac{122}{10}=12.2$$

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$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{559.6}{10-1}} = 7.89$$

Largest margin 24: $z = \frac{x - \bar{x}}{s} = \frac{24 - 12.2}{7.89} = 1.50$. No outliers.

35. a. $\bar{x} = \frac{\sum x_i}{n} = \frac{1050}{14} = 75$

b. \$75.00 - \$72.20 = \$2.80

\$2.80/\$72.20 = .0388 Ticket price increased 3.88% during the one-year period.

c. 7th position Green Bay Packers 63

8th position Pittsburgh Steelers 67

$$\text{Median} = \frac{63 + 67}{2} = 65$$

d. $i = .25(14) = 3.5$ Use 4th position

$Q_1 = 61$ (Tennessee Titans)

$i = .75(14) = 10.5$ Use 11th position

$Q_1 = 83$ (Indianapolis Colts)

e. $s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{9504}{13}} = 27.04$

f. Dallas Cowboys: $z = \frac{x - \mu}{\sigma} = \frac{160 - 75}{27.04} = 3.14$

With $z > 3$, this is an outlier. The Dallas Cowboys have an unusually high ticket price compared to the other NFL teams.

36. 15, 20, 25, 25, 27, 28, 30, 34

Smallest = 15

$i = \frac{25}{100}(8) = 2$ $Q_1 = \frac{20 + 25}{2} = 22.5$

$\text{Median} = \frac{25 + 27}{2} = 26$

$i = \frac{75}{100}(8) = 8$ $Q_3 = \frac{28 + 30}{2} = 29$

Largest = 34

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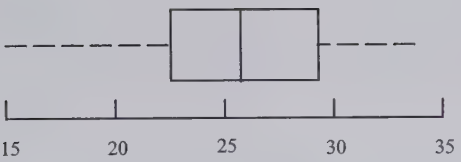
37.



38. 5, 6, 8, 10, 10, 12, 15, 16, 18

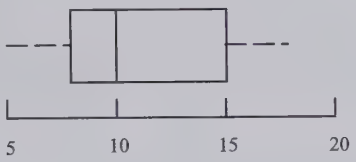
Smallest = 5

$i = \frac{25}{100}(9) = 2.25$ $Q_1 = 8$ (3rd position)

Median = 10

$i = \frac{75}{100}(9) = 6.75$ $Q_3 = 15$ (7th position)

Largest = 18



39. IQR = 50 - 42 = 8

Lower Limit: Q_1 - 1.5 IQR = 42 - 12 = 30

Upper Limit: Q_3 + 1.5 IQR = 50 + 12 = 62

65 is an outlier

40. a. Subway with 29,612 locations.

b. Ordered data:

1,397 2,558 2,615 4,516 5,110 5,619 5,889 8,053 8,082 10,238 11,553 24,799 29,612

Median

n = 13; median is the middle value or 7th position

Median = 5,889

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c. Smallest value = 1,397

$i = \frac{25}{100}(n) = \frac{25}{100}(13) = 3.25$ Round up to 4th position

Q_1 = 4,516

$i = \frac{75}{100}(n) = \frac{75}{100}(13) = 9.75$ Round up to 10th position

Q_3 10,238

Largest value = 29,612

5-number summary: 1,397 4,516 5,889 10,238 29,612

d. Limits:

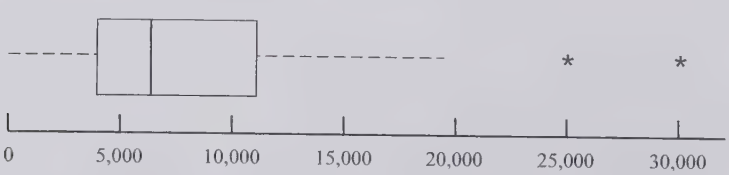
IQR = Q_3 - Q_1 = 10,238 - 4,516 = 5,722

Lower Limit: Q_1 - 1.5 (IQR) = 4,516 - 1.5(5,722) = -4,067

Upper Limit: Q_3 + 1.5 (IQR) = 10,238 + 1.5(5,722) = 18,821

Both Subway and McDonald's are outliers with a greater number of locations than other well-known food franchises.

e.



41. a. The first place runner in the men's group finished $109.03 - 65.30 = 43.73$ minutes ahead of the first place runner in the women's group. Lauren Wald would have finished in 11th place for the combined groups.

b. Men: $i = \left(\frac{50}{100}\right)22 = .50(22) = 11$. Use the 11th and 12th place finishes.

Median = $\frac{109.05 + 110.23}{2} = 109.64$

Women: $i = \left(\frac{50}{100}\right)31 = .50(31) = 15.5$. Use the 16th place finish. Median = 131.67.

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Using the median finish times, the men's group finished $131.67 - 109.64 = 22.03$ minutes ahead of the women's group.
Also note that the fastest time for a woman runner, 109.03 minutes, is approximately equal to the median time of 109.64 minutes for the men's group.

c. Men: Lowest time = 65.30; Highest time = 148.70

Q_1: $i = \left(\frac{25}{100}\right)n = .25(22) = 5.5$ Use 6^{th} position. $Q_1 = 87.18$

Q_3: $i = \left(\frac{75}{100}\right)n = .75(22) = 16.5$ Use 17^{th} position. $Q_3 = 128.40$

Five number summary for men: 65.30, 87.18, 109.64, 128.40, 148.70

Women: Lowest time = 109.03; Highest time = 189.28

Q_1: $i = \left(\frac{25}{100}\right)n = .25(31) = 7.75$ Use 8^{th} position. $Q_1 = 122.08$

Q_3: $i = \left(\frac{75}{100}\right)n = .75(31) = 23.25$ Use 24^{th} position. $Q_3 = 147.18$

Five number summary for women: 109.03, 122.08, 131.67, 147.18, 189.28

d. Men: IQR = $128.40 - 87.18 = 41.22$

Lower Limit = $Q_1 - 1.5(IQR) = 87.18 - 1.5(41.22) = 25.35$

Upper Limit = $Q_3 + 1.5(IQR) = 128.40 + 1.5(41.22) = 190.23$

There are no outliers in the men's group.

Women: IQR = $147.18 - 122.08 = 25.10$

Lower Limit = $Q_1 - 1.5(IQR) = 122.08 - 1.5(25.10) = 84.43$

Upper Limit = $Q_3 + 1.5(IQR) = 147.18 + 1.5(25.10) = 184.83$

The two slowest women runners with times of 189.27 and 189.28 minutes are outliers in the women's group.

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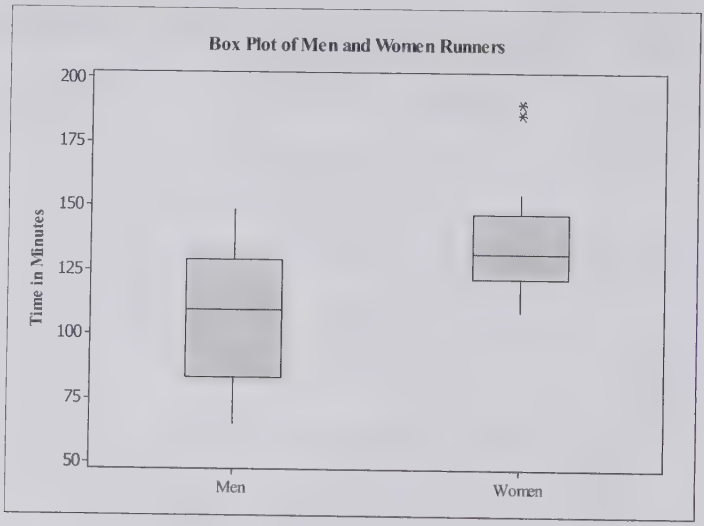
e.



The box plots show the men runners with the faster or lower finish times. However, the box plots show the women runners with the lower variation in finish times. The interquartile ranges of 41.22 minutes for men and 25.10 minutes for women support this conclusion.

42. a. Median $n = 20$; 10th and 11th positions

$$\text{Median} = \frac{73+74}{2} = 73.5$$

b. Smallest 68

Q_1: $i = \left(\frac{25}{100}\right)20 = 5$; 5th and 6th positions

$$Q_1 = \frac{71+72}{2} = 71.5$$

Q_3: $i = \left(\frac{75}{100}\right)20 = 15$; 15th and 16th positions

$$Q_3 = \frac{74+75}{2} = 74.5$$

Largest 77

5- number summary: 68, 71.5, 73.5, 74.5, 77

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c. IQR $= Q_3 - Q_1 = 74.5 - 71.5 = 3$

Lower Limit $= Q_1 - 1.5(\text{IQR})$

$= 71.5 - 1.5(3) = 67$

Upper Limit $= Q_3 + 1.5(\text{IQR})$

$= 74.5 + 1.5(3) = 79$

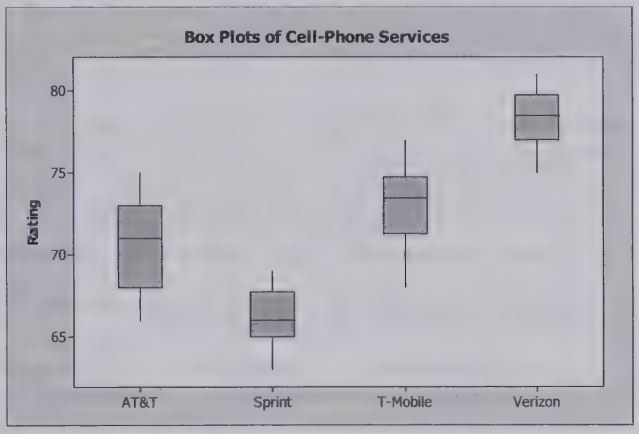
All ratings are between 67 and 79. There are no outliers for the T-Mobile service.

d. Using the solution procedures shown in parts a, b, and c, the five number summaries and outlier limits for the other three cell-phone services are as follows.

AT&T	66, 68, 71, 73, 75	Limits: 60.5 and 80.5
Sprint	63, 65, 66, 67.5, 69	Limits: 61.25 and 71.25
Verizon	75, 77, 78.5, 79.5, 81	Limits: 73.25 and 83.25

There are no outliers for any of the cell-phone services.

e.



The box plots show that Verizon is the best cell-phone service provider in terms of overall customer satisfaction. Verizon's lowest rating is better than the highest AT&T and Sprint ratings and is better than 75% of the T-Mobile ratings. Sprint shows the lowest customer satisfaction ratings among the four services.

43. a. Total Salary for the Philadelphia Phillies = $96,870,000

Median $n = 28$; 14th and 15th positions

$$\text{Median} = \frac{900 + 1700}{2} = 1300$$

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Smallest 390

Q_1: $i = \left(\frac{25}{100}\right)28 = 7$; 7th and 8th positions

$$Q_1 = \frac{425 + 440}{2} = 432.5$$

Q_3: $i = \left(\frac{75}{100}\right)28 = 21$; 21st and 22nd positions

$$Q_3 = \frac{6000 + 6350}{2} = 6175$$

Largest 14250

5- number summary for the Philadelphia Phillies: 390, 432.5, 1300, 6175, 14250

Using the 5-number summary, the lower quartile shows salaries closely bunched between 390 and 432.5. The median is 1300. The most variation is in the upper quartile where the salaries are spread between 6175 and 14250, or between $6,175,000 and $14,250,000.

b. IQR = $Q_3 - Q_1$ = 6175 – 432.5 = 5742.5

Lower Limit = Q_1 – 1.5(IQR)

= 432.5 –1.5(5742.5) = – 8181.25; Use 0

Upper Limit = Q_3 + 1.5(IQR)

= 6175 + 1.5(5742.5) = 14788.75

All salaries are between 0 and 14788.75. There are no salary outliers for the Philadelphia Phillies.

c. Using the solution procedures shown in parts a and b, the total salary, the five-number summaries, and the outlier limits for the other teams are as follows.

Los Angeles Dodgers $136,373,000
390, 403, 857.5, 9125, 19000 Limits: 0 and 22208

Tampa Bay Rays $ 42,334,000
390, 399, 415, 2350, 6000 Limits: 0 and 5276.5

Boston Red Sox $120,460,000
396, 439.5, 2500, 8166.5, 14000 Limits: 0 and 19757

The Los Angeles Dodgers had the highest payroll while the Tampa Bay Rays clearly had the lowest payroll among the four teams. With the lower salaries, the Rays had two outlier salaries compared to other salaries on the team. But these top two salaries are substantially below the top salaries for the other three teams. There are no outliers for the Phillies, Dodgers and Red Sox.

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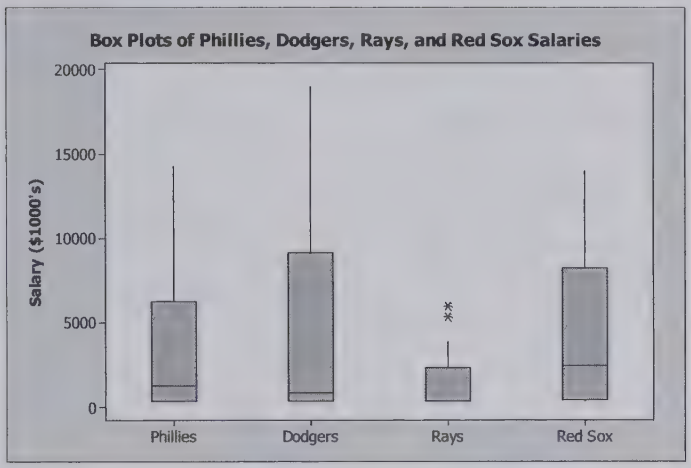
d.



The box plots show that the lowest salaries for the four teams are very similar. The Red Sox have the highest median salary. Of the four teams the Dodgers have the highest upper end salaries and highest total payroll, while the Rays are clearly the lowest paid team.

For this data, we would conclude that paying higher salaries do not always bring championships. In the National League Championship, the lower paid Phillies beat the higher paid Dodgers. In the American League Championship, the lower paid Rays beat the higher paid Red Sox. The biggest surprise was how the Tampa Bay Rays over achieved based on their salaries and made it to the World Series. Teams with the highest salaries do not always win the championships.

44. a. $\bar{x} = \dfrac{\Sigma x_i}{n} = \dfrac{837.5}{46} = 18.2$

Median 23rd position 15.1
24th position 15.6

$$\text{Median} = \frac{15.1+15.6}{2} = 15.35$$

b. Q_1: $i = \left(\dfrac{25}{100}\right)n = .25(46) = 11.5$

12th position: $Q_1 = 11.7$

Q_3: $i = \left(\dfrac{75}{100}\right)n = .75(46) = 34.5$

35th position: $Q_3 = 23.5$

c. 3.4, 11.7, 15.35, 23.5, 41.3

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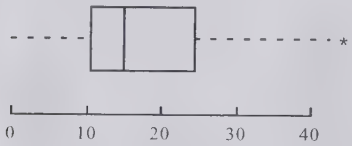
d. IQR = 23.5 - 11.7 = 11.8

Lower Limit = Q_1 - 1.5(IQR)
= 11.7 - 1.5(11.8) = -6 Use 0

Upper Limit = Q_3 + 1.5(IQR)
= 23.5 + 1.5(11.8) = 41.2

Yes, one: Alger Small Cap 41.3



45. a.

b. Negative relationship

c/d. $\Sigma x_i = 40 \quad \bar{x} = \frac{40}{5} = 8 \quad \Sigma y_i = 230 \quad \bar{y} = \frac{230}{5} = 46$

$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = -240 \quad \Sigma(x_i - \bar{x})^2 = 118 \quad \Sigma(y_i - \bar{y})^2 = 520$

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$$s_{xy} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{n-1} = \frac{-240}{5-1} = -60$$

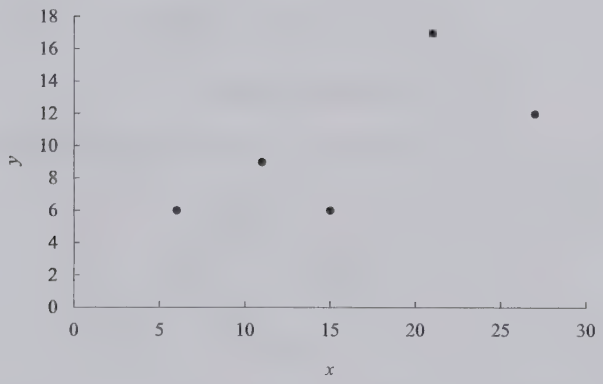
$$s_x = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{118}{5-1}} = 5.4314$$

$$s_y = \sqrt{\frac{\Sigma(y_i - \bar{y})^2}{n-1}} = \sqrt{\frac{520}{5-1}} = 11.4018$$

$$r_{xy} = \frac{s_{xy}}{s_x s_y} = \frac{-60}{(5.4314)(11.4018)} = -.969$$

There is a strong negative linear relationship.

46. a.



b. Positive relationship

c/d. $\Sigma x_i = 80 \quad \bar{x} = \frac{80}{5} = 16 \quad \Sigma y_i = 50 \quad \bar{y} = \frac{50}{5} = 10$

$$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 106 \quad \Sigma(x_i - \bar{x})^2 = 272 \quad \Sigma(y_i - \bar{y})^2 = 86$$

$$s_{xy} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{n-1} = \frac{106}{5-1} = 26.5$$

$$s_x = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{272}{5-1}} = 8.2462$$

$$s_y = \sqrt{\frac{\Sigma(y_i - \bar{y})^2}{n-1}} = \sqrt{\frac{86}{5-1}} = 4.6368$$

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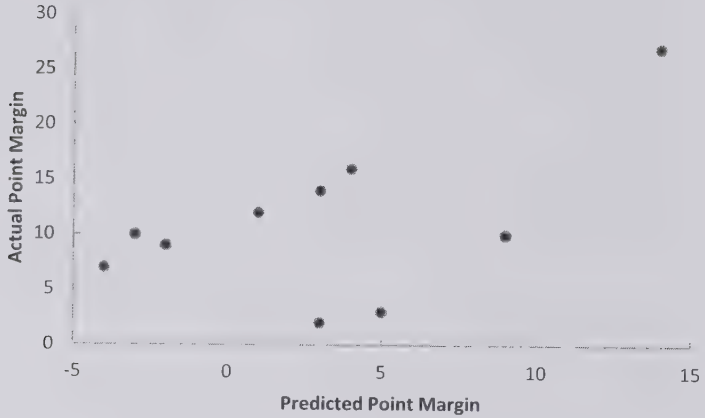
$$r_{xy} = \frac{s_{xy}}{s_x s_y} = \frac{26.5}{(8.2462)(4.6368)} = .693$$

A positive linear relationship

47. a.



b. The scatter diagram shows a positive relationship with higher predicted point margins associated with higher actual point margins.

c. Let x = predicted point margin and y = actual point margin

$$\Sigma x_i = 30 \quad \bar{x} = \frac{30}{10} = 3 \quad \Sigma y_i = 110 \quad \bar{y} = \frac{110}{10} = 11$$

$$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 201 \quad \Sigma(x_i - \bar{x})^2 = 276 \quad \Sigma(y_i - \bar{y})^2 = 458$$

$$s_{xy} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{n-1} = \frac{201}{10-1} = 22.3333$$

A positive covariance shows a positive relationship between predicted point margins and actual point margins.

d.

$$s_x = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{276}{10-1}} = 5.5377$$

$$s_y = \sqrt{\frac{\Sigma(y_i - \bar{y})^2}{n-1}} = \sqrt{\frac{458}{10-1}} = 7.1336$$

$$r_{xy} = \frac{s_{xy}}{s_x s_y} = \frac{22.3333}{(5.5377)(7.1336)} = .565$$

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The modest positive correlation shows that the Las Vegas predicted point margin is a general, but not a perfect, indicator of the actual point margin in college football bowl games.

Note: The Las Vegas odds makers set the point margins so that someone betting on a favored team has to have the team *win by more than the point margin* to win the bet. For example, someone betting on Auburn to win the Outback Bowl would have to have Auburn win by more than five points to win the bet. Since Auburn beat Northwestern by only three points, the person betting on Auburn would have lost the bet.

A review of the predicted and actual point margins shows that the favorites won by more than the predicted point margin in five bowl games: Gator, Sugar, Cotton, Alamo, and the Championship bowl game. The underdog either won its game or kept the actual point margin less than the predicted point margin in the other five bowl games. In this case, betting on the underdog would have provided winners in the Outback, Capital One, Rose, Fiesta and Orange bowls. In this example, the Las Vegas odds point margins made betting on the favored team a 50-50 probability of winning the bet.

48. Let x = miles per hour and y = miles per gallon

$$\Sigma x_i = 420 \quad \bar{x} = \frac{420}{10} = 42 \quad \Sigma y_i = 270 \quad \bar{y} = \frac{270}{10} = 27$$

$$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = -475 \quad \Sigma(x_i - \bar{x})^2 = 1660 \quad \Sigma(y_i - \bar{y})^2 = 164$$

$$s_{xy} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{n-1} = \frac{-475}{10-1} = -52.7778$$

$$s_x = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{1660}{10-1}} = 13.5810$$

$$s_y = \sqrt{\frac{\Sigma(y_i - \bar{y})^2}{n-1}} = \sqrt{\frac{164}{10-1}} = 4.2687$$

$$r_{xy} = \frac{s_{xy}}{s_x s_y} = \frac{-52.7778}{(13.5810)(4.2687)} = -.91$$

A strong negative linear relationship exists. For driving speeds between 25 and 60 miles per hour, higher speeds are associated with lower miles per gallon.

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49. a. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{184}{27} = 6.81$ $\bar{y} = \frac{\Sigma y_i}{n} = \frac{170.93}{27} = 6.33$

x_i	y_i	$(x_i - \bar{x})$	$(y_i - \bar{y})$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
7.1	7.02	0.2852	0.6893	0.0813	0.4751	0.1966
5.2	5.31	-1.6148	-1.0207	2.6076	1.0419	1.6483
7.8	5.38	0.9852	-0.9507	0.9706	0.9039	-0.9367
7.8	5.40	0.9852	-0.9307	0.9706	0.8663	-0.9170
5.8	5.00	-1.0148	-1.3307	1.0298	1.7709	1.3505
5.8	4.07	-1.0148	-2.2607	1.0298	5.1109	2.2942
9.3	6.53	2.4852	0.1993	6.1761	0.0397	0.4952
5.7	5.57	-1.1148	-0.7607	1.2428	0.5787	0.8481
7.3	6.99	0.4852	0.6593	0.2354	0.4346	0.3199
7.6	11.12	0.7852	4.7893	0.6165	22.9370	3.7605
8.2	7.56	1.3852	1.2293	1.9187	1.5111	1.7028
7.1	12.11	0.2852	5.7793	0.0813	33.3998	1.6482
6.3	4.39	-0.5148	-1.9407	0.2650	3.7665	0.9991
6.6	4.78	-0.2148	-1.5507	0.0461	2.4048	0.3331
6.2	5.78	-0.6148	-0.5507	0.3780	0.3033	0.3386
6.3	6.08	-0.5148	-0.2507	0.2650	0.0629	0.1291
7.0	10.05	0.1852	3.7193	0.0343	13.8329	0.6888
6.2	4.75	-0.6148	-1.5807	0.3780	2.4987	0.9719
5.5	7.22	-1.3148	0.8893	1.7287	0.7908	-1.1692
6.5	3.79	-0.3148	-2.5407	0.0991	6.4554	0.7999
6.0	3.62	-0.8148	-2.7107	0.6639	7.3481	2.2088
8.3	9.24	1.4852	2.9093	2.2058	8.4638	4.3208
7.5	4.40	0.6852	-1.9307	0.4695	3.7278	-1.3229
7.1	6.91	0.2852	0.5793	0.0813	0.3355	0.1652
6.8	5.57	-0.0148	-0.7607	0.0002	0.5787	0.0113
5.5	3.87	-1.3148	-2.4607	1.7287	6.0552	3.2354
7.5	8.42	0.6852	2.0893	0.4695	4.3650	1.4315
			Total	25.77407	130.0594	25.5517

$$s_{xy} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{n-1} = \frac{25.5517}{26} = .9828$$

$$s_x = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{25.7741}{26}} = .9956$$

$$s_y = \sqrt{\frac{\Sigma(y_i - \bar{y})^2}{n-1}} = \sqrt{\frac{130.0594}{26}} = 2.2366$$

$$r_{xy} = \frac{s_{xy}}{s_x s_y} = \frac{.9828}{(.9956)(2.2366)} = .4413$$

There is evidence of a modest positive linear association between the jobless rate and the delinquent housing loan percentage. If the jobless rate were to increase, it is likely that an increase in the percentage of delinquent housing loans would also occur.

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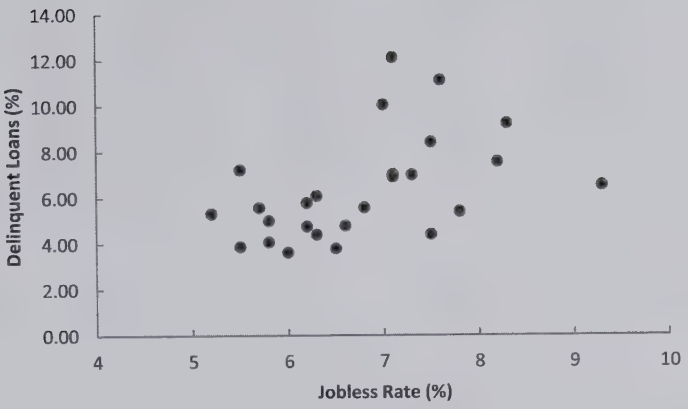
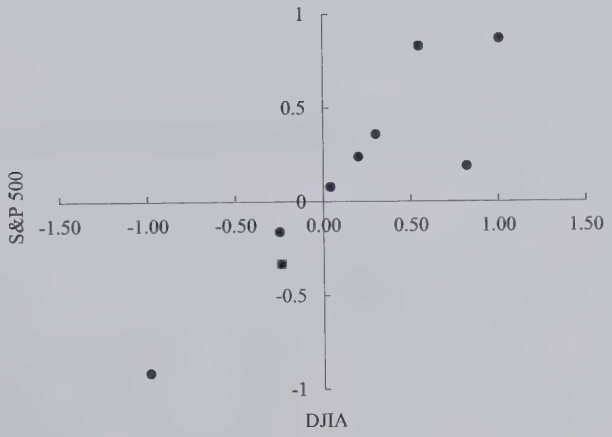
b.



50. a.



b. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{1.44}{9} = .16$

$\bar{y} = \frac{\Sigma x_i}{n} = \frac{1.17}{9} = .13$

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x_i	y_i	$(x_i-\bar{x})$	$(y_i-\bar{y})$	$(x_i-\bar{x})^2$	$(y_i-\bar{y})^2$	$(x_i-\bar{x})(y_i-\bar{y})$
0.20	0.24	0.04	0.11	0.0016	0.0121	0.0044
0.82	0.19	0.66	0.06	0.4356	0.0036	0.0396
-0.99	-0.91	-1.15	-1.04	1.3225	1.0816	1.1960
0.04	0.08	-0.12	-0.05	0.0144	0.0025	0.0060
-0.24	-0.33	-0.40	-0.46	0.1600	0.2166	0.1840
1.01	0.87	0.85	0.74	0.7225	0.5476	0.6290
0.30	0.36	0.14	0.23	0.0196	0.0529	0.0322
0.55	0.83	0.39	0.70	0.1521	0.4900	0.2730
-0.25	-0.16	-0.41	-0.29	0.1681	0.0841	0.1189
			Total	2.9964	2.4860	2.4831

$$s_{xy}=\frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})}{n-1}=\frac{2.4831}{8}=.3104$$

$$s_x=\sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n-1}}=\sqrt{\frac{2.9964}{8}}=.6120$$

$$s_y=\sqrt{\frac{\Sigma(y_i-\bar{y})^2}{n-1}}=\sqrt{\frac{2.4860}{8}}=.5574$$

$$r_{xy}=\frac{s_{xy}}{s_x s_y}=\frac{.3104}{(.6120)(.5574)}=.910$$

c. There is a strong positive linear association between DJIA and S&P 500. If you know the change in either, you will have a good idea of the stock market performance for the day.

51. a. $\bar{x}=\frac{\Sigma x_i}{n}=\frac{945}{14}=67.5$

b. $\bar{y}=\frac{\Sigma y_i}{n}=\frac{706}{14}=50.4286\approx 50.4$

c.

x_i	y_i	$(x_i-\bar{x})$	$(y_i-\bar{y})$	$(x_i-\bar{x})^2$	$(y_i-\bar{y})^2$	$(x_i-\bar{x})(y_i-\bar{y})$
68	50	.5	-.4286	.25	.1837	-.2143
70	49	2.5	-1.4286	6.25	2.0408	-3.5714
65	44	-2.5	-6.4286	6.25	41.3265	16.0714
96	64	28.5	13.5714	812.25	184.1837	386.7857
57	46	-10.5	-4.4286	110.25	19.6122	46.5000
70	45	2.5	-5.4286	6.25	29.4694	-13.5714
80	73	12.5	22.5714	156.25	509.4694	282.1429
67	45	-.5	-5.4286	.25	29.4694	2.7143
44	29	-23.5	-21.4286	552.25	459.1837	503.5714
69	44	1.5	-6.4286	2.25	41.3265	-9.6429
76	69	8.5	18.5714	72.25	344.8980	157.8571
69	51	1.5	.5714	2.25	.3265	.8571
70	58	2.5	7.5714	6.25	57.3265	18.9286
44	39	-23.5	-11.4286	552.25	130.6122	268.5714
			Total	2285.5	1849.4286	1657.0000

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$$s_{xy} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{n-1} = \frac{1657}{14-1} = 127.4615$$

$$s_x = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{2285.5}{14-1}} = 13.2592$$

$$s_y = \sqrt{\frac{\Sigma(y_i - \bar{y})^2}{n-1}} = \sqrt{\frac{1849.4286}{14-1}} = 11.9274$$

$$r_{xy} = \frac{s_{xy}}{s_x s_y} = \frac{127.4615}{13.2592(11.9274)} = +.806$$

High positive correlation as should be expected.

52. a. $\bar{x} = \frac{\Sigma w_i x_i}{\Sigma w_i} = \frac{6(3.2) + 3(2) + 2(2.5) + 8(5)}{6+3+2+8} = \frac{70.2}{19} = 3.69$

b. $\frac{3.2 + 2 + 2.5 + 5}{4} = \frac{12.7}{4} = 3.175$

53.

f_i	M_i	$f_i M_i$
4	5	20
7	10	70
9	15	135
5	20	100
25		325

$$\bar{x} = \frac{\Sigma f_i M_i}{n} = \frac{325}{25} = 13$$

f_i	M_i	$M_i - \bar{x}$	$(M_i - \bar{x})^2$	$f_i(M_i - \bar{x})^2$
4	5	-8	64	256
7	10	-3	9	63
9	15	+2	4	36
5	20	+7	49	245
				600

$$s^2 = \frac{\Sigma f_i (M_i - \bar{x})^2}{n-1} = \frac{600}{24} = 25$$

$$s = \sqrt{25} = 5$$

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54. a.

Grade x_i	Weight w_i
4 (A)	9
3 (B)	15
2 (C)	33
1 (D)	3
0 (F)	0
	60 Credit Hours

$$\bar{x} = \frac{\Sigma w_i x_i}{\Sigma w_i} = \frac{9(4)+15(3)+33(2)+3(1)}{9+15+33+3} = \frac{150}{60} = 2.50$$

b. Yes; satisfies the 2.5 grade point average requirement

55. a. $$\bar{x} = \frac{\Sigma f_i M_i}{N} = \frac{9191(4.65)+2621(18.15)+1419(11.36)+2900(6.75)}{9191+2621+1419+2900}$$

$$= \frac{126,004.14}{16,131} = 7.81$$

The weighted average total return for the Morningstar funds is 7.81%.

b. If the amount invested in each fund was available, it would be better to use those amounts as weights. The weighted return computed in part (a) will be a good approximation, if the amount invested in the various funds is approximately equal.

c. Portfolio Return $= \frac{2000(4.65)+4000(18.15)+3000(11.36)+1000(6.75)}{2000+4000+3000+1000}$

$$= \frac{122,730}{10,000} = 12.27$$

The portfolio return would be 12.27%.

56.

Assessment	Deans	f_iM_i	Recruiters	f_iM_i
5	44	220	31	155
4	66	264	34	136
3	60	180	43	129
2	10	20	12	24
1	0	0	0	0
Total	180	684	120	444

Deans: $\bar{x} = \frac{\Sigma f_i M_i}{n} = \frac{684}{180} = 3.8$

Recruiters: $\bar{x} = \frac{\Sigma f_i M_i}{n} = \frac{444}{120} = 3.7$

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57. a.

Price per Share	Frequency	Midpoint	f_iM_i
\$0-9	4	4.5	18.0
\$10-19	5	14.5	72.5
\$20-29	7	24.5	171.5
\$30-39	3	34.5	103.5
\$40-49	4	44.5	178.0
\$50-59	4	54.5	218.0
\$60-69	0	64.5	0.0
\$70-79	2	74.5	149.0
\$80-89	0	84.5	0.0
\$90-99	1	94.5	94.5
Total	30		1005.0

$$\bar{x} = \frac{\Sigma_i f M_i}{n} = \frac{1005.0}{30} = 33.50$$

Price per Share	Frequency	Midpoint	$M_i - \bar{x}$	$(M_i - \bar{x})^2$	$f_i(M_i - \bar{x})^2$
\$0-9	4	4.5	-29	841	3364
\$10-19	5	14.5	-19	361	1805
\$20-29	7	24.5	-9	81	567
\$30-39	3	34.5	1	1	3
\$40-49	4	44.5	11	121	484
\$50-59	4	54.5	21	441	1764
\$60-69	0	64.5	31	961	0
\$70-79	2	74.5	41	1681	3362
\$80-89	0	84.5	51	2601	0
\$90-99	1	94.5	61	3721	3721
				Total	15070

$$s = \sqrt{\frac{\Sigma f_i (M_i - \bar{x})^2}{n-1}} = \sqrt{\frac{15070}{29}} = 22.80$$

b. The mean price per share had decreased (\$45.83-\$33.50)=\$12.33, or (12.33/45.83)(100) = 26.9% over the three-year period. The standard deviation had increased from \$18.14 to \$22.80 over the same three-year period. In January 2009, the stock market as measured by the Dow Jones Industrial Average companies had declined and was showing more variability.

58. a. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{27000}{15} = 1800$

Median 8th position = 1351

b. Q_1: $i = \left(\frac{25}{100}\right)15 = 3.75$

4th position: $Q_1 = 387$

Q_3: $i = \left(\frac{75}{100}\right)15 = 11.25$

12th position: $Q_3 = 1710$

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c. Range = 7450 - 170 = 7280

IQR = Q_3 - Q_1 = 1710 - 387 = 1323

d. $s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1} = \frac{51,454,242}{15-1} = 3,675,303$

$s = \sqrt{3,675,303} = 1917$

e. High positive skewness. This seems reasonable. A relatively few people will have large monthly expenditures causing the right tail of the distribution to become longer.

f. $z = \frac{x - \bar{x}}{s} = \frac{4135 - 1800}{1917} = 1.22$

$z = \frac{x - \bar{x}}{s} = \frac{7450 - 1800}{1917} = 2.95$ do not indicate outliers.

These values of z do not indicate outliers.

However, the upper limit for outliers is

Q_3 + 1.5(IQR) = 1710 + 1.5(1323) = 3695

Thus, both $4135 and $7450 are outliers.

59. a. Arrange the data in order

Men

21 23 24 25 25 26 26 27 27 27 27 28 28 29 30 30 32 35

Median i = .5(18) = 9

Use 9th and 10th positions

Median = 27

Women

19 20 22 22 23 23 24 25 25 26 26 27 28 29 30

Median i = .5(15) = 7.5

Use 8th position

Median = 25

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b.

	Men	Women
Q_1	$i = .25(18) = 4.5$ Use 5th position $Q_1 = 25$	$i = .25(15) = 3.75$ Use 4th position $Q_1 = 22$
Q_3	$i = .75(18) = 13.5$ Use 14th position $Q_3 = 29$	$i = .75(15) = 11.25$ Use 12th position $Q_3 = 27$

c. Young people today are waiting longer to get married than young people did 25 years ago. The median age for men has increased from 25 to 27. The median age for women has increased from 22 to 25.

60. a. $\bar{x} = \dfrac{\Sigma x_i}{n} = \dfrac{23}{10} = 2.3$

Median: 5th and 6th positions

$$\text{Median} = \frac{1.8+1.9}{2} = 1.85$$

b. $s^2 = \dfrac{\Sigma(x_i - \bar{x})^2}{n-1} = \dfrac{17.08}{10-1} = 1.90$

$s = \sqrt{1.90} = 1.38$

c. Altria Group at 5%

d. $z = \dfrac{x - \bar{x}}{s} = \dfrac{1.6 - 2.3}{1.38} = -.51$

McDonald's is about 1/2 a standard deviation below the mean dividend yield.

e. $z = \dfrac{x - \bar{x}}{s} = \dfrac{3.7 - 2.3}{1.38} = +1.02$

General Motors is about one standard deviation above the mean dividend yield.

f. Altria Group $z = \dfrac{x - \bar{x}}{s} = \dfrac{5.0 - 2.3}{1.38} = +1.96$

Wal-Mart $z = \dfrac{x - \bar{x}}{s} = \dfrac{0.7 - 2.3}{1.38} = -1.16$

No outliers.

61. a. $\bar{x} = \dfrac{\Sigma x_i}{n} = \dfrac{100}{10} = \$10,000$

Mean debt upon graduation is $10,000.

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b. $s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1} = \frac{221.78}{9} = 24.64$

$s = \sqrt{24.64} = 4.96$

62. a. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{13,400}{20} = 670$

b. $s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{3,949,200}{20-1}} = \$456$

c. $z = \frac{x - \bar{x}}{s} = \frac{2040 - 670}{456} = 3.00$

Yes it is an outlier.

d. First of all, the employee payroll service will be up to date on tax regulations. This will save the small business owner the time and effort of learning tax regulations. This will enable the owner greater time to devote to other aspects of the business. In addition, a correctly filed employment tax return will reduce the potential of a tax penalty.

63. a. Public Transportation: $\bar{x} = \frac{320}{10} = 32$

Automobile: $\bar{x} = \frac{320}{10} = 32$

b. Public Transportation: $s = 4.64$

Automobile: $s = 1.83$

c. Prefer the automobile. The mean times are the same, but the auto has less variability.

d. Data in ascending order:

Public: 25 28 29 29 32 32 33 34 37 41

Auto: 29 30 31 31 32 32 33 33 34 35

Five number Summaries

Public: 25 29 32 34 41

Auto: 29 31 32 33 35

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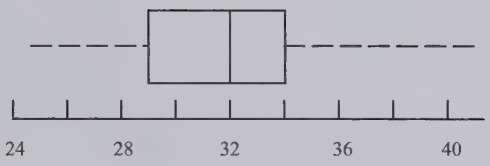
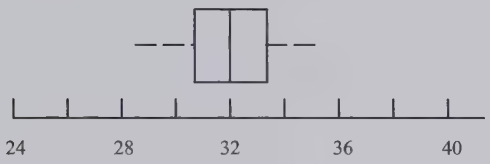
Box Plots:

Public:



Auto:



The box plots do show lower variability with automobile transportation and support the conclusion in part c.

64. a. Arrange the data in ascending order

48.8 92.6 111.0 ..... 958.0 995.9 2325.0

With $n = 14$, the median is the average of home prices in position 7 and 8.

$$\text{Median home price} = \frac{212.9 + 218.9}{2} = 215.9$$

Median home price = $215,900

b. $$\frac{215{,}900 - 139{,}300}{139{,}300} = .55$$

55% increase over the five-year period

c. $n = 14$

$$i = \frac{25}{100}(n) = 3.5$$

Use the 4th position

$Q_1 = 175.0$

$$i = \frac{75}{100}(n) = 10.5$$

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Use the 11^{th} position

$Q_3 = 628.3$

$$Q_1 = \frac{362.5 + 628.3}{2} = 495.4$$

d. Lowest price = 48.8 and highest price = 2324.0.

Five-number summary: 48.8, 175.0, 215.9, 628.3, 2325.0

e. IQR = Q_3 - Q_1 = 628.3 – 175.0 = 453.3

Upper limit = Q_3 + 1.5IQR = 628.3 + 1.5(679.95) = 1308.25

Any price over $1,308,250 is an outlier.

Yes, the price $2,325,000 is an outlier.

f. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{6749.4}{14} = 482.1$

The mean is sensitive to extremely high home prices and tends to overstate the more typical midrange home price. The sample mean of $482,100 has 79% of home prices below this value and 21% of the home prices above this value while the sample median $215,900 has 50% above and 50% below. The median is more stable and not influenced by the extremely high home prices. Using the sample mean $482,100 would overstate the more typical or middle home price.

65. a. Median for $n = 50$; Use 25th and 26th positions

25^{th} – South Dakota 16.8

26^{th} – Pennsylvania 16.9

$$\text{Median} = \frac{16.8 + 16.9}{2} = 16.85\%$$

b. Q_1: $i = \left(\frac{25}{100}\right)50 = 12.5$

13th position: Q_1 = 13.7% (Iowa)

Q_3: $i = \left(\frac{75}{100}\right)50 = 37.5$

38^{th} position: Q_3 = 20.2% (North Carolina & Georgia)

25% of the states have a poverty level less than or equal to 13.7% and 25% of the states have a poverty level greater than or equal to 20.2%

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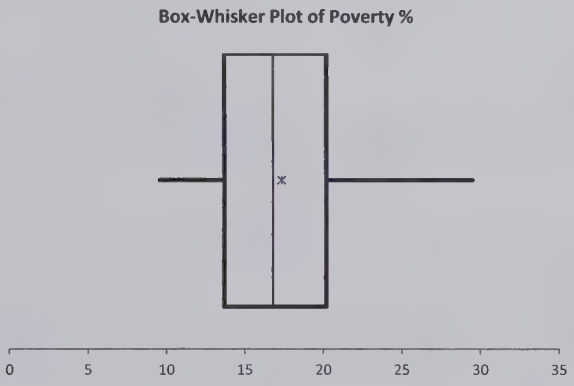
c. IQR = $Q_3 - Q_1$ = 20.2 – 13.7 = 6.5

Upper Limit = Q_3 + 1.5(IQR)

= 20.2 + 1.5(6.5) = 29.95

Lower Limit = Q_1 - 1.5(IQR)

= 13.7 - 1.5(6.5) = 3.95



The StatTools box plot shows the distribution of poverty levels is skewed to the right (positive). There are no states considered outliers. Mississippi with 29.5% is closest to being an outlier on the high poverty rate side. New Hampshire has the lowest poverty level with 9.6%. The five-number summary is 9.6, 13.7, 16.85, 20.2 and 29.95.

d. The states in the lower quartile are the states with the lowest percentage of children who have lived below the poverty level in the last 12 months. These states are as follows.

State	Region	Poverty %
New Hampshire	NE	9.6
Maryland	NE	9.7
Connecticut	NE	11.0
Hawaii	W	11.4
New Jersey	NE	11.8
Utah	W	11.9
Wyoming	W	12.0
Minnesota	MW	12.2
Virginia	SE	12.2
Massachusetts	NE	12.4
North Dakota	MW	13.0
Vermont	NE	13.2

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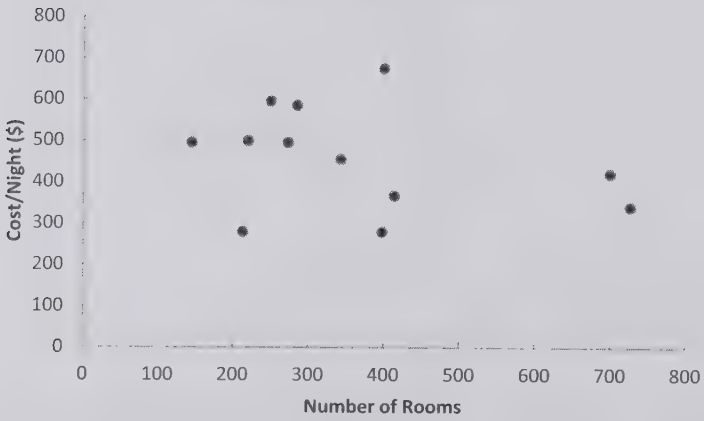
Generally, these states are the states with better economic conditions and less poverty. The Northeast region with 6 of the 12 states in this quartile appears to be the best economic region of the country. The West region was second with 3 of the 12 states in this group.

66. a. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{4368}{12} = 364$ rooms

b. $\bar{y} = \frac{\Sigma y_i}{n} = \frac{5484}{12} = \$457$

c.



It is difficult to see much of a relationship. When the number of rooms becomes larger, there is no indication that the cost per night increases. The cost per night may even decrease slightly.

d.

x_i	y_i	$(x_i - \bar{x})$	$(y_i - \bar{y})$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
220	499	-144	42	20,736	1,764	-6,048
727	340	363	-117	131,769	13,689	-42,471
285	585	-79	128	6,241	16,384	-10,112
273	495	-91	38	8,281	1,444	-3,458
145	495	-219	38	47,961	1,444	-8,322
213	279	-151	-178	22,801	31,684	26,878
398	279	34	-178	1,156	31,684	-6,052
343	455	-21	-2	441	4	42
250	595	-114	138	12,996	19,044	-15,732
414	367	50	-90	2,500	8,100	-4,500
400	675	36	218	1,296	47,524	7,848
700	420	336	-37	112,896	1,369	-12,432
			Total	69,074	174,134	-74,359

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$$s_{xy} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{n-1} = \frac{-74,350}{11} = -6759.91$$

$$s_x = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{369,074}{11}} = 183.17$$

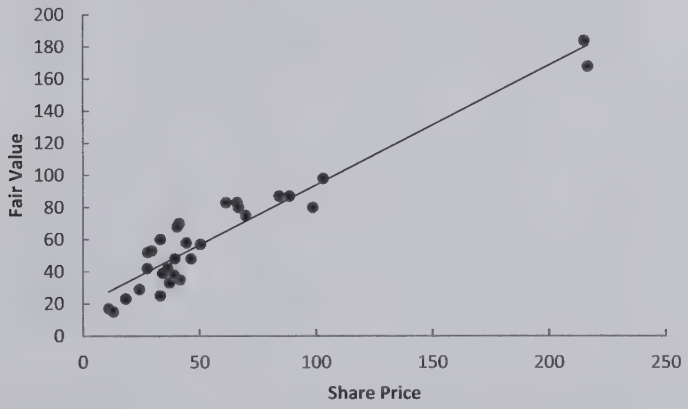
$$s_y = \sqrt{\frac{\Sigma(y_i - \bar{y})^2}{n-1}} = \sqrt{\frac{174,134}{11}} = 125.82$$

$$r_{xy} = \frac{s_{xy}}{s_x s_y} = \frac{-6759.91}{(183.17)(125.82)} = -.293$$

There is evidence of a slightly negative linear association between the number of rooms and the cost per night for a double room. Although this is not a strong relationship, it suggests that the higher room rates tend to be associated with the smaller hotels.

This tends to make sense when you think about the economies of scale for the larger hotels. Many of the amenities in terms of pools, equipment, spas, restaurants, and so on exist for all hotels in the *Travel + Leisure* top 50 hotels in the world. The smaller hotels tend to charge more for the rooms. The larger hotels can spread their fixed costs over many room and may actually be able to charge less per night and still achieve and nice profit. The larger hotels may also charge slightly less in an effort to obtain a higher occupancy rate. In any case, it appears that there is a slightly negative linear association between the number of rooms and the cost per night for a double room at the top hotels.

67. a. The scatter diagram is shown below.



The sample correlation coefficient is .954. This indicates a strong positive linear relationship between Morningstar's Fair Value estimate per share and the most recent price per share for the stock.

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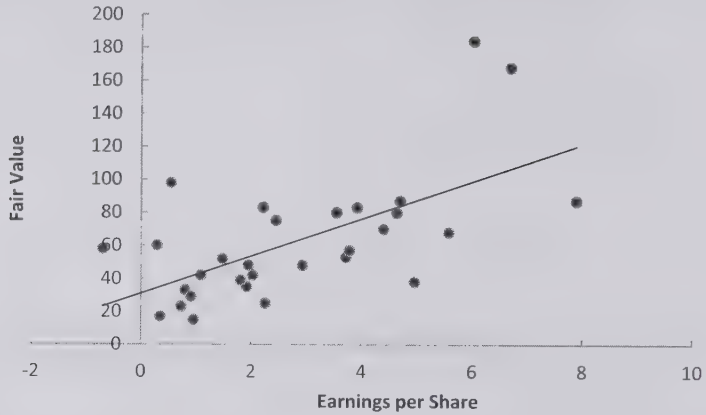
b. The scatter diagram is shown below:



The sample correlation coefficient is .624. While not a strong of a relationship as shown in part a, this indicates a positive linear relationship between Morningstar's Fair Value estimate per share and the earnings per share for the stock.

68. a.

x_i	y_i	$(x_i-\bar{x})$	$(y_i-\bar{y})$	$(x_i-\bar{x})^2$	$(y_i-\bar{y})^2$	$(x_i-\bar{x})(y_i-\bar{y})$
.407	.422	-.1458	-.0881	.0213	.0078	.0128
.429	.586	-.1238	.0759	.0153	.0058	-.0094
.417	.546	-.1358	.0359	.0184	.0013	-.0049
.569	.500	.0162	-.0101	.0003	.0001	-.0002
.569	.457	.0162	-.0531	.0003	.0028	-.0009
.533	.463	-.0198	-.0471	.0004	.0022	.0009
.724	.617	.1712	.1069	.0293	.0114	.0183
.500	.540	-.0528	.0299	.0028	.0009	-.0016
.577	.549	.0242	.0389	.0006	.0015	.0009
.692	.466	.1392	-.0441	.0194	.0019	-.0061
.500	.377	-.0528	-.1331	.0028	.0177	.0070
.731	.599	.1782	.0889	.0318	.0079	.0158
.643	.488	.0902	-.0221	.0081	.0005	-.0020
.448	.531	-.1048	.0209	.0110	.0004	-.0022
			Total	.1617	.0623	.0287

$$s_{xy} = \frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})}{n-1} = \frac{.0287}{14-1} = .0022$$

$$s_x = \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n-1}} = \sqrt{\frac{.1617}{14-1}} = .1115$$

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$$s_y = \sqrt{\frac{\Sigma(y_i - \bar{y})^2}{n-1}} = \sqrt{\frac{.0623}{14-1}} = .0692$$

$$r_{xy} = \frac{s_{xy}}{s_x s_y} = \frac{.0022}{.1115(.0692)} = +.286$$

b. There is a low positive correlation between a major league baseball team's winning percentage during spring training and its winning percentage during the regular season. The spring training record should not be expected to be a good indicator of how a team will play during the regular season.

Spring training consists of practice games between teams with the outcome as to who wins or who loses not counting in the regular season standings or affecting the chances of making the playoffs. Teams use spring training to help players regain their timing and evaluate new players. Substitutions are frequent with the regular or better players rarely playing an entire spring training game. Winning is not the primary goal in spring training games. A low correlation between spring training winning percentage and regular season winning percentage should be anticipated.

69. $$\bar{x} = \frac{\Sigma w_i x_i}{\Sigma w_i} = \frac{20(20)+30(12)+10(7)+15(5)+10(6)}{20+30+10+15+10} = \frac{965}{85} = 11.4 \text{ days}$$

70.

f_i	M_i	$f_i M_i$	$M_i - \bar{x}$	$(M_i - \bar{x})^2$	$f_i(M_i - \bar{x})^2$
10	47	470	-13.68	187.1424	1871.42
40	52	2080	-8.68	75.3424	3013.70
150	57	8550	-3.68	13..5424	2031.36
175	62	10850	+1.32	1.7424	304.92
75	67	5025	+6.32	39.9424	2995.68
15	72	1080	+11.32	128.1424	1922.14
10	77	770	+16.32	266.3424	2663.42
475		28,825			14,802.64

a. $$\bar{x} = \frac{28,825}{475} = 60.68$$

b. $$s^2 = \frac{14,802.64}{474} = 31.23$$

$$s = \sqrt{31.23} = 5.59$$

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Chapter 4
Introduction to Probability

Learning Objectives

1. Obtain an appreciation of the role probability information plays in the decision making process.
2. Understand probability as a numerical measure of the likelihood of occurrence.
3. Know the three methods commonly used for assigning probabilities and understand when they should be used.
4. Know how to use the laws that are available for computing the probabilities of events.
5. Understand how new information can be used to revise initial (prior) probability estimates using Bayes' theorem.

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Solutions:

1. Number of experimental Outcomes = (3)(2)(4) = 24

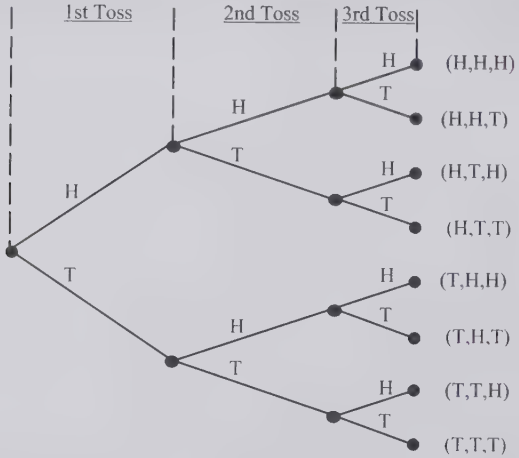
2. $\binom{6}{3} = \frac{6!}{3!3!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(3 \cdot 2 \cdot 1)(3 \cdot 2 \cdot 1)} = 20$

ABC	ACE	BCD	BEF
ABD	ACF	BCE	CDE
ABE	ADE	BCF	CDF
ABF	ADF	BDE	CEF
ACD	AEF	BDF	DEF

3. $P_3^6 = \frac{6!}{(6-3)!} = (6)(5)(4) = 120$

BDF BFD DBF DFB FBD FDB

4. a.



b. Let: H be head and T be tail

(H,H,H) (T,H,H)
(H,H,T) (T,H,T)
(H,T,H) (T,T,H)
(H,T,T) (T,T,T)

c. The outcomes are equally likely, so the probability of each outcome is 1/8.

5. $P(E_i) = 1/5$ for i = 1, 2, 3, 4, 5

$P(E_i) \geq 0$ for i = 1, 2, 3, 4, 5

$P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) = 1/5 + 1/5 + 1/5 + 1/5 + 1/5 = 1$

The classical method was used.

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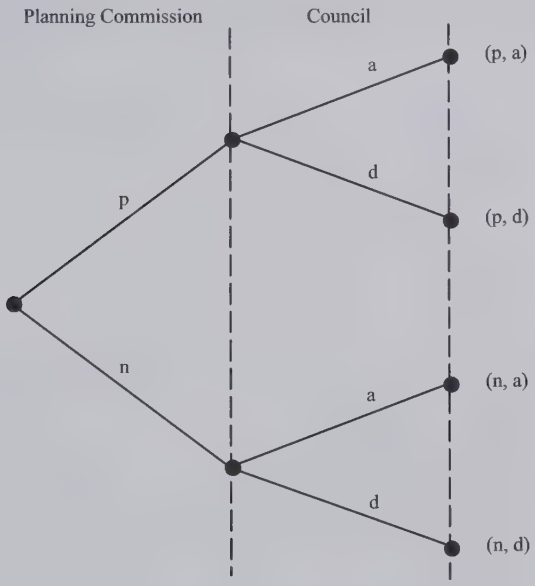
6. $P(E_1) = .40,\ P(E_2) = .26,\ P(E_3) = .34$

   The relative frequency method was used.

7. No. Requirement (4.4) is not satisfied; the probabilities do not sum to 1. $P(E_1) + P(E_2) + P(E_3) + P(E_4) = .10 + .15 + .40 + .20 = .85$

8. a. There are four outcomes possible for this 2-step experiment; planning commission positive - council approves; planning commission positive - council disapproves; planning commission negative - council approves; planning commission negative - council disapproves.

   b. Let p = positive, n = negative, a = approves, and d = disapproves



9. $\binom{50}{4} = \frac{50!}{4!46!} = \frac{50 \cdot 49 \cdot 48 \cdot 47}{4 \cdot 3 \cdot 2 \cdot 1} = 230{,}300$

10. a. Using the table provided, 94% of students graduating from Morehouse College have debt.

    $P(\text{Debt}) = .94$

    b. Five of the 8 institutions have over 60% of their graduates with debt.

    $P(\text{over } 60\%) = 5/8 = .625$

    c. Two of the 8 institutions have graduates with debt who have an average debt more than $30,000.

    $P(\text{more than } \$30{,}000) = 2/8 = .25$

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d. $P(\text{No debt}) = 1 - P(\text{Debt}) = 1 - .72 = .28$

e. This is a weighted average calculation. 72% graduate with an average debt of $32,980 and 28% graduate with a debt of $0.

$$\text{Average debt per graduate} = \frac{.72(\$32,980) + .28(\$0)}{.72 + .28} = \$23,746$$

11. a. Total motorcyclists = 350 + 170 = 520

$$P(\text{DOT-Compliant Helmet}) = \frac{350}{520} = .6731$$

b. Yes, the overall probability has been increasing from .48 five years ago, to .63 one year ago, and is now approximately .67. The probability that a motorcyclist wears a DOT-compliant helmet appears to be increasing.

c. Northeast: $\frac{96}{158} = .6076$

Midwest: $\frac{86}{129} = .6667$

South: $\frac{92}{141} = .6525$

West: $\frac{76}{92} = .8261$

The West region shows the highest probability (.8261) of DOT-compliant helmet use.

12. a. Use the counting rule for combinations:

$$\binom{55}{5} = \frac{55!}{5!50!} = \frac{(55)(54)(53)(52)(51)}{(5)(4)(3)(2)(1)} = 3,478,761$$

One chance in 3,489,761

b. Very small: 1/3,478,761 = .000000287

c. Multiply the answer in part (a) by 42 to get the number of choices for the six numbers.

Number of Choices = (3,478,761)(42) = 146,107,962

Probability of Winning = 1/146,107,962 = .00000000684

13. Initially a probability of .20 would be assigned if selection is equally likely. Data does not appear to confirm the belief of equal consumer preference. For example using the relative frequency method we would assign a probability of 5/100 = .05 to the design 1 outcome, .15 to design 2, .30 to design 3, .40 to design 4, and .10 to design 5.

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14. a. $P(E_2) = 1/4$

    b. P(any 2 outcomes) = 1/4 + 1/4 = 1/2

    c. P(any 3 outcomes) = 1/4 + 1/4 + 1/4 = 3/4

15. a. S = {ace of clubs, ace of diamonds, ace of hearts, ace of spades}

    b. S = {2 of clubs, 3 of clubs, . . . , 10 of clubs, J of clubs, Q of clubs, K of clubs, A of clubs}

    c. There are 12; jack, queen, or king in each of the four suits.

    d. For a: 4/52 = 1/13 = .08

       For b: 13/52 = 1/4 = .25

       For c: 12/52 = .23

16. a. (6)(6) = 36 sample points

    b.

Die 1 \ Die 2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

← Total for Both

    c. 6/36 = 1/6

    d. 10/36 = 5/18

    e. No. P(odd) = 18/36 = P(even) = 18/36 or 1/2 for both.

    f. Classical. A probability of 1/36 is assigned to each experimental outcome.

17. a. (4,6), (4,7), (4,8)

    b. .05 + .10 + .15 = .30

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c. (2,8), (3,8), (4,8)

d. .05 + .05 + .15 = .25

e. .15

18. a. $P(\text{no meals}) = \frac{11}{496} = .0222$

b. $P(\text{at least four meals}) = P(4) + P(5) + P(6) + P(7 \text{ or more})$

$$= \frac{36}{496} + \frac{119}{496} + \frac{114}{496} + \frac{139}{496} = .8226$$

c. $P(\text{two or fewer meals}) = P(2) + P(1) + P(0)$

$$= \frac{30}{496} + \frac{11}{496} + \frac{11}{496} \quad .1048$$

19. a. A summary of the data provided in the exercise follows:

Response	United States	Great Britain	Total
Yes	187	197	384
No	334	411	745
Unsure	256	213	469
Total	777	821	1598

Probability = 334/777 = .4299

b. Probability = (411 + 213)/821 = .76

c. Probability = (334 + 411) /1598 = .4662

d. The probability that an investor in the United States thinks the government is adequately protecting investors is 187/777 =.2407; for investors in Great Britain the probability is 197/821 = .24. The two probabilities are almost identical; thus, there does not appear to be a difference between the perceptions of investors in these two countries with regard to the "Yes" response.

However, in part (a) we showed that the probability that an investor in the United States does not think the government is adequately protecting investors is.4299, or approximately .43; for investors in Great Britain the probability is 411/821 = .5006 or approximately .50. These results show a slightly higher probability that an investor in Great Britain will say that the government is not protecting investors adequately.

20. a. $P(N) = 54/500 = .108$

b. $P(T) = 48/500 = .096$

c. Total in 5 states = 54 + 52 + 48 + 33 + 30 = 217

$P(B) = 217/500 = .434$

Almost half the Fortune 500 companies are headquartered in these five states.

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21. a. Use the relative frequency method. Divide by the total adult population of 227.6 million.

Age	Number	Probability
18 to 24	29.8	0.1309
25 to 34	40.0	0.1757
35 to 44	43.4	0.1907
45 to 54	43.9	0.1929
55 to 64	32.7	0.1437
65 and over	37.8	0.1661
Total	227.6	1.0000

b. $P(18 \text{ to } 24) = .1309$

c. $P(18 \text{ to } 34) = .1309 + .1757 = .3066$

d. $P(45 \text{ or older}) = .1929 + .1437 + .1661 = .5027$

22. a. $P(A) = .40$, $P(B) = .40$, $P(C) = .60$

b. $P(A \cup B) = P(E_1, E_2, E_3, E_4) = .80$. Yes $P(A \cup B) = P(A) + P(B)$.

c. $A^c = \{E_3, E_4, E_5\}$ $C^c = \{E_1, E_4\}$ $P(A^c) = .60$ $P(C^c) = .40$

d. $A \cup B^c = \{E_1, E_2, E_5\}$ $P(A \cup B^c) = .60$

e. $P(B \cup C) = P(E_2, E_3, E_4, E_5) = .80$

23. a. $P(A) = P(E_1) + P(E_4) + P(E_6) = .05 + .25 + .10 = .40$

$P(B) = P(E_2) + P(E_4) + P(E_7) = .20 + .25 + .05 = .50$

$P(C) = P(E_2) + P(E_3) + P(E_5) + P(E_7) = .20 + .20 + .15 + .05 = .60$

b. $A \cup B = \{E_1, E_2, E_4, E_6, E_7\}$

$$P(A \cup B) = P(E_1) + P(E_2) + P(E_4) + P(E_6) + P(E_7)$$
$$= .05 + .20 + .25 + .10 + .05 = .65$$

c. $A \cap B = \{E_4\}$ $P(A \cap B) = P(E_4) = .25$

d. Yes, they are mutually exclusive.

e. $B^c = \{E_1, E_3, E_5, E_6\}$; $P(B^c) = P(E_1) + P(E_3) + P(E_5) + P(E_6)$
$= .05 + .20 + .15 + .10 = .50$

24. Let E = experience exceeded expectations
M = experience met expectations

a. Percentage of respondents that said their experience exceeded expectations
$= 100 - (4 + 26 + 65) = 5\%$

$P(E) = .05$

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b. $P(M \cup E) = P(M) + P(E) = .65 + .05 = .70$

25. Let M = male young adult living in his parents' home
F = female young adult living in her parents' home

a. $P(M \cup F) = P(M) + P(F) - P(M \cap F)$
$= .56 + .42 - .24 = .74$

b. $1 - P(M \cup F) = 1 - .74 = .26$

26. a. Let D = Domestic Equity Fund

$P(D) = 16/25 = .64$

b. Let A = 4- or 5-star rating

13 funds were rated 3-star of less; thus, 25 - 13 = 12 funds must be 4-star or 5-star.

$P(A) = 12/25 = .48$

c. 7 Domestic Equity funds were rated 4-star and 2 were rated 5-star. Thus, 9 funds were Domestic Equity funds and were rated 4-star or 5-star

$P(D \cap A) = 9/25 = .36$

d. $P(D \cup A) = P(D) + P(A) - P(D \cap A)$

$= .64 + .48 - .36 = .76$

27. Let A = the event the ACC has a team in the championship game
S = the event the SEC has a team in the championship game

a. $P(A) = \dfrac{10}{20} = .50$

b. $P(S) = \dfrac{8}{20} = .40$

c. $P(A \cap S) = \dfrac{1}{20} = .05$

There is a low probability that teams from both the ACC and SEC will be in the championship game.

d. $P(A \cup S) = P(A) + P(S) - P(A \cap S) = .50 + .40 - .05 = .85$

There is a high probability that a team from the ACC or SEC will be in the championship game.

e. $P(\text{Neither conference}) = 1 - P(A \cup S) = 1 - .85 = .15$

In this case, teams will most likely come from the Big Ten (6), Big East (4), Pac-10 (4), or Big 12 (3). Numbers shown are the number of times teams from these conferences have played in the national championship game over the previous 20 years.

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28. Let: B = rented a car for business reasons
P = rented a car for personal reasons

a. $P(B \cup P) = P(B) + P(P) - P(B \cap P)$
$= .54 + .458 - .30 = .698$

b. $P(\text{Neither}) = 1 - .698 = .302$

29. a. $P(E) = \dfrac{1033}{2851} = .3623$

$P(R) = \dfrac{854}{2851} = .2995$

$P(D) = \dfrac{964}{2851} = .3381$

b. Yes; $P(E \cap D) = 0$

c. Probability $= \dfrac{1033}{2375} = .4349$

d. Let F denote the event that a student who applies for early admission is deferred and later admitted during the regular admission process.

Events E and F are mutually exclusive and the addition law applies.

$P(E \cup F) = P(E) + P(F)$

$P(E) = .3623$ from part (a)

Of the 964 early applicants who were deferred, we expect 18%, or .18(964) students, to be admitted during the regular admission process. Thus, for the total of 2851 early admission applicants

$P(F) = \dfrac{.18(964)}{2851} = .0609$

$P(E \cup F) = P(E) + P(F) = .3623 + .0609 = .4232$

Note: .18(964) = 173.52. Some students may round this to 174 students. If rounding is done, the answer becomes .4233. Either approach is acceptable.

30. a. $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.40}{.60} = .6667$

b. $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{.40}{.50} = .80$

c. No because $P(A \mid B) \neq P(A)$

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31. a. $P(A \cap B) = 0$

b. $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0}{.4} = 0$

c. No. $P(A \mid B) \neq P(A)$; $\therefore$ the events, although mutually exclusive, are not independent.

d. Mutually exclusive events are dependent.

32. a. Row and column sums are shown.

	Car	Light Truck	Total
U.S.	87.4	193.1	280.5
Non U.S.	228.5	148.0	376.5
Total	315.9	341.1	657.0

A total of 657.0 thousand vehicles were sold.

Dividing each entry in the table by 657.0 provides the following joint probability table.

	Car	Light Truck	Total
U.S.	.1330	.2939	.4269
Non U.S.	.3478	.2253	.5731
Total	.4808	.5192	1.0000

b. Let U = U. S. manufacturer
N = Non U.S. manufacturer
C = Car
L = Light Truck

Marginal probabilities: $P(U) = .4269$ $P(B) = .5731$
There is a higher probability that the vehicle was not manufactured by a U. S. auto maker. In terms of market share, non U.S. auto makers lead with a 57.3% share of vehicle sales.

Marginal probabilities: $P(C) = .4808$ $P(L) = .5192$
The light truck category which includes pickup, minivans, SUVs and crossover models has a slightly higher probability. But the types of vehicles are fairly even split.

c. $P(C|U) = \dfrac{P(C \cap U)}{P(U)} = \dfrac{.1330}{.4269} = .3115$ $\quad P(L|U) = \dfrac{P(L \cap U)}{P(L)} = \dfrac{.2939}{.4269} = .6885$

If a vehicle was manufactured by one of the U.S. auto makers, there is a higher probability it will be in the light truck category.

d. $P(C|N) = \dfrac{P(C \cap N)}{P(N)} = \dfrac{.3478}{.5731} = .6069$ $\quad P(L|N) = \dfrac{P(L \cap N)}{P(L)} = \dfrac{.2253}{.5731} = .3931$

If a vehicle was not manufactured by one of the U.S. auto makers, there is a higher probability it will be a car.

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e. $P(U|L) = \dfrac{P(U \cap L)}{P(L)} = \dfrac{.2939}{.5192} = .5661$

If a vehicle was a light truck, there is better than a 50-50 chance that it was manufactured by one of the U.S. auto makers.

f. There is a higher probability, and thus a larger market share for non U.S. auto makers. However, the U. S. auto makers are leaders in sales for the light truck category.

33. a.

	Reason for Applying			
	Quality	Cost/Convenience	Other	Total
Full Time	.218	.204	.039	.461
Part Time	.208	.307	.024	.539
	.426	.511	.063	1.000

b. It is most likely a student will cite cost or convenience as the first reason - probability = .511. School quality is the first reason cited by the second largest number of students - probability = .426.

c. P(Quality | full time) = .218/.461 = .473

d. P(Quality | part time) = .208/.539 = .386

e. For independence, we must have $P(A)P(B) = P(A \cap B)$.

From the table, $P(A \cap B) = .218$, $P(A) = .461$, $P(B) = .426$

$P(A)P(B) = (.461)(.426) = .196$

Because $P(A)P(B) \neq P(A \cap B)$, the events are not independent.

34. a. Let O = flight arrives on time
O^c = flight arrives late
S = Southwest flight
U = US Airways flight
J = JetBlue flight

Given: $P(O \mid S) = .834$ $P(O \mid U) = .751$ $P(O \mid J) = .701$

$P(S) = .40$ $P(U) = .35$ $P(J) = .25$

$$P(O \mid S) = \frac{P(O \cap S)}{P(S)}$$

$\therefore P(O \cap S) = P(O \mid S)P(S) = (.834)(.4) = .3336$

Similarly

$P(O \cap U) = P(O \mid U)P(U) = (.751)(.35) = .2629$

$P(O \cap J) = P(O \mid J)P(J) = (.701)(.25) = .1753$

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Joint probability table

	On time	Late	Total
Southwest	.3336	.0664	.40
US Airways	.2629	.0871	.35
JetBlue	.1753	.0747	.25
Total:	.7718	.2282	1.00

b. Southwest Airlines; $P(S) = .40$

c. $P(O) = P(S \cap O) + P(U \cap O) + P(J \cap O) = .3336 + .2629 + .1753 = .7718$

d. $P(S|O^c) = \dfrac{P(S \cap O^c)}{P(O^c)} = \dfrac{.0664}{.2282} = .2910$

Similarly, $P(U|O^c) = \dfrac{.0871}{.2282} = .3817$

$P(J|O^c) = \dfrac{.0747}{.2282} = .3273$

Most likely airline is US Airways; least likely is Southwest

35. a. The total sample size is 200. Dividing each entry by 200 provides the following joint probability table.

		Pay Rent		
		Yes	No	
Buy a Car	Yes	.28	.26	.54
	No	.07	.39	.46
		.35	.65	

b. Let C = the event of financial assistance to buy a car
R = the event of financial assistance to pay rent

Using the marginal probabilities, $P(C) = .54$ and $P(R) = .35$. Parents are more likely to provide their adult children with financial assistance to buy a car. The probability of financial assistance to buy a car is .54 and the probability of financial assistance to pay rent is .35.

c. $P(R|C) = \dfrac{P(R \cap C)}{P(C)} = \dfrac{.28}{.54} = .5185$

d. $P(R|C^c) = \dfrac{P(R \cap C^c)}{P(C^c)} = \dfrac{.07}{.46} = .1522$

e. Financial assistance to buy a car is not independent of financial assistance to pay rent, $P(R|C) \neq P(R)$.

If there is financial assistance to buy a car, the probability of financial assistance to pay rent increases from .35 to .5185. However, if there is no financial assistance to buy a car, the probability of financial assistance to pay rent decreases from .35 to .1522.

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f. $P(C \cup R) = P(C) + P(R) - P(R \cap C) = .54 + .35 - .28 = .61$

36. a. Let A = makes 1st free throw
B = makes 2nd free throw

Assuming independence, $P(A \cap B) = P(A)P(B) = (.89)(.89) = .7921$

b. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = (.89)(.89) - .7921 = .9879$

c. P(Miss Both) = 1 - P(at least one) = 1 - .9878 = .0121

d. For this player use $P(A) = .58$

$P(A \cap B) = (.58)(.58) = .3364$

$P(A \cup B) = .58 + .58 - .3364 = .8236$

P(Miss Both) = 1 - .8236 = .1764

The probability Jerry Stackhouse makes both free throws is .7921, while the center's probability is .3364. The probability Jerry Stackhouse misses both free throws is only .0121, while the center's probability is a much higher, .1764. The opponent's strategy should be to foul the center and not Jerry Stackhouse.

37. Let C = event consumer uses a plastic card
B = event consumer is 18 to 24 years old
B^c = event consumer is over 24 years old

Given information:

$P(C) = .37$

$P(B|C) = .19$

$P(B^c|C) = .81$

$P(B) = .14$

a. $P(C|B) = \dfrac{P(C \cap B)}{P(B)}$

but $P(C \cap B)$ is unknown. So first compute

$$P(C \cap B) = P(C)P(B|C) = .37(.19) = .0703$$

Then

$$P(C|B) = \frac{P(C \cap B)}{P(B)} = \frac{.0703}{.14} = .5021$$

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b. $P(C|B^c) = \dfrac{P(C \cap B^c)}{P(B^c)}$

but $P(C \cap B^c)$ and $P(B^c)$ are unknown. However, they can be computed as follows.

$$P(C \cap B^c) = P(C)P(B^c|C)$$
$$= .37(.81) = .2997$$

$$P(B^c) = 1 - P(B) = 1 - .14 = .86$$

Then

$$P(C|B^c) = \frac{P(C \cap B^c)}{P(B^c)} = \frac{.2997}{.86} = .3485$$

c. There is a higher probability that the younger consumer, age 18 to 24, will use plastic when making a purchase. The probability that the 18 to 24 year old consumer uses plastic is .5021 and the probability that the older than 24 year old consumer uses plastic is .3485. Note that there is greater than .50 probability that the 18 to 24 years old consumer will use plastic.

d. Companies such as Visa, Mastercard and Discovery want their cards in the hands of consumers who will have a high probability of using the card. So yes, these companies should get their cards in the hands of young consumers even before these consumers have established a credit history. The companies should place a low limit of the amount of credit charges until the young consumer has demonstrated the responsibility to handle higher credit limits.

38. a. The data table with row and column totals is shown below. Note that a total of 423,392 students took the state mathematics exam.

	Met Proficiency Standards?		
Grade	Yes	No	Total
3	47,401	23,975	71,376
4	35,020	34,740	69,760
5	36,062	33,540	69,602
6	36,361	32,929	69,290
7	40,945	29,768	70,713
8	40,720	31,931	72,651
Total	236,509	186,883	423,392

The joint probability table follows. The probabilities were computed by dividing each entry in the above table by the total number of students taking the state mathematics exam: 423,392.

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Grade	Met Proficiency Standards Yes	No	Total
3	.11196	.05663	0.16858
4	.08271	.08205	0.16476
5	.08517	.07922	0.16439
6	.08588	.07777	0.16365
7	.09671	.07031	0.16702
8	.09618	.07542	0.17159
Total	.55861	.44139	1.00000

For example, let $G3$ = event that a student is in the third grade and S = event that the student met the proficiency standards on the exam. The joint probability in the Grade 3 row and the Yes column is

$$P(G3 \cap S) = 47,401 / 423,392 = .11196$$

b. The column marginal probabilities are .55861 and .44139. The marginal probability of .55861 is the probability that a randomly selected student met the proficiency standards on the exam and the marginal probability of .44139 is the probability that a randomly selected student did not meet the proficiency standards on the exam. There is a slightly better than 50-50 chance that a student met the proficiency standards on the exam.

The row marginal probabilities of .16858, .16476, and so on show the probability that a random selected student is in each grade. Because the number of students in each grade is approximately the same, the probabilities are very similar. Note that the highest probability of .17159 is the probability that a randomly selected student who took the exam is in the 8th grade.

c. $$P(S|G3) = \frac{P(S \cap G3)}{P(G3)} = \frac{.11196}{.16858} = .6641$$

Let $G4$ = event that a randomly student is in the fourth grade.

$$P(S|G4) = \frac{P(S \cap G4)}{P(G4)} = \frac{.08271}{.16476} = .5020$$

d. $$P(G3|S) = \frac{P(G3 \cap S)}{P(S)} = \frac{.11196}{.55861} = .2004$$

$$P(G4|S) = \frac{P(G4 \cap S)}{P(S)} = \frac{.08271}{.55861} = .1481$$

39. a. Yes, since $P(A_1 \cap A_2) = 0$

b. $P(A_1 \cap B) = P(A_1)P(B \mid A_1) = .40(.20) = .08$

$P(A_2 \cap B) = P(A_2)P(B \mid A_2) = .60(.05) = .03$

c. $P(B) = P(A_1 \cap B) + P(A_2 \cap B) = .08 + .03 = .11$

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d. $P(A_1|B) = \dfrac{.08}{.11} = .7273$

$P(A_2|B) = \dfrac{.03}{.11} = .2727$

40. a. $P(B \cap A_1) = P(A_1)P(B \mid A_1) = (.20)(.50) = .10$

$P(B \cap A_2) = P(A_2)P(B \mid A_2) = (.50)(.40) = .20$

$P(B \cap A_3) = P(A_3)P(B \mid A_3) = (.30)(.30) = .09$

b. $P(A_2|B) = \dfrac{.20}{.10+.20+.09} = .51$

c.

Events	$P(A_i)$	$P(B \mid A_i)$	$P(A_i \cap B)$	$P(A_i \mid B)$
A_1	.20	.50	.10	.26
A_2	.50	.40	.20	.51
A_3	.30	.30	.09	.23
	1.00		.39	1.00

41. S_1 = successful, S_2 = not successful and B = request received for additional information.

a. $P(S_1) = .50$

b. $P(B \mid S_1) = .75$

c. $P(S_1|B) = \dfrac{(.50)(.75)}{(.50)(.75)+(.50)(.40)} = \dfrac{.375}{.575} = .65$

42. M = missed payment
D_1 = customer defaults
D_2 = customer does not default

$P(D_1) = .05 \quad P(D_2) = .95 \quad P(M \mid D_2) = .2 \quad P(M \mid D_1) = 1$

a. $P(D_1|M) = \dfrac{P(D_1)P(M|D_1)}{P(D_1)P(M|D_1)+P(D_2)P(M|D_2)} = \dfrac{(.05)(1)}{(.05)(1)+(.95)(.2)} = \dfrac{.05}{.24} = .21$

b. Yes, the probability of default is greater than .20.

43. a. Let M = event that a putt is made

$P(M)$ = 983,764/1,613,234 = .610

Note: The probability that a putt is missed is $P(M^C) = 1 - P(M) = 1 - .610 = .390$

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b. Let A= event that a PGA Tour player has a par putt

$P(A|M) = .640$

$P(A|M^C) = .203$

$$P(M|A) = \frac{P(M)P(A|M)}{P(M)P(A|M)+P(M^C)P(A|M^C)} = \frac{(.610)(.640)}{(.610)(.640)+(.390)(.203)} = .831$$

c. Let B = event that a PGA Tour player has a birdie putt

$P(B|M) = .188$

$P(B|M^C) = .734$

$$P(M|B) = \frac{P(M)P(B|M)}{P(M)P(B|M)+P(M^C)P(B|M^C)} = \frac{(.610)(.188)}{(.610)(.188)+(.390)(.734)} = .286$$

d. These probabilities indicate that there is a much higher probability of making a par put than a birdie putt. The authors of the article referenced in this exercise state that "even the best players … show evidence of loss aversion." In other words, PGA Tour players tend to view missing a par putt as a loss and making a birdie putt as a gain. With regard to putting, their behavior indicates that they prefer avoiding losses to making gains. "On average this bias costs the best golfers over $1.2 million in tournament winnings per year."

44. a. $P(A_1) = .47$ $P(W \mid A_1) = .50$

$P(A_2) = .53$ $P(W \mid A_2) = .45$

b. Using tabular approach

	Prior Probabilities	Conditional Probability	Joint Probability	Posterior Probability
Events	$P(A_i)$	$P(W \mid A_i)$	$P(A_i \cap W)$	$P(A_i \mid W)$
Graduate	.47	.50	.2350	.4963
Not a Graduate	.53	.45	.2385	.5037
		$P(W)$ =	.4735	1.0000

$P(A_i \mid W) = .4963$

c.

	Prior Probabilities	Conditional Probability	Joint Probability	Posterior Probability
Events	$P(A_i)$	$P(M \mid A_i)$	$P(A_i \cap W)$	$P(A_i \mid M)$
Graduate	.47	.50	.2350	.4463
Not a Graduate	.53	.45	.2915	.5537
		$P(M)$ =	.5265	1.0000

$P(A_i \mid M) = .4463$

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About a .05 higher probability a woman student will graduate compared to a man.

d. $P(\text{W}) = .4735$

$P(\text{M}) = .5265$

Approximately 47% women and 53% men.

45. a. Let A = age 65 or older

$P(A) = 1 - .835 = .165$

b. Let D = takes drugs regularly

$$P(A|D) = \frac{P(A)P(D|A)}{P(A)P(D|A) + P(A^C)P(D|A^C)}$$

$$= \frac{.165(.82)}{.165(.82) + .835(.49)}$$

$$\frac{.1353}{.1353 + .4092} = .2485$$

46. a. Let A = a respondent owns a home

$P(A) = 1249/2082 = .60$

b. Let B = a respondent aged 18 to 34 owns a home

$P(B) = 117/450 = .26$

c. Let A^C = a respondent does not own a home

$P(A^C) = 1 - P(A) = 1 - .60 = .40$

d. Let B^C = a respondent aged 18 to 34 does not own a home

$P(B^C) = 1 - P(B) = 1 - .26 = .74$

47. a. $(2)(2) = 4$

b. Let S = successful
U = unsuccessful

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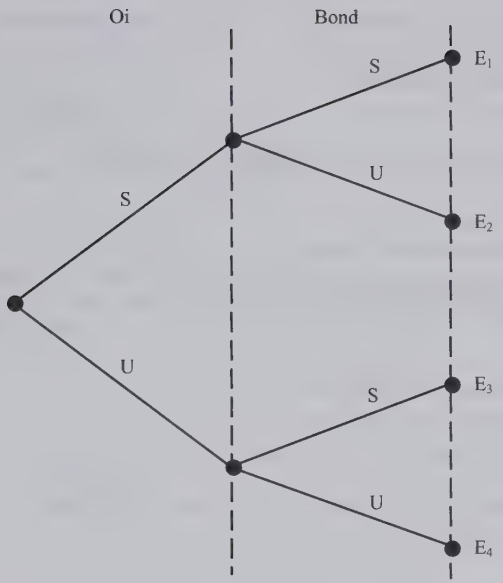

c. $O = \{E_1, E_2\}$

$M = \{E_1, E_3\}$

d. $O \cup M = \{E_1, E_2, E_3\}$

e. $O \cap M = \{E_1\}$

f. No; since $O \cap M$ has a sample point.

48. a. Let A = event that a randomly selected player had a batting average of .300 or higher.

$P(A) = 42/157 = .2675$

b. Let B = event that a randomly selected player hit 25 or more home runs.

$P(B) = 53/157 = .3376$

c. $P(A \cap B) = 14/157 = .0892$

Events A and B are not mutually exclusive because they can occur simultaneously.

d. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .2675 + .3376 - .0892 = .5159$

e. Let C = event that a randomly selected player had 200 or more hits.

$P(C) = 4/157 = .0255$

The probability of event C is extremely low compared to the probability of event B indicating that it is much harder to get 200 or more hits in a season than it is to hit 25 or more home runs.

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49. Let I = treatment-caused injury
D = death from injury
N = injury caused by negligence
M = malpractice claim filed
$ = payment made in claim

We are given $P(I) = 0.04$, $P(N \mid I) = 0.25$, $P(D \mid I) = 1/7$, $P(M \mid N) = 1/7.5 = 0.1333$, and $P(\$ \mid M) = 0.50$

a. $P(N) = P(N \mid I)\,P(I) + P(N \mid I^c)\,P(I^c)$
$= (0.25)(0.04) + (0)(0.96) = 0.01$

b. $P(D) = P(D \mid I)\,P(I) + P(D \mid I^c)\,P(I^c)$
$= (1/7)(0.04) + (0)(0.96) = 0.006$

c. $P(M) = P(M \mid N)\,P(N) + P(M \mid N^c)\,P(N^c)$
$= (0.1333)(0.01) + (0)(0.99) = 0.001333$

$P(\$) = P(\$ \mid M)\,P(M) + P(\$ \mid M^c)\,P(M^c)$
$= (0.5)(0.001333) + (0)(0.9987) = 0.00067$

50. a. Probability of the event $= P(\text{average}) + P(\text{above average}) + P(\text{excellent})$

$$= \frac{11}{50} + \frac{14}{50} + \frac{13}{50} = .22 + .28 + .26 = .76$$

b. Probability of the event $= P(\text{poor}) + P(\text{below average})$

$$= \frac{4}{50} + \frac{8}{50} = .24$$

51. a.

	Household Income ($1000)					
Education Level	Under 25	25-49.9	50-74.9	75-99.9	100 or More	Total
Not H.S. Graduate	.0571	.0469	.0188	.0073	.0050	.1351
H.S. Graduate	.0667	.0929	.0682	.0358	.0362	.2997
Some College	.0381	.0713	.0634	.0441	.0553	.2721
Bachelor's Degree	.0120	.0284	.0386	.0350	.0729	.1870
Beyond Bach. Degree	.0039	.0112	.0173	.0168	.0568	.1061
Total	.1777	.2508	.2064	.1390	.2262	1.0000

b. This is a marginal probability.

$P(\text{Not H.S. graduate}) = .1351$

c. This is the sum of 2 marginal probabilities.

$P(\text{Bachelor's Degree} \cup \text{Beyond Bachelor's Degree}) = .1870 + .1061 = .2931$

d. This is a conditional probability.

$$P(\text{100 or More} \mid BD) = \frac{P(\text{100 or More} \cap BD)}{P(BD)} = \frac{.0729}{.1870} = .3898$$

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e. This is a marginal probability.

$P(\text{Under } 25) = .1777$

f. This is a conditional probability.

$$P(\text{Under } 25 \mid BD) = \frac{P(\text{Under } 25 \cap BD)}{P(BD)} = \frac{.0120}{.1870} = .0642$$

g. No. $P(100 \text{ or More} \mid BD) = .3898$ which is not equal to $P(100 \text{ or More}) = .2262$. This is also shown by comparing the probabilities in parts (e) and (f). Household income is not independent of education level. Individuals with a Bachelor's Degree have a higher probability of having a higher household income.

52. a.

	Yes	No	Total
23 and Under	.1026	.0996	.2022
24 - 26	.1482	.1878	.3360
27 - 30	.0917	.1328	.2245
31 - 35	.0327	.0956	.1283
36 and Over	.0253	.0837	.1090
Total	.4005	.5995	1.0000

b. .2022

c. .2245 + .1283 + .1090 = .4618

d. .4005

53. a. $P(24 \text{ to } 26 \mid \text{Yes}) = .1482/.4005 = .3700$

b. $P(\text{Yes} \mid 36 \text{ and over}) = .0253/.1090 = .2321$

c. .1026 + .1482 + .1878 + .0917 + .0327 + .0253 = .5883

d. $P(31 \text{ or more} \mid \text{No}) = (.0956 + .0837)/.5995 = .2991$

e. No, because the conditional probabilities do not all equal the marginal probabilities. For instance,

$P(24 \text{ to } 26 \mid \text{Yes}) = .3700 \neq P(24 \text{ to } 26) = .3360$

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54. Let I = important or very important
M = male
F = female

a. $P(I) = .49$ (a marginal probability)

b. $P(I \mid M) = .22/.50 = .44$ (a conditional probability)

c. $P(I \mid F) = .27/.50 = .54$ (a conditional probability)

d. It is not independent

$$P(I) = .49 \neq P(I \mid M) = .44$$

and

$$P(I) = .49 \neq P(I \mid F) = .54$$

e. Since level of importance is dependent on gender, we conclude that male and female respondents have different attitudes toward risk.

55. a. $$P(B|S) = \frac{P(B \cap S)}{P(S)} = \frac{.12}{.40} = .30$$

We have $P(B \mid S) > P(B)$.

Yes, continue the ad since it increases the probability of a purchase.

b. Estimate the company's market share at 20%. Continuing the advertisement should increase the market share since $P(B \mid S) = .30$.

c. $$P(B|S) = \frac{P(B \cap S)}{P(S)} = \frac{.10}{.30} = .333$$

The second ad has a bigger effect.

56. a. $P(A) = 200/800 = .25$

b. $P(B) = 100/800 = .125$

c. $P(A \cap B) = 10/800 = .0125$

d. $P(A \mid B) = P(A \cap B)/P(B) = .0125/.125 = .10$

e. No, $P(A \mid B) \neq P(A) = .25$

57. Let A = lost time accident in current year
B = lost time accident previous year

Given: $P(B) = .06$, $P(A) = .05$, $P(A \mid B) = .15$

a. $P(A \cap B) = P(A \mid B)P(B) = .15(.06) = .009$

b. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .06 + .05 - .009 = .101$ or 10.1%

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58. Let: B = blogger
B^c = non blogger
Y = young adult (18-29)
Y^c = older adult

Given: $P(B) = .08$ $P(Y \mid B) = .54$ $P(Y \mid B^c) = .24$

$$P(Y \mid B) = \frac{P(Y \cap B)}{P(B)}$$

$\therefore P(Y \cap B) = P(Y \mid B)P(B) = (.54)(.08) = .0432$

$$P(Y \mid B^c) = \frac{P(Y \cap B^c)}{P(B^c)}$$

$\therefore P(Y \cap B^c) = P(Y \mid B^c)P(B^c) = (.24)(.92) = .2208$

	Young Adult	Older Adult	Total
Blogger	.0432	.0368	.08
Non Blogger	.2208	.6992	.92
Total:	.2640	.7360	1.00

b. $P(Y) = P(B \cap Y) + P(B^c \cap Y) = .0432 + .2208 = .2640$

c. $P(Y \cap C) = .0432$

d. $$P(B \mid Y) = \frac{P(B \cap Y)}{P(Y)} = \frac{.0432}{.2640} = .1636$$

59. a. $P(\text{Oil}) = .50 + .20 = .70$

b. Let S = Soil test results

Events	$P(A_i)$	$P(S \mid A_i)$	$P(A_i \cap S)$	$P(A_i \mid S)$
High Quality (A_1)	.50	.20	.10	.31
Medium Quality (A_2)	.20	.80	.16	.50
No Oil (A_3)	.30	.20	.06	.19
	1.00		$P(S) = .32$	1.00

$P(\text{Oil}) = .81$ which is good; however, probabilities now favor medium quality rather than high quality oil.

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60. a. Let F = female. Using past history as a guide, $P(\text{F}) = .40$.

b. Let D = Dillard's

$$P(\text{F}|\text{D}) = \frac{.40\left(\frac{3}{4}\right)}{.40\left(\frac{3}{4}\right)+.60\left(\frac{1}{4}\right)} = \frac{.30}{.30+.15} = .67$$

The revised (posterior) probability that the visitor is female is .67.

We should display the offer that appeals to female visitors.

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Chapter 5
Discrete Probability Distributions

Learning Objectives

1. Understand the concepts of a random variable and a probability distribution.
2. Be able to distinguish between discrete and continuous random variables.
3. Be able to compute and interpret the expected value, variance, and standard deviation for a discrete random variable.
4. Be able to compute and work with probabilities involving a binomial probability distribution.
5. Be able to compute and work with probabilities involving a Poisson probability distribution.
6. Know when and how to use the hypergeometric probability distribution.

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Solutions:

1. a. Head, Head (H,H)
   Head, Tail (H,T)
   Tail, Head (T,H)
   Tail, Tail (T,T)

   b. x = number of heads on two coin tosses

   c.

Outcome	Values of x
(H,H)	2
(H,T)	1
(T,H)	1
(T,T)	0

   d. Discrete. It may assume 3 values: 0, 1, and 2.

2. a. Let x = time (in minutes) to assemble the product.

   b. It may assume any positive value: $x > 0$.

   c. Continuous

3. Let Y = position is offered
   N = position is not offered

   a. S = {(Y,Y,Y), (Y,Y,N), (Y,N,Y), (N,Y,Y), (Y,N,N), (N,Y,N), (N,N,Y), (N,N,N)}

   b. Let N = number of offers made; N is a discrete random variable.

   c.

Experimental Outcome	(Y,Y,Y)	(Y,Y,N)	(Y,N,Y)	(N,Y,Y)	(Y,N,N)	(N,Y,N)	(N,N,Y)	(N,N,N)
Value of N	3	2	2	2	1	1	1	0

4. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

5. a. S = {(1,1), (1,2), (1,3), (2,1), (2,2), (2,3)}

   b.

Experimental Outcome	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)
Number of Steps Required	2	3	4	3	4	5

6. a. values: 0,1,2,...,20
   discrete

   b. values: 0,1,2,...
   discrete

   c. values: 0,1,2,...,50
   discrete

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d. values: $0 \le x \le 8$
continuous

e. values: $x > 0$
continuous

7. a. $f(x) \ge 0$ for all values of x.

$\Sigma f(x) = 1$ Therefore, it is a proper probability distribution.

b. Probability $x = 30$ is $f(30) = .25$

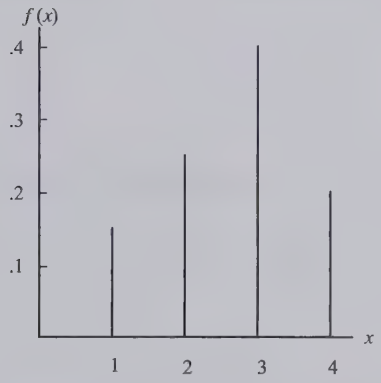
c. Probability $x \le 25$ is $f(20) + f(25) = .20 + .15 = .35$

d. Probability $x > 30$ is $f(35) = .40$

8. a.

x	$f(x)$
1	3/20 = .15
2	5/20 = .25
3	8/20 = .40
4	4/20 = .20
	Total 1.00

b.



c. $f(x) \ge 0$ for $x = 1,2,3,4$.

$\Sigma f(x) = 1$

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9. a. There are a total of 26,975 unemployed persons in the data set. Each probability $f(x)$ is computed by dividing the number of months of unemployment by 26,975. For example, $f(1) = 1029/26,975 = .0381$. The complete probability distribution is as follows.

x	$f(x)$
1	.0381
2	.0625
3	.0841
4	.0992
5	.1293
6	.1725
7	.1537
8	.1330
9	.0862
10	.0415

b. $f(x) \geq 0$ and $\sum f(x) = 1$

c. Probability 2 months or less = $f(1) + f(2) = .0381 + .0625 = .1006$

Probability more than 2 months = 1 - .1006 = .8994

d. Probability more than 6 months = $f(7) + f(8) + f(9) + f(10) = .1537 + .1330 + .0862 + .0415 = .4144$

10. a.

x	$f(x)$
1	0.05
2	0.09
3	0.03
4	0.42
5	0.41
	1.00

b.

x	$f(x)$
1	0.04
2	0.10
3	0.12
4	0.46
5	0.28
	1.00

c. P(4 or 5) = $f(4) + f(5) = 0.42 + 0.41 = 0.83$

d. Probability of very satisfied: 0.28

e. Senior executives appear to be more satisfied than middle managers. 83% of senior executives have a score of 4 or 5 with 41% reporting a 5. Only 28% of middle managers report being very satisfied.

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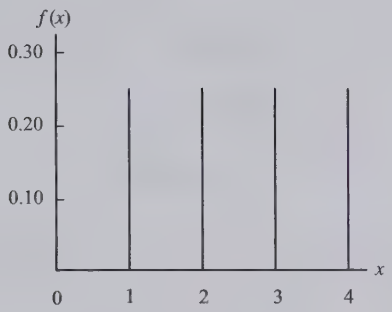
11. a.

Duration of Call

x	$f(x)$
1	0.25
2	0.25
3	0.25
4	0.25
	1.00

b.



c. $f(x) \geq 0$ and $f(1) + f(2) + f(3) + f(4) = 0.25 + 0.25 + 0.25 + 0.25 = 1.00$

d. $f(3) = 0.25$

e. P(overtime) $= f(3) + f(4) = 0.25 + 0.25 = 0.50$

12. a. Yes; $f(x) \geq 0$. $\Sigma f(x) = 1$

b. $f(500{,}000) + f(600{,}000) = .10 + .05 = .15$

c. $f(100{,}000) = .10$

13. a. Yes, since $f(x) \geq 0$ for $x = 1,2,3$ and $\Sigma f(x) = f(1) + f(2) + f(3) = 1/6 + 2/6 + 3/6 = 1$

b. $f(2) = 2/6 = .333$

c. $f(2) + f(3) = 2/6 + 3/6 = .833$

14. a. $f(200) = 1 - f(-100) - f(0) - f(50) - f(100) - f(150)$

$= 1 - .95 = .05$

This is the probability MRA will have a $200,000 profit.

b. P(Profit) $= f(50) + f(100) + f(150) + f(200)$

$= .30 + .25 + .10 + .05 = .70$

c. P(at least 100) $= f(100) + f(150) + f(200)$

$= .25 + .10 + .05 = .40$

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15. a.

x	$f(x)$	$xf(x)$
3	.25	.75
6	.50	3.00
9	.25	2.25
	1.00	6.00

$E(x) = \mu = 6$

b.

x	$x - \mu$	$(x - \mu)^2$	$f(x)$	$(x - \mu)^2 f(x)$
3	-3	9	.25	2.25
6	0	0	.50	0.00
9	3	9	.25	2.25
				4.50

$\text{Var}(x) = \sigma^2 = 4.5$

c. $\sigma = \sqrt{4.50} = 2.12$

16. a.

y	$f(y)$	$yf(y)$
2	.2	.4
4	.3	1.2
7	.4	2.8
8	.1	.8
	1.0	5.2

$E(y) = \mu = 5.2$

b.

y	$y - \mu$	$(y - \mu)^2$	$f(y)$	$(y - \mu)^2 f(y)$
2	-3.20	10.24	.20	2.048
4	-1.20	1.44	.30	.432
7	1.80	3.24	.40	1.296
8	2.80	7.84	.10	.784
				4.560

$\text{Var}(y) = 4.56$

$\sigma = \sqrt{4.56} = 2.14$

17. a. Total Student = 1,518,859

$x = 1$ $f(1) = 721{,}769/1{,}518{,}859 = .4752$
$x = 2$ $f(2) = 601{,}325/1{,}518{,}859 = .3959$
$x = 3$ $f(3) = 166{,}736/1{,}518{,}859 = .1098$
$x = 4$ $f(4) = 22{,}299/1{,}518{,}859 = .0147$
$x = 5$ $f(5) = 6730/1{,}518{,}859 = .0044$

b. $P(x > 1) = 1 \quad f(1) = 1 - .4752 = .5248$

Over 50% of the students take the SAT more than 1 time.

c. $P(x \geq 3) = f(3) + f(4) + f(5) = .1098 + .0147 + .0044 = .1289$

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d./e.

x	$f(x)$	$xf(x)$	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2 f(x)$
1	.4752	.4752	-.6772	.4586	.2179
2	.3959	.7918	.3228	.1042	.0412
3	.1098	.3293	1.3228	1.7497	.1921
4	.0147	.0587	2.3228	5.3953	.0792
5	.0044	.0222	3.3228	11.0408	.0489
		1.6772			.5794

$E(x) = \Sigma x f(x) = 1.6772$

The mean number of times a student takes the SAT is 1.6772, or approximately 1.7 times.

$\sigma^2 = \Sigma(x-\mu)^2 f(x) = .5794$

$\sigma = \sqrt{\sigma^2} = \sqrt{.5794} = .7612$

18. a/b.

x	$f(x)$	$xf(x)$	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2 f(x)$
0	0.04	0.00	-1.84	3.39	0.12
1	0.34	0.34	-0.84	0.71	0.24
2	0.41	0.82	0.16	0.02	0.01
3	0.18	0.53	1.16	1.34	0.24
4	0.04	0.15	2.16	4.66	0.17
Total	1.00	1.84			0.79
		↑			↑
		$E(x)$			Var(x)

c/d.

y	$f(y)$	$yf(y)$	$y - \mu$	$(y - \mu)^2$	$(y - \mu)^2 f(y)$
0	0.00	0.00	-2.93	8.58	0.01
1	0.03	0.03	-1.93	3.72	0.12
2	0.23	0.45	-0.93	0.86	0.20
3	0.52	1.55	0.07	0.01	0.00
4	0.22	0.90	1.07	1.15	0.26
Total	1.00	2.93			0.59
		↑			↑
		$E(y)$			Var(y)

e. The number of bedrooms in owner-occupied houses is greater than in renter-occupied houses. The expected number of bedrooms is 2.93 - 1.84 = 1.09 greater. And, the variability in the number of bedrooms is less for the owner-occupied houses.

19. a. $E(x) = \Sigma x f(x) = 0(.56) + 2(.44) = .88$

b. $E(x) = \Sigma x f(x) = 0(.66) + 3(.34) = 1.02$

c. The expected value of a 3 - point shot is higher. So, if these probabilities hold up, the team will make more points in the long run with the 3 - point shot.

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20. a.

x	$f(x)$	$xf(x)$
0	.85	0
500	.04	20
1000	.04	40
3000	.03	90
5000	.02	100
8000	.01	80
10000	.01	100
Total	1.00	430

The expected value of the insurance claim is $430. If the company charges $430 for this type of collision coverage, it would break even.

b. From the point of view of the policyholder, the expected gain is as follows:

Expected Gain = Expected claim payout – Cost of insurance coverage
– $430 - $520 = -$90

The policyholder is concerned that an accident will result in a big repair bill if there is no insurance coverage. So even though the policyholder has an expected annual loss of $90, the insurance is protecting against a large loss.

21. a. $E(x) = \Sigma x f(x) = 0.05(1) + 0.09(2) + 0.03(3) + 0.42(4) + 0.41(5) = 4.05$

b. $E(x) = \Sigma x f(x) = 0.04(1) + 0.10(2) + 0.12(3) + 0.46(4) + 0.28(5) = 3.84$

c. Executives: $\sigma^2 = \Sigma (x - \mu)^2 f(x) = 1.25$

Middle Managers: $\sigma^2 = \Sigma (x - \mu)^2 f(x) = 1.13$

d. Executives: $\sigma = 1.12$

Middle Managers: $\sigma = 1.07$

e. The senior executives have a higher average score: 4.05 vs. 3.84 for the middle managers. The executives also have a slightly higher standard deviation.

22. a. $E(x) = \Sigma x f(x) = 300 (.20) + 400 (.30) + 500 (.35) + 600 (.15) = 445$

The monthly order quantity should be 445 units.

b. Cost: 445 @ $50 = $22,250
Revenue: 300 @ $70 = 21,000
$ 1,250 Loss

23. a. Rent Controlled: $E(x) = 1(.61) + 2(.27) + 3(.07) + 4(.04) + 5(.01) = 1.57$

Rent Stabilized: $E(x) = 1(.41) + 2(.30) + 3(.14) + 4(.11) + 5(.03) + 6(.01) = 2.08$

b. Rent Controlled:
$\text{Var}(x) = (-.57)^2.61 + (.43)^2.27 + (1.43)^2.07 + (2.43)^2.04 + (3.43)^2.01 = .75$

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Rent Stabilized:
$\text{Var}(x) = (-1.08)^2.41 + (-.08)^2.30 + (.92)^2.14 + (1.92)^2.11 + (2.92)^2.03 + (3.92)^2.01 = 1.41$

c. From the expected values in part (a), it is clear that the expected number of persons living in rent stabilized units is greater than the number of persons living in rent controlled units. For example, comparing a building that contained 10 rent controlled units to a building that contained 10 rent stabilized units, the expected number of persons living in the rent controlled building would be 1.57(10) = 15.7 or approximately 16. For the rent stabilized building, the expected number of persons is approximately 21. There is also more variability in the number of persons living in rent stabilized units.

24. a. Medium $E(x) = \Sigma x f(x)$

$= 50(.20) + 150(.50) + 200(.30) = 145$

Large: $E(x) = \Sigma x f(x)$

$= 0(.20) + 100(.50) + 300(.30) = 140$

Medium preferred.

b. Medium

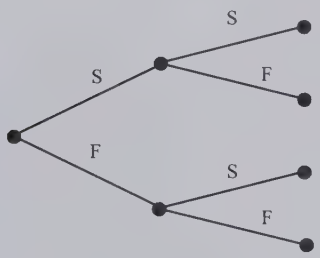
x	$f(x)$	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2 f(x)$
50	.20	-95	9025	1805.0
150	.50	5	25	12.5
200	.30	55	3025	907.5
				$\sigma^2 = 2725.0$

Large

y	$f(y)$	$y - \mu$	$(y - \mu)^2$	$(y - \mu)^2 f(y)$
0	.20	-140	19600	3920
100	.50	-40	1600	800
300	.30	160	25600	7680
				$\sigma^2 = 12{,}400$

Medium preferred due to less variance.

25. a.



b. $f(1) = \binom{2}{1}(.4)^1(.6)^1 = \frac{2!}{1!1!}(.4)(.6) = .48$

Using Excel: BINOM.DIST(1,2,.4,FALSE) = .48

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c. $f(0) = \binom{2}{0}(.4)^0(.6)^2 = \frac{2!}{0!2!}(1)(.36) = .36$

Using Excel: BINOM.DIST(0,2,.4,FALSE) = .36

d. $f(2) = \binom{2}{2}(.4)^2(.6)^0 = \frac{2!}{2!0!}(.16)(1) = .16$

Using Excel: BINOM.DIST(2,2,.4,FALSE) = .16

e. $P(x \geq 1) = f(1) + f(2) = .48 + .16 = .64$

f. $E(x) = np = 2(.4) = .8$

$\text{Var}(x) = np(1-p) = 2(.4)(.6) = .48$

$\sigma = \sqrt{.48} = .6928$

26. a. $f(0)$ = BINOM.DIST(0,10,.1,FALSE) = .3487

b. $f(2)$ = BINOM.DIST(2,10,.1,FALSE) = .1937

c. $P(x \leq 2) = f(0) + f(1) + f(2) = .3487 + .3874 + .1937$ = BINOM.DIST(2,10,.1,TRUE) = .9298

d. $P(x \geq 1) = 1 - f(0) = 1 - .3487 = .6513$

e. $E(x) = np = 10(.1) = 1$

f. $\text{Var}(x) = np(1-p) = 10(.1)(.9) = .9$

$\sigma = \sqrt{.9} = .9487$

27. a. $f(12)$ = BINOM.DIST(12,20,.7,FALSE) = .1144

b. $f(16)$ = BINOM.DIST(16,20,.7,FALSE) = .1304

c. $P(x \geq 16)$ = 1 - BINOM.DIST(15,20,.7,TRUE) = .2375

d. $P(x \leq 15) = 1 - P(x \geq 16) = 1 - .2375 = .7625$

e. $E(x) = np = 20(.7) = 14$

f. $\text{Var}(x) = np(1-p) = 20(.7)(.3) = 4.2$

$\sigma = \sqrt{4.2} = 2.0494$

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28. a. $f(2) = \binom{6}{2}(.23)^2(.77)^4 = .2789$

b. $P(\text{at least } 2) = 1 - f(0) - f(1)$

$$= 1 - \binom{6}{0}(.23)^0(.77)^6 - \binom{6}{1}(.23)^1(.77)^5$$

$$= 1 - .2084 - .3735 = .4181$$

c. $f(0) = \binom{10}{0}(.23)^0(.77)^{10} = .0733$

29. a. $f(x) = \binom{n}{x}(p)^x(1-p)^{n-x}$

$$f(3) = \frac{10!}{3!(10-3)!}(.30)^3(1-.30)^{10-3}$$

$$f(3) = \frac{10(9)(8)}{3(2)(1)}(.30)^3(1-.30)^7 = .2668$$

b. $P(x \geq 3) = 1 - f(0) - f(1) - f(2)$

$$f(0) = \frac{10!}{0!(10)!}(.30)^0(1-.30)^{10} = .0282$$

$$f(1) = \frac{10!}{1!(9)!}(.30)^1(1-.30)^9 = .1211$$

$$f(2) = \frac{10!}{2!(8)!}(.30)^2(1-.30)^8 = .2335$$

$P(x \geq 3) = 1 - .0282 - .1211 - .2335 = .6172$

30. a. Probability of a defective part being produced must be .03 for each part selected; parts must be selected independently.

b. Let: D = defective
G = not defective

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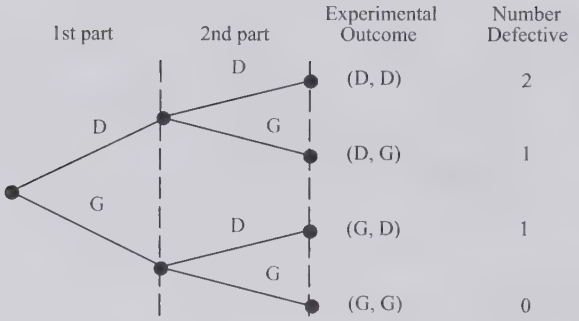

c. 2 outcomes result in exactly one defect.

d. $P(\text{no defects}) = (.97)(.97) = .9409$

$P(1 \text{ defect}) = 2(.03)(.97) = .0582$

$P(2 \text{ defects}) = (.03)(.03) = .0009$

31. a. Yes. Since the employees are selected randomly, p is the same from trial to trial and the trials are independent. The two outcomes per trial are loyal and not loyal.

Binomial $n = 10$ and $p = .25$

$$f(x) = \frac{10!}{x!(10-x)!}(.25)^x(1-.25)^{10-x}$$

b. $$f(0) = \frac{10!}{0!(10-0)!}(.25)^0(1-.25)^{10-0} = \text{BINOM.DIST(0,10,.25,FALSE)} = .0563$$

c. $$f(4) = \frac{10!}{4!(10-4)!}(.25)^4(1-.25)^{10-4} = \text{BINOM.DIST(4,10,.25,FALSE)} = .1460$$

d. Probability $(x \geq 2) = 1 - f(0) - f(1)$

From part (b), $f(0) = .0563$

$$f(1) = \frac{10!}{1!(10-1)!}(.25)^1(1-.25)^{10-1} = \text{BINOM.DIST(1,10,.25,FALSE)} = .1877$$

Probability $(x \geq 2) = 1 - f(0) - f(1) = 1 - (.0563 + .1877) = .7560$

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32. a. .90

b. $P(\text{at least 1}) = f(1) + f(2)$

$$f(1) = \frac{2!}{1!\,1!}(.9)^1(.1)^1$$

$$= 2(.9)(.1) = .18$$

$$f(2) = \frac{2!}{2!\,0!}(.9)^1(.1)^0$$

$$= 1(.81)(1) = .81$$

$\therefore P(\text{at least 1}) = .18 + .81 = .99$

Alternatively

$P(\text{at least 1}) = 1 - f(0)$

$$f(0) = \frac{2!}{0!\,2!}(.9)^0(.1)^2 = .01$$

Therefore, $P(\text{at least 1}) = 1 - .01 = .99$

c. $P(\text{at least 1}) = 1 - f(0)$

$$f(0) = \frac{3!}{0!\,3!}(.9)^0(.1)^3 = .001$$

Therefore, $P(\text{at least 1}) = 1 - .001 = .999$

d. Yes; $P(\text{at least 1})$ becomes very close to 1 with multiple systems and the inability to detect an attack would be catastrophic.

33. a. Using the 20 golfers in the Hazeltine PGA Championship, the probability that a PGA professional golfer uses a Titleist brand golf ball is $p = 14/20 = .6$

For the sample of 15 PGA Tour players, use a binomial distribution with $n = 15$ and $p = .6$.

$$f(10) = \frac{15!}{10!5!}(.6)^{10}(1-.6)^{15-10} = \text{BINOM.DIST(10,15,.6,FALSE)} = .1859$$

b. $P(x > 10) = 1 - P(x \leq 10)$

$P(x \leq 10) = \text{BINOM.DIST(10,15,.6,FALSE)} = .7827$

$P(x > 10) = 1 - P(x \leq 10) = 1 - .7828 = .2173$

c. $E(x) = np = 15(.6) = 9$

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d. $\text{Var}(x) = \sigma^2 = np(1 - p) = 15(.6)(1 - .6) = 3.6$

$\sigma = \sqrt{3.6} = 1.8974$

34. a. $f(x) = \binom{n}{x}(p)^x(1-p)^{n-x}$

$f(4) = \dfrac{15!}{4!(15-4)!}(.28)^4(1-.28)^{15-4}$

$f(4) = \text{BINOM.DIST}(4,15,.28,\text{FALSE}) = .2262$

b. $P(x \geq 3) = 1 - f(0) - f(1) - f(2)$

$f(0) = \text{BINOM.DIST}(0,15,.28,\text{FALSE}) = .0072$

$f(1) = \text{BINOM.DIST}(1,15,.28,\text{FALSE}) = .0423$

$f(2) = \text{BINOM.DIST}(2,15,.28,\text{FALSE}) = .1150$

$P(x \geq 3) = 1 - .0072 - .0423 - .1150 = .8355$

35. a. $f(0) + f(1) + f(2) = \text{BINOM.DIST}(2,20,.2,\text{TRUE}) = .2061$

b. $f(4) = \text{BINOM.DIST}(4,20,.2,\text{FALSE}) = .2182$

c. $1 - [f(0) + f(1) + f(2) + f(3)] = 1 - \text{BINOM.DIST}(3,20,.2,\text{TRUE}) = .5886$

d. $\mu = np = 20(.20) = 4$

36. a. $p = 1/4 = .25$

$f(x) = \binom{n}{x}(p)^x(1-p)^{n-x}$

$f(4) = \binom{20}{4}(.25)^4(1-.25)^{20-4}$

$f(4) = \text{BINOM.DIST}(4,20,.25,\text{FALSE}) = .1897$

b. $P(x \geq 2) = 1 - f(0) - f(1)$

$P(x \geq 2) = 1 \quad \text{BINOM.DIST}(1,20,.25,\text{TRUE}) = .9757$

c. $f(12) = \text{BINOM.DIST}(12,20,.25,\text{FALSE}) = .0008$

And, with $f(13) = .0002$, $f(14) = .0000$, and so on, the probability of finding that 12 or more investors have exchange-traded funds in their portfolio is so small that it is highly unlikely that $p = .25$. In such a case, we would doubt the accuracy of the results and conclude that p must be greater than .25.

d. $\mu = np = 20(.25) = 5$

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37. $E(x) = np = 35(.23) = 8.05$ (8 automobiles)

$\text{Var}(x) = np(1 - p) = 35(.23)(1-.23) = 6.2$

$\sigma = \sqrt{6.2} = 2.49$

38. a. $f(x) = \dfrac{3^x e^{-3}}{x!}$

b. $f(2) = \dfrac{3^2 e^{-3}}{2!} = \dfrac{9(.0498)}{2} = .2241$

Using Excel: POISSON.DIST(2,3,FALSE) = .2240

c. $f(1) = \dfrac{3^1 e^{-3}}{1!} = 3(.0498) = .1494$

Using Excel: POISSON.DIST(1,3,FALSE) = .1494

d. $P(x \geq 2) = 1 - f(0) - f(1) = 1 - .0498 - .1494 = .8008$

Using Excel: 1 - POISSON.DIST(1,3,TRUE) = .8009

39. a. $f(x) = \dfrac{2^x e^{-2}}{x!}$

b. $\mu = 6$ for 3 time periods

c. $f(x) = \dfrac{6^x e^{-6}}{x!}$

d. $f(2) = \dfrac{2^2 e^{-2}}{2!} = \dfrac{4(.1353)}{2} = .2706$

Using Excel: POISSON.DIST(2,2,FALSE) = .2707

e. $f(6) = \dfrac{6^6 e^{-6}}{6!} = .1606$

Using Excel: POISSON.DIST(6,6,FALSE) = .1606

f. $f(5) = \dfrac{4^5 e^{-4}}{5!} = .1563$

Using Excel: POISSON.DIST(5,4,FALSE) = .1563

40. a. $\mu = 48\ (5/60) = 4$

$f(3) = \dfrac{4^3 e^{-4}}{3!} = \dfrac{(64)\ (.0183)}{6} = .1952$

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b. $\mu = 48\,(15/60) = 12$

$$f(10) = \frac{12^{10}e^{-12}}{10!} = \text{POISSON.DIST(10,12,FALSE)} = .1048$$

c. $\mu = 48\,(5/60) = 4$ I expect 4 callers to be waiting after 5 minutes.

$$f(0) = \frac{4^{0}e^{-4}}{0!} = \text{POISSON.DIST(0,4,FALSE)} = .0183$$

The probability none will be waiting after 5 minutes is .0183.

d. $\mu = 48\,(3/60) = 2.4$

$$f(0) = \frac{2.4^{0}e^{-2.4}}{0!} = \text{POISSON.DIST (0,2.4,FALSE)} = .0907$$

The probability of no interruptions in 3 minutes is .0907.

41. a. 30 per hour

b. $\mu = 1\,(5/2) = 5/2$

$$f(3) = \frac{(5/2)^{3}e^{-(5/2)}}{3!} = \text{POISSON.DIST(3,2.5,FALSE)} = .2138$$

c. $$f(0) = \frac{(5/2)^{0}e^{-(5/2)}}{0!} = \text{POISSON.DIST(0,2.5,FALSE)} = .0821$$

42. a. $$f(0) = \frac{7^{0}e^{-7}}{0!} = \text{POISSON.DIST(0,7,FALSE)} = .0009$$

b. probability $= 1 - [f(0) + f(1)]$

$= 1 - \text{POISSON.DIST(1,7,TRUE)} = .9927$

c. $\mu = 3.5$

$$f(0) = \frac{3.5^{0}e^{-3.5}}{0!} = \text{POISSON.DIST(0,3.5,FALSE)} = .0302$$

probability $= 1 - f(0) = 1 - .0302 = .9698$

d. probability $= 1 - [f(0) + f(1) + f(2) + f(3) + f(4)] = 1 - \text{POISSON.DIST(4,7,TRUE)} = .8270$

43. a. $$f(0) = \frac{10^{0}e^{-10}}{0!} = \text{POISSON.DIST(0,10,FALSE)} = .000045$$

b. $f(0) + f(1) + f(2) + f(3) = \text{POISSON.DIST(3,10,TRUE)} = .0103$

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c. 2.5 arrivals / 15 sec. period Use $\mu = 2.5$

$$f(0) = \frac{2.5^0 e^{-2.5}}{0!} = \text{POISSON.DIST(0,2.5,FALSE)} = .0821$$

d. $1 - f(0) = 1 - .0821 = .9179$

44. Poisson distribution applies

a. $\mu = 1.25$ per month

b. $f(0) = \dfrac{1.25^0 e^{-1.25}}{0!} = \text{POISSON.DIST(0,1.25,FALSE)} = .2865$

c. $f(1) = \dfrac{1.25^1 e^{-1.25}}{1!} = \text{POISSON.DIST(1,1.25,FALSE)} = .3581$

d. $P(\text{More than 1}) = 1 - f(0) - f(1) = 1 - 0.2865 - 0.3581 = 0.3554$

45. a. $f(x) = \dfrac{\mu^x e^{-\mu}}{x!}$

$$f(0) = \frac{3^0 e^{-3}}{0!} = e^{-3} = .0498$$

b. $P(x \geq 2) = 1 - f(0) - f(1)$

$P(x \geq 2) = 1 - \text{POISSON.DIST(1,3,TRUE)} = .8009$

c. $\mu = 3$ per year

$\mu = 3/2 = 1.5$ per 6 months

d. $f(0) = \dfrac{1.5^0 e^{-1.5}}{0!} = .2231$

46. a. $$f(1) = \frac{\binom{3}{1}\binom{10-3}{4-1}}{\binom{10}{4}} = \frac{\left(\frac{3!}{1!2!}\right)\left(\frac{7!}{3!4!}\right)}{\frac{10!}{4!6!}} = \frac{(3)(35)}{210} = .50$$

Using Excel: HYPGEOM.DIST(1,4,3,10,FALSE) = .5000

b. $$f(2) = \frac{\binom{3}{2}\binom{10-3}{2-2}}{\binom{10}{2}} = \frac{(3)(1)}{45} = .0667$$

Using Excel: HYPGEOM.DIST(2,2,3,10,FALSE) = .0667

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c. $f(0) = \dfrac{\binom{3}{0}\binom{10-3}{2-0}}{\binom{10}{2}} = \dfrac{(1)(21)}{45} = .4667$

Using Excel: HYPGEOM.DIST(0,2,3,10,FALSE) = .4667

d. $f(2) = \dfrac{\binom{3}{2}\binom{10-3}{4-2}}{\binom{10}{4}} = \dfrac{(3)(21)}{210} = .30$

Using Excel: HYPGEOM.DIST (2,4,3,10,FALSE) = .3000

47. $f(3) = \dfrac{\binom{4}{3}\binom{15-4}{10-3}}{\binom{15}{10}} = \dfrac{(4)(330)}{3003} = .4396$

48. Hypergeometric with $N = 10$ and $r = 7$

a. $f(2) = \dfrac{\binom{7}{2}\binom{3}{1}}{\binom{10}{3}} = \dfrac{(21)(3)}{120} =$ HYPGEOM.DIST(2,3,7,10,FALSE) = .5250

b. In part (a) we computed the probability that 2 prefer football. Now compute the probability that 3 prefer football and add to the part (a) answer.

$f(3) = \dfrac{\binom{7}{3}\binom{3}{0}}{\binom{10}{3}} =$ HYPGEOM.DIST(3,3,7,10,FALSE) = .2917

P(majority football) = $f(2) + f(3)$ = .5250 + .2917 = .8167

49. Parts a, b & c involve the hypergeometric distribution with $N = 52$ and $n = 2$

a. $r = 20, x = 2$

$f(2) = \dfrac{\binom{20}{2}\binom{32}{0}}{\binom{52}{2}} = \dfrac{(190)(1)}{1326} =$ HYPGEOM.DIST(2,2,20,52,FALSE) = .1433

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b. $r = 4, x = 2$

$$f(2) = \frac{\binom{4}{2}\binom{48}{0}}{\binom{52}{2}} = \frac{(6)(1)}{1326} = \text{HYPGEOM.DIST(2,2,4,52,FALSE)} = .0045$$

c. $r = 16, x = 2$

$$f(2) = \frac{\binom{16}{2}\binom{36}{0}}{\binom{52}{2}} = \frac{(120)(1)}{1326} = \text{HYPGEOM.DIST(2,2,16,52,FALSE)} = .0905$$

d. Part (a) provides the probability of blackjack plus the probability of 2 aces plus the probability of two 10s. To find the probability of blackjack we subtract the probabilities in (b) and (c) from the probability in (a).

$P(\text{blackjack}) = .1433 - .0045 - .0905 = .0483$

50. $N = 60 \quad n = 10$

a. $r = 20 \quad x = 0$

$$f(0) = \frac{\binom{20}{0}\binom{40}{10}}{\binom{60}{10}} = \frac{(1)\left(\frac{40!}{10!30!}\right)}{\frac{60!}{10!50!}} = \text{HYPGEOM.DIST(0,10,20,60,FALSE)} = .0112$$

b. $r = 20 \quad x = 1$

$$f(1) = \frac{\binom{20}{1}\binom{40}{9}}{\binom{60}{10}} = \text{HYPGEOM.DIST(1,10,20,60,FALSE)} = .0725$$

c. $1 - f(0) - f(1) = 1 - .0112 - .0725 = .9163 \approx .92$

d. Same as the probability one will be from Hawaii. In part b that was found to equal approximately .07.

51. a. $$f(0) = \frac{\binom{5}{0}\binom{10}{3}}{\binom{15}{3}} = \frac{(1)(120)}{455} = \text{HYPGEOM.DIST(0,3,5,15,FALSE)} = .2637$$

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b. $f(1) = \dfrac{\binom{5}{1}\binom{10}{2}}{\binom{15}{3}} = \dfrac{(5)(45)}{455} =$ HYPGEOM.DIST(1,3,5,15,FALSE) = .4945

c. $f(2) = \dfrac{\binom{5}{2}\binom{10}{1}}{\binom{15}{3}} = \dfrac{(10)(10)}{455} =$ HYPGEOM.DIST(2,3,5,15,FALSE) = .2198

d. $f(3) = \dfrac{\binom{5}{3}\binom{10}{0}}{\binom{15}{3}} = \dfrac{(10)(1)}{455} =$ HYPGEOM.DIST(3,3,5,15,FALSE) = .0220

52. Hypergeometric with $N = 10$ and $r = 3$.

a. $n \quad 3, x \quad 0$

$$f(0) = \frac{\binom{3}{0}\binom{7}{3}}{\binom{10}{3}} = \frac{\left(\frac{3!}{0!3!}\right)\left(\frac{7!}{3!4!}\right)}{\frac{10!}{3!7!}} =$$ HYPGEOM.DIST(0,3,3,10,FALSE) = .2917

This is the probability there will be no banks with increased lending in the study.

b. $n - 3, x \quad 3$

$$f(3) = \frac{\binom{3}{3}\binom{7}{0}}{\binom{10}{3}} = \frac{\left(\frac{3!}{3!0!}\right)\left(\frac{7!}{0!7!}\right)}{\frac{10!}{3!7!}} =$$ HYPGEOM.DIST(3,3,3,10,FALSE) = .0083

This is the probability there all three banks with increased lending will be in the study. This has a very low probability of happening.

c. $n \cdot 3, x - 1$

$$f(1) = \frac{\binom{3}{1}\binom{7}{2}}{\binom{10}{3}} = \frac{\left(\frac{3!}{1!2!}\right)\left(\frac{7!}{2!5!}\right)}{\frac{10!}{3!7!}} =$$ HYPGEOM.DIST(1,3,3,10,FALSE) = .5250

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$n = 3, x = 2$

$$f(2) = \frac{\binom{3}{2}\binom{7}{1}}{\binom{10}{3}} = \frac{\left(\frac{3!}{2!1!}\right)\left(\frac{7!}{1!6!}\right)}{\frac{10!}{3!7!}} = \text{HYPGEOM.DIST(2,3,3,10,FALSE)} = .1750$$

x	$f(x)$
0	0.2917
1	0.5250
2	0.1750
3	0.0083
Total	1.0000

$f(1) = .5250$ has the highest probability showing that there is over a .50 chance that there will be exactly one bank that had increased lending in the study.

d. $P(x \geq 1) = 1 - f(0) = 1 - .2917 = .7083$

There is a reasonably high probability of .7083 that there will be at least one bank that had increased lending in the study.

e. $E(x) = n\left(\frac{r}{N}\right) = 3\left(\frac{3}{10}\right) = .90$

$$\sigma^2 = n\left(\frac{r}{N}\right)\left(1-\frac{r}{N}\right)\left(\frac{N-n}{N-1}\right) = 3\left(\frac{3}{10}\right)\left(1-\frac{3}{10}\right)\left(\frac{10-3}{10-1}\right) = .49$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{.49} = .70$$

53. a/b/c.

x	$f(x)$	$xf(x)$	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2 f(x)$
1	0.07	0.07	-2.12	4.49	0.31
2	0.21	0.42	-1.12	1.25	0.26
3	0.29	0.87	-0.12	0.01	0.00
4	0.39	1.56	0.88	0.77	0.30
5	0.04	0.20	1.88	3.53	0.14
Total	1.00	3.12			1.03
		↑			↑
		$E(x)$			Var(x)

$\sigma = \sqrt{1.03} = 1.01$

d. The expected level of optimism is 3.12. This is a bit above neutral and indicates that investment managers are somewhat optimistic. Their attitudes are centered between neutral and bullish with the consensus being closer to neutral.

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54. a/b.

x	$f(x)$	$xf(x)$	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2 f(x)$
1	0.24	0.24	-2.00	4.00	0.97
2	0.21	0.41	-1.00	1.00	0.21
3	0.10	0.31	0.00	0.00	0.00
4	0.21	0.83	1.00	1.00	0.21
5	0.24	1.21	2.00	4.00	0.97
Total	1.00	3.00			2.34
		↑			↑
		$E(x)$			Var(x)

c. For the bond fund categories: $E(x) = 1.36$ Var(x) = .23

For the stock fund categories: $E(x) = 4$ Var(x) = 1.00

The total risk of the stock funds is much higher than for the bond funds. It makes sense to analyze these separately. When you do the variances for both groups (stocks and bonds), they are reduced.

55. a.

x	$f(x)$
9	.30
10	.20
11	.25
12	.05
13	.20

b. $E(x) = \Sigma x f(x)$

$= 9(.30) + 10(.20) + 11(.25) + 12(.05) + 13(.20) = 10.65$

Expected value of expenses: $10.65 million

c. $\text{Var}(x) = \Sigma(x - \mu)^2 f(x)$

$= (9 - 10.65)^2 (.30) + (10 - 10.65)^2 (.20) + (11 - 10.65)^2 (.25)$

$+ (12 - 10.65)^2 (.05) + (13 - 10.65)^2 (.20) = 2.13$

d. Looks Good: $E(\text{Profit}) = 12 - 10.65 = 1.35$ million

However, there is a .20 probability that expenses will equal $13 million and the college will run a deficit.

56. a. $n = 20$ and $x = 3$

$$f(3) = \binom{20}{3}(.05)^3(.95)^{17} = \text{BINOM.DIST(3,20,.50,FALSE)} = .0596$$

b. $n = 20$ and $x = 0$

$$f(0) = \binom{20}{0}(.05)^0(.95)^{20} = \text{BINOM.DIST(0,20,.05,FALSE)} = .3585$$

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c. $E(x) = np = 2000(.05) = 100$

The expected number of employees is 100.

d. $\sigma^2 = np(1 - p) = 2000(.05)(.95) = 95$

$\sigma = \sqrt{95} = 9.7468$

57. a. We must have $E(x) = np \geq 20$

For the 18-34 age group, $p = .26$.

$n(.26) \geq 20$

$n \geq 76.92$

Sample at least 77 people to have an expected number of home owners at least 20 for this age group.

b. For the 35-44 age group, $p = .50$.

$n(.50) \geq 20$

$n \geq 40$

Sample at least 40 people to have an expected number of home owners at least 20 for this age group.

c. For the 55 and over age group, $p = .88$.

$n(.88) \geq 20$

$n \geq 22.72$

Sample at least 23 people to have an expected number of home owners at least 20 for this age group.

d. $\sigma = \sqrt{np(1-p)} = \sqrt{77(.26)(.74)} = 3.85$

e. $\sigma = \sqrt{np(1-p)} = \sqrt{40(.50)(.50)} = 3.16$

58. Since the shipment is large we can assume that the probabilities do not change from trial to trial and use the binomial probability distribution.

a. $n = 5$

$$f(0) = \binom{5}{0}(0.01)^0(0.99)^5 = \text{BINOM.DIST(0,5,.01,FALSE)} = .9510$$

b. $$f(1) = \binom{5}{1}(0.01)^1(0.99)^4 = \text{BINOM.DIST(1,5,.01,FALSE)} = .0480$$

c. $1 - f(0) = 1 - .9510 = .0490$

d. No, the probability of finding one or more items in the sample defective when only 1% of the items in the population are defective is small (only .0490). I would consider it likely that more than 1% of the items are defective.

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59. a. $E(x) = np = 100(.041) = 4.1$

b. $\text{Var}(x) = np(1 - p) = 100(.041)(.959) = 3.93$

$\sigma = \sqrt{3.93} = 1.98$

60. a. $E(x) = 800(.30) = 240$

b. $\sigma = \sqrt{np(1-p)} = \sqrt{800(.30)(.70)} = 12.9615$

c. For this one $p = .70$ and $(1-p) = .30$, but the answer is the same as in part (b). For a binomial probability distribution, the variance for the number of successes is the same as the variance for the number of failures. Of course, this also holds true for the standard deviation.

61. $\mu = 15$

Probability of 20 or more arrivals $= f(20) + f(21) + \cdots$

$= .0418 + .0299 + .0204 + .0133 + .0083 + .0050 + .0029$
$+ .0016 + .0009 + .0004 + .0002 + .0001 + .0001 = .1249$

Using Excel: 1 – POISSON.DIST(19,15,TRUE) = .1248

62. $\mu = 1.5$

Probability of 3 or more breakdowns is $1 - [f(0) + f(1) + f(2)]$.

$1 - [f(0) + f(1) + f(2)]$

$= 1 - [.2231 + .3347 + .2510]$

$= 1 - .8088 = .1912$

Using Excel: 1 - BINOM.DIST(2,1.5,TRUE) = .1912

63. $\mu = 10 \quad f(4) = \text{POISSON.DIST}(4,10,\text{FALSE}) = .0189$

64. a. $f(3) = \dfrac{3^3 e^{-3}}{3!} = \text{POISSON.DIST}(3,3,\text{FALSE}) = .2240$

b. $f(3) + f(4) + \cdots = 1 - [f(0) + f(1) + f(2)]$

$f(0) = \dfrac{3^0 e^{-3}}{0!} = e^{-3} = .0498$

Similarly, $f(1) = .1494, f(2) = .2240$

$\therefore 1 - [.0498 + .1494 + .2241] = .5767$

Using Excel: 1 - POISSON.DIST(2,3,TRUE) = .5768

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65. Hypergeometric $N = 52, n = 5$ and $r = 4$.

a. $\dfrac{\binom{4}{2}\binom{48}{3}}{\binom{52}{5}} = \dfrac{6(17296)}{2{,}598{,}960} = \text{HYPGEOM.DIST(2,5,4,52,FALSE)} = .0399$

b. $\dfrac{\binom{4}{1}\binom{48}{4}}{\binom{52}{5}} = \dfrac{4(194580)}{2{,}598{,}960} = \text{HYPGEOM.DIST(1,5,4,52,FALSE)} = .2995$

c. $\dfrac{\binom{4}{0}\binom{48}{5}}{\binom{52}{5}} = \dfrac{1{,}712{,}304}{2{,}598{,}960} = \text{HYPGEOM.DIST(0,5,4,52,FALSE)} = .6588$

d. $1 - f(0) = 1 - .6588 = .3412$

66. a. Hypergeometric distribution with $N = 10$, $n = 2$, and $r = 7$.

$$f(1) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}} = \frac{\binom{7}{1}\binom{10-7}{2-1}}{\binom{10}{2}} = \text{HYPGEOM.DIST(1,2,7,10,FALSE)} = .4667$$

b. $f(1) = \dfrac{\binom{7}{2}\binom{3}{0}}{\binom{10}{2}} = \text{HYPGEOM.DIST(2,2,7,10,FALSE)} = .4667$

c. $f(1) = \dfrac{\binom{7}{0}\binom{3}{2}}{\binom{10}{2}} = \text{HYPGEOM.DIST(0,2,7,10,FALSE)} = .0667$

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Chapter 6
Continuous Probability Distributions

Learning Objectives

1. Understand the difference between how probabilities are computed for discrete and continuous random variables.

2. Know how to compute probability values for a continuous uniform probability distribution and be able to compute the expected value and variance for such a distribution.

3. Be able to compute probabilities using a normal probability distribution. Understand the role of the standard normal distribution in this process.

4. Be able to compute probabilities using an exponential probability distribution.

5. Understand the relationship between the Poisson and exponential probability distributions.

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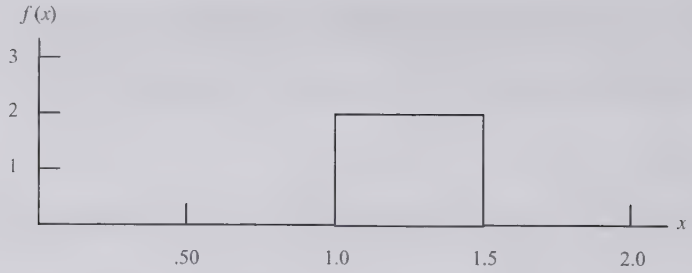
Solutions:

1. a.

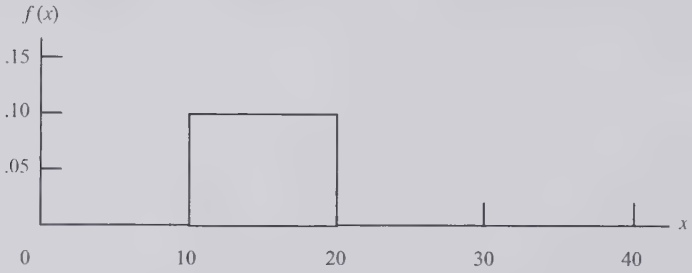


b. $P(x = 1.25) = 0$. The probability of any single point is zero since the area under the curve above any single point is zero.

c. $P(1.0 \le x \le 1.25) = 2(.25) = .50$

d. $P(1.20 < x < 1.5) = 2(.30) = .60$

2. a.



b. $P(x < 15) = .10(5) = .50$

c. $P(12 \le x \le 18) = .10(6) = .60$

d. $E(x) = \dfrac{10+20}{2} = 15$

e. $\text{Var}(x) = \dfrac{(20-10)^2}{12} = 8.33$

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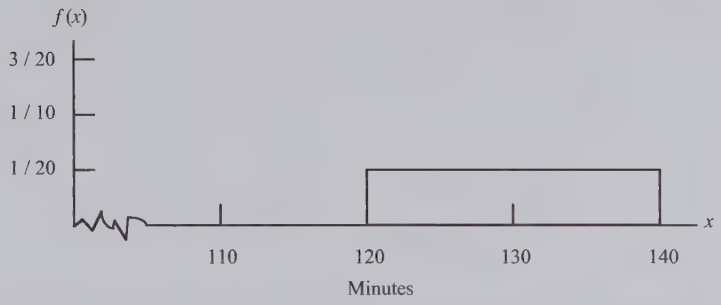
3. a.

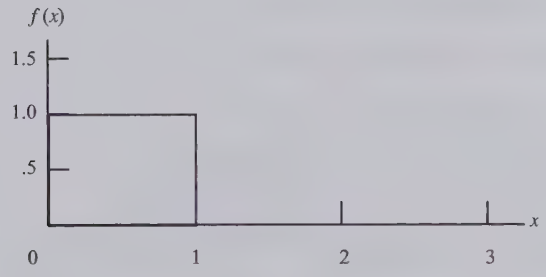


b. $P(x \leq 130) = (1/20)(130 - 120) = 0.50$

c. $P(x > 135) = (1/20)(140 - 135) = 0.25$

d. $E(x) = \frac{120+140}{2} = 130$ minutes

4. a.



b. $P(.25 < x < .75) = 1(.50) = .50$

c. $P(x \leq .30) = 1(.30) = .30$

d. $P(x > .60) = 1(.40) = .40$

5. a. Length of Interval = 310.6 - 284.7 = 25.9

$$f(x) = \begin{cases} \frac{1}{25.9} & \text{for } 284.7 \leq x \leq 310.6 \\ 0 & \text{elsewhere} \end{cases}$$

b. Note: 1/25.9 = .0386

$P(x < 290) = .0386(290 - 284.7) = .2046$

c. $P(x \geq 300) = .0386(310.6 - 300) = .4092$

d. $P(290 \leq x \leq 305) = .0386(305 - 290) = .5790$

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e. $P(x \geq 290) = .0386(310.6 - 290) = .7952$

Rounding up, we conclude that 80 of the top 100 golfers drive the ball this far.

6. a. $P(x \geq 25) = \frac{1}{8}(26 - 25) = .125$

b. $P(21 \leq x \leq 25) = \frac{1}{8}(25 - 21) = .50$

c. This occurs when programming is 20 minutes or less

$P(x \leq 20) = \frac{1}{8}(20 - 18) = .25$

7. a. $P(10{,}000 \leq x < 12{,}000) = 2000\,(1 / 5000) = .40$

The probability your competitor will bid lower than you, and you get the bid, is .40.

b. $P(10{,}000 \leq x < 14{,}000) = 4000\,(1 / 5000) = .80$

c. A bid of \$15,000 gives a probability of 1 of getting the property.

d. Yes, the bid that maximizes expected profit is \$13,000.

The probability of getting the property with a bid of \$13,000 is

$$P(10{,}000 \leq x < 13{,}000) = 3000\,(1 / 5000) = .60.$$

The probability of not getting the property with a bid of \$13,000 is .40.

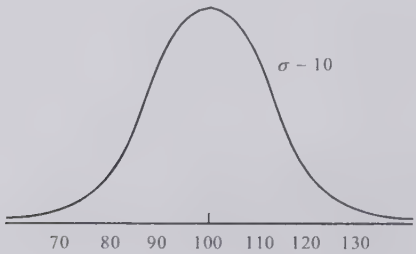
The profit you will make if you get the property with a bid of \$13,000 is \$3000 = \$16,000 - 13,000. So your expected profit with a bid of \$13,000 is

$$\text{EP}(\$13{,}000) = .6\,(\$3000) + .4\,(0) = \$1800.$$

If you bid \$15,000 the probability of getting the bid is 1, but the profit if you do get the bid is only \$1000 = \$16,000 - 15,000. So your expected profit with a bid of \$15,000 is

$$\text{EP}(\$15{,}000) = 1\,(\$1000) + 0\,(0) = \$1{,}000.$$

8.



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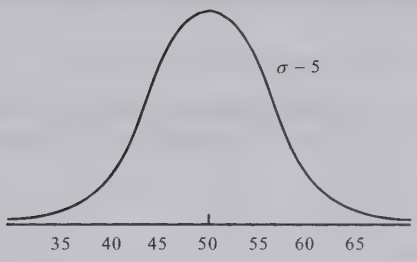
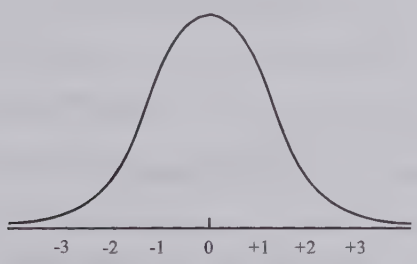
9. a.



b. .683 since 45 and 55 are within plus or minus 1 standard deviation from the mean of 50 (Use the table or see characteristic 7a of the normal distribution).

c. .954 since 40 and 60 are within plus or minus 2 standard deviations from the mean of 50 (Use the table or see characteristic 7b of the normal distribution).

10.



These probabilities can be obtained using Excel's NORM.S.DIST function or the standard normal probability table in the text.

a. $P(z \leq 1.5) = .9332$

b. $P(z \leq 1.0) = .8413$

c. $P(1 \leq z \leq 1.5) = P(z \leq 1.5) - P(z < 1) = .9932 - .8413 = .0919$

d. $P(0 < z < 2.5) = P(z < 2.5) - P(z \leq 0) = .9938 - .5000 = .4938$

11. These probabilities can be obtained using Excel's NORM.S.DIST function or the standard normal probability table in the text.

a. $P(z \leq -1) = .1587$

b. $P(z \geq -1) = 1 - P(z < -1) = 1 - .1587 = .8413$

c. $P(z \geq -1.5) = 1 - P(z < -1.5) = 1 - .0668 = .9332$

d. $P(-2.5 \leq z) = 1 - P(z < -2.5) = 1 - .0062 = .9938$

e. $P(-3 < z \leq 0) = P(z \leq 0) - P(z \leq -3) = .5000 - .0013 = .4987$

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12. These probabilities can be obtained using Excel's NORM.S.DIST function or the standard normal probability table in the text.

    a. $P(0 \le z \le .83) = .7967 - .5000 = .2967$

    b. $P(-1.57 \le z \le 0) = .5000 - .0582 = .4418$

    c. $P(z > .44) = 1 - .6700 = .3300$

    d. $P(z \ge -.23) = 1 - .4090 = .5910$

    e. $P(z < 1.20) = .8849$

    f. $P(z < -.71)$ .2389

13. These probabilities can be obtained using Excel's NORM.S.DIST function or the standard normal probability table in the text.

    a. $P(-1.98 \le z \le .49) = P(z \le .49) - P(z < -1.98) = .6879 - .0239 = .6640$

    b. $P(.52 \le z \le 1.22) = P(z \le 1.22) - P(z < .52) = .8888 - .6985 = .1903$

    c. $P(-1.75 \le z \le -1.04) = P(z \le -1.04) - P(z < -1.75) = .1492 - .0401 = .1091$

14. These z values can be obtained using Excel's NORM.S.INV function or by using the standard normal probability table in the text.

    a. The z value corresponding to a cumulative probability of .9750 is $z = 1.96$.

    b. The z value here also corresponds to a cumulative probability of .9750: $z = 1.96$.

    c. The z value corresponding to a cumulative probability of .7291 is $z = .61$.

    d. Area to the left of z is 1 - .1314 = .8686. So $z = 1.12$.

    e. The z value corresponding to a cumulative probability of .6700 is $z = .44$.

    f. The area to the left of z is .6700. So $z = .44$.

15. These z values can be obtained using Excel's NORM.S.INV function or by using the standard normal probability table in the text.

    a. The z value corresponding to a cumulative probability of .2119 is $z = -.80$.

    b. Compute .9030/2 = .4515; z corresponds to a cumulative probability of .5000 + .4515 = .9515. So z = 1.66.

    c. Compute .2052/2 = .1026; z corresponds to a cumulative probability of .5000 + .1026 = .6026. So z = .26.

    d. The z value corresponding to a cumulative probability of .9948 is z = 2.56.

    e. The area to the left of z is 1 - .6915 = .3085. So $z = -.50$.

16. These z values can be obtained using Excel's NORM.S.INV function or the standard normal probability table in the text.

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a. The area to the left of z is 1 - .0100 = .9900. The z value in the table with a cumulative probability closest to .9900 is $z = 2.33$.

b. The area to the left of z is .9750. So $z = 1.96$.

c. The area to the left of z is .9500. Since .9500 is exactly halfway between .9495 ($z = 1.64$) and .9505($z = 1.65$), we select $z = 1.645$. However, $z = 1.64$ or $z = 1.65$ are also acceptable answers.

d. The area to the left of z is .9000. So $z = 1.28$ is the closest z value.

17. Let x = debt amount

$\mu = 15{,}015, \quad \sigma = 3540$

a. $z = \dfrac{18{,}000 - 15{,}015}{3540} = .84$

$P(x > 18{,}000) = 1 - P(z \leq .84) = 1 - .7995 = .2005$

Using Excel: 1-NORM.DIST(18000,15015,3540,TRUE) = .1996

b. $z = \dfrac{10{,}000 - 15{,}015}{3540} = -1.42$

$P(x < 10{,}000) = P(z < -1.42) = .0778$

Using Excel: NORM.DIST(10000,15015,3540,TRUE) = .0783

c. At 18,000, $z = .84$ from part (a)

At 12,000, $z = \dfrac{12{,}000 - 15{,}015}{3540} = -.85$

$P(12{,}000 < x < 18{,}000) = P(-.85 < z < .84) = .7995 - .1977 = .6018$

Using Excel: NORM.DIST(18000,15015,3540,TRUE) –
NORM.DIST(12000,15015,3540,TRUE) = .6033

d. $z = \dfrac{14{,}000 - 15{,}015}{3540} = -.29$

$P(x \leq 14{,}000) = P(z \leq -.29) = .3859$

Using Excel: NORM.DIST(14000,15015,3540,TRUE) = .3872

18. $\mu = 30$ and $\sigma = 8.2$

a. At $x = 40$, $z = \dfrac{40 - 30}{8.2} = 1.22$

$P(z \leq 1.22) = .8888$

$P(x \geq 40) = 1 - .8888 = .1112$

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Using Excel: 1-NORM.DIST(40,30,8.2,TRUE) = .1113

b. At $x = 20$, $z = \dfrac{20-30}{8.2} = -1.22$

$P(z \leq -1.22) = .1112$

So, $P(x \leq 20) = .1112$

Using Excel: =NORM.DIST(20,30,8.2,TRUE) = .1113

c. A z-value of 1.28 cuts off an area of approximately 10% in the upper tail.

$x = 30 + 8.2(1.28) = 40.50$

A stock price of $40.50 or higher will put a company in the top 10%

Using Excel: =NORM.INV(.9,30,8.2) = 40.51

19. $\mu = 328$ and $\sigma = 92$

a. $z = \dfrac{x-\mu}{\sigma} = \dfrac{500-328}{92} = 1.87$

$P(x > 500) = P(z > 1.87) = 1 - P(z \leq 1.87) = 1 - .9693 = .0307$

The probability that the emergency room visit will cost more than $500 is .0307.

Using Excel: 1-NORM.DIST(500,328,92,TRUE) = .0308

b. $z = \dfrac{x-\mu}{\sigma} = \dfrac{250-328}{92} = -.85$

$P(x < 250) = P(z < -.85) = .1977$

The probability that the emergency room visit will cost less than $250 is .1977.

Using Excel: NORM.DIST(250,328,92,TRUE) = .1983

c. For $x = 400$, $z = \dfrac{x-\mu}{\sigma} = \dfrac{400-328}{92} = .78$

For $x = 300$, $z = \dfrac{x-\mu}{\sigma} = \dfrac{300-328}{92} = -.30$

$P(300 < x < 400) = P(z \leq .78) - P(z \leq -.30) = .7823 - .3821 = .4002$

The probability that the emergency room visit will cost between $300 and $400 is .4002.

Using Excel: NORM.DIST(400,328,92,TRUE) - NORM.DIST(300,328,92,TRUE) = .4026

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d. The lower 8%, or area = .08, occurs for $z = -1.41$

$x = \mu + z\sigma = 328 - 1.41(92) = \$198.28$

For a patient to have a charge in the lower 8%, the cost of the visit must have been $198.28 or less.

Using Excel: NORM.INV(.08,328,92) = 198.73

20. $\mu = 77$ and $\sigma = 20$

a. At $x = 50$, $z = \dfrac{50-77}{20} = -1.35$

$P(z < -1.35) = .0885$

So, $P(x < 50) = .0885$

Using Excel: NORM.DIST(50,77,20,TRUE) = .0885

b. At $x = 100$, $z = \dfrac{100-77}{20} = 1.15$

$P(z > 1.15) = 1 - .8749 = .1251$

So, $P(x > 100) = .1251$

Using Excel: 1-NORM.DIST(100,77,20,TRUE) = .1251

12.51% of workers logged on over 100 hours.

c. A z-value of .84 cuts off an area of approximately .20 in the upper tail.

$x = \mu + z\sigma = 77 + 20(.84) = 93.8$

Using Excel: NORM.INV(.8,77,20) = 93.8

A worker must spend 93.8 or more hours logged on to be classified a heavy user.

21. From the normal probability tables, a z-value of 2.05 cuts off an area of approximately .02 in the upper tail of the distribution.

$x = \mu + z\sigma = 100 + 2.05(15) = 130.75$

Using Excel: NORM.INV(.98,100,15) = 130.8

A score of 131 or better should qualify a person for membership in Mensa.

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22. Use $\mu = 8.35$ and $\sigma = 2.5$

a. We want to find $P(5 \le x \le 10)$

At $x = 10$,

$$z = \frac{x-\mu}{\sigma} = \frac{10-8.35}{2.5} = .66$$

At $x = 5$,

$$z = \frac{x-\mu}{\sigma} = \frac{5-8.35}{2.5} = -1.34$$

$P(5 \le x \le 10) = P(-1.34 \le z \le .66) = P(z \le .66) - P(z \le -1.34)$
$= .7454 - .0901$
$= .6553$

Using Excel: NORM.DIST(10,8.35,2.5,TRUE) - NORM.DIST(5,8.35,2.5,TRUE) = .6553

The probability of a household viewing television between 5 and 10 hours a day is .6553.

b. Find the z-value that cuts off an area of .03 in the upper tail. Using a cumulative probability of 1 - .03 = .97, $z = 1.88$ provides an area of .03 in the upper tail of the normal distribution.

$$x = \mu + z\sigma = 8.35 + 1.88(2.5) = 13.05 \text{ hours}$$

Using Excel: NORM.INV(.97,8.35,2.5) = 13.05

A household must view slightly over 13 hours of television a day to be in the top 3% of television viewing households.

c. At $x = 3$, $z = \frac{x-\mu}{\sigma} = \frac{3-8.35}{2.5} = -2.14$

$P(x>3) = 1 - P(z \le -2.14) = 1 - .0162 = .9838$

The probability a household views more than 3 hours of television a day is .9838.

Using Excel: 1-NORM.DIST(3,8.35,2.5,TRUE) = .9838

23. a. $z = \frac{60-80}{10} = -2$ $P(z \le -2) = .0228$. So $P(x < 60) = .0228$

Using Excel: NORM.DIST(60,80,10,TRUE) =.0228

b. At $x = 60$

$$z = \frac{60-80}{10} = -2 \quad \text{Area to left is .0228}$$

At $x = 75$

$$z = \frac{75-80}{10} = -.5 \quad \text{Area to left is .3085}$$

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$P(60 \leq x \leq 75) = .3085 - .0228 = .2857$

Using Excel: NORM.DIST(75,80,10,TRUE)-NORM.DIST(60,80,10,TRUE) =.2858

c. $z = \dfrac{90-80}{10} = 1 \qquad P(z \leq 1) = P(x \leq 90) = .1587$

Using Excel: 1-NORM.DIST(90,80,10,TRUE) =.1587

Therefore 15.87% of students will not complete on time.

$$(60)\,(.1587) \;=\; 9.522$$

We would expect 9.522 students to be unable to complete the exam in time.

24. a. $\bar{x} = \sum \dfrac{x_i}{n} = 200$

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = 26.04$$

We will use $\bar{x}$ as an estimate of μ and s as an estimate of σ in parts (b) - (d) below.

b. Remember the data are in thousands of shares.

At $x = 180$

$$z = \frac{180-200}{26.04} = -.77$$

$$P(x \leq 180) = P(z \leq -.77) = .2206$$

The probability trading volume will be less than 180 million shares is .2206.

c. At $x = 230$

$$z = \frac{230-200}{26.04} = 1.15$$

$P(x > 230) = P(z > 1.15) = 1 - P(z \leq 1.15) = 1 - .8749 = .1251$

The probability trading volume will exceed 230 million shares is .1251.

d. A z-value of 1.645 cuts off an area of .05 in the upper tail

$x = \mu + z\sigma = 200 + 1.645(26.04) = 242.84$

If the early morning trading volume exceeds 242.84 million shares, the day is among the busiest 5%.

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25. μ 6.8, σ .6

a. At x 8, $z = \dfrac{8-6.8}{.6} = 2.00$

$P(x > 8) = P(z > 2.0) = 1 - .9772 = .0228$

b. At $x = 6$, $z = \dfrac{6-6.8}{.6} = -1.33$

$P(x \le 6) = P(z \le -1.33) = .0918$

c. At $x = 9$, $z = \dfrac{9-6.8}{.6} = 3.67$

At $x = 7$, $z = \dfrac{7-6.8}{.6} = .33$

$P(7 < x < 9) = P(.33 < z < 3.67) = 1 - .6293 = .3707$

Only 37.07 percent of the population gets the amount of sleep recommended by doctors. Most get less.

26. a. $P(x \le 6) = 1 - e^{-6/8} = 1 - .4724 = .5276$

Using Excel: EXPON.DIST(6,1/8,TRUE) = .5276

b. $P(x \le 4) = 1 - e^{-4/8} = 1 - .6065 = .3935$

Using Excel: EXPON.DIST(4,1/8,TRUE) = .3935

c. $P(x \ge 6) = 1 - P(x \le 6) = 1 - .5276 = .4724$

Using Excel: =1-EXPON.DIST(6,1/8,TRUE) = .4724

d. $P(4 \le x \le 6) = P(x \le 6) - P(x \le 4) = .5276 - .3935 = .1341$

Using Excel: EXPON.DIST(6,1/8,TRUE)-EXPON.DIST(4,1/8,TRUE) = .1342

27. a. $P(x \le x_0) = 1 - e^{-x_0/3}$

b. $P(x \le 2) = 1 - e^{-2/3} = 1 - .5134 = .4866$

Using Excel: EXPON.DIST(2,1/3,TRUE) = .4866

c. $P(x \ge 3) = 1 - P(x \le 3) = 1 - (1 - e^{-3/3}) = e^{-1} = .3679$

Using Excel: =1-EXPON.DIST(3,1/3,TRUE) = .3679

d. $P(x \le 5) = 1 - e^{-5/3} = 1 - .1889 = .8111$

Using Excel: EXPON.DIST(5,1/3,TRUE) = .8111

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e. $P(2 \le x \le 5) = P(x \le 5) - P(x \le 2) = .8111 - .4866 = .3245$

Using Excel: EXPON.DIST(5,1/3,TRUE)-EXPON.DIST(2,1/3,TRUE) = .3245

28. a. $P(x < 10) = 1 - e^{-10/12.1} = .5624$

b. $P(x > 20) = 1 - P(x \le 20) = 1 - (1 - e^{-20/12.1}) = e^{-20/12.1} = .1915$

c. $P(10 \le x \le 20) = P(x \le 20) - P(x \le 10)$

$= (1 - e^{-20/12.1}) - (1 - e^{-10/12.1}) = e^{-10/12.1} - e^{-20/12.1} =$

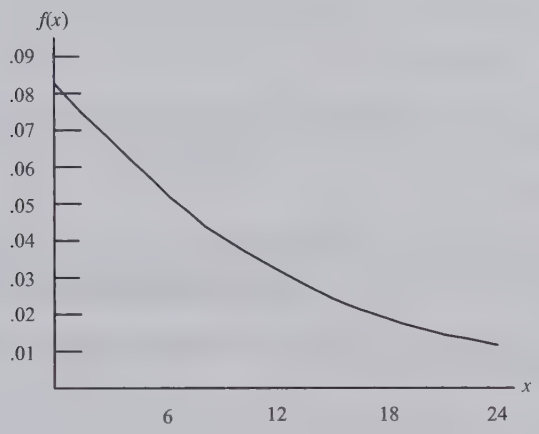
$= .4376 - .1915 = .2461$

d. To make your flight you must get through security within 18 minutes.

$P(x < 18) = 1 - e^{-18/12.1} = .7741$

Thus, the probability you will not make your flight is 1 - .7741 = .2259.

29. a.



b. $P(x \le 12) = 1 - e^{-12/12} = 1 - .3679 = .6321$

Using Excel: EXPON.DIST(12,1/12,TRUE) = .6321

c. $P(x \le 6) = 1 - e^{-6/12} = 1 - .6065 = .3935$

Using Excel: EXPON.DIST(6,1/12,TRUE) = .3935

d. $P(x \ge 30) = 1 - P(x < 30)$
$= 1 - (1 - e^{-30/12})$
$= .0821$

Using Excel: =1-EXPON.DIST(30,1/12,TRUE) = .0821

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30. a. $f(x) = \frac{1}{\mu} e^{-x/\mu} = \frac{1}{2} e^{-x/2}$ for $x \geq 0$

$P(x \leq x_0) = 1 - e^{-x_0/\mu}$

$P(x \leq 1) = 1 - e^{-1/2} = 1 - e^{-.5} = 1 - .6065 = .3935$

b. $P(x \leq 2) = 1 - e^{-2/2} = 1 - e^{-1.0} = 1 - .3679 = .6321$

$P(1 \leq x \leq 2) = P(x \leq 2) - P(x \leq 1) = .6321 - .3935 = .2386$

c. For this customer, the cable service repair would have to take longer than 4 hours.

$P(x > 4) = 1 - P(x \leq 4) = 1 - (1 - e^{-4/2}) = e^{-2.0} = .1353$

31. a. $f(x) = \frac{1}{\mu} e^{-x/\mu} = \frac{1}{25} e^{-x/25}$ for $x \geq 0$

$P(x \leq x_0) = 1 - e^{-x_0/\mu}$

$P(x \leq 20) = 1 - e^{-20/25} = 1 - e^{-.80} = 1 - .4493 = .5507$

b. $P(x > 30) = 1 - P(x \leq 30) = 1 - (1 - e^{-30/25}) = e^{-1.2} = .3012$

c. For the customer to make the 15-minute return trip home by 6:00 p.m., the order must be ready by 5:45 p.m. Since the order was placed at 5:20 p.m., the order must to be ready within 25 minutes.

$P(x \leq 25) = 1 - e^{-25/25} = 1 - e^{-1} = 1 - .3679 = .6321$

This may seem surprising high since the mean time is 25 minutes. But, for the exponential distribution, the probability x being greater than the mean is significantly less than the probability of x being less than the mean. This is because the exponential distribution is skewed to the right.

32. a. If the mean number of interruptions per hour follows the Poisson distribution, the time between interruptions follows the exponential distribution. So,

$\mu = \frac{1}{5.5}$ of an hour and $\frac{1}{\mu} = \frac{1}{1/5.5} = 5.5$

Thus, $f(x) = 5.5e^{-5.5x}$.

Here x is the time between interruptions in hours.

b. Fifteen minutes is 1/4 of an hour so,

$$P\left(x > \frac{1}{4}\right) = 1 - P\left(x \leq \frac{1}{4}\right) = 1 - (1 - e^{-5.5/4}) = e^{-5.5/4} = .2528$$

The probability of no interruptions during a15-minute period is .2528.

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c. Since 10 minutes is 1/6 of an hour, we compute,

$$P\left(x \le \frac{1}{6}\right) = 1 - e^{-5.5/6} = 1 - .3998 = .6002$$

Thus, the probability of being interrupted within 10 minutes is .6002.

33. a. Let x = sales price ($1000s)

$$f(x) = \begin{cases} \frac{1}{25} & \text{for } 200 \le x \le 225 \\ 0 & \text{elsewhere} \end{cases}$$

b. $P(x \ge 215) = (1 / 25)(225 - 215) = 0.40$

c. $P(x < 210) = (1 / 25)(210 - 200) = 0.40$

d. $E(x) = (200 + 225)/2 = 212{,}500$

If she waits, her expected sale price will be $2,500 higher than if she sells it back to her company now. However, there is a 0.40 probability that she will get less. It's a close call. But, the expected value approach to decision making would suggest she should wait.

34. a. Find the z value that cuts off an area of .10 in the lower tail.

From the standard normal table $z \approx -1.28$. Solve for x,

$$-1.28 = \frac{x - 5700}{1500}$$

$$x = 5700 - 1.28(1500) = 3780$$

10% of families spend $3780 or less.

b. $$z = \frac{7000 - 5700}{1500} = .87$$

$P(x > 7000) = 1 - P(z \le .87) = 1 - .8078 = .1922$

19.22% of families spend more than $7000 annually for food and drink.

c. Find the z value that cuts off an area of .05 in the upper tail: $z = 1.645$. Solve for x,

$$1.645 = \frac{x - 5700}{1500}$$

$$x = 5700 + 1.645(1500) = 8167.5$$

5% of families spend more than $8,167.50 annually on food and drink.

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35. a. $P(defect) = 1 - P(9.85 \le x \le 10.15)$

$= 1 - P(-1 \le z \le 1) = 1 - .6826 = .3174$

Expected number of defects $= 1000(.3174) = 317.4$

b. $P(defect) = 1 - P(9.85 \le x \le 10.15)$

$= 1 - P(-3 \le z \le 3) = 1 - .9974 = .0026$

Expected number of defects $= 1000(.0026) = 2.6$

c. Reducing the process standard deviation causes a substantial reduction in the number of defects.

36. $\mu = 6{,}312$

a. $z = -1.645$ cuts off .05 in the lower tail

So,

$$-1.645 = \frac{1000 - 6312}{\sigma}$$

$$\sigma = \frac{1000 - 6312}{-1.645} = 3229$$

b. At 6000, $z = \dfrac{6000 - 6312}{3229} = -.10$

At 4000, $z = \dfrac{4000 - 6312}{3229} = -.72$

$P(4000 < x < 6000) = P(-.72 < z < -.10) = .4602 - .2358 = .2244$

c. $z = 1.88$ cuts off approximately .03 in the upper tail

$x = 6312 + 1.88(3229) = 12{,}382.52$

The households with the highest 3% of expenditures spent more than $12,382.

37. a. $\mu = 670 \quad \sigma = 30$

All rooms will be rented if demand is at least 700.

At $x = 700$

$$z = \frac{x - \mu}{\sigma} = \frac{700 - 670}{30} = 1$$

$P(x \ge 700) = P(z \ge 1) = 1 - P(z < 1) = 1 - .8413 = .1587$

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b. 50 or more rooms will be unrented is demand falls to 650 or less.

At $x = 650$

$$z = \frac{650 - 670}{30} = -.67$$

$P(x \leq 650) = P(z \leq -.67) = .2514$

c. A promotion might be a good idea if it isn't too expensive. Things to consider:

- The probability of renting all the rooms without a promotion is approximately .16.
- The probability is about .25 that 50 or more rooms will go unrented. This is significant lost revenue.
- To be successful, a promotion should increase the expected value of demand above 670.

38. a. At $x = 200$

$$z = \frac{200 - 150}{25} = 2$$

$P(x > 200) = P(z > 2) = 1 - P(z \leq 2) = 1 - .9772 = .0228$

b. Expected Profit = Expected Revenue - Expected Cost

$= 200 - 150 = \$50$

39. $\mu = 1550 \quad \sigma = 300$

a. At $x = 1000$,

$$z = \frac{x - \mu}{\sigma} = \frac{1000 - 1550}{300} = -1.83$$

$P(x < 1000) = P(z < -1.83) = .0336$

Using Excel: NORM.DIST(1000,1550,300,TRUE) =.0334

b. At $x = 2000$,

$$z = \frac{2000 - 1550}{300} = 1.50$$

$P(x < 2000) = P(z < 1.50) = .9332$

$P(x < 1000) = .0336$ (from part (a))

$P(1000 < x < 2000) = .9332 - .0336 = .8996$

Using Excel: NORM.DIST(2000,1550,300,TRUE) - NORM.DIST(1000,1550,300,TRUE) =.8998

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c. Find the z value that cuts off an area of .05 in the upper tail: $z = 1.645$

Solve for x,

$$1.645 = \frac{x - 1550}{300}$$

$x = 1550 + 1.645(300) = 2043.5$

Using Excel: NORM.INV(.95,1550,300) = 2043.5

Rounding up, we would say that 2044 or more crashes would put a year in the top 5% for fatal crashes. It would be a bad year.

40. a. At 400,

$$z = \frac{400 - 450}{100} = -.500$$

Area to left is .3085

At 500,

$$z = \frac{500 - 450}{100} = +.500$$

Area to left is .6915

$P(400 \leq x \leq 500) = .6915 - .3085 = .3830$

38.3% will score between 400 and 500.

b. At 630,

$$z = \frac{630 - 450}{100} = 1.80$$

96.41% do worse and 3.59% do better .

c. At 480,

$$z = \frac{480 - 450}{100} = .30$$

Area to left is .6179

38.21% are acceptable.

41. a. At 100,000

$$z = \frac{100,000 - 88,592}{19,900} = .57$$

$P(x > 100{,}000) = P(z > .57) = 1 - P(z \leq .57) = 1 - .7157 = .2843$

The probability of a Houston brand manager having a base salary in excess of $100,000 is .2843.

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b. At 100,000

$$z = \frac{100{,}000 - 97{,}417}{21{,}800} \approx .12$$

$$P(x > 100{,}000) = P(z > .12) = 1 - P(z \leq .12) = 1 - .5478 = .4522$$

The probability of a Los Angeles brand manager having a base salary in excess of $100,000 is .4522

c. At $x = 75{,}000$

$$z = \frac{75{,}000 - 97{,}417}{21{,}800} \approx -1.03$$

$$P(x < 75{,}000) = P(z < -1.03) = .1515$$

The probability of a Los Angeles brand manager receiving a base salary below $75,000 is small: .1515

d. The answer to this is the Houston brand manager base salary that cuts off an area of .01 in the upper tail of the distribution for Houston brand managers.

Use $z = 2.33$

$$x = 88{,}592 + 2.33(19{,}900) = 134{,}959$$

A Los Angeles brand manager who makes $134,959 or more will earn more than 99% of the Houston brand managers.

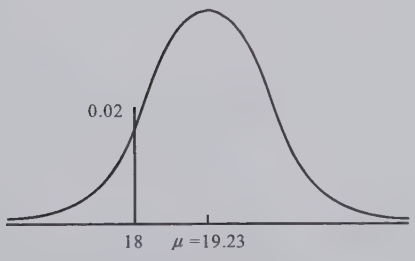
42. $\sigma = .6$

At 2%

$$z \approx -2.05 \quad x = 18$$

$$z = \frac{x - \mu}{\sigma} \qquad \therefore -2.05 = \frac{18 - \mu}{.6}$$

$$\mu = 18 + 2.05\,(.6) = 19.23 \text{ oz.}$$



The mean filling weight must be 19.23 oz.

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43. a. Given $P(x \le 5) = 1 - e^{-x/\mu} = 1 - e^{-5/\mu} = .53$

Compute $P(x \le 5)$ for μ= 5.8, 6.2 and 7.0

$P(x \le 5) = 1 - e^{-5/5.8} = .5777$

$P(x \le 5) = 1 - e^{-5/5.8} = .5777$

$P(x \le 5) = 1 - e^{-5/6.2} = .5536$

$P(x \le 5) = 1 - e^{-5/7} = .5105$

Using Excel:

μ	$P(x \le 5)$
5.8	EXPON.DIST(5,1/5.8,TRUE) =.5777
6.2	EXPON.DIST(5,1/6.2,TRUE) =.5536
6.6	EXPON.DIST(5,1/6.6,TRUE) =.5312
7.0	EXPON.DIST(5,1/7,TRUE) =.5105

μ = 6.6 provides the closest probability to $P(x \le 5) = .53$

b. $P(x > 10) = 1 - P(x \le 10) = 1 - (1 - e^{-10/6.6}) = 1 - .7802 = .2198$

Using Excel: 1-EXPON.DIST(10,1/6.6,TRUE) = .2198

c. $P(x \le 4) = 1 - e^{-4/6.6} = .4545$

$P(x \le 8) = 1 - e^{-8/6.6} = .7024$

The probability that a worker uses the office computer between four and eight hours is .7024 – .4545 = .2479

Using Excel: EXPON.DIST(8,1/6.6,TRUE) - EXPON.DIST(4,1/6.6,TRUE) = .2479

44. a. Mean time between arrivals = 1/7 minutes

b. $f(x) = 7e^{-7x}$

c. $P(x > 1) = 1 - P(x < 1) = 1 - [1 - e^{-7(1)}] = e^{-7} = .0009$

d. 12 seconds is .2 minutes

$P(x > .2) = 1 - P(x < .2) = 1 - [1 - e^{-7(.2)}] = e^{-1.4} = .2466$

45. a. $\frac{1}{36.5} e^{-x/36.5} \approx .0274e^{-.0274x}$

b. $P(x < 40) = 1 - e^{-.0274(40)} = 1 - .3342 = .6658$

$P(x < 20) = 1 - e^{-.0274(20)} = 1 - .5781 = .4219$

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$P(20 < x < 40) = .6658 - .4219 = .2439$

c. From part (b), $P(x < 40) = .6658$

$P(x > 40) = P(x \geq 40) = 1 - P(x < 40) = 1 - .6658 = .3342$

46. a. $\frac{1}{\mu} = 0.5$ therefore μ = 2 minutes = mean time between telephone calls

b. Note: 30 seconds = .5 minutes

$P(x \leq .5) = 1 - e^{-.5/2} = 1 - .7788 = .2212$

c. $P(x \leq 1) = 1 - e^{-1/2} = 1 - .6065 = .3935$

d. $P(x \geq 5) = 1 - P(x < 5) = 1 - (1 - e^{-5/2}) = .0821$

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Chapter 7
Sampling and Sampling Distributions

Learning Objectives

1. Understand the importance of sampling and how results from samples can be used to provide estimates of population characteristics such as the population mean, the population standard deviation and / or the population proportion.

2. Know what simple random sampling from a finite population is and how simple random samples are selected.

3. Understand how to select a random sample from an infinite population.

4. Understand the concept of a sampling distribution.

5. Understand the central limit theorem and the important role it plays in sampling.

6. Specifically know the characteristics of the sampling distribution of the sample mean ($\bar{x}$) and the sampling distribution of the sample proportion ($\bar{p}$).

7. Learn about a variety of sampling methods including stratified random sampling, cluster sampling, systematic sampling, convenience sampling and judgment sampling.

8. Know the definition of the following terms:

parameter	target population
sampled population	sampling distribution
sample statistic	finite population correction factor
simple random sampling	standard error
point estimator	central limit theorem
point estimate	unbiased

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Solutions:

1. a. AB, AC, AD, AE, BC, BD, BE, CD, CE, DE

   b. With 10 samples, each has a 1/10 probability.

   c. B and D because the two smallest random numbers are .0476 and .0957.

2. The 4 smallest random numbers are .0341, .0729, .0936, and .1449. So elements 2, 3, 5, and 10 are the simple random sample.

3. The simple random sample consists of New York, Detroit, Oakland, Boston, and Kansas City.

4. Step 1: Generate a random number using the RAND() function for each of the 10 stocks.

   Step 2: Sort the list of stocks with respect to the random numbers. The first 3 stocks in the sorted list make up the simple random sample.

5. Use the data disk accompanying the book and the EAI file. Generate a random number using the RAND() function for each of the 2500 managers. Then sort the list of managers with respect to the random numbers. The first 50 managers are the sample.

6. a. Finite population. A frame could be constructed obtaining a list of licensed drivers from the New York state driver's license bureau.

   b. Sampling from an infinite population. The sample is taken from the production line producing boxes of cereal.

   c. Sampling from an infinite population. The sample is taken from the ongoing arrivals to the Golden Gate Bridge.

   d. Finite population. A frame could be constructed by obtaining a listing of students enrolled in the course from the professor.

   e. Sampling from an infinite population. The sample is taken from the ongoing orders being processed by the mail-order firm.

7. a. $\bar{x} = \Sigma x_i / n = \frac{54}{6} = 9$

   b. $s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$

   $\Sigma(x_i - \bar{x})^2 = (-4)^2 + (-1)^2 + 1^2 + (-2)^2 + 1^2 + 5^2 = 48$

   $s = \sqrt{\frac{48}{6-1}} = 3.1$

8. a. $\bar{p} = 75/150 = .50$

   b. $\bar{p} = 55/150 = .3667$

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9. a. $\bar{x} = \Sigma x_i / n = \dfrac{465}{5} = 93$

b.

	x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
	94	+1	1
	100	+7	49
	85	-8	64
	94	+1	1
	92	-1	1
Totals	465	0	116

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{116}{4}} = 5.39$$

10. a. Eighteen of the 40 funds in the sample are load funds. Our point estimate is

$$\bar{p} = \frac{18}{40} = .45$$

b. Six of the 40 funds in the sample are high risk funds. Our point estimate is

$$\bar{p} = \frac{6}{40} = .15$$

c. The below average fund ratings are low and very low. Twelve of the funds have a rating of low and 6 have a rating of very low. Our point estimate is

$$\bar{p} = \frac{18}{40} = .45$$

11. a. $\bar{x} = \Sigma x_i / n = \dfrac{\$45,500}{10} = \$4,550$

b. $s = \sqrt{\dfrac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\dfrac{9,068,620}{10-1}} = \$1003.80$

12. a. We would use the sample proportion for the estimate.

$$\bar{p} = \frac{5}{50} = .10$$

(Authors' note: The actual proportion from New York is $p = \dfrac{52}{500} = .104.$)

b. The sample proportion from Minnesota is

$$\bar{p} = \frac{2}{50} = .04$$

Our estimate of the number of Fortune 500 companies from Minnesota is (.04)500 = 20.

(Authors' note: The actual number from Minnesota is 18.)

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c. Fourteen of the 50 in the sample come from these 4 states. So 36 do not.

$$\bar{p} = \frac{36}{50} = .72$$

(Authors' note: The actual proportion not based in these 4 states is $p = \frac{366}{500} = .732.$)

13. a. 409/999 = .41

b. 299/999 = .30

c. 291/999 = .29

14. a. Use the data disk accompanying the book and the EAI file. Generate a random number for each manager and select managers associated with the 50 smallest random numbers as the sample.

b. Use Excel's AVERAGE function to compute the mean for the sample.

c. Use Excel's STDEV function to compute the sample standard deviation.

d. Use the sample proportion as a point estimate of the population proportion.

15. a. The sampling distribution is normal with

$E(\bar{x}) = \mu = 200$

$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 50/\sqrt{100} = 5$

For ± 5, $195 \le \bar{x} \le 205$

Using Standard Normal Probability Table:

At $\bar{x} = 205$, $z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{5}{5} = 1$ $P(z \le 1) = .8413$

At $\bar{x} = 195$, $z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{-5}{5} = -1$ $P(z < -1) = .1587$

$P(195 \le \bar{x} \le 205) = .8413 - .1587 = .6826$

Using Excel: NORM.DIST(205,200,5,TRUE)-NORM.DIST(195,200,5,TRUE) = .6827

b. For ± 10, $190 \le \bar{x} \le 210$

Using Standard Normal Probability Table:

At $\bar{x} = 210$, $z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{10}{5} = 2$ $P(z \le 2) = .9772$

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At $\bar{x} = 190$, $z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \dfrac{-10}{5} = -2$ $P(z < -2) = .0228$

$P(190 \le \bar{x} \le 210) = .9772 - .0228 = .9544$

Using Excel: NORM.DIST(210,200,5,TRUE)-NORM.DIST(190,200,5,TRUE) = .9545

16. $\sigma_{\bar{x}} = \sigma / \sqrt{n}$

$\sigma_{\bar{x}} = 25 / \sqrt{50} = 3.54$

$\sigma_{\bar{x}} = 25 / \sqrt{100} = 2.50$

$\sigma_{\bar{x}} = 25 / \sqrt{150} = 2.04$

$\sigma_{\bar{x}} = 25 / \sqrt{200} = 1.77$

The standard error of the mean decreases as the sample size increases.

17. a. $\sigma_{\bar{x}} = \sigma / \sqrt{n} = 10 / \sqrt{50} = 1.41$

b. $n / N = 50 / 50{,}000 = .001$

Use $\sigma_{\bar{x}} = \sigma / \sqrt{n} = 10 / \sqrt{50} = 1.41$

c. $n / N = 50 / 5000 = .01$

Use $\sigma_{\bar{x}} = \sigma / \sqrt{n} = 10 / \sqrt{50} = 1.41$

d. $n / N = 50 / 500 = .10$

Use $\sigma_{\bar{x}} = \sqrt{\dfrac{N-n}{N-1}} \dfrac{\sigma}{\sqrt{n}} = \sqrt{\dfrac{500-50}{500-1}} \dfrac{10}{\sqrt{50}} = 1.34$

Note: Only case (d) where $n / N = .10$ requires the use of the finite population correction factor.

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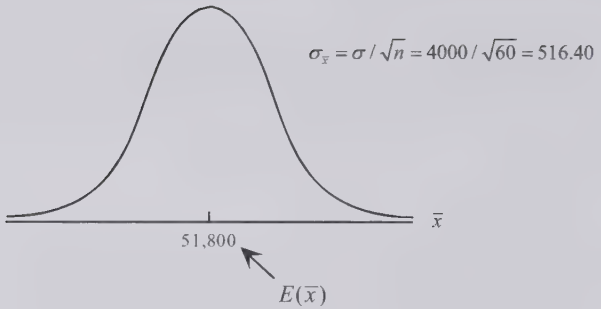
18. a.



The normal distribution for $\bar{x}$ is based on the Central Limit Theorem.

b. For $n = 120$, $E(\bar{x})$ remains \$51,800 and the sampling distribution of $\bar{x}$ can still be approximated by a normal distribution. However, $\sigma_{\bar{x}}$ is reduced to $4000/\sqrt{120} = 365.15$.

c. As the sample size is increased, the standard error of the mean, $\sigma_{\bar{x}}$, is reduced. This appears logical from the point of view that larger samples should tend to provide sample means that are closer to the population mean. Thus, the variability in the sample mean, measured in terms of $\sigma_{\bar{x}}$, should decrease as the sample size is increased.

19. a. With a sample of size 60 $\sigma_{\bar{x}} = \dfrac{4000}{\sqrt{60}} = 516.40$

At $\bar{x} = 52{,}300$, $z = \dfrac{52{,}300 - 51{,}800}{516.40} = .97$

$P(\bar{x} \le 52{,}300) = P(z \le .97) = .8340$

At $\bar{x} = 51{,}300$, $z = \dfrac{51{,}300 - 51{,}800}{516.40} = -.97$

$P(\bar{x} < 51{,}300) = P(z < -.97) = .1660$

$P(51{,}300 \le \bar{x} \le 52{,}300) = .8340 - .1660 = .6680$

Using Excel: NORM.DIST(52300,51800,516.40,TRUE)-
NORM.DIST(51300,51800,516.40,TRUE) = .6671

b. $\sigma_{\bar{x}} = \dfrac{4000}{\sqrt{120}} = 365.15$

At $\bar{x} = 52{,}300$, $z = \dfrac{52{,}300 - 51{,}800}{365.15} = 1.37$

$P(\bar{x} \le 52{,}300) = P(z \le 1.37) = .9147$

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At $\bar{x} = 51{,}300$, $z = \dfrac{51{,}300 - 51{,}800}{365.15} = -1.37$

$P(\bar{x} < 51{,}300) = P(z < -1.37) = .0853$

$P(51{,}300 \le \bar{x} \le 52{,}300) = .9147 - .0853 = .8294$

Using Excel: NORM.DIST(52300,51800,365.15,TRUE)-
NORM.DIST(51300,51800,365.15,TRUE) = .8291

20. a. Normal distribution, $E(\bar{x}) = 17.5$

$\sigma_{\bar{x}} = \sigma / \sqrt{n} = 4 / \sqrt{50} = .57$

b. Within 1 week means $16.5 \le \bar{x} \le 18.5$

At $\bar{x} = 18.5$, $z = \dfrac{18.5 - 17.5}{.57} = 1.75$ $P(z \le 1.75) = .9599$

At $\bar{x} = 16.5$, $z = -1.75$. $P(z < -1.75) = .0401$

So $P(16.5 \le \bar{x} \le 18.5) = .9599 - .0401 = .9198$

c. Within 1/2 week means $17.0 \le \bar{x} \le 18.0$

At $\bar{x} = 18.0$, $z = \dfrac{18.0 - 17.5}{.57} = .88$ $P(z \le .88) = .8106$

At $\bar{x} = 17.0$, $z = -.88$ $P(z < -.88) = .1894$

$P(17.0 \le \bar{x} \le 18.0) = .8106 - .1894 = .6212$

21. $\sigma_{\bar{x}} = \sigma / \sqrt{n} = 100 / \sqrt{90} = 10.54$ This value for the standard error can be used for parts (a) and (b) below.

a. $z = \dfrac{512 - 502}{10.54} = .95$ $P(z \le .95) = .8289$

$z = \dfrac{492 - 502}{10.54} = -.95$ $P(z < -.95) = .1711$

probability = .8289 - .1711 = .6578

Using Excel: NORM.DIST(512,502,10.54,TRUE)-NORM.DIST(492,502,10.54,TRUE) = .6573

b. $z = \dfrac{525 - 515}{10.54} = .95$ $P(z \le .95) = .8289$

$z = \dfrac{505 - 515}{10.54} = -.95$ $P(z < -.95) = .1711$

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probability = .8289 - .1711 = .6578

Using Excel: NORM.DIST(525,515,10.94,TRUE)-NORM.DIST(505,515,10.54,TRUE) = .6573

The probability of being within 10 of the mean on the Mathematics portion of the test is exactly the same as the probability of being within 10 on the Critical Reading portion of the SAT. This is because the standard error is the same in both cases. The fact that the means differ does not affect the probability calculation.

22. a. $z = \dfrac{\bar{x} - 939}{\sigma / \sqrt{n}}$

Within $\pm$ 25 means $\bar{x}$ - 939 must be between -25 and +25.

The z value for $\bar{x}$ - 939 = -25 is just the negative of the z value for $\bar{x}$ - 939 = 25. So we just show the computation of z for $\bar{x}$ - 939 = 25.

$n = 30$ $\quad z = \dfrac{25}{245/\sqrt{30}} = .56$ $\quad P(-.56 \le z \le .56) = .7123 - .2877 = .4246$

Using Excel: NORM.DIST(964,939,245/SQRT(30),TRUE)-
NORM.DIST(914,939,245/SQRT(30),TRUE) = .4238

$n = 50$ $\quad z = \dfrac{25}{245/\sqrt{50}} = .72$ $\quad P(-.72 \le z \le .72) = .7642 - .2358 = .5284$

Using Excel: NORM.DIST(964,939,245/SQRT(50),TRUE)-
NORM.DIST(914,939,245/SQRT(50),TRUE) = .5294

$n = 100$ $\quad z = \dfrac{25}{245/\sqrt{100}} = 1.02$ $\quad P(-1.02 \le z \le 1.02) = .8461 - .1539 = .6922$

Using Excel: NORM.DIST(964,939,245/SQRT(100),TRUE)-
NORM.DIST(914,939,245/SQRT(100),TRUE) = .6925

$n = 400$ $\quad z = \dfrac{25}{245/\sqrt{400}} = 2.04$ $\quad P(-2.04 \le z \le 2.04) = .9793 - .0207 = .9586$

Using Excel: NORM.DIST(964,939,245/SQRT(400),TRUE)-
NORM.DIST(914,939,245/SQRT(400),TRUE) = .9587

b. A larger sample increases the probability that the sample mean will be within a specified distance of the population mean. In the automobile insurance example, the probability of being within ±25 of μ ranges from .4246 for a sample of size 30 to .9586 for a sample of size 400.

23. a. $\sigma_{\bar{x}} = \sigma / \sqrt{n} = 40{,}000 / \sqrt{40} = 6324.56$

At $\bar{x}$ = 178,000, $z = \dfrac{178{,}000 - 168{,}000}{6324.56} = 1.58$ $\quad P(z \le 1.58) = .9429$

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At $\bar{x} = 158{,}000$, $z = -1.58$

$P(z < -1.58) = .0571$, thus

$P(158{,}000 \le \bar{x} \le 178{,}000) = .9429 - .0571 = .8858$

Using Excel: NORM.DIST(178000,168000,6324.56,TRUE)-
NORM.DIST(158000,168000,6324.56,TRUE) = .8861

b. $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 25{,}000/\sqrt{40} = 3952.85$

At $\bar{x} = 127{,}000$, $z = \dfrac{127{,}000 - 117{,}000}{3952.85} = 2.53$ $P(z \le 2.53) = .9943$

At $\bar{x} = 107{,}000$, $z = -2.53$, $P(z < -2.53) = .0057$, thus

$P(107{,}000 \le \bar{x} \le 127{,}000) = .9943 - .0057 = .9886$

Using Excel: NORM.DIST(127000,117000,3952.85,TRUE)-
NORM.DIST(107000,117000,3952.85,TRUE) = .9886

c. In part (b) we have a higher probability of obtaining a sample mean within $10,000 of the population mean because the standard error is smaller.

d. With $n = 100$, $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 40{,}000/\sqrt{100} = 4000$

At $\bar{x} = 164{,}000$, $z = \dfrac{164{,}000 - 168{,}000}{4000} = -1$

$P(\bar{x} < 164{,}000) = P(z < -1) = .1587$

Using Excel: NORM.DIST(164000,168000,6324.56,TRUE) = .1587

24. a. This is a graph of a normal distribution with $E(\bar{x}) = 95$ and

$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 14/\sqrt{30} = 2.56$

b. Within 3 strokes means $92 \le \bar{x} \le 98$

$z = \dfrac{98 - 95}{2.56} = 1.17$ $\quad$ $z = \dfrac{92 - 95}{2.56} = -1.17$

$P(92 \le \bar{x} \le 98) = P(-1.17 \le z \le 1.17) = .8790 - .1210 = .7580$

The probability the sample means will be within 3 strokes of the population mean of 95 is .7580.

Using Excel: NORM.DIST(98,95,14/SQRT(30),TRUE)-
NORM.DIST(92,95,14/SQRT(30),TRUE) = .7595

c. $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 14/\sqrt{45} = 2.09$

Within 3 strokes means $103 \le \bar{x} \le 109$

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$$z = \frac{109 - 106}{2.09} = 1.44 \qquad z = \frac{103 - 106}{2.09} = -1.44$$

$P(103 \le \bar{x} \le 109) = P(-1.44 \le z \le 1.44) = .9251 - .0749 = .8502$

The probability the sample means will be within 3 strokes of the population mean of 106 is .8502.

Using Excel: NORM.DIST(98,95,14/SQRT(45),TRUE)-
NORM.DIST(92,95,14/SQRT(45),TRUE) = .8494

d. The probability of being within 3 strokes for female golfers is higher because the sample size is larger.

25. $\mu = 2.34 \quad \sigma = .20$

a. $n = 30$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{.03}{.20 / \sqrt{30}} = .82$$

$P(2.31 \le \bar{x} \le 2.37) = P(-.82 \le z \le .82) = .7939 - .2061 = .5878$

b. $n = 50$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{.03}{.20 / \sqrt{50}} = 1.06$$

$P(2.31 \le \bar{x} \le 2.37) = P(-1.06 \le z \le 1.06) = .8554 - .1446 = .7108$

c. $n = 100$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{.03}{.20 / \sqrt{100}} = 1.50$$

$P(2.31 \le \bar{x} \le 2.37) = P(-1.50 \le z \le 1.50) = .9332 - .0668 = .8664$

d. None of the sample sizes in parts (a), (b), and (c) are large enough. At $z = 1.96$ we find $P(-1.96 \le z \le 1.96) = .95$. So, we must find the sample size corresponding to $z = 1.96$. Solve

$$\frac{.03}{.20 / \sqrt{n}} = 1.96$$

$$\sqrt{n} = 1.96\left(\frac{.20}{.03}\right) = 13.0667$$

$$n = 170.73$$

Rounding up, we see that a sample size of 171 will be needed to ensure a probability of .95 that the sample mean will be within ± $.03 of the population mean.

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26. a. $n / N = 40 / 4000 = .01 < .05$; therefore, the finite population correction factor is not necessary.

b. With the finite population correction factor

$$\sigma_{\bar{x}} = \sqrt{\frac{N-n}{N-1}}\frac{\sigma}{\sqrt{n}} = \sqrt{\frac{4000-40}{4000-1}}\frac{8.2}{\sqrt{40}} = 1.29$$

Without the finite population correction factor

$$\sigma_{\bar{x}} = \sigma / \sqrt{n} = 1.30$$

Including the finite population correction factor provides only a slightly different value for $\sigma_{\bar{x}}$ than when the correction factor is not used.

c. $z = \dfrac{\bar{x}-\mu}{1.30} = \dfrac{2}{1.30} = 1.54 \quad P(z \le 1.54) = .9382$

$P(z < -1.54) = .0618$

Probability = .9382 - .0618 = .8764

Using Excel:

No value for the population mean is given, but we can use Excel's NORM.S.DIST function here.

NORM.S.DIST(2/1.3)-NORM.S.DIST(-2/1.3) = .8761

27. a. $E(\bar{p}) = p = .40$

b. $\sigma_{\bar{p}} = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{.40(.60)}{100}} = .0490$

c. Normal distribution with $E(\bar{p}) = .40$ and $\sigma_{\bar{p}} = .0490$

d. It shows the probability distribution for the sample proportion $\bar{p}$.

28. a. $E(\bar{p}) = .40$

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.40(.60)}{200}} = .0346$$

Within ± .03 means $.37 \le \bar{p} \le .43$

$$z = \frac{\bar{p}-p}{\sigma_{\bar{p}}} = \frac{.03}{.0346} = .87 \quad P(z \le .87) = .8078$$

$P(z < -.87) = .1922$

$P(.37 \le \bar{p} \le .43) = .8078 - .1922 = .6156$

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Using Excel: NORM.DIST(.43,.40,.0346,TRUE)-NORM.DIST(.37,.40,.0346,TRUE) = .6141

b. $z = \dfrac{\bar{p} - p}{\sigma_{\bar{p}}} = \dfrac{.05}{.0346} = 1.44 \quad P(z < 1.44) = .9251$

$P(z < -1.44) = .0749$

$P(.35 \le \bar{p} \le .45) = .9251 - .0749 = .8502$

Using Excel: NORM.DIST(.45,.40,.0346,TRUE)-NORM.DIST(.35,.40,.0346,TRUE) = .8516

29. $\sigma_{\bar{p}} = \sqrt{\dfrac{p(1-p)}{n}}$

$\sigma_{\bar{p}} = \sqrt{\dfrac{(.55)(.45)}{100}} = .0497$

$\sigma_{\bar{p}} = \sqrt{\dfrac{(.55)(.45)}{200}} = .0352$

$\sigma_{\bar{p}} = \sqrt{\dfrac{(.55)(.45)}{500}} = .0222$

$\sigma_{\bar{p}} = \sqrt{\dfrac{(.55)(.45)}{1000}} = .0157$

$\sigma_{\bar{p}}$ decreases as n increases

30. a. $\sigma_{\bar{p}} = \sqrt{\dfrac{(.30)(.70)}{100}} = .0458$

Within ± .04 means $.26 \le \bar{p} \le .34$

$z = \dfrac{\bar{p} - p}{\sigma_{\bar{p}}} = \dfrac{.04}{.0458} = .87 \quad P(z \le .87) = .8078$

$P(z < -.87) = .1922$

$P(.26 \le \bar{p} \le .34) = .8078 - .1922 = .6156$

Using Excel: NORM.DIST(.34,.30,.0458,TRUE)-NORM.DIST(.26,.30,.0458,TRUE) = .6175

b. $\sigma_{\bar{p}} = \sqrt{\dfrac{(.30)(.70)}{200}} = .0324$

$z = \dfrac{\bar{p} - p}{\sigma_{\bar{p}}} = \dfrac{.04}{.0324} = 1.23 \quad P(z < 1.23) \quad .8907$

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$P(z < -1.23) = .1093$

$P(.26 \le \bar{p} \le .34) = .8907 - .1093 = .7814$

Using Excel: NORM.DIST(.34,.30,.0324,TRUE)-NORM.DIST(.26,.30,.0324,TRUE) = .7830

c. $\sigma_{\bar{p}} = \sqrt{\frac{(.30)(.70)}{500}} = .0205$

$z = \frac{\bar{p} - p}{\sigma_{\bar{p}}} = \frac{.04}{.0205} = 1.95 \quad P(z \le 1.95) = .9744$

$P(z < -1.95) = .0256$

$P(.26 \le \bar{p} \le .34) = .9744 - .0256 = .9488$

Using Excel: NORM.DIST(.34,.30,.0205,TRUE)-NORM.DIST(.26,.30,.0205,TRUE) = .9490

d. $\sigma_{\bar{p}} = \sqrt{\frac{(.30)(.70)}{1000}} = .0145$

$z = \frac{\bar{p} - p}{\sigma_{\bar{p}}} = \frac{.04}{.0145} = 2.76 \quad P(z \le 2.76) = .9971$

$P(z < -2.76) = .0029$

$P(.26 \le \bar{p} \le .34) = .9971 - .0029 = .9942$

Using Excel: NORM.DIST(.34,.30,.0145,TRUE)-NORM.DIST(.26,.30,.0145,TRUE) = .9942

e. With a larger sample, there is a higher probability $\bar{p}$ will be within ± .04 of the population proportion p.

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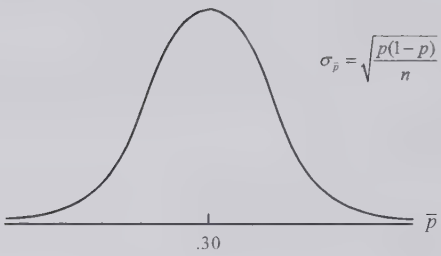
31. a.



The normal distribution is appropriate because $np = 100(.30) = 30$ and $n(1 - p) = 100(.70) = 70$ are both greater than 5.

b. $P(.20 \le \bar{p} \le .40) = ?$

$$z = \frac{.40 - .30}{.0458} = 2.18 \quad P(z \le 2.18) = .9854$$

$P(z < -2.18) = .0146$

$P(.20 \le \bar{p} \le .40) = .9854 - .0146 = .9708$

Using Excel: NORM.DIST(.40,.30,.0458,TRUE)-NORM.DIST(.20,.30,.0458,TRUE) = .9710

c. $P(.25 \le \bar{p} \le .35) = ?$

$$z = \frac{.35 - .30}{.0458} = 1.09 \quad P(z \le 1.09) = .8621$$

$P(z < -1.09) = .1379$

$P(.25 \le \bar{p} \le .35) = .8621 - .1379 = .7242$

Using Excel: NORM.DIST(.35,.30,.0458,TRUE)-NORM.DIST(.25,.30,.0458,TRUE) = .7250

32. a. This is a graph of a normal distribution with a mean of $E(\bar{p}) = .66$ and

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.66(1-.66)}{300}} = .0273$$

b. Within ± .04 means $.62 \le \bar{p} \le .70$

$$z = \frac{.70 - .66}{.0273} = 1.47 \qquad z = \frac{.62 - .66}{.0273} = -1.47$$

$P(.62 \le \bar{p} \le .70) = P(-1.47 \le z \le 1.47) = .9292 - .0708 = .8584$

Using Excel: NORM.DIST(.70,.66,.0273,TRUE)-NORM.DIST(.62,.66,.0273,TRUE) = .8571

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c. $\sigma_{\bar{p}} = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{.87(1-.87)}{300}} = .0194$

Within ± .04 means $.83 \le \bar{p} \le .91$

$z = \dfrac{.91-.87}{.0194} = 2.06 \qquad z = \dfrac{.83-.87}{.0194} = -2.06$

$P(.83 \le \bar{p} \le .91) = P(-2.06 \le z \le 2.06) = .9803 - .0197 = .9606$

Using Excel: NORM.DIST(.91,.87,.0194,TRUE)-NORM.DIST(.83,.87,.0194,TRUE) = .9608

d. Yes, the probability of being within ± .04 is higher for the sample of youth users. This is because the standard error is smaller for the population proportion as it gets closer to 1.

e. For $n = 600$, $\sigma_{\bar{p}} = \sqrt{\dfrac{.66(1-.66)}{600}} = .0193$

Within ± .04 means $.62 \le \bar{p} \le .70$

$z = \dfrac{.70-.66}{.0193} = 2.07 \qquad z = \dfrac{.62-.66}{.0193} = -2.07$

$P(.62 \le \bar{p} \le .70) = P(-2.07 \le z \le 2.07) = .9808 - .0192 = .9616$

Using Excel: NORM.DIST(.70,.66,.0193,TRUE)-NORM.DIST(.62,.66,.0193,TRUE) = .9618

The probability is larger than in part (b). This is because the larger sample size has reduced the standard error.

33. a. Normal distribution

$E(\bar{p}) = .12$

$\sigma_{\bar{p}} = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{(.12)(1-.12)}{540}} = .0140$

b. $z = \dfrac{\bar{p}-p}{\sigma_{\bar{p}}} = \dfrac{.03}{.0140} = 2.14 \qquad P(z \le 1.94) = .9838$

$P(z < -2.14) = .0162$

$P(.09 \le \bar{p} \le .15) = .9838 - .0162 = .9676$

Using Excel: NORM.DIST(.15,.12,.0140,TRUE)-NORM.DIST(.09,.12,.0140,TRUE) = .9679

c. $z = \dfrac{\bar{p}-p}{\sigma_{\bar{p}}} = \dfrac{.015}{.0140} = 1.07 \qquad P(z \le 1.07) = .8577$

$P(z < -1.07) = .1423$

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$P(.105 \le \bar{p} \le .135) = .8577 - .1423 = .7154$

Using Excel: NORM.DIST(.135,.12,.0140,TRUE)-NORM.DIST(.105,.12,.0140,TRUE) = .7160

34. a. Normal distribution

$E(\bar{p}) = .56$

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(.56)(.44)}{400}} = .0248$$

b. $z = \dfrac{.02}{.0248} = .81$ $P(z \le .81) = .7910$

$P(z < -.81) = .2090$

$P(.54 \le \bar{p} \le .58) = .7910 - .2090 = .5820$

Using Excel: NORM.DIST(.58,.56,.0248,TRUE)-NORM.DIST(.54,.56,.0248,TRUE) = .5800

c. $z = \dfrac{.04}{.0248} = 1.61$ $P(z \le 1.61) = .9463$

$P(z < -1.61) = .0537$

$P(.52 \le \bar{p} \le .60) = .9463 - .0537 = .8926$

Using Excel: NORM.DIST(.60,.56,.0248,TRUE)-NORM.DIST(.52,.56,.0248,TRUE) = .8932

35. a. Normal distribution with $E(\bar{p}) = p = .75$ and

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.75(1-.75)}{450}} = .0204$$

b. $z = \dfrac{\bar{p} - p}{\sigma_{\bar{p}}} = \dfrac{.04}{.0204} = 1.96$ $P(z < 1.96)$ .9750

$P(z < -1.96) = .0250$

$P(.71 \le \bar{p} \le .79) = P(-1.96 \le z \le 1.96) = .9750 - .0275 = .9500$

Using Excel: NORM.DIST(.79,.75,.0204,TRUE)-NORM.DIST(.71,.75,.0204,TRUE) = .9501

c. Normal distribution with $E(\bar{p}) = p = .75$ and

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.75(1-.75)}{200}} = .0306$$

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d. $z = \dfrac{\bar{p} - p}{\sqrt{\dfrac{.75(1-.75)}{200}}} = \dfrac{.04}{.0306} = 1.31 \qquad P(z \le 1.31) = .9049$

$P(z < -1.31) = .0951$

$P(.71 \le \bar{p} \le .79) = P(-1.31 \le z \le 1.31) = .9049 - .0951 = .8098$

Using Excel: NORM.DIST(.79,.75,.0306,TRUE)-NORM.DIST(.71,.75,.0306,TRUE) = .8089

e. The probability of the sample proportion being within .04 of the population mean was reduced from .9501 to .8089. So there is a gain in precision by increasing the sample size from 200 to 450. If the extra cost of using the larger sample size is not too great, we should probably do so.

36. a. $E(\bar{p}) = .76$

$\sigma_{\bar{p}} = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{.76(1-.76)}{400}} = .0214$

Normal distribution because $np = 400(.76) = 304$ and $n(1 - p) = 400(.24) = 96$

b. $z = \dfrac{.79 - .76}{.0214} = 1.40 \qquad P(z \le 1.40) = .9192$

$P(z < -1.40) = .0808$

$P(.73 \le \bar{p} \le .79) = P(-1.40 \le z \le 1.40) = .9192 - .0808 = .8384$

Using Excel: NORM.DIST(.79,.76,.0214,TRUE)-NORM.DIST(.73,.76,.0214,TRUE) = .8390

c. $\sigma_{\bar{p}} = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{.76(1-.76)}{750}} = .0156$

$z = \dfrac{.79 - .76}{.0156} = 1.92 \qquad P(z \le 1.92) = .9726$

$P(z < -1.92) = .0274$

$P(.73 \le \bar{p} \le .79) = P(-1.92 \le z \le 1.92) = .9726 - .0274 = .9452$

Using Excel: NORM.DIST(.79,.76,.0156,TRUE)-NORM.DIST(.73,.76,.0156,TRUE) = .9455

37. a. $E(\bar{p}) = .17$

$\sigma_{\bar{p}} = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{(.17)(1-.17)}{800}} = .0133$

Distribution is approximately normal because $np = 800(.17) = 136 > 5$

and $n(1 - p) = 800(.83) = 664 > 5$

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b. $z = \dfrac{.19 - .17}{.0133} = 1.51 \qquad P(z \le 1.51) = .9345$

$P(z < -1.51) = .0655$

$P(.15 \le \bar{p} \le .19) = P(-1.51 \le z \le 1.51) = .9345 - .0655 = .8690$

Using Excel: NORM.DIST(.19,.17,.0133,TRUE)-NORM.DIST(.15,.17,.0133,TRUE) = .8674

c. $\sigma_{\bar{p}} = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{(.17)(1-.17)}{1600}} = .0094$

$z = \dfrac{.19 - .17}{.0094} = 2.13 \qquad P(z \le 2.13) = .9834$

$P(z < -2.13) = .0166$

$P(.15 \le \bar{p} \le .19) = P(-2.13 \le z \le 2.13) = .9834 - .0166 = .9668$

Using Excel: NORM.DIST(.19,.17,.0094,TRUE)-NORM.DIST(.15,.17,.0094,TRUE) = .9666

38. a. The restaurants associated with the five smallest random numbers are selected as the simple random sample. They are:

Restaurant	Random Number
Thoroughbreds	.012825
The Marker	.020815
Officers Club	.03058
SeaBlue	.062942
Crickets	.06914

b. Step 1: Generate a new set of random numbers in column B. Step 2: Sort the random numbers and corresponding restaurant names into ascending order and then select the restaurants associated with the five smallest random numbers. It is extremely unlikely that you will get the same restaurants as in part (a).

39. a. Normal distribution because $n = 50$

$E(\bar{x}) = 6883$

$\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{2000}{\sqrt{50}} = 282.84$

b. $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{300}{2000/\sqrt{50}} = 1.06 \qquad P(z \le 1.06) = .8554$

$P(z < -1.06) = .1446$

$P(6583 \le \bar{x} \le 7183) = P(-1.06 \le z \le 1.06) = .8554 - .1446 = .7108$

Using Excel: NORM.DIST(7183,6883,282.84,TRUE)-
NORM.DIST(6583,6883,282.84,TRUE) = .7112

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c. At 7500, $z = \dfrac{7500 - 6883}{2000/\sqrt{50}} = 2.18$

$P(\bar{x} \geq 7500) = P(z \geq 2.18) = 1 - P(z < 2.18) = 1 - .9854 = .0146$

Yes, I would question the consulting firm. A sample mean this large is unlikely if the population mean is $6883.

Using Excel: 1-NORM.DIST(7500,6883,282.84,TRUE) = .0146

40. a. Normal distribution because of central limit theorem ($n > 30$)

$E(\bar{x}) = 115.50$

$\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{35}{\sqrt{40}} = 5.53$

b. $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{10}{35/\sqrt{40}} = 1.81$ $\quad P(z \leq 1.81) = .9649$

$P(z < -1.81) = .0351$

$P(105.50 \leq \bar{x} \leq 125.50) = P(-1.81 \leq z \leq 1.81) = .9649 - .0351 = .9298$

Using Excel: NORM.DIST(125.5,115.5,35/SQRT(40),TRUE)-
NORM.DIST(105.5,115.5,35/SQRT(40),TRUE) = .9292

c. At $\bar{x} = 100$, $z = \dfrac{100 - 115.50}{35/\sqrt{40}} = -2.80$

$P(\bar{x} \leq 100) = P(z \leq -2.80) = .0026$

Yes, this is an unusually low spending group of 40 alums. The probability of spending this much or less is only .0026.

Using Excel: NORM.DIST(100,115.5,35/SQRT(40),TRUE) = .0025

41. With $n = 60$ the central limit theorem allows us to conclude the sampling distribution is approximately normal.

a. This means $14 \leq \bar{x} \leq 16$

At $\bar{x} = 16$, $z = \dfrac{16 - 15}{4/\sqrt{60}} = 1.94$ $\quad P(z \leq 1.94) = .9738$

$P(z < -1.94) = .0262$
$P(14 \leq \bar{x} \leq 16) = P(-1.94 \leq z \leq 1.94) = .9738 - .0262 = .9476$

Using Excel: NORM.DIST(16,15,4/SQRT(60),TRUE)-
NORM.DIST(14,15,4/SQRT(60),TRUE) = .9472

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b. This means $14.25 \le \bar{x} \le 15.75$

At $\bar{x} = 15.75$, $z = \dfrac{15.75 - 15}{4/\sqrt{60}} = 1.45$ $\quad P(z \le 1.45) = .9265$

$P(z < -1.45) = .0735$

$P(14.25 \le \bar{x} \le 15.75) = P(-1.45 \le z \le 1.45) = .9265 - .0735 = .8530$

Using Excel: NORM.DIST(15.75,15,4/SQRT(60),TRUE)-
NORM.DIST(14.25,15,4/SQRT(60),TRUE) = .8536

42. $\mu = 27{,}175$ $\sigma = 7400$

a. $\sigma_{\bar{x}} = 7400/\sqrt{60} = 955$

b. $z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \dfrac{0}{955} = 0$

$P(\bar{x} > 27{,}175) = P(z > 0) = .50$

Note: This could have been answered easily without any calculations; 27,175 is the expected value of the sampling distribution of $\bar{x}$.

c. $z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \dfrac{1000}{955} = 1.05$ $\quad P(z \le 1.05) = .8531$

$P(z < -1.05) = .1469$

$P(26{,}175 \le \bar{x} \le 28{,}175) = P(-1.05 \le z \le 1.05) = .8531 - .1469 = .7062$

Using Excel: NORM.DIST(28175,27175,955,TRUE)-
NORM.DIST(26175,27175,955,TRUE) = .7050

d. $\sigma_{\bar{x}} = 7400/\sqrt{100} = 740$

$z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \dfrac{1000}{740} = 1.35$ $\quad P(z \le 1.35) = .9115$

$P(z < -1.35) = .0885$

$P(26{,}175 \le \bar{x} \le 28{,}175) = P(-1.35 \le z \le 1.35) = .9115 - .0885 = .8230$

Using Excel: NORM.DIST(28175,27175,740,TRUE)-
NORM.DIST(26175,27175,740,TRUE) = .8234

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43. a. $\sigma_{\bar{x}} = \sqrt{\frac{N-n}{N-1}}\frac{\sigma}{\sqrt{n}}$

$N = 2000$

$$\sigma_{\bar{x}} = \sqrt{\frac{2000-50}{2000-1}}\frac{144}{\sqrt{50}} = 20.11$$

$N = 5000$

$$\sigma_{\bar{x}} = \sqrt{\frac{5000-50}{5000-1}}\frac{144}{\sqrt{50}} = 20.26$$

$N = 10{,}000$

$$\sigma_{\bar{x}} = \sqrt{\frac{10{,}000-50}{10{,}000-1}}\frac{144}{\sqrt{50}} = 20.31$$

Note: With $n / N \leq .05$ for all three cases, common statistical practice would be to ignore the finite population correction factor and use $\sigma_{\bar{x}} = \frac{144}{\sqrt{50}} = 20.36$ for each case.

b. $N = 2000$

$$z = \frac{25}{20.11} = 1.24 \quad P(z \leq 1.24) = .8925$$

$P(z < -1.24) = .1075$

Probability = $P(-1.24 \leq z \leq 1.24) = .8925 - .1075 = .7850$

$N = 5000$

$$z = \frac{25}{20.26} = 1.23 \quad P(z \leq 1.23) = .8907$$

$P(z < -1.23) = .1093$

Probability = $P(-1.23 \leq z \leq 1.23) = .8907 - .1093 = .7814$

$N = 10{,}000$

$$z = \frac{25}{20.31} = 1.23 \quad P(z \leq 1.23) = .8907$$

$P(z < -1.23) = .1093$

Probability = $P(-1.23 \leq z \leq 1.23) = .8907 - .1093 = .7814$

All probabilities are approximately .78 indicating that a sample of size 50 will work well for all 3 firms.

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44. a. $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{500}{\sqrt{n}} = 20$

$\sqrt{n} = 500/20 = 25$ and $n = (25)^2 = 625$

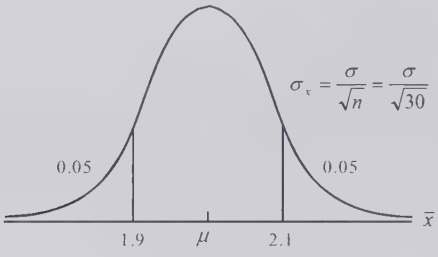
b. For ± 25,

$z = \dfrac{25}{20} = 1.25$ $\qquad P(z \le 1.25) = .8944$

$P(z < -1.25) = .1056$

Probability = $P(-1.25 \le z \le 1.25) = .8944 - .1056 = .7888$

45. Sampling distribution of $\bar{x}$



μ $\quad \dfrac{1.9 + 2.1}{2} \quad 2$

The area below $\bar{x}$ 2.1 must be 1 - .05 = .95. An area of .95 in the standard normal table shows $z = 1.645$.

Thus,

$z = \dfrac{2.1 - 2.0}{\sigma/\sqrt{30}} = 1.645$

Solve for σ.

$\sigma = \dfrac{(.1)\sqrt{30}}{1.645} = .33$

46. $p = .28$

a. This is the graph of a normal distribution with $E(\bar{p}) = p = .28$ and

$\sigma_{\bar{p}} = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{.28(1-.28)}{240}} = .0290$

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b. Within ± .04 means $.24 \le \bar{p} \le .32$

$$z = \frac{.32 - .28}{.0290} = 1.38 \qquad z = \frac{.24 - .28}{.0290} = -1.38$$

$P(.24 \le \bar{p} \le .32) = P(-1.38 \le z \le 1.38) = .9162 - .0838 = .8324$

Using Excel: NORM.DIST(.32,.28,.0290,TRUE)-NORM.DIST(.24,.28,.0290,TRUE) = .8322

c. Within ± .02 means $.26 \le \bar{p} \le .30$

$$z = \frac{.30 - .28}{.0290} = .69 \qquad z = \frac{.26 - .28}{.0290} = -.69$$

$P(.26 \le \bar{p} \le .30) = P(-.69 \le z \le .69) = .7549 - .2451 = .5098$

Using Excel: NORM.DIST(.30,.28,.0290,TRUE)-NORM.DIST(.26,.28,.0290,TRUE) = .5096

47. $\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(.40)(.60)}{400}} = .0245$

$P(\bar{p} \ge .375) = ?$

$$z = \frac{.375 - .40}{.0245} = -1.02 \quad P(z < -1.02) = .1539$$

$P(\bar{p} \ge .375) = 1 - .1539 = .8461$

Using Excel: 1-NORM.DIST(.375,.40,0245,TRUE) = .8462

48. a. $\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(.40)(1-.40)}{380}} = .0251$

Within ± .04 means $.36 \le \bar{p} \le .44$

$$z = \frac{.44 - .40}{.0251} = 1.59 \qquad z = \frac{.36 - .40}{.0251} = -1.59$$

$P(.36 \le \bar{p} \le .44) = P(-1.59 \le z \le 1.59) = .9441 - .0559 = .8882$

Using Excel: NORM.DIST(.44,.40,.0251,TRUE)-NORM.DIST(.36,.40,.0251,TRUE) = .8900

b. We want $P(\bar{p} \ge .45)$

$$z = \frac{\bar{p} - p}{\sigma_{\bar{p}}} = \frac{.45 - .40}{.0251} = 1.99$$

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$P(\bar{p} \geq .45) = P(z \geq 1.99) = 1 - .9767 = .0233$

Using Excel: 1-NORM.DIST(.45,.28,.0251,TRUE) = .0232

49. a. Normal distribution with $E(\bar{p}) = .15$ and

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(.15)(.85)}{150}} = .0292$$

b. $P(.12 \leq \bar{p} \leq .18) = ?$

$$z = \frac{.18 - .15}{.0292} = 1.03 \qquad P(z \leq 1.03) \quad .8485$$

$P(z < -1.03) = .1515$

$P(.12 \leq \bar{p} \leq .18) = P(-1.03 \leq z \leq 1.03) = .8485 - .1515 = .6970$

Using Excel: NORM.DIST(.18,.15,0292,TRUE)-NORM.DIST(.12,.15,.0292,TRUE) = .6958

50. a. $\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.25(.75)}{n}} = .0625$

Solve for n

$$n = \frac{.25(.75)}{(.0625)^2} = 48$$

b. Normal distribution with $E(\bar{p}) = .25$ and $\sigma_{\bar{p}} = .0625$

(Note: (48)(.25) = 12 > 5, and (48)(.75) = 36 > 5)

c. $P(\bar{p} \geq .30)$?

$$z = \frac{.30 - .25}{.0625} = .80 \qquad P(z \leq .80) = .7881$$

$P(\bar{p} \geq .30) = 1 - .7881 = .2119$

Using Excel: 1-NORM.DIST(.30,.25,.0625,TRUE) = .2119

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Chapter 8
Interval Estimation

Learning Objectives

1. Know how to construct and interpret an interval estimate of a population mean and / or a population proportion.

2. Understand and be able to compute the margin of error.

3. Learn about the t distribution and its use in constructing an interval estimate for a population mean when σ is unknown.

4. Be able to determine the size of a simple random sample necessary to estimate a population mean and/or a population proportion with a specified level of precision.

5. Know the definition of the following terms:

confidence interval	margin of error
confidence coefficient	degrees of freedom
confidence level	

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Solutions:

1. a. $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 5/\sqrt{40} = .79$

   b. At 95%, $z\sigma/\sqrt{n} = 1.96(5/\sqrt{40}) = 1.55$

2. a. $32 \pm 1.645\ (6/\sqrt{50})$

   32 ± 1.4 or 30.6 to 33.4

   b. $32 \pm 1.96\ (6/\sqrt{50})$

   32 ± 1.66 or 30.34 to 33.66

   c. $32 \pm 2.576\ (6/\sqrt{50})$

   32 ± 2.19 or 29.81 to 34.19

3. a. $80 \pm 1.96\ (15/\sqrt{60})$

   80 ± 3.8 or 76.2 to 83.8

   b. $80 \pm 1.96\ (15/\sqrt{120})$

   80 ± 2.68 or 77.32 to 82.68

   c. Larger sample provides a smaller margin of error.

4. Sample mean $\bar{x} = \dfrac{160+152}{2} = 156$

   Margin of Error = 160 156 = 4

   $1.96(\sigma/\sqrt{n}) = 4$

   $\sqrt{n} = 1.96\sigma/4 = 1.96(15)/4 = 7.35$

   $n = (7.35)^2 = 54$

5. a. $1.96\sigma/\sqrt{n} = 1.96(5/\sqrt{49}) = 1.40$

   b. 24.80 ± 1.40 or 23.40 to 26.20

6. $\bar{x} \pm z_{.025}(\sigma/\sqrt{n})$

   $8.5 \pm 1.96(3.5/\sqrt{300})$

   $8.5 \pm .4$ or 8.1 to 8.9

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7. Margin of error = $z_{.025}(\sigma/\sqrt{n})$

$z_{.025} = 1.96 \qquad \sigma = \$600 \qquad n = 50$

Margin of error = $1.96(600/\sqrt{50}) = 166.31$

A larger sample size would be needed to reduce the margin of error. Section 8.3 can be used to show that the sample size would need to be increased to $n = 62$.

$1.96(600/\sqrt{n}) = 150$

Solving for n shows $n = 62$

8. a. Since n is small, an assumption that the population is at least approximately normal is required.

b. $z_{.025}(\sigma/\sqrt{n}) = 1.96(5/\sqrt{10}) = 3.1$

c. $z_{.005}(\sigma/\sqrt{n}) = 2.576(5/\sqrt{10}) = 4.1$

9. $\bar{x} \pm z_{.025}(\sigma/\sqrt{n})$

$33.5 \pm 1.96(9/\sqrt{40})$

33.5 ± 2.8 or 30.7 to 36.3 hours

10. a. $\bar{x} \pm z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}}$

$119{,}155 \pm 1.645(30{,}000/\sqrt{80})$

$119{,}155 \pm 5517$ or \$113,638 to \$124,672

b. $119{,}155 \pm 1.96\ (30{,}000/\sqrt{80})$

$119{,}155 \pm 6574$ or \$112,581 to \$125,729

c. $119{,}155 \pm 2.576\ (30{,}000/\sqrt{80})$

$119{,}155 \pm 8640$ or \$110,515 to \$127,795

d. The confidence interval gets wider as we increase our confidence level. We need a wider interval to be more confident that it will contain the population mean.

11. a. .025

b. 1 - .10 = .90

c. .05

d. .01

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e. 1 - 2(.025) = .95

f. 1 2(.05) .90

12. a. 2.179

b. -1.676

c. 2.457

d. Use .05 column, -1.708 and 1.708

e. Use .025 column, -2.014 and 2.014

13. a. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{80}{8} = 10$

b.

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
10	0	0
8	-2	4
12	2	4
15	5	25
13	3	9
11	1	1
6	-4	16
5	-5	25
		84

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{84}{7}} = 3.464$$

c. $t_{.025}(s/\sqrt{n}) = 2.365(3.464/\sqrt{8}) = 2.9$

d. $\bar{x} \pm t_{.025}(s/\sqrt{n})$

10 ± 2.9 or 7.1 to 12.9

14. $\bar{x} \pm t_{\alpha/2}(s/\sqrt{n})$ $df = 53$

a. $22.5 \pm 1.674\,(4.4/\sqrt{54})$

22.5 ± 1 or 21.5 to 23.5

b. $22.5 \pm 2.006\,(4.4/\sqrt{54})$

22.5 ± 1.2 or 21.3 to 23.7

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c. $22.5 \pm 2.672\,(4.4/\sqrt{54})$

22.5 ± 1.6 or 20.9 to 24.1

d. As the confidence level increases, there is a larger margin of error and a wider confidence interval.

15. $\bar{x} \pm t_{\alpha/2}(s/\sqrt{n})$

90% confidence $df = 64$ $t_{.05} = 1.669$

$19.5 \pm 1.669\,(5.2/\sqrt{65})$

19.5 ± 1.08 or 18.42 to 20.58

95% confidence $df = 64$ $t_{.025} = 1.998$

$19.5 \pm 1.998\,(5.2/\sqrt{65})$

19.5 ± 1.29 or 18.21 to 20.79

16. a. $t_{.025}(s/\sqrt{n})$ $df = 99$ $t_{.025} = 1.984$

$1.984\,(8.5/\sqrt{100}) = 1.69$

b. $\bar{x} \pm t_{.025}(s/\sqrt{n})$

49 ± 1.69 or 47.31 to 50.69

c. At 95% confidence, the population mean flying time for Continental pilots is between 47.31 and 50.69 hours per month. This is clearly more flying time than the 36 hours for United pilots. With the greater flying time, Continental will use fewer pilots and have lower labor costs. United will require relatively more pilots and can be expected to have higher labor costs.

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17. For the Miami data set, the output obtained using Excel's Descriptive Statistics tool follows:

Rating	
Mean	6.34
Standard Error	0.3059
Median	6.5
Mode	8
Standard Deviation	2.1629
Sample Variance	4.6780
Kurtosis	-1.1806
Skewness	-0.1445
Range	8
Minimum	2
Maximum	10
Sum	317
Count	50
Confidence Level(95.0%)	0.6147

The 95% confidence interval is $\overline{x} \pm$ margin of error

6.34 ± 0.6147 or 5.73 to 6.95

18. For the JobSearch data set, the output obtained using Excel's Descriptive Statistics tool follows:

Job Search Time (Weeks)	
Mean	22
Standard Error	1.8794
Median	20
Mode	17
Standard Deviation	11.8862
Sample Variance	141.2821
Kurtosis	0.9030
Skewness	1.0062
Range	52
Minimum	0
Maximum	52
Sum	880
Count	40
Confidence Level (95.0%)	3.8014

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a. $\bar{x} = 22$ weeks

b. margin of error = 3.8014

c. The 95% confidence interval is $\bar{x} \pm$ margin of error

22 ± 3.8014 or 18.20 to 25.80

d. The descriptive statistics output shows that the skewness is 1.0062. There is a moderate positive skewness in this data set. This can be expected to exist in a population such as this. While the above results are acceptable, considering a slightly larger sample next time would be a good strategy.

19. a. $t_{.025}(s/\sqrt{n})$ $\quad df = 44$

$t_{.025} = 2.015$ $\quad s = 65$

$2.015\,(65/\sqrt{45}) = 19.52$ or approximately \$20

b. $\bar{x} \pm t_{.025}(s/\sqrt{n})$

273 ± 20 or 253 to 293

c. At 95% confidence, the population mean is between \$253 and \$293. This is definitely above the \$229 level of 2 years ago. Hotel room rates are increasing.

The point estimate of the increase is \$273 - \$229 = \$44 or 19%.

20. $\bar{x} = \Sigma x_i / n = 22$ minutes

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = 1.12 \text{ minutes}$$

$\bar{x} \pm t_{.025}(s/\sqrt{n})$ $\quad df = 19$

$22.00 \pm 2.093\ (1.12/\sqrt{20})$

$22.00 \pm .52$ or 21.48 to 22.52 minutes

21. $$\bar{x} = \frac{\Sigma x_i}{n} = \frac{2600}{20} = 130 \text{ liters of alcoholic beverages}$$

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{81244}{20-1}} = 65.39$$

$t_{.025} = 2.093$ $\quad df = 19$

95% confidence interval: $\bar{x} \pm t_{.025}(s/\sqrt{n})$

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$130 \pm 2.093(65.39/\sqrt{20})$

130 ± 30.60 or 99.40 to 160.60 liters per year

22. a. The Excel output from using the Descriptive Statistics analysis tool with the TicketSales file is shown:

TicketSales ($)	
Mean	10905
Standard Error	792.42
Median	10750
Mode	#N/A
Standard Deviation	3962.11
Sample Variance	15698350
Kurtosis	1.6567
Skewness	1.0147
Range	17200
Minimum	4200
Maximum	21400
Sum	272625
Count	25
Confidence Level(95.0%)	1635.48

The sample mean is 10,905 and the margin of error (Confidence Level) is 1635.48.

The 95% confidence interval for the population mean is $9269.52 to $12,540.48. We are 95% confident that the population mean three-day ticket sales revenue per theater is between $9,269.52 and $12,540.48.

b. Mean number of customers per theater = 10,905/7.16 = 1523

c. Total number of customers = 3118(1523) = 4,748,714

Total box office ticket sales for the three-day weekend = 3118(10,905) ≈ $34 million

23. $n = \dfrac{z_{.025}^2\sigma^2}{E^2} = \dfrac{(1.96)^2(40)^2}{10^2} = 61.47$ Use $n = 62$

24. a. Planning value of σ = Range/4 = 36/4 = 9

b. $n = \dfrac{z_{.025}^2\sigma^2}{E^2} = \dfrac{(1.96)^2(9)^2}{3^2} = 34.57$ Use $n = 35$

c. $n = \dfrac{(1.96)^2(9)^2}{2^2} = 77.79$ Use $n = 78$

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25. $n = \dfrac{(1.96)^2(6.84)^2}{(1.5)^2} = 79.88$ Use $n = 80$

$n = \dfrac{(1.645)^2(6.84)^2}{2^2} = 31.65$ Use $n = 32$

26. a. $n = \dfrac{z^2_{.025}\sigma^2}{E^2} = \dfrac{(1.96)^2(.15)^2}{(.07)^2} = 17.64$ Use 18.

If the normality assumption for the population appears questionable, this should be adjusted upward.

b. $n = \dfrac{(1.96)^2(.15)^2}{(.05)^2} = 34.6$ Use 35

c. $n = \dfrac{(1.96)^2(.15)^2}{(.03)^2} = 96.04$ Use 97

For reporting purposes, the newspaper might decide to round up to a sample size of 100.

27. Planning value $\sigma = \dfrac{45{,}000 - 30{,}000}{4} = 3750$

a. $n = \dfrac{z^2_{.025}\sigma^2}{E^2} = \dfrac{(1.96)^2(3750)^2}{(500)^2} = 216.09$ Use $n = 217$

b. $n = \dfrac{(1.96)^2(3750)^2}{(200)^2} = 1350.56$ Use $n = 1351$

c. $n = \dfrac{(1.96)^2(3750)^2}{(100)^2} = 5402.25$ Use $n = 5403$

d. Sampling 5403 college graduates to obtain the $100 margin of error would be viewed as too expensive and too much effort by most researchers.

28. a. $n = \dfrac{z^2_{\alpha/2}\sigma^2}{E^2} = \dfrac{(1.645)^2(1100)^2}{(100)^2} = 327.43$ Use $n = 328$

b. $n = \dfrac{(1.96)^2(1100)^2}{(100)^2} = 464.83$ Use $n = 465$

c. $n = \dfrac{(2.576)^2(1100)^2}{(100)^2} = 802.93$ Use $n = 803$

d. The sample size gets larger as the confidence is increased. We would not recommend 99% confidence. The sample size must be increased by 137 = 465 - 328 to go from 90% to 95%. This may be reasonable. However, increasing the sample size by 338 = 803 - 465 to go from 95% to 99% would probably be viewed as too expensive and time consuming for the 4% gain in confidence.

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29. a. $n = \dfrac{(1.96)^2(6.25)^2}{2^2} = 37.52$ Use $n = 38$

b. $n = \dfrac{(1.96)^2(6.25)^2}{1^2} = 150.06$ Use $n = 151$

30. Planning value $\sigma = \dfrac{60-5}{4} = 13.75$

$$n = \frac{z_{.025}^2\sigma^2}{E^2} = \frac{(1.96)^2(13.75)^2}{(3)^2} = 80.70 \quad \text{Use } n = 81$$

31. a. $\bar{p} = 100/400 = .25$

b. $\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\dfrac{.25(.75)}{400}} = .0217$

c. $\bar{p} \pm z_{.025}\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}}$

$.25 \pm 1.96\,(.0217)$

$.25 \pm .0424$ or .2076 to .2924

32. a. $.70 \pm 1.645\sqrt{\dfrac{.70(.30)}{800}}$

$.70 \pm .0267$ or .6733 to .7267

b. $.70 \pm 1.96\sqrt{\dfrac{.70(.30)}{800}}$

$.70 \pm .0318$ or .6682 to .7318

33. $n = \dfrac{z_{.025}^2 p^*(1-p^*)}{E^2} = \dfrac{(1.96)^2(.35)(.65)}{(.05)^2} = 349.59$ Use $n = 350$

34. Use planning value $p = .50$

$$n = \frac{(1.96)^2(.50)(.50)}{(.03)^2} = 1067.11 \quad \text{Use } n = 1068$$

35. a. $\bar{p} = 1760/2000 = .88$

b. Margin of Error

$$z_{.05}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 1.645\sqrt{\frac{.88(1-.88)}{2000}} = .0120$$

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c. Confidence interval:

$.88 \pm .0120$

or .868 to .892

d. Margin of Error

$$z_{.025}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 1.96\sqrt{\frac{.88(1-.88)}{2000}} = .0142$$

95% Confidence Interval

$.88 \pm .0142$ or .8658 to .8942

36. a. $\bar{p} = 46/200 = .23$

b. $$\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\frac{.23(1-.23)}{200}} = .0298$$

$$\bar{p} \pm z_{.025}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$.23 \pm 1.96(.0298)$

$.23 \pm .0584$ or .1716 to .2884

37. a. $\bar{p} = 473/1100 = .43$

b. $$z_{.025}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 1.96\sqrt{\frac{.43(1-.43)}{1100}} = .0293$$

c. $\bar{p} \pm .0293$

$.43 \pm .0293$ or .4007 to .4593

d. With roughly 40% to 46% of employees surveyed indicating strong dissatisfaction and with the high cost of finding successors, employers should take steps to improve employee satisfaction. The survey suggested employers may anticipate high employee turnover costs if employee dissatisfaction remains at the current level.

38. a. $\bar{p} = 29/162 = .1790$

b. $\bar{p} = 104/162 = .6420$

Margin of error $= 1.96\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 1.96\sqrt{\frac{(.642)(.358)}{162}} = .0738$

Confidence interval: $.6420 \pm .0738$ or .5682 to .7158

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c. $n = \dfrac{1.96^2(.642)(.358)}{(.05)^2} = 353.18$ Use $n = 354$

39. a. $n = \dfrac{z^2_{.025}p^*(1-p^*)}{E^2} = \dfrac{(1.96)^2(.156)(1-.156)}{(.03)^2} = 562$

b. $n = \dfrac{z^2_{.005}p^*(1-p^*)}{E^2} = \dfrac{(2.576)^2(.156)(1-.156)}{(.03)^2} = 970.77$ Use 971

40. Margin of error: $z_{.025}\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}} = 1.96\sqrt{\dfrac{.52(1-.52)}{800}} = .0346$

95% Confidence interval: $\bar{p} \pm .0346$

$.52 \pm .0346$ or .4854 to .5546

41. Margin of error = $z_{.025}\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}} = 1.96\sqrt{\dfrac{.09(1-.09)}{1400}} = .0150$

$.09 \pm .0150$ or .075 to .1050

42. a. $\sqrt{\dfrac{p^*(1-p^*)}{n}} = \sqrt{\dfrac{.50(1-.50)}{491}} = .0226$

$z_{.025}\sqrt{\dfrac{p^*(1-p^*)}{n}} = 1.96(.0226) = .0442$

b. $n = \dfrac{z^2_{.025}p^*(1-p^*)}{E^2}$

September $n = \dfrac{1.96^2(.50)(1-.50)}{.04^2} = 600.25$ Use 601

October $n = \dfrac{1.96^2(.50)(1-.50)}{.03^2} = 1067.11$ Use 1068

November $n = \dfrac{1.96^2(.50)(1-.50)}{.02^2} = 2401$

Pre-Election $n = \dfrac{1.96^2(.50)(1-.50)}{.01^2} = 9604$

43. a. Margin of Error = $z_{\alpha/2}\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}} = 1.96\sqrt{\dfrac{(.53)(.47)}{1500}} = .0253$

95% Confidence Interval: $.53 \pm .0253$ or .5047 to .5553

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b. Margin of Error $= 1.96\sqrt{\frac{(.31)(.69)}{1500}} = .0234$

95% Confidence Interval: .31 ± .0234 or .2866 to .3334

c. Margin of Error $= 1.96\sqrt{\frac{(.05)(.95)}{1500}} = .0110$

95% Confidence Interval: .05 ± .0110 or .039 to .061

d. The margin of error decreases as $\bar{p}$ gets smaller. If the margin of error for all of the interval estimates must be less than a given value (say .03), an estimate of the largest proportion should be used as a planning value. Using $p^* = .50$ as a planning value guarantees that the margin of error for all the interval estimates will be small enough.

44. a. Margin of error: $z_{.025}\frac{\sigma}{\sqrt{n}} = 1.96\frac{15}{\sqrt{54}} = 4.00$

b. Confidence interval: $\bar{x} \pm$ margin of error

33.77 ± 4.00 or \$29.77 to \$37.77

45. a. $\bar{x} \pm t_{.025}(s/\sqrt{n})$ $df = 63$ $t_{.025} = 1.998$

$252.45 \pm 1.998(74.50/\sqrt{64})$

252.45 ± 18.61 or \$233.84 to \$271.06

b. Yes. the lower limit for the population mean at Niagara Falls is \$233.84 which is greater than \$215.60.

46. a. Margin of error $= t_{.025}(s/\sqrt{n})$

$df = 79$ $t_{.025} = 1.990$ $s = 550$

$1.990(550/\sqrt{80}) = 122$

b. $\bar{x} \pm$ margin of error

1873 ± 122 or \$1751 to \$1995

c. As of March, 2008, 92 million Americans were of age 50 and over

Estimate of total expenditures = 92(1873) = 172,316

In dollars, we estimate that \$172,316 million dollars are spent annually by Americans of age 50 and over on restaurants and carryout food.

d. We would expect the median to be less than the mean. The few individuals that spend much more than the average cause the mean to be larger than the median. This is typical for data of this type.

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47. For the NYSEStocks data set, the output obtained using Excel's Descriptive Statistics tool follows:

P/E Ratio	
Mean	21.9
Standard Error	2.7132
Median	19
Mode	21
Standard Deviation	14.8611
Sample Variance	220.8517
Kurtosis	6.3311
Skewness	2.4470
Range	66
Minimum	6
Maximum	72
Sum	657
Count	30
Confidence Level(95.0%)	5.5492

a. Point estimate = $\bar{x} = 21.9$

The 95% confidence interval is $\bar{x} \pm$ margin of error

21.9 ± 5.5492 or 16.35 to 27.45

b. The point estimate is greater than 20 but the 95% confidence interval goes down to 16.4. So we would be hesitant to conclude that the population mean P/E ratio was greater than 20. Perhaps taking a larger sample would be in order.

c. From the sample of 30 stocks, we find $\bar{p} = 21/30 = .70$

A point estimate of the proportion of NYSE stocks paying dividends is .70.

With $n\bar{p} = 30(.7) = 21$ and $n(1-\bar{p}) = 30(.3) = 9$, we would be justified in using a normal distribution to construct a confidence interval. A 95% confidence interval is

$$\bar{p} \pm z_{.025}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$.7 \pm 1.96\sqrt{\frac{(.7)(.3)}{30}}$$

$.7 \pm .16$ or .54 to .86

While the sample size is large enough to use the normal distribution approximation, the sample size is not large enough to provide much precision. The margin of error is larger than most people would like.

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48. For the Flights data set, the output obtained using Excel's Descriptive Statistics tool follows:

Time	
Mean	14
Standard Error	0.3133
Median	14.25
Mode	16
Standard Deviation	3.8376
Sample Variance	14.7268
Kurtosis	1.0245
Skewness	0.1473
Range	23.2
Minimum	4.8
Maximum	28
Sum	2100
Count	150
Confidence Level(95.0%)	0.6192

a. $\bar{x} = 14$ minutes

b. The 95% confidence interval is $\bar{x} \pm$ margin of error

$14 \pm .6192$ or 13.38 to 14.62

c. 7.5 hours = 7.5(60) = 450 minutes per day

An average of 450/14 = 32 reservations per day if no idle time. Assuming perhaps 15% idle time or time on something other than reservations, this could be reduced to 27 reservations per day.

d. For large airlines, there are many telephone calls such as these per day. Using the online reservations would reduce the telephone reservation staff and payroll. Adding in a reduction in total benefit costs, a change to online reservations could provide a sizeable cost reduction for the airline.

49. a. Using Excel: $\bar{x} = 66.93$

The amount of playing time is approximately 11 minutes. So the time standing around is over 6 times as much. You may find this difference surprising; the authors did.

b. Using Excel: $s = 4.4943$ minutes

c. $\bar{x} \pm t_{.025} s/\sqrt{n}$ $\quad df = 59$ $\quad t_{.025} = 2.001$

$66.93 \pm 2.001\ (4.4943/\sqrt{60}\ )$

66.93 ± 1.16 or 65.77 to 68.09

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50. $n = \dfrac{(2.33)^2(2.6)^2}{1^2} = 36.7$ Use $n = 37$

51. $n = \dfrac{(1.96)^2(8)^2}{2^2} = 61.47$ Use $n = 62$

$n = \dfrac{(2.576)^2(8)^2}{2^2} = 106.17$ Use $n = 107$

52. $n = \dfrac{(1.96)^2(675)^2}{100^2} = 175.03$ Use $n = 176$

53. a. $\bar{p} \pm 1.96\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}}$

$.47 \pm 1.96\sqrt{\dfrac{(.47)(.53)}{450}}$

$.47 \pm .0461$ or .4239 to .5161

b. $.47 \pm 2.576\sqrt{\dfrac{(.47)(.53)}{450}}$

$.47 \pm .0606$ or .4094 to .5306

c. The margin of error becomes larger.

54. a. $\bar{p} = 200/369 = .5420$

b. $1.96\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}} = 1.96\sqrt{\dfrac{(.5420)(.4580)}{369}} = .0508$

c. $.5420 \pm .0508$ or .4912 to .5928

55. a. $\bar{p} = .74$

Margin of error $= z_{.025}\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}} = 1.96\sqrt{\dfrac{(.74)(.26)}{1677}} = .02$

95% Confidence Interval: $.74 \pm .02$ or .72 to .76

b. $\bar{p} = .48$

Margin of error $= z_{.005}\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}} = 2.576\sqrt{\dfrac{(.48)(.52)}{1677}} = .03$

95% Confidence Interval: $.48 \pm .03$ or .45 to .51

c. The margin of error is larger in part b for two reasons. With $\bar{p} = .48$, the estimate of the standard error is larger. And $z_{.005} = 2.576$ is larger than $z_{.025} = 1.96$

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56. a. $\bar{p} = 455/550 = .8273$

b. Margin of error $= 1.96\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}} = 1.96\sqrt{\dfrac{.8273(1-.8273)}{550}} = .0316$

95% Confidence interval: .8273 ± .0316 or .7957 to .8589

57. a. $n = \dfrac{(1.96)^2(.3)(.7)}{(.02)^2} = 2016.84$ Use $n = 2017$

b. $\bar{p} = 520/2017 = .2578$

c. $\bar{p} \pm 1.96\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}}$

$.2578 \pm 1.96\sqrt{\dfrac{(.2578)(.7422)}{2017}}$

.2578 ± .0191 or .2387 to .2769

58. a. $n = \dfrac{(2.33)^2(.70)(.30)}{(.03)^2} = 1266.74$ Use $n = 1267$

b. $n = \dfrac{(2.33)^2(.50)(.50)}{(.03)^2} = 1508.03$ Use $n = 1509$

59. a. Government industry 95% margin of error: $1.96\sqrt{\dfrac{.37(1-.37)}{200}} = .0669$

95% Confidence interval: $.37 \pm .0669$ or .3031 to .4369

Health care industry 95% margin of error: $1.96\sqrt{\dfrac{.33(1-.33)}{200}} = .0652$

95% Confidence interval: $.33 \pm .0652$ or .2648 to .3952

Education industry 95% margin of error: $1.96\sqrt{\dfrac{.28(1-.28)}{200}} = .0622$

95% Confidence interval: $.28 \pm .0622$ or .2178 to .3422

b. The government industry has the largest margin of error. So the sample size must be large enough to reduce the margin of error for the government industry to .05 or less. Using the proportions found in the study reported in *USA Today* as a planning value $p^* = .37$.

$n = \dfrac{(1.96)^2(.37)(.63)}{(.05)^2} = 358.1908$ Use $n = 359$

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This is an increase of 159 workers surveyed in each industry. The added cost of the larger sample size would have to be taken into account before deciding whether the smaller margin of error is worth the added cost of obtaining the data.

60. a. $\bar{p} = 618/1993 = .3101$

b. $\bar{p} \pm 1.96\sqrt{\dfrac{\bar{p}(1-\bar{p})}{1993}}$

$.3101 \pm 1.96\sqrt{\dfrac{(.3101)(.6899)}{1993}}$

$.3101 \pm .0203$ or .2898 to .3304

c. $n = \dfrac{z^2 p^*(1-p^*)}{E^2}$

$z = \dfrac{(1.96)^2(.3101)(.6899)}{(.01)^2} = 8218.64 \quad \text{Use } n = 8219$

No; the sample appears unnecessarily large. The .02 margin of error reported in part (b) should provide adequate precision.

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Chapter 9
Hypothesis Tests

Learning Objectives

1. Learn how to formulate and test hypotheses about a population mean and/or a population proportion.

2. Understand the types of errors possible when conducting a hypothesis test.

3. Understand the level of significance and be able to determine the probability of making a Type I error in hypothesis tests.

4. Know how to compute and interpret *p*-values.

5. Be able to use critical values to draw hypothesis testing conclusions.

6. Know the definition of the following terms:

null hypothesis	two-tailed test
alternative hypothesis	*p*-value
Type I error	level of significance
Type II error	critical value
one-tail test	

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Solutions:

1. a. H_0: $\mu \leq 600$ Manager's claim, the assumption being challenged

   H_a: $\mu > 600$

   b. We are not able to conclude that the manager's claim is wrong.

   c. The manager's claim can be rejected. We can conclude that $\mu > 600$.

2. a. H_0: $\mu \leq 14$

   H_a: $\mu > 14$ Research hypothesis

   b. There is no statistical evidence that the new bonus plan increases sales volume.

   c. The research hypothesis that $\mu > 14$ is supported. We can conclude that the new bonus plan increases the mean sales volume.

3. a. H_0: $\mu = 32$ Specified filling weight, the assumption being challenged

   H_a: $\mu \neq 32$ Overfilling or underfilling exists

   b. There is no evidence that the production line is not operating properly. Allow the production process to continue.

   c. Conclude $\mu \neq 32$ and that overfilling or underfilling exists. Shut down and adjust the production line.

4. a. H_0: $\mu \geq 220$

   H_a: $\mu < 220$ Research hypothesis to see if mean cost is less than \$220.

   b. We are unable to conclude that the new method reduces costs.

   c. Conclude $\mu < 220$. Consider implementing the new method based on the conclusion that it lowers the mean cost per hour.

5. a. The Type I error is rejecting H_0 when it is true. This error occurs if the researcher concludes that young men in Germany spend more than 56.2 minutes per day watching prime-time TV when the national average for Germans is not greater than 56.2 minutes.

   b. The Type II error is accepting H_0 when it is false. This error occurs if the researcher concludes that the national average for German young men is $\leq$ 56.2 minutes when in fact it is greater than 56.2 minutes.

6. a. H_0: $\mu \leq 1$ The label claim or assumption.

   H_a: $\mu > 1$

   b. Claiming $\mu > 1$ when it is not. This is the error of rejecting the product's claim when the claim is true.

   c. Concluding $\mu \leq 1$ when it is not. In this case, we miss the fact that the product is not meeting its label specification.

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7. a. H_0: $\mu \leq 8000$

   H_a: $\mu > 8000$ Research hypothesis to see if the plan increases average sales.

   b. Claiming $\mu > 8000$ when the plan does not increase sales. A mistake could be implementing the plan when it does not help.

   c. Concluding $\mu \leq 8000$ when the plan really would increase sales. This could lead to not implementing a plan that would increase sales.

8. a. H_0: $\mu \geq 220$

   H_a: $\mu < 220$

   b. Claiming $\mu < 220$ when the new method does not lower costs. A mistake could be implementing the method when it does not help.

   c. Concluding $\mu \geq 220$ when the method really would lower costs. This could lead to not implementing a method that would lower costs.

9. a. $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{19.4 - 20}{2/\sqrt{50}} = -2.12$

   b. p-value is the area in the lower tail

   Using normal table with z = -2.12: p-value =.0170

   Using Excel: p-value = NORM.S.DIST(-2.12,TRUE) = .0170

   c. p-value $\leq$ .05, reject H_0

   d. Reject H_0 if $z \leq$ -1.645

   -2.12 $\leq$ -1.645, reject H_0

10. a. $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{26.4 - 25}{6/\sqrt{40}} = 1.48$

    b. p-value is the area in the upper tail: $P(z \geq 1.48)$

    Using normal table with z = 1.48: p-value = 1.0000 - .9306 = .0694

    Using Excel: p-value = 1 – NORM.S.DIST(1.48,TRUE) = .0694

    c. p-value > .01, do not reject H_0

    d. Reject H_0 if $z \geq 2.33$

    1.48 < 2.33, do not reject H_0

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11. a. $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{14.15 - 15}{3/\sqrt{50}} = -2.00$

b. Because $z < 0$, p-value is two times the lower tail area

Using normal table with z = -2.00: p-value = 2(.0228) = .0456

Using Excel: p-value = 2*NORM.S.DIST(-2.00,TRUE) = .0455

c. p-value ≤ .05, reject H_0

d. Reject H_0 if $z \leq -1.96$ or $z \geq 1.96$

-2.00 ≤ -1.96, reject H_0

12. a. $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{78.5 - 80}{12/\sqrt{100}} = -1.25$

p-value is the lower-tail area

Using normal table with z = -1.25: p-value =.1056

Using Excel: p-value = NORM.S.DIST(-1.25,TRUE) = .1056

p-value > .01, do not reject H_0

b. $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{77 - 80}{12/\sqrt{100}} = -2.50$

p-value is the lower-tail area

Using normal table with z = -2.50: p-value =.0062

Using Excel: p-value = NORM.S.DIST(-2.50,TRUE) = . 0062

p-value ≤ .01, reject H_0

c. $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{75.5 - 80}{12/\sqrt{100}} = -3.75$

p-value is the lower-tail area

Using normal table with z = -3.75: p-value ≈ 0

Using Excel: p-value = NORM.S.DIST(-3.75,TRUE) ≈ 0

p-value ≤ .01, reject H_0

d. $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{81 - 80}{12/\sqrt{100}} = .83$

p-value is the lower-tail area

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Using normal table with $z = .83$: p-value =.7967

Using Excel: p-value = NORM.S.DIST(.83,TRUE) = .7967

p-value > .01, do not reject H_0

13. Reject H_0 if $z \geq 1.645$

a. $$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{52.5 - 50}{8/\sqrt{60}} = 2.42$$

$2.42 \geq 1.645$, reject H_0

b. $$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{51 - 50}{8/\sqrt{60}} = .97$$

$.97 < 1.645$, do not reject H_0

c. $$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{51.8 - 50}{8/\sqrt{60}} = 1.74$$

$1.74 \geq 1.645$, reject H_0

14. a. $$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{23 - 22}{10/\sqrt{75}} = .87$$

Because $z > 0$, p-value is two times the upper tail area

Using normal table with $z = .87$: p-value = 2(1 - .8078) = .3844

Using Excel: p-value = 2*(1-NORM.S.DIST(.87,TRUE)) = .3843

p-value > .01, do not reject H_0

b. $$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{25.1 - 22}{10/\sqrt{75}} = 2.68$$

Because $z > 0$, p-value is two times the upper tail area

Using normal table with $z = 2.68$: p-value = 2(1 - .9963) = .0074

Using Excel: p-value = 2*(1-NORM.S.DIST(2.68,TRUE)) = .0074

p-value ≤ .01, reject H_0

c. $$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{20 - 22}{10/\sqrt{75}} = -1.73$$

Because $z < 0$, p-value is two times the lower tail area

Using normal table with $z = -1.73$: p-value = 2(.0418) = .0836

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Using Excel: p-value = 2*(NORM.S.DIST(-1.73,TRUE)) = .0836

p-value > .01, do not reject H_0

15. a. H_0: $\mu \geq 1056$

H_a: $\mu < 1056$

b. $$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{910 - 1056}{1600 / \sqrt{400}} = -1.83$$

p-value is the lower-tail area

Using normal table with z = -1.83: p-value =.0336

Using Excel: p-value = NORM.S.DIST(-1.83,TRUE) = .0336

c. p-value ≤ .05, reject H_0. Conclude the mean refund of "last minute" filers is less than $1056.

d. Reject H_0 if $z \leq$ -1.645

-1.83 ≤ -1.645, reject H_0

16. a. H_0: $\mu \leq 3173$

H_a: $\mu > 3173$

b. $$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{3325 - 3173}{1000 / \sqrt{180}} = 2.04$$

p-value is $P(z \geq 2.04)$

Using normal table with z = 2.04: p-value = 1.0000 - .9793 = .0207

Using Excel: p-value = 1-NORM.S.DIST(2.04,TRUE) = .0207

c. p-value ≤ .05. Reject H_0. The current population mean credit card balance for undergraduate students has increased compared to the previous all-time high of $3173 reported in April 2009.

17. a. H_0: $\mu = 125{,}500$

H_a: $\mu \neq 125{,}500$

b. $$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{118{,}000 - 125{,}500}{30{,}000 / \sqrt{40}} = -1.58$$

Because $z < 0$, p-value is two times the lower tail area

Using normal table with z = -1.58: p-value = 2(.0571) = .1142

Using Excel: p-value = 2*NORM.S.DIST(-1.58,TRUE) = .1141

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c. p-value > .05, do not reject H_0. We cannot conclude that the year-end bonuses paid by Jones & Ryan differ significantly from the population mean of $125,500.

d. Reject H_0 if $z \leq -1.96$ or $z \geq 1.96$

$z = -1.58$; cannot reject H_0

18. a. H_0: $\mu = 4.1$

H_a: $\mu \neq 4.1$

b. $$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{3.4 - 4.1}{2/\sqrt{40}} = -2.21$$

Because $z < 0$, p-value is two times the lower tail area

Using normal table with $z = -2.21$: p-value = 2(.0136) = .0272

Using Excel: p-value = 2*NORM.S.DIST(-2.21,TRUE) = .0271

c. p-value = .0272 < .05

Reject H_0 and conclude that the return for Mid-Cap Growth Funds differs significantly from that for U.S. Diversified funds.

19. H_0: $\mu \geq 12$

H_a: $\mu < 12$

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{10 - 12}{8/\sqrt{50}} = -1.77$$

p-value is the area in the lower tail

Using normal table with $z = -1.77$: p-value = .0384

Using Excel: p-value = NORM.S.DIST(-1.77,TRUE) = .0384

p-value ≤ .05, reject H_0. Conclude that the actual mean waiting time is significantly less than the claim of 12 minutes made by the taxpayer advocate.

20. a. H_0: $\mu \geq 32.79$

H_a: $\mu < 32.79$

b. $$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{30.63 - 32.79}{5.6/\sqrt{50}} = -2.73$$

c. p-value is lower-tail area

Using normal table with $z = -2.73$: p-value = .0032.

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Using Excel: p-value = NORM.S.DIST(-2.73,TRUE) = .0032

d. p-value $\leq$ .01; reject H_0. Conclude that the mean monthly internet bill is less in the southern state.

21. a. H_0: $\mu \leq 15$

H_a: $\mu > 15$

b. $$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{17 - 15}{4 / \sqrt{35}} = 2.96$$

c. p-value is upper-tail area

Using normal table with $z = 2.96$: p-value = 1.0000 - .9985 = .0015

Using Excel: p-value = 1-NORM.S.DIST(2.96,TRUE) = .0015

d. p-value $\leq$ .01; reject H_0; the premium rate should be charged.

22. a. H_0: $\mu = 8$

H_a: $\mu \neq 8$

b. $$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{8.4 - 8.0}{3.2 / \sqrt{120}} = 1.37$$

b. Because $z > 0$, p-value is two times the upper tail area

Using normal table with $z = 1.37$: p-value = 2(1 - .9147) = .1706

Using Excel: p-value = 2*(1-NORM.S.DIST(1.37,TRUE)) = .1707

c. Do not reject H_0. Cannot conclude that the population mean waiting time differs from 8 minutes.

d. $\bar{x} \pm z_{.025}(\sigma / \sqrt{n})$

$8.4 \pm 1.96(3.2 / \sqrt{120})$

$8.4 \pm .57$ (7.83 to 8.97)

Yes; $\mu = 8$ is in the interval. Do not reject H_0.

23. a. $$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} = \frac{14 - 12}{4.32 / \sqrt{25}} = 2.31$$

b. Degrees of freedom = $n - 1 = 24$

p-value is upper-tail area

Using t table: p-value is between .01 and .025

Using Excel: p-value = 1-T.DIST(2.31,24,TRUE) = .0149

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c. p-value $\leq .05$, reject H_0.

d. With $df = 24$, $t_{.05} = 1.711$

Reject H_0 if $t \geq 1.711$

$2.31 > 1.711$, reject H_0.

24. a. $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{17 - 18}{4.5/\sqrt{48}} = -1.54$

b. Degrees of freedom $= n - 1 = 47$

Because $t < 0$, p-value is two times the lower tail area

Using t table: area in lower tail is between .05 and .10; therefore, p-value is between .10 and .20.

Using Excel: p-value = 2*T.DIST(-1.54,47,TRUE) = .1303

c. p-value $> .05$, do not reject H_0.

d. With $df = 47$, $t_{.025} = 2.012$

Reject H_0 if $t \leq -2.012$ or $t \geq 2.012$

$t = -1.54$; do not reject H_0

25. a. $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{44 - 45}{5.2/\sqrt{36}} = -1.15$

Degrees of freedom $= n - 1 = 35$

p-value is lower-tail area

Using t table: p-value is between .10 and .20

Using Excel: p-value = T.DIST(-1.15,35,TRUE) = .1290

p-value $> .01$, do not reject H_0

b. $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{43 - 45}{4.6/\sqrt{36}} = -2.61$

p-value is lower-tail area

Using t table: p-value is between .005 and .01

Using Excel: p-value = T.DIST(-2.61,35,TRUE) = .0066

p-value $\leq .01$, reject H_0

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c. $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{46 - 45}{5/\sqrt{36}} = 1.20$

p-value is lower-tail area

Using t table: p-value is between .80 and .90

Using Excel: p-value = T.DIST(1.20,35,TRUE) = .8809

p-value > .01, do not reject H_0

26. a. $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{103 - 100}{11.5/\sqrt{65}} = 2.10$

Degrees of freedom = $n - 1 = 64$

Because $t > 0$, p-value is two times the upper tail area

Using t table; area in upper tail is between .01 and .025; therefore, p-value is between .02 and .05.

Using Excel: p-value = 2*(1-T.DIST(2.10,64,TRUE)) = .0397

p-value ≤ .05, reject H_0

b. $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{96.5 - 100}{11/\sqrt{65}} = -2.57$

Because $t < 0$, p-value is two times the lower tail area

Using t table: area in lower tail is between .005 and .01; therefore, p-value is between .01 and .02.

Using Excel: p-value = 2*T.DIST(-2.57,64,TRUE) = .0125

p-value ≤ .05, reject H_0

c. $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{102 - 100}{10.5/\sqrt{65}} = 1.54$

Because $t > 0$, p-value is two times the upper tail area

Using t table: area in upper tail is between .05 and .10; therefore, p-value is between .10 and .20.

Using Excel: p-value = 2*(1-T.DIST(1.54,64,TRUE)) = .1285

p-value > .05, do not reject H_0

27. a. H_0: $\mu \geq 238$

H_a: $\mu < 238$

b. $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{231 - 238}{80/\sqrt{100}} = -.88$

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Degrees of freedom = $n - 1 = 99$

p-value is lower-tail area

Using t table: p-value is between .10 and .20

Using Excel: p-value = T.DIST(-.88,99,TRUE) = .1905

c. p-value > .05; do not reject H_0. Cannot conclude mean weekly benefit in Virginia is less than the national mean.

d. $df = 99$ $t_{.05} = -1.66$

Reject H_0 if $t \leq -1.66$

-.88 > -1.66; do not reject H_0

28. a. H_0: $\mu \geq 9$

H_a: $\mu < 9$

b. $$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{7.27 - 9}{6.38/\sqrt{85}} = -2.50$$

Degrees of freedom = $n - 1 = 84$

p-value is lower-tail area

Using t table: p-value is between .005 and .01

Using Excel: p-value = T.DIST(-2.50,84,TRUE) = .0072

c. p-value ≤ .01; reject H_0. The mean tenure of a CEO is significantly shorter than 9 years. The claim of the shareholders group is not valid.

29. a. H_0: $\mu = 5600$

H_a: $\mu \neq 5600$

b. $$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{5835 - 5600}{520/\sqrt{25}} = 2.26$$

Degrees of freedom = $n - 1 = 24$

Because $t > 0$, p-value is two times the upper tail area

Using t table: area in upper tail is between .01 and .025; therefore, p-value is between .02 and .05.

Using Excel: p-value = 2*(1-T.DIST(2.26,24,TRUE)) = .0332

c. p-value ≤ .05; reject H_0. The mean diamond price in New York City differs.

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d. $df = 24$ $t_{.025} = 2.064$

Reject H_0 if $t < -2.064$ or $t > 2.064$

$2.26 > 2.064$; reject H_0

30. a. H_0: $\mu = 600$

H_a: $\mu \neq 600$

b. $$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{612 - 600}{65/\sqrt{40}} = 1.17$$

$df = n - 1 = 39$

Because $t > 0$, p-value is two times the upper tail area

Using t table: area in upper tail is between .10 and .20; therefore, p-value is between .20 and .40.

Using Excel: p-value = 2*(1-T.DIST(1.17,39,TRUE)) = .2491

c. With $\alpha = .10$ or less, we cannot reject H_0. We are unable to conclude there has been a change in the mean CNN viewing audience.

d. The sample mean of 612 thousand viewers is encouraging but not conclusive for the sample of 40 days. Recommend additional viewer audience data. A larger sample should help clarify the situation for CNN.

31. H_0: $\mu \leq 423$

H_a: $\mu > 423$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{460.4 - 423.0}{101.9/\sqrt{36}} = 2.20$$

Degrees of freedom = $n - 1 = 35$

p-value is upper-tail area

Using t table: p-value is between .01 and .025.

Using Excel: p-value = 1-T.DIST(2.20,35,TRUE) = .0173

Because p-value = .0173 < α, reject H_0; Atlanta customers have a higher annual rate of consumption of Coca Cola beverages.

32. a. H_0: $\mu = 10{,}192$

H_a: $\mu \neq 10{,}192$

b. $$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{9750 - 10{,}192}{1400/\sqrt{50}} = -2.23$$

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Degrees of freedom = $n - 1 = 49$

Because $t < 0$, p-value is two times the lower tail area

Using t table: area in lower tail is between .01 and .025; therefore, p-value is between .02 and .05.

Using Excel: p-value = 2*T.DIST(-2.23,49,TRUE) = .0304

c. p-value ≤ .05; reject H_0. The population mean price at this dealership differs from the national mean price $10,192.

33. a. H_0: $\mu \leq 21.6$

H_a: $\mu > 21.6$

b. $24.1 - 21.6 = 2.5$ gallons

c. $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{24.1 - 21.6}{4.8/\sqrt{16}} = 2.08$

Degrees of freedom = $n - 1 = 15$

p-value is upper-tail area

Using t table: p-value is between .025 and .05

Using Excel: p-value = 1-T.DIST(2.08,15,TRUE) = .0275

d. p-value ≤ .05; reject H_0. The population mean consumption of milk in Webster City is greater than the National mean.

34. a. H_0: $\mu = 2$

H_a: $\mu \neq 2$

b. $\bar{x} = \dfrac{\Sigma x_i}{n} = \dfrac{22}{10} = 2.2$

c. $s = \sqrt{\dfrac{\Sigma(x_i - \bar{x})^2}{n-1}} = .516$

d. $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{2.2 - 2}{.516/\sqrt{10}} = 1.22$

Degrees of freedom = n - 1 = 9

Because $t > 0$, p-value is two times the upper tail area

Using t table: area in upper tail is between .10 and .20; therefore, p-value is between .20 and .40.

Using Excel: p-value = 2*(1-T.DIST(1.22,9,TRUE)) = .2535

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e. p-value > .05; do not reject H_0. No reason to change from the 2 hours for cost estimating purposes.

35. a. $$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{.175 - .20}{\sqrt{\frac{.20(1-.20)}{400}}} = -1.25$$

b. Because $z < 0$, p-value is two times the lower tail area

Using normal table with z = -1.25: p-value = 2(.1056) = .2112

Using Excel: p-value = 2*NORM.S.DIST(-1.25,TRUE) = .2113

c. p-value > .05; do not reject H_0

d. $z_{.025} = 1.96$

Reject H_0 if $z \leq -1.96$ or $z \geq 1.96$

$z = -1.25$; do not reject H_0

36. a. $$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{.68 - .75}{\sqrt{\frac{.75(1-.75)}{300}}} = -2.80$$

p-value is lower-tail area

Using normal table with z = -2.80: p-value =.0026

Using Excel: p-value = NORM.S.DIST(-2.80,TRUE) = .0026

b. $$z = \frac{.72 - .75}{\sqrt{\frac{.75(1-.75)}{300}}} = -1.20$$

p-value is lower-tail area

Using normal table with z = -1.20: p-value =.1151

Using Excel: p-value = NORM.S.DIST(-1.20,TRUE) = .1151

c. $$z = \frac{.70 - .75}{\sqrt{\frac{.75(1-.75)}{300}}} = -2.00$$

p-value is lower-tail area

Using normal table with z = -2.00: p-value =.0228

Using Excel: p-value = NORM.S.DIST(-2.00,TRUE) = . 0228

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d. $$z = \frac{.77 - .75}{\sqrt{\frac{.75(1-.75)}{300}}} = .80$$

p-value is lower-tail area

Using normal table with $z = .80$: p-value =.7881

Using Excel: p-value = NORM.S.DIST(.80,TRUE) = . 7881

37. a. H_0: $p \leq .125$

H_a: $p > .125$

b. $$\bar{p} = \frac{52}{400} = .13$$

$$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{.13 - .125}{\sqrt{\frac{.125(1-.125)}{400}}} = .30$$

p-value is upper-tail area

Using normal table with $z = .30$: p-value = 1.0000 - .6179 = .3821

Using Excel: p-value = 1-NORM.S.DIST(.30,TRUE) = .3821

c. p-value > .05; do not reject H_0. We cannot conclude that there has been an increase in union membership.

38. a. H_0: $p = .64$

H_a: $p \neq .64$

b. $$\bar{p} = \frac{52}{100} = .52$$

$$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{.52 - .64}{\sqrt{\frac{.64(1-.64)}{100}}} = -2.50$$

Because $z < 0$, p-value is two times the lower tail area

Using normal table with $z = -2.50$: p-value = 2(.0062) = .0124

Using Excel: p-value = 2*NORM.S.DIST(-2.50,TRUE) = . 0124

c. p-value ≤ .05; reject H_0. Proportion differs from the reported .64.

d. Yes. Since $\bar{p} = .52$, it indicates that fewer than 64% of the shoppers believe the supermarket brand is as good as the name brand.

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39. a. H_0: $p = .75$

H_a: $p \neq .75$

b. 30 – 49 Age Group $\bar{p} = \dfrac{85}{100} = .85$

$$z = \frac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} = \frac{.85 - .75}{\sqrt{\dfrac{.75(1-.75)}{100}}} = 2.31$$

Because $z > 0$, p-value is two times the upper tail area

Using normal table with $z = 2.31$: p-value = 2(.0104) = .0208

Using Excel: p-value = 2*(1-NORM.S.DIST(2.31,TRUE)) = . 0209

Reject H_0. Conclude that the proportion of users in the 30 – 49 age group is higher than the overall proportion of .75.

c. 50 – 64 Age Group $\bar{p} = \dfrac{144}{200} = .72$

$$z = \frac{.72 - .75}{\sqrt{\dfrac{.75(1-.75)}{200}}} = -.98$$

Because $z < 0$, p-value is two times the lower tail area

Using the normal table with z = -.98: p-value = 2(.1635) = .3270

Using Excel: p-value = 2*NORM.S.DIST(-.98,TRUE) = . 3271

Do not reject H_0. The proportion for the 50 – 64 age group does not differ significantly from the overall proportion.

d. The proportion of internet users increases from .72 to .85 as we go from the 50 – 64 age group to the younger 30 – 49 age group. So we might expect the proportion to increase further for the even younger 18 – 29 age group. Indeed, the Pew project found the proportion of users in the 18 – 29 age group to be .92.

40. a. Sample proportion: $\bar{p} = .35$

Number planning to provide holiday gifts: $n\bar{p} = 60(.35) = 21$

b. H_0: $p \geq .46$

H_a: $p < .46$

$$z = \frac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} = \frac{.35 - .46}{\sqrt{\dfrac{.46(1-.46)}{60}}} = -1.71$$

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p-value is area in lower tail

Using normal table with z = -1.71: p-value = .0436

Using Excel: p-value = NORM.S.DIST(-1.71,TRUE) = . 0436

c. Using a .05 level of significance, we can conclude that the proportion of business owners providing gifts has decreased from 2008 to 2009. The smallest level of significance for which we could draw this conclusion is .0436; this corresponds to the p-value = .0436. This is why the p-value is often called the observed level of significance.

41. a. H_0: $p \geq .70$

H_a: $p < .70$

b. $$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{.67 - .70}{\sqrt{\frac{.70(1-.70)}{300}}} = -1.13$$

p-value is lower-tail area

Using normal table with z = -1.13: p-value =.1292

Using Excel: p-value = NORM.S.DIST(-1.13,TRUE) = .1292

c. p-value > .05; do not reject H_0. The executive's claim cannot be rejected.

42. a. $\bar{p}$ = 12/80 = .15

b. $$\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\frac{.15(.85)}{80}} = .0399$$

$$\bar{p} \pm z_{.025}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

.15 ± 1.96 (.0399)

.15 ± .0782 or .0718 to .2218

c. We can conduct a hypothesis test concerning whether the return rate for the Houston store is equal to .06 at an α = .05 level of significance using the 95% confidence interval in part (b). Since the confidence interval does not include .06, we conclude that the return rate for the Houston store is different than the U.S. national return rate.

43. a. H_0: $p \leq .10$

H_a: $p > .10$

b. There are 13 "Yes" responses in the Eagle data set.

$$\bar{p} = \frac{13}{100} = .13$$

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c. $$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}} = \frac{.13 - .10}{\sqrt{\frac{.10(1 - .10)}{100}}} = 1.00$$

p-value is upper-tail area

Using normal table with $z = 1.00$: p-value = 1 - .8413 = .1587

Using Excel: p-value = 1-NORM.S.DIST(1,TRUE) = .1587

p-value > .05; do not reject H_0.

The statistical results do not allow us to conclude that $p > .10$. But, given that $\bar{p} = .13$, management may want to authorize a larger study before deciding not to go national.

44. a. H_0: $p \leq .51$

H_a: $p > .51$

b. $$\bar{p} = \frac{232}{400} = .58$$

$$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}} = \frac{.58 - .51}{\sqrt{\frac{(.51)(.49)}{400}}} = 2.80$$

p-value is upper-tail area

Using normal table with $z = 2.80$: p-value = 1 – .9974 = .0026

Using Excel: p-value = 1-NORM.S.DIST(2.80,TRUE) = .0026

c. Since p-value = .0026 ≤ .01, we reject H_0 and conclude that people working the night shift get drowsy while driving more often than the average for the entire population.

45. a. H_0: $p = .30$

H_a: $p \neq .30$

b. $$\bar{p} = \frac{24}{50} = .48$$

c. $$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}} = \frac{.48 - .30}{\sqrt{\frac{.30(1 - .30)}{50}}} = 2.78$$

Because $z > 0$, p-value is two times the upper tail area

Using normal table with $z = 2.78$: p-value = 2(.0027) = .0054

Using Excel: p-value = 2*[1-NORM.S.DIST(2.78,TRUE)] = .0054

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p-value $\leq .01$; reject H_0.

We would conclude that the proportion of stocks going up on the NYSE is not 30%. This would suggest not using the proportion of DJIA stocks going up on a daily basis as a predictor of the proportion of NYSE stocks going up on that day.

46. a. H_0: $\mu = 16$

H_a: $\mu \neq 16$

b. $$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{16.32 - 16}{.8/\sqrt{30}} = 2.19$$

Because $z > 0$, p-value is two times the upper tail area

Using normal table with $z = 2.19$: p-value = 2(.0143) = .0286

Using Excel: p-value = 2*(1-NORM.S.DIST(2.19,TRUE)) = . 0285

p-value $\leq .05$; reject H_0. Readjust production line.

c. $$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{15.82 - 16}{.8/\sqrt{30}} = -1.23$$

Because $z < 0$, p-value is two times the lower tail area

Using normal table with z = -1.23: p-value = 2(.1093) = .2186

Using Excel: p-value = 2*NORM.S.DIST(-1.23,TRUE) = . 2187

p-value > .05; do not reject H_0. Continue the production line.

d. Reject H_0 if $z \leq$ -1.96 or $z \geq 1.96$

For $\bar{x} = 16.32$, $z = 2.19$; reject H_0

For $\bar{x} = 15.82$, z = -1.23; do not reject H_0

Yes, same conclusion.

47. a. H_0: $\mu = 900$

H_a: $\mu \neq 900$

b. $$\bar{x} \pm z_{.025}\frac{\sigma}{\sqrt{n}}$$

$$935 \pm 1.96\frac{180}{\sqrt{200}}$$

935 ± 25 (910 to 960)

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c. Reject H_0 because $\mu = 900$ is not in the interval.

d. $$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} - \frac{935 - 900}{180 / \sqrt{200}} = 2.75$$

Because $z > 0$, p-value is two times the upper tail area

Using normal table with $z = 2.75$: p-value = 2(.0030) = .0060

Using Excel: p-value = 2*(1-NORM.S.DIST(2.75,TRUE)) = . 0060

48. a. H_0: $\mu \leq 119{,}155$

H_a: $\mu > 119{,}155$

b. $$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{126{,}100 - 119{,}155}{20{,}700 / \sqrt{60}} = 2.60$$

p-value is upper-tail area

Using normal table with $z = 2.60$: p-value = 1.0000 - .9953 = .0047

Using Excel: p-value = 1-NORM.S.DIST(2.60,TRUE) = .0047

c. p-value $\leq .01$, reject H_0. We can conclude that the mean annual household income for theater goers in the San Francisco Bay area is higher than the mean for all *Playbill* readers.

49. The hypothesis test that will allow us to conclude that the consensus estimate has increased is given below.

H_0: $\mu \leq 250{,}000$

H_a: $\mu > 250{,}000$

$$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} = \frac{266{,}000 - 250{,}000}{24{,}000 / \sqrt{20}} = 2.981$$

Degrees of freedom = $n - 1 = 19$

p-value is upper-tail area

Using t table: p-value is less than .005

Using Excel: p-value = 1-T.DIST(2.981,19,TRUE) = .0038

p-value $\leq .01$; reject H_0. The consensus estimate has increased.

50. H_0: $\mu = 25$

H_a: $\mu \neq 25$

$$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} = \frac{24.0476 - 25.0}{5.8849 / \sqrt{42}} = -1.05$$

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Degrees of freedom = $n - 1 = 41$

Because $t < 0$, p-value is two times the lower tail area

Using t table: area in lower tail is between .10 and .20; therefore, p-value is between .20 and .40.

Using Excel: p-value = 2*T.DIST(-1.05,41,TRUE) = .2999

Because p-value $> \alpha = .05$, do not reject H_0. There is no evidence to conclude that the mean age at which women had their first child has changed.

51. a. H_0: $\mu \leq 520$

H_a: $\mu > 520$

b. Sample mean: 637.94

Sample standard deviation: 148.4694

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{637.94 - 520}{148.4694/\sqrt{50}} = 5.62$$

Degrees of freedom = $n - 1 = 49$

p-value is the area in the upper tail

Using t table: p-value is $< .005$

Using Excel: p-value = 1-T.DIST(5.62,49,TRUE) ≈ 0

c. We can conclude that the mean weekly pay for all women is higher than that for women with only a high school degree.

d. Using the critical value approach we would:

Reject H_0 if $t \geq t_{.05} = 1.677$

Since $t = 5.62 > 1.677$, we reject H_0.

52. H_0: $\mu \leq 125{,}000$

H_a: $\mu > 125{,}000$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{130{,}000 - 125{,}000}{12{,}500/\sqrt{32}} = 2.26$$

Degrees of freedom = 32 – 1 = 31

p-value is upper-tail area

Using t table: p-value is between .01 and .025

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Using Excel: p-value = 1-T.DIST(2.26,31,TRUE) = .0155

p-value ≤ .05; reject H_0. Conclude that the mean cost is greater than $125,000 per lot.

53. H_0: $\mu = 2.357$

H_a: $\mu \neq 2.357$

$$\bar{x} = \frac{\Sigma x_i}{n} = 2.3496$$

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = .0444$$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{2.3496 - 2.3570}{.0444/\sqrt{50}} = -1.18$$

Degrees of freedom = 50 - 1 = 49

Because $t < 0$, p-value is two times the lower tail area

Using t table: area in lower tail is between .10 and .20; therefore, p-value is between .20 and .40.

Using Excel: p-value = 2*T.DIST(-1.18,49,TRUE) = .2437

p-value > .05; do not reject H_0.

There is not a statistically significant difference between the National mean price per gallon and the mean price per gallon in the Lower Atlantic states.

54. a. H_0: $p \leq .80$

H_a: $p > .80$

$$\bar{p} = \frac{455}{542} = .84$$

$$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{.84 - .80}{\sqrt{\frac{.80(1-.80)}{542}}} = 2.33$$

p-value is the area in the upper tail
Using normal table with $z = 2.33$: p-value = 1.0000 - .9901 = .0099

Using Excel: p-value = 1-NORM.S.DIST(2.33,TRUE) = .0099

p-value ≤ .05; reject H_0. We conclude that over 80% of airline travelers feel that use of the full body scanners will improve airline security.

b. H_0: $p \leq .75$

H_a: $p > .75$

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$$\bar{p} = \frac{423}{542} = .78$$

$$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{.78 - .75}{\sqrt{\frac{.75(1-.75)}{542}}} = 1.61$$

p-value is the area in the upper tail

Using normal table with $z = 1.61$: p-value = 1.0000 - .9463 = .0537

Using Excel: p-value = 1-NORM.S.DIST(1.61,TRUE) = .0537

p-value > .01; we cannot reject H_0. Thus, we cannot conclude that over 75% of airline travelers approve of using full body scanners. Mandatory use of full body scanners is not recommended.

Author's note: The TSA is also considering making the use of full body scanners optional. Travelers would be given a choice of a full body scan or a pat down search.

55. a. H_0: $p \leq .39$

H_a: $p > .39$

b. $$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{.41 - .39}{\sqrt{\frac{.39(1-.39)}{450}}} = .87$$

p-value is the area in the upper tail

Using normal table with $z = .87$: p-value = 1 - .8078 = .1922

Using Excel: p-value = 1-NORM.S.DIST(.87,TRUE) = .1922

c. With .1922 > .10, we cannot reject H_0. We cannot conclude that the proportion of bullishness is greater than the long term average. The difference is not significant at the .10 level.

56. a. H_0: $p \leq .80$

H_a: $p > .80$

b. $$\bar{p} = \frac{252}{300} = .84 \quad (84\%)$$

c. $$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{.84 - .80}{\sqrt{\frac{.80(1-.80)}{300}}} = 1.73$$

p-value is the upper-tail area

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Using normal table with $z = 1.73$: p-value = 1.0000 - .9582 = .0418

Using Excel: p-value = 1-NORM.S.DIST(1.73,TRUE) = .0418

d. p-value ≤ .05; reject H_0. Conclude that more than 80% of the customers are satisfied with the service provided by the home agents. Regional Airways should consider implementing the home agent system.

57. a. $\bar{p} = \dfrac{503}{910} = .553$

b. H_0: $p \leq .50$

H_a: $p > .50$

c. $$z = \frac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} = \frac{.553 - .500}{\sqrt{\dfrac{(.5)(.5)}{910}}} = 3.19$$

p-value is the upper-tail area

Using normal table with $z = 3.19$: p-value ≈ 0

Using Excel: p-value = 1 NORM.S.DIST(3.19,TRUE) = . 0007

You can tell the manager that the observed level of significance is very close to zero and that this means the results are highly significant. Any reasonable person would reject the null hypotheses and conclude that the proportion of adults who are optimistic about the national outlook is greater than .50

58. H_0: $p \geq .90$

H_a: $p < .90$

$$\bar{p} = \frac{49}{58} = .8448$$

$$z = \frac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} = \frac{.8448 - .90}{\sqrt{\dfrac{.90(1-.90)}{58}}} = -1.40$$

p-value is the lower-tail area

Using normal table with z = -1.40: p-value =.0808

Using Excel: p-value = NORM.S.DIST(-1.40,TRUE) = . 0808

p-value > .05; do not reject H_0. Claim of at least 90% cannot be rejected.

59. a. H_0: $p \geq .24$

H_a: $p < .24$

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b. $\bar{p} = \dfrac{81}{400} = .2025$

c. $z = \dfrac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} = \dfrac{.2025 - .24}{\sqrt{\dfrac{.24(1-.24)}{400}}} = -1.76$

p-value is the lower-tail area

Using normal table with z = -1.76: p-value =.0392

Using Excel: p-value = NORM.S.DIST(-1.76,TRUE) = .0392

p-value $\leq .05$; reject H_0.

The proportion of workers not required to contribute to their company sponsored health care plan has declined. There seems to be a trend toward companies requiring employees to share the cost of health care benefits.

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Chapter 10
Comparisons Involving Means, Experimental Design, and Analysis of Variance

Learning Objectives

1. Be able to develop interval estimates and conduct hypothesis tests about the difference between two population means when σ_1 and σ_2 are known.

2. Know the properties of the sampling distribution of $\bar{x}_1 - \bar{x}_2$.

3. Be able to use the t distribution to conduct statistical inferences about the difference between two population means when σ_1 and σ_2 are unknown.

4. Learn how to analyze the difference between two population means when the samples are independent and when the samples are matched.

5. Understand the basic principles of an experimental study and how they are used in a completely randomized design.

6. Understand how the analysis of variance (ANOVA) procedure can be used to test for the equality of k population means using data from a completely randomized design as well as data from an observational study.

7. Know the assumptions necessary to use the analysis of variance procedure.

8. Know how to set up an ANOVA table and interpret the entries in the table.

9. Be able to use output from Excel to solve experimental design problems.

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Solutions:

1. a. $\bar{x}_1 - \bar{x}_2 = 13.6 - 11.6 = 2$

   b. $z_{\alpha/2} = z_{.05} = 1.645$

   $$\bar{x}_1 - \bar{x}_2 \pm 1.645\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

   $$2 \pm 1.645\sqrt{\frac{(2.2)^2}{50} + \frac{(3)^2}{35}}$$

   $2 \pm .98$ (1.02 to 2.98)

   c. $z_{\alpha/2} = z_{.025} = 1.96$

   $$2 \pm 1.96\sqrt{\frac{(2.2)^2}{50} + \frac{(3)^2}{35}}$$

   2 ± 1.17 (.83 to 3.17)

2. a. $$z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(25.2 - 22.8) - 0}{\sqrt{\frac{(5.2)^2}{40} + \frac{6^2}{50}}} = 2.03$$

   b. p-value = 1.0000 - .9788 = .0212

   c. p-value $\leq$ .05, reject H_0.

3. a. $$z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(104 - 106) - 0}{\sqrt{\frac{(8.4)^2}{80} + \frac{(7.6)^2}{70}}} = -1.53$$

   b. p-value = 2(.0630) = .1260

   c. p-value > .05, do not reject H_0.

4. a. μ_1 = Population mean for smaller cruise ships

   μ_2 = Population mean for larger cruise ships

   $\bar{x}_1 - \bar{x}_2 = 85.36 - 81.40 = 3.96$

   b. $z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

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$$1.96\sqrt{\frac{(4.55)^2}{37}+\frac{(3.97)^2}{44}}=1.88$$

c. 3.96 ± 1.88 (2.08 to 5.84)

5. a. $\bar{x}_1-\bar{x}_2=135.67-68.64=67.03$

b. $z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}=2.576\sqrt{\frac{(35)^2}{40}+\frac{(20)^2}{30}}=17.08$

c. 67.03 ± 17.08 (49.95 to 84.11) We estimate that men spend \$67.03 more than women on Valentine's Day with a margin of error of \$17.08.

6. μ_1 = Mean hotel price in Atlanta

μ_2 = Mean hotel price in Houston

H_0: $\mu_1-\mu_2 \geq 0$

H_a: $\mu_1-\mu_2 < 0$

$$z=\frac{(\bar{x}_1-\bar{x}_2)-D_0}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}=\frac{(91.71-101.13)-0}{\sqrt{\frac{20^2}{35}+\frac{25^2}{40}}}=-1.81$$

p-value = .0351

p-value $\leq$ .05; reject H_0. The mean price of a hotel room in Atlanta is lower than the mean price of a hotel room in Houston.

7. a. μ_1 = Population mean 2002

μ_2 = Population mean 2003

H_0: $\mu_1-\mu_2 \leq 0$

H_a: $\mu_1-\mu_2 > 0$

b. With time in minutes, $\bar{x}_1-\bar{x}_2$ = 172 - 166 = 6 minutes

c. $$z=\frac{(\bar{x}_1-\bar{x}_2)-D_0}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}=\frac{(172-166)-0}{\sqrt{\frac{12^2}{60}+\frac{12^2}{50}}}=2.61$$

p-value = 1.0000 - .9955 = .0045

p-value $\leq$.05; reject H_0. The population mean duration of games in 2003 is less than the population mean in 2002.

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d. $\bar{x}_1 - \bar{x}_2 \pm z_{.025}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

$(172 - 166) \pm 1.96\sqrt{\frac{12^2}{60} + \frac{12^2}{50}}$

6 ± 4.5 (1.5 to 10.5)

e. Percentage reduction: 6/172 = 3.5%. Management should be encouraged by the fact that steps taken in 2003 reduced the population mean duration of baseball games. However, the statistical analysis shows that the reduction in the mean duration is only 3.5%. The interval estimate shows the reduction in the population mean is 1.5 minutes (.9%) to 10.5 minutes (6.1%). Additional data collected by the end of the 2003 season would provide a more precise estimate. In any case, most likely the issue will continue in future years. It is expected that major league baseball would prefer that additional steps be taken to further reduce the mean duration of games.

8. a. This is an upper tail hypothesis test.

$H_0 : \mu_1 \leq \mu_2$

$H_a : \mu_1 > \mu_2$

$$z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(76 - 73)}{\sqrt{\frac{6^2}{60} + \frac{6^2}{60}}} = 2.74$$

p-value = area in upper tail at $z = 2.74$

p-value = 1.0000 - .9969 = .0031

Since .0031 $\leq \alpha = .05$, we reject the null hypothesis. The difference is significant. We can conclude that customer service has improved for Rite Aid.

b. This is another upper tail test but it only involves one population.

$H_0 : \mu \leq 75.7$

$H_a : \mu > 75.7$

$$z = \frac{(\bar{x}_1 - 75.7)}{\sqrt{\frac{\sigma^2}{n_1}}} = \frac{(76 - 75.7)}{\sqrt{\frac{6^2}{60}}} = .39$$

p-value = area in upper tail at $z = .39$

p-value = 1.0000 - .6517 = .3483

Since .3483 > $\alpha = .05$, we cannot reject the null hypothesis. The difference is not statistically significant.

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c. This is an upper tail test similar to the one in part (a).

$H_0 : \mu_1 \leq \mu_2$

$H_a : \mu_1 > \mu_2$

$$z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(77-75)}{\sqrt{\frac{6^2}{60} + \frac{6^2}{60}}} = 1.83$$

p-value = area in upper tail at $z = 1.83$

p-value = 1.0000 - .9664 = .0336

Since .0336 $\leq \alpha = .05$, we reject the null hypothesis. The difference is significant. We can conclude that customer service has improved for Expedia.

d. We will reject the null hypothesis of "no increase" if the p-value $\leq .05$. For an upper tail hypothesis test, the p-value is the area in the upper tail at the value of the test statistic. A value of $z = 1.645$ provides an upper tail area of .05. So, we must solve the following equation for $\bar{x}_1 - \bar{x}_2$.

$$z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{6^2}{60} + \frac{6^2}{60}}} = 1.645$$

$$\bar{x}_1 - \bar{x}_2 = 1.645\sqrt{\frac{6^2}{60} + \frac{6^2}{60}} = 1.80$$

This tells us that as long as the 2008 score for a company exceeds the 2007 score by 1.80 or more the difference will be statistically significant.

e. The increase from 2007 to 2008 for J.C. Penney is not statistically significant because it is less than 1.80. We cannot conclude that customer service has improved for J.C. Penney.

9. a. $\bar{x}_1 - \bar{x}_2 = 22.5 - 20.1 = 2.4$

b. $$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2} = \frac{\left(\frac{2.5^2}{20} + \frac{4.8^2}{30}\right)^2}{\frac{1}{19}\left(\frac{2.5^2}{20}\right)^2 + \frac{1}{29}\left(\frac{4.8^2}{30}\right)^2} = 45.8$$

Use $df = 45$.

c. $t_{.025} = 2.014$

$$t_{.025}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 2.014\sqrt{\frac{2.5^2}{20} + \frac{4.8^2}{30}} = 2.1$$

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d. 2.4 ± 2.1 (.3 to 4.5)

10. a. $$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(13.6 - 10.1) - 0}{\sqrt{\frac{5.2^2}{35} + \frac{8.5^2}{40}}} = 2.18$$

b. $$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2} = \frac{\left(\frac{5.2^2}{35} + \frac{8.5^2}{40}\right)^2}{\frac{1}{34}\left(\frac{5.2^2}{35}\right)^2 + \frac{1}{39}\left(\frac{8.5^2}{40}\right)^2} = 65.7$$

Use $df = 65$

c. Using t table, area in tail is between .01 and .025

$\therefore$ two-tail p-value is between .02 and .05.

Exact p-value corresponding to $t = 2.18$ is .0329

d. p-value $\leq .05$, reject H_0.

11. a. $\bar{x}_1 = \frac{54}{6} = 9 \quad \bar{x}_2 = \frac{42}{6} = 7$

b. $$s_1 = \sqrt{\frac{\Sigma(x_i - \bar{x}_1)^2}{n_1 - 1}} = 2.28$$

$$s_2 = \sqrt{\frac{\Sigma(x_i - \bar{x}_2)^2}{n_2 - 1}} = 1.79$$

c. $\bar{x}_1 - \bar{x}_2 = 9 - 7 = 2$

d. $$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2} = \frac{\left(\frac{2.28^2}{6} + \frac{1.79^2}{6}\right)^2}{\frac{1}{5}\left(\frac{2.28^2}{6}\right)^2 + \frac{1}{5}\left(\frac{1.79^2}{6}\right)^2} = 9.5$$

Use $df = 9$, $t_{.05} = 1.833$

$$\bar{x}_1 - \bar{x}_2 \pm 1.833\sqrt{\frac{2.28^2}{6} + \frac{1.79^2}{6}}$$

2 ± 2.17 (-.17 to 4.17)

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12. a. $\bar{x}_1 - \bar{x}_2 = 22.5 - 18.6 = 3.9$

b. $$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2} = \frac{\left(\frac{8.4^2}{50} + \frac{7.4^2}{40}\right)^2}{\frac{1}{49}\left(\frac{8.4^2}{50}\right)^2 + \frac{1}{39}\left(\frac{7.4^2}{40}\right)^2} = 87.1$$

Use $df = 87$, $t_{.025} = 1.988$

$$3.9 \pm 1.988\sqrt{\frac{8.4^2}{50} + \frac{7.4^2}{40}}$$

3.9 ± 3.3 (.6 to 7.2)

13. a. $$\bar{x}_1 = \frac{\Sigma x_i}{n_1} = \frac{111.6}{12} = 9.3$$

$$s_1 = \sqrt{\frac{\Sigma(x_i - \bar{x}_1)^2}{n_1 - 1}} = \sqrt{\frac{71.12}{12 - 1}} = 2.54$$

$$\bar{x}_2 = \frac{\Sigma x_i}{n_2} = \frac{42}{10} = 4.2$$

$$s_2 = \sqrt{\frac{\Sigma(x_i - \bar{x}_2)^2}{n_2}} = \sqrt{\frac{18.4}{10.1}} = 1.43$$

b. $\bar{x}_1 - \bar{x}_2 = 9.3 - 4.2 = 5.1$ tons

Memphis is the higher volume airport and handled an average of 5.1 tons per day more than Louisville. Memphis handles more than twice the volume of Louisville.

c. $$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2} = \frac{\left(\frac{2.54^2}{12} + \frac{1.43^2}{10}\right)^2}{\frac{1}{11}\left(\frac{2.54^2}{12}\right)^2 + \frac{1}{9}\left(\frac{1.43^2}{10}\right)^2} = 17.8$$

Use $df = 17$, $t_{.025} = 2.110$

$$(\bar{x}_1 - \bar{x}_2) \pm t_{.025}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$5.1 \pm 2.110\sqrt{\frac{2.54^2}{12} + \frac{1.43^2}{10}}$$

5.1 ± 1.82 (3.28 to 6.92)

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14. a. H_0: $\mu_1 - \mu_2 \geq 0$

H_a: $\mu_1 - \mu_2 < 0$

b. $$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(56{,}100 - 59{,}400) - 0}{\sqrt{\frac{(6000)^2}{40} + \frac{(7000)^2}{50}}} = -2.41$$

c. $$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2} = \frac{\left(\frac{6000^2}{40} + \frac{7000^2}{50}\right)^2}{\frac{1}{40-1}\left(\frac{6000^2}{40}\right)^2 + \frac{1}{50-1}\left(\frac{7000^2}{50}\right)^2} = 87.55$$

Rounding down, we will use a t distribution with 87 degrees of freedom. From the t table we see that t = -2.41 corresponds to a p-value between .005 and .01.

Exact p-value corresponding to t = -2.41 is .009.

d. p-value $\leq$ .05, reject H_0. We conclude that the salaries of staff nurses are lower in Tampa than in Dallas.

15. a. μ_1 = mean annual lease rate for Hong Kong

μ_2 = mean annual lease rate for Paris

H_0: $\mu_1 - \mu_2 \leq 0$

H_a: $\mu_1 - \mu_2 > 0$

b. $\bar{x}_1 - \bar{x}_2$ = \$1114 - 989 = \$125

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(1114 - 989) - 0}{\sqrt{\frac{230^2}{30} + \frac{195^2}{40}}} = 2.40$$

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2} = \frac{\left(\frac{230^2}{30} + \frac{195^2}{40}\right)^2}{\frac{1}{29}\left(\frac{230^2}{30}\right)^2 + \frac{1}{39}\left(\frac{195^2}{40}\right)^2} = 56.5$$

Rounding down, use df = 56

Using t table, p-value is between .005 and .01.

Exact p-value corresponding to t = 2.40 is .0099

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p-value $\leq .01$, reject H_0. Conclusion: the annual lease rate in Hong Kong is significantly higher than in Paris.

16. a. μ_1 = population mean road-test score for midpriced family sedans

μ_2 = population mean road-test score for affordable family sedans

H_0: $\mu_1 - \mu_2 \leq 0$

H_a: $\mu_1 - \mu_2 > 0$

b. $\bar{x}_1 = \dfrac{\Sigma x_i}{n} = \dfrac{1074}{15} = 71.6$

$\bar{x}_2 = \dfrac{\Sigma x_i}{n} = \dfrac{1029}{15} = 68.6$

$\bar{x}_1 - \bar{x}_2 = 71.6 - 68.6 = 3$ points higher for midpriced family sedans

c. $s_1 = \sqrt{\dfrac{\Sigma(x_i - \bar{x}_1)^2}{n_1 - 1}} = 14.51$

$s_2 = \sqrt{\dfrac{\Sigma(x_i - \bar{x}_2)^2}{n_2 - 1}} = 11.36$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \frac{(71.6 - 68.6) - 0}{\sqrt{\dfrac{14.51^2}{15} + \dfrac{11.36^2}{15}}} = .63$$

$$df = \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{1}{n_1 - 1}\left(\dfrac{s_1^2}{n_1}\right)^2 + \dfrac{1}{n_2 - 1}\left(\dfrac{s_2^2}{n_2}\right)^2} = \frac{\left(\dfrac{14.51^2}{15} + \dfrac{11.36^2}{15}\right)^2}{\dfrac{1}{14}\left(\dfrac{14.51^2}{15}\right)^2 + \dfrac{1}{14}\left(\dfrac{11.36^2}{15}\right)^2} = 26.5$$

Use $df = 26$

Using t table, p-value is greater than .20

Exact p-value corresponding to $t = .63$ is .2671

d. p-value > .05; we cannot reject H_0. We cannot conclude that the mean road-test score for midpriced sedans is greater than the mean road-test score for family sedans.

17. a. H_0: $\mu_1 - \mu_2 \leq 0$

H_a: $\mu_1 - \mu_2 > 0$

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b. $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(6.82 - 6.25) - 0}{\sqrt{\dfrac{.64^2}{16} + \dfrac{.75^2}{10}}} = 1.99$

c. $df = \dfrac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{1}{n_1 - 1}\left(\dfrac{s_1^2}{n_1}\right)^2 + \dfrac{1}{n_2 - 1}\left(\dfrac{s_2^2}{n_2}\right)^2} = \dfrac{\left(\dfrac{.64^2}{16} + \dfrac{.75^2}{10}\right)^2}{\dfrac{1}{15}\left(\dfrac{.64^2}{16}\right)^2 + \dfrac{1}{9}\left(\dfrac{.75^2}{10}\right)^2} = 16.9$

Use $df = 16$

Using t table, p-value is between .025 and .05

Exact p-value corresponding to $t = 1.99$ is .0320

d. p-value $\leq .05$, reject H_0. The consultant with more experience has a higher population mean rating.

18. a. H_0: $\mu_1 - \mu_2 \geq 120$

H_a: $\mu_1 - \mu_2 < 120$

b. $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(1058 - 983) - 120}{\sqrt{\dfrac{90^2}{35} + \dfrac{105^2}{48}}} = -2.10$

$df = \dfrac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{1}{n_1 - 1}\left(\dfrac{s_1^2}{n_1}\right)^2 + \dfrac{1}{n_2 - 1}\left(\dfrac{s_2^2}{n_2}\right)^2} = \dfrac{\left(\dfrac{90^2}{35} + \dfrac{105^2}{48}\right)^2}{\dfrac{1}{34}\left(\dfrac{90^2}{35}\right)^2 + \dfrac{1}{47}\left(\dfrac{105^2}{48}\right)^2} = 78.8$

Use $df = 78$

Using t table, p-value is between .01 and .025

Exact p-value corresponding to $t = -2.10$ is .0195

p-value $\leq .05$, reject H_0. The improvement is less than the stated average of 120 points.

c. $\bar{x}_1 - \bar{x}_2 \pm t_{.025}\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$

$df = 78$

$(1058 - 983) \pm 1.991\sqrt{\dfrac{90^2}{35} + \dfrac{105^2}{48}}$

75 ± 43 (32 to 118)

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d. This is a wide interval. A larger sample should be used to reduce the margin of error.

19. a. 1, 2, 0, 0, 2

b. $\bar{d} = \sum d_i / n = 5/5 = 1$

c. $s_d = \sqrt{\frac{\sum(d_i - \bar{d})^2}{n-1}} = \sqrt{\frac{4}{5-1}} = 1$

d. $t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}} = \frac{1-0}{1/\sqrt{5}} = 2.24$

$df = n - 1 = 4$

Using t table, p-value is between .025 and .05

Exact p-value corresponding to $t = 2.24$ is .0443

Reject H_0; conclude $\mu_d > 0$.

20. a. 3, -1, 3, 5, 3, 0, 1

b. $\bar{d} = \sum d_i / n = 14/7 = 2$

c. $s_d = \sqrt{\frac{\sum(d_i - \bar{d})^2}{n-1}} = \sqrt{\frac{26}{7-1}} = 2.08$

d. $\bar{d} = 2$

e. With 6 degrees of freedom $t_{.025} = 2.447$

$2 \pm 2.447\left(2.082/\sqrt{7}\right)$

2 ± 1.93 (.07 to 3.93)

21. Difference = rating after - rating before

H_0: $\mu_d \leq 0$

H_a: $\mu_d > 0$

$\bar{d} = .625$ and $s_d = 1.30$

$t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}} = \frac{.625 - 0}{1.30/\sqrt{8}} = 1.36$

$df = n - 1 = 7$

Using t table, p-value is between .10 and .20

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Exact p-value corresponding to $t = 1.36$ is .1080

Do not reject H_0; we cannot conclude that seeing the commercial improves the mean potential to purchase.

22. Let d_i = current qtr. per share earnings previous quarter per share earnings

$\bar{d} = \sum d_i / n = .2064$

$$s_d = \sqrt{\frac{\sum(d_i - \bar{d})^2}{n-1}} = .2654$$

With $df = 24$, $t_{.025} = 2.064$

$$\bar{d} \pm t_{.025} \frac{s_d}{\sqrt{n}}$$

$$.2064 \pm 2.064 \left(\frac{.2654}{\sqrt{25}} \right)$$

Confidence interval: $.21 \pm $.11 ($.10 to $.32)

Earnings have increased. The point estimate of the increase in earnings per share is $.21 with a margin of error of $.11.

23. a. μ_1 = population mean grocery expenditures

μ_2 = population mean dining-out expenditures

H_0: $\mu_d = 0$

H_a: $\mu_d \neq 0$

b. $$t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}} = \frac{850 - 0}{1123 / \sqrt{42}} = 4.91$$

$df = n - 1 = 41$

p-value ≈ 0

Conclude that there is a difference between the annual population mean expenditures for groceries and for dining-out.

c. Groceries has the higher mean annual expenditure by an estimated $850.

$$\bar{d} \pm t_{.025} \frac{s_d}{\sqrt{n}}$$

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$$850 \pm 2.020\frac{1123}{\sqrt{42}}$$

850 ± 350 (500 to 1200)

24. H_0: $\mu_d \leq 0$

H_a: $\mu_d > 0$

Differences 177, -21, 186, -131, 22, 212, -5, 14

$$\bar{d} = \sum d_i / n = 454/8 = 56.75$$

$$s_d = \sqrt{\frac{\sum(d_i - \bar{d})^2}{n-1}} = 121.5445$$

$$t = \frac{\bar{d} - 0}{\frac{s_d}{\sqrt{n}}} = \frac{56.75}{\frac{121.5445}{\sqrt{8}}} = 1.32$$

$df = n - 1 = 7$

Using t table, p-value is greater than .10

Exact p-value corresponding to $t = 1.32$ is .1142

Since p-value > .10, do not reject H_0. We cannot conclude that airfares from Dayton are higher than those from Louisville at a $\alpha = .05$ level of significance.

25. a. H_0: $\mu_d = 0$

H_a: $\mu_d \neq 0$

Use difference data: -3, -2, -4, 3, -1, -2, -1, -2, 0, 0, -1, -4, -3, 1, 1

$$\bar{d} = \frac{\sum d_i}{n} = \frac{-18}{15} = -1.2$$

$$s_d = \sqrt{\frac{\sum(d_i - \bar{d})^2}{n-1}} = \sqrt{\frac{54.4}{15-1}} = 1.97$$

$$t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}} = \frac{-1.2 - 0}{1.97/\sqrt{15}} = -2.36$$

$df = n - 1 = 14$

Using t table, area is between .01 and .025.

Two-tail p-value is between .02 and .05.

Exact p-value corresponding to $t = -2.36$ is .0333

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p-value $\leq .05$, reject H_0. Conclude that there is a difference between the population mean weekly usage for the two media.

b. $\bar{x}_{TV} = \dfrac{\sum x_i}{n} = \dfrac{282}{15} = 18.8$ hours per week for cable television.

$\bar{x}_R = \dfrac{\sum x_i}{n} = \dfrac{300}{15} = 20$ hours per week for radio.

Radio has greater usage.

26. a. H_0: $\mu_d = 0$

H_a: $\mu_d \neq 0$

Differences: -2, -1, -5, 1, 1, 0, 4, -7, -6, 1, 0, 2, -3, -7, -2, 3, 1, 2, 1, -4

$\bar{d} = \sum d_i / n = -21/20 = -1.05$

$s_d = \sqrt{\dfrac{\sum (d_i - \bar{d})^2}{n-1}} = 3.3162$

$t - \dfrac{\bar{d} - \mu_d}{s_d / \sqrt{n}} = \dfrac{-1.05 - 0}{3.3162/\sqrt{20}} = -1.42$

$df = n - 1 = 19$

Using t table, area in tail is between .05 and .10

Two-tail p-value must be between .10 and .20

Exact p-value corresponding to t = -1.42 is .1718

Cannot reject H_0. There is no significant difference between the mean scores for the first and fourth rounds.

b. $\bar{d}$ = -1.05; First round scores were lower than fourth round scores.

c. $\alpha = .05$ $df = 19$ $t = 1.729$

Margin of error $= t_{.025} \dfrac{s_d}{\sqrt{n}} - 1.729 \dfrac{3.3162}{\sqrt{20}} = 1.28$

Yes, just check to see if the 90% confidence interval includes a difference of zero. If it does, the difference is not statistically significant.

90% Confidence interval: -1.05 ± 1.28 (-2.33, .23)

The interval does include 0, so the difference is not statistically significant.

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27. a. Difference = Price deluxe - Price Standard

H_0: $\mu_d = 10$

H_a: $\mu_d \neq 10$

$\bar{d} = 8.86$ and $s_d = 2.61$

$$t = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}} = \frac{8.86 - 10}{2.61/\sqrt{7}} = -1.16$$

$df = n - 1 = 6$

Using t table, area is between .10 and .20

Two-tail p-value is between .20 and .40

Exact p-value corresponding to t = -1.16 is .2901

Do not reject H_0; we cannot reject the hypothesis that a $10 price differential exists.

b. $$\bar{d} \pm t_{.025} \frac{s_d}{\sqrt{n}}$$

$$8.86 \pm 2.447 \frac{2.61}{\sqrt{7}}$$

6.45 to 11.27

28. a. $\bar{\bar{x}} = (156 + 142 + 134)/3 = 144$

$$\text{SSTR} = \sum_{j=1}^{k} n_j \left(\bar{x}_j - \bar{\bar{x}}\right)^2 = 6(156 - 144)^2 + 6(142 - 144)^2 + 6(134 - 144)^2 = 1{,}488$$

b. MSTR = SSTR /(k - 1) = 1488/2 = 744

c. $s_1^2 = 164.4$ $s_2^2 = 131.2$ $s_3^2 = 110.4$

$$\text{SSE} = \sum_{j=1}^{k} (n_j - 1)s_j^2 = 5(164.4) + 5(131.2) + 5(110.4) = 2030$$

d. MSE = SSE /(n_T - k) = 2030/(12 - 3) = 135.3

e.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Treatments	1488	2	744	5.50	.0162
Error	2030	15	135.3		
Total	3518	17			

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f. F = MSTR /MSE = 744/135.3 = 5.50

Using F table (2 degrees of freedom numerator and 15 denominator), p-value is between .01 and .025

Using Excel, the p-value corresponding to $F = 5.50$ is .0162.

Because p-value $\leq \alpha = .05$, we reject the hypothesis that the means for the three treatments are equal.

29.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Treatments	300	4	75	14.07	.0000
Error	160	30	5.33		
Total	460	34			

30. a. H_0: $u_1 = u_2 = u_3 = u_4 = u_5$

H_a: Not all the population means are equal

b. Using F table (4 degrees of freedom numerator and 30 denominator), p-value is less than .01

Using Excel, the p-value corresponding to $F = 14.07$ is .0000.

Because p-value $\leq \alpha = .05$, we reject H_0

31.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Treatments	150	2	75	4.80	.0233
Error	250	16	15.63		
Total	400	18			

Using F table (2 degrees of freedom numerator and 16 denominator), p-value is between .01 and .025

Using Excel, the p-value corresponding to $F = 4.80$ is .0233.

Because p-value $\leq \alpha = .05$, we reject the null hypothesis that the means of the three treatments are equal.

32.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Treatments	1200	2	600	43.99	.0000
Error	600	44	13.64		
Total	1800	46			

Using F table (2 degrees of freedom numerator and 44 denominator), p-value is less than .01

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Using Excel, the p-value corresponding to $F = 43.99$ is .0000.

Because p-value $\leq \alpha = .05$, we reject the hypothesis that the treatment means are equal.

33.

	A	B	C
Sample Mean	119	107	100
Sample Variance	146.89	96.43	173.78

$$\bar{\bar{x}} = \frac{8(119)+10(107)+10(100)}{28} = 107.93$$

$$\text{SSTR} = \sum_{j=1}^{k} n_j \left(\bar{x}_j - \bar{\bar{x}}\right)^2 = 8(119 - 107.93)^2 + 10(107 - 107.93)^2 + 10(100 - 107.93)^2 = 1617.9$$

MSTR = SSTR /(k - 1) = 1617.9 /2 = 809.95

$$\text{SSE} = \sum_{j=1}^{k} (n_j - 1)s_j^2 = 7(146.86) + 9(96.44) + 9(173.78) = 3{,}460$$

MSE = SSE /(n_T - k) = 3,460 /(28 - 3) = 138.4

F = MSTR /MSE = 809.95 /138.4 = 5.85

Using F table (2 degrees of freedom numerator and 25 denominator), p-value is less than .01

Using Excel, the p-value corresponding to $F = 5.85$ is .0082.
Because p-value $\leq \alpha = .05$, we reject the null hypothesis that the means of the three treatments are equal.

34. a.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Treatments	4560	2	2280	9.87	.0006
Error	6240	27	231.11		
Total	10800	29			

b. Using F table (2 degrees of freedom numerator and 27 denominator), p-value is less than .01

Using Excel, the p-value corresponding to $F = 9.87$ is .0006.

Because p-value $\leq \alpha = .05$, we reject the null hypothesis that the means of the three assembly methods are equal.

35. $\bar{\bar{x}} = (79 + 74 + 66)/3 = 73$

$$\text{SSTR} = \sum_{j=1}^{k} n_j \left(\bar{x}_j - \bar{\bar{x}}\right)^2 = 6(79 - 73)^2 + 6(74 - 73)^2 + 6(66 - 73)^2 = 516$$

MSTR = SSTR /(k - 1) = 516/2 = 258

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$s_1^2 = 34 \quad s_2^2 = 20 \quad s_3^2 = 32$

$$SSE = \sum_{j=1}^{k} (n_j - 1)s_j^2 = 5(34) + 5(20) + 5(32) = 430$$

$MSE = SSE/(n_T - k) = 430/(18 - 3) = 28.67$

$F = MSTR/MSE = 258/28.67 = 9.00$

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Treatments	516	2	258	9.00	.003
Error	430	15	28.67		
Total	946	17			

Using F table (2 degrees of freedom numerator and 15 denominator), p-value is less than .01

Using Excel the p-value corresponding to $F = 9.00$ is .003.

Because p-value $\leq \alpha = .05$, we reject the null hypothesis that the means for the three plants are equal. In other words, analysis of variance supports the conclusion that the population mean examination score at the three NCP plants are not equal.

36.

	50°	60°	70°
Sample Mean	33	29	28
Sample Variance	32	17.5	9.5

$\bar{\bar{x}} = (33 + 29 + 28)/3 = 30$

$$SSTR = \sum_{j=1}^{k} n_j \left(\bar{x}_j - \bar{\bar{x}}\right)^2 = 5(33 - 30)^2 + 5(29 - 30)^2 + 5(28 - 30)^2 = 70$$

$MSTR = SSTR/(k - 1) = 70/2 = 35$

$$SSE = \sum_{j=1}^{k} (n_j - 1)s_j^2 = 4(32) + 4(17.5) + 4(9.5) = 236$$

$MSE = SSE/(n_T - k) = 236/(15 - 3) = 19.67$

$F = MSTR/MSE = 35/19.67 = 1.78$

Using F table (2 degrees of freedom numerator and 12 denominator), p-value is greater than .10

Using Excel the p-value corresponding to $F = 1.78$ is .2104.

Because p-value $> \alpha = .05$, we cannot reject the null hypothesis that the mean yields for the three temperatures are equal.

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37.

	Direct Experience	Indirect Experience	Combination
Sample Mean	17.0	20.4	25.0
Sample Variance	5.01	6.26	4.01

$\bar{\bar{x}} = (17 + 20.4 + 25)/3 = 20.8$

$$\text{SSTR} = \sum_{j=1}^{k} n_j \left(\bar{x}_j - \bar{\bar{x}}\right)^2 = 7(17 - 20.8)^2 + 7(20.4 - 20.8)^2 + 7(25 - 20.8)^2 = 225.68$$

MSTR = SSTR /(k - 1) = 225.68 /2 = 112.84

$$\text{SSE} = \sum_{j=1}^{k} (n_j - 1)s_j^2 = 6(5.01) + 6(6.26) + 6(4.01) = 91.68$$

MSE = SSE /(n_T - k) = 91.68 /(21 - 3) = 5.09

F = MSTR /MSE = 112.84 /5.09 = 22.17

Using F table (2 degrees of freedom numerator and 18 denominator), p-value is less than .01

Using Excel the p-value corresponding to F = 22.17 is .0000.

Because p-value $\leq \alpha$ = .05, we reject the null hypothesis that the means for the three groups are equal.

38.

	Paint 1	Paint 2	Paint 3	Paint 4
Sample Mean	13.3	139	136	144
Sample Variance	47.5	.50	21	54.5

$\bar{\bar{x}} = (133 + 139 + 136 + 144)/3 = 138$

$$\text{SSTR} = \sum_{j=1}^{k} n_j \left(\bar{x}_j - \bar{\bar{x}}\right)^2 = 5(133 - 138)^2 + 5(139 - 138)^2 + 5(136 - 138)^2 + 5(144 - 138)^2 = 330$$

MSTR = SSTR /(k - 1) = 330 /3 = 110

$$\text{SSE} = \sum_{j=1}^{k} (n_j - 1)s_j^2 = 4(47.5) + 4(50) + 4(21) + 4(54.5) = 692$$

MSE = SSE /(n_T - k) = 692 /(20 - 4) = 43.25

F = MSTR /MSE = 110 /43.25 = 2.54

Using F table (3 degrees of freedom numerator and 16 denominator), p-value is between .05 and .10

Using Excel the p-value corresponding to F = 2.54 is .0931.

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Because p-value $> \alpha = .05$, we cannot reject the null hypothesis that the mean drying times for the four paints are equal.

39.

	Italian	Seafood	Steakhouse
Sample Mean	17	19	24
Sample Variance	14.857	13.714	14.000

$\bar{\bar{x}} = (17 + 19 + 24)/3 = 20$

$$\text{SSTR} = \sum_{j=1}^{k} n_j \left(\bar{x}_j - \bar{\bar{x}}\right)^2 = 8(17 - 20)^2 + 8(19 - 20)^2 + 8(24 - 20)^2 = 208$$

MSTR = SSTR /(k - 1) = 208/2 = 104

$$\text{SSE} = \sum_{j=1}^{k} (n_j - 1)s_j^2 = 7(14.857) + 7(13.714) + 7(14.000) = 298$$

MSE = SSE /(n_T - k) = 298 /(24 - 3) = 14.19

F = MSTR /MSE = 104 /14.19 = 7.33

Using the F table (2 degrees of freedom numerator and 21 denominator), the p-value is less than .01.

Using Excel, the p-value corresponding to $F = 7.33$ is .0038.
Because p-value $\leq \alpha = .05$, we reject the null hypothesis that the mean meal prices are the same for the three types of restaurants.

40. a. $\bar{x}_1 - \bar{x}_2 = 5260 - 4848 = 412$

Our point estimate of the cost increase is $412.

b. Margin of error = $z_{.025}\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$

$$1.96\sqrt{\frac{(800)^2}{80} + \frac{(800)^2}{60}} = 1.96(136.63) = 267.79$$

c. 412 ± 267.79 (144.21 to 679.79)

A 95% confidence interval for the size of the increase is $144.21 to $679.79.

41. a. $\bar{x}_1 = \dfrac{\Sigma x_i}{n_1} = \dfrac{6,776,900}{30} = 225,897$ Mean resale price in 2006

$\bar{x}_2 = \dfrac{\Sigma x_i}{n_2} = \dfrac{6,839,735}{40} = 170,993$ Mean resale price in 2009

Difference = 225,897 - 170,993 = 54,904

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Using sample mean prices, the 2009 resale prices are $54,904 less than in 2006.

b. $s_1 = \sqrt{\frac{\Sigma(x_i - \bar{x}_1)^2}{n_1 - 1}} = 55,207$

$s_2 = \sqrt{\frac{\Sigma(x_i - \bar{x}_2)^2}{n_2}} = 44,958$

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2} = \frac{\left(\frac{55207^2}{30} + \frac{44958^2}{40}\right)^2}{\frac{1}{29}\left(\frac{55207^2}{30}\right)^2 + \frac{1}{39}\left(\frac{44958^2}{40}\right)^2} = 54.92$$

Use $df = 54$, $t_{.005} = 2.670$

$$(\bar{x}_1 - \bar{x}_2) \pm t_{.005}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$54904 \pm 2.670\sqrt{\frac{55207^2}{30} + \frac{44958^2}{40}}$$

54904 ± 32931 (21,973 to 87,835)

We are 99% confident that home prices have declined by between $21,973 and $87,835.

c. To answer this question we need to conduct a one-tailed hypothesis test. No value for the level of significance (α) has been given. But, most people would agree that a p-value $\leq .01$ would justify concluding that prices have declined from 2006 to 2009.

$$H_0 : \mu_1 \leq \mu_2$$
$$H_a : \mu_1 > \mu_2$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{54,904}{\sqrt{\frac{55,207^2}{30} + \frac{44,958^2}{40}}} = 4.45$$

For $t = 4.45$ and $df = 54$, we find p-value ≈ 0.00. Thus, we are justified in concluding that existing home prices have declined between 2006 and 2009.

42. a. H_0: $\mu_1 - \mu_2 \leq 0$

H_a: $\mu_1 - \mu_2 > 0$

b. $n_1 = 30$ $n_2 = 30$

$\bar{x}_1 = 16.23$ $\bar{x}_2 = 15.70$

$s_1 = 3.52$ $s_2 = 3.31$

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$$t = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(16.23 - 15.70) - 0}{\sqrt{\frac{(3.52)^2}{30} + \frac{(3.31)^2}{30}}} = .60$$

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2} = \frac{\left(\frac{3.52^2}{30} + \frac{3.31^2}{30}\right)^2}{\frac{1}{29}\left(\frac{3.52^2}{30}\right)^2 + \frac{1}{29}\left(\frac{3.31^2}{30}\right)^2} = 57.8$$

Use $df = 57$

Using t table, p-value is greater than .20

Exact p-value corresponding to $t = .60$ is .2754

p-value > .05, do not reject H_0. Cannot conclude that the mutual funds with a load have a greater mean rate of return.

43. a. $n_1 = 10$ $n_2 = 8$

$\bar{x}_1 = 21.2$ $\bar{x}_2 = 22.8$

$s_1 = 2.70$ $s_2 = 3.55$

$\bar{x}_1 - \bar{x}_2 = 21.2 - 22.8 = -1.6$

Kitchens are less expensive by $1600.

b. $$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2} = \frac{\left(\frac{2.70^2}{10} + \frac{3.55^2}{8}\right)^2}{\frac{1}{9}\left(\frac{2.70^2}{10}\right)^2 + \frac{1}{7}\left(\frac{3.55^2}{8}\right)^2} = 12.9$$

Use $df = 12$, $t_{.05} = 1.782$

$$-1.6 \pm 1.782\sqrt{\frac{2.70^2}{10} + \frac{3.55^2}{8}}$$

-1.6 + 2.7 (-4.3 to 1.1)

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44. a.

January 1	April 30	d_i	$(d_i - \bar{d})$	$(d_i - \bar{d})^2$
10.13	12.21	-2.08	-4.53	20.5209
28.33	25.48	2.85	0.40	0.1600
73.97	66.10	7.87	5.42	29.3764
16.30	19.32	-3.02	-5.47	29.9209
45.27	43.05	2.22	-0.23	0.0529
16.88	15.46	1.42	-1.03	1.0609
2.29	5.98	-3.69	-6.14	37.6996
16.20	12.65	3.55	1.10	1.2100
59.83	52.36	7.47	5.02	25.2004
31.53	33.00	-1.47	-3.92	15.3664
19.44	20.26	-0.82	-3.27	10.6929
17.73	19.34	-1.61	-4.06	16.4836
17.71	13.36	4.35	1.90	3.6100
43.51	36.18	7.33	4.88	23.8144
61.82	49.44	12.38	9.93	98.6049
	Sum	36.75		313.7742

$\bar{d} = \frac{\sum d_i}{n} = \frac{36.75}{15} = 2.45$ The mean price per share declined $2.45 over the four months.

b. $s_d = \sqrt{\frac{\sum (d_i - \bar{d})^2}{n-1}} = \sqrt{\frac{313.7742}{14}} = 4.73$

$df = n - 1 = 14$, $t_{.05} = 1.761$

$\bar{d} \pm t_{.05} \frac{s_d}{\sqrt{n}} = 2.45 \pm 1.761 \frac{4.73}{\sqrt{15}}$

2.45 ± 2.15 ($.30 to $4.60)

We are 90% confident that the population mean price per share has decreased between $.30 and $4.60 over the four month period.

c. Sample mean price per share January 1: $\bar{x} = \frac{\sum x_i}{n} = \frac{460.94}{15} = \$30.73$

Percentage decrease over the 4 months: $\frac{2.45}{30.73}(100) = 8\%$

d. Mean price per share December 31, 2009 = $30.73(.92)(.92)(.92) = $23.93. This is a decline of $30.73 – 23.93 = $6.80 per share for the year.

45.

	x	y	z
Sample Mean	92	97	84
Sample Variance	30	6	35.33

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$\bar{\bar{x}} = (92 + 97 + 44)/3 = 91$

$$SSTR = \sum_{j=1}^{k} n_j \left(\bar{x}_j - \bar{\bar{x}}\right)^2 = 4(92 - 91)^2 + 4(97 - 91)^2 + 4(84 - 91)^2 = 344$$

$MSTR = SSTR/(k - 1) = 344/2 = 172$

$$SSE = \sum_{j=1}^{k} (n_j - 1)s_j^2 = 3(30) + 3(6) + 3(35.33) = 213.99$$

$MSE = SSE/(n_T - k) = 213.99/(12 - 3) = 23.78$

$F = MSTR/MSE = 172/23.78 = 7.23$

Using F table (2 degrees of freedom numerator and 9 denominator), p-value is between .01 and .025

Using Excel, the p-value corresponding to $F = 7.23$ is .0134.

Because p-value $\leq \alpha = .05$, we reject the null hypothesis that the mean absorbency ratings for the three brands are equal.

46.

	Lawyer	Physical Therapist	Cabinet Maker	Systems Analyst
Sample Mean	50.0	63.7	69.1	61.2
Sample Variance	124.22	164.68	105.88	136.62

$$\bar{\bar{x}} = \frac{50.0 + 63.7 + 69.1 + 61.2}{4} = 61$$

$$SSTR = \sum_{j=1}^{k} n_j \left(\bar{x}_j - \bar{\bar{x}}\right)^2 = 10(50.0 - 61)^2 + 10(63.7 - 61)^2 + 10(69.1 - 61)^2 + 10(61.2 - 61)^2 = 1939.4$$

$MSTR = SSTR/(k - 1) = 1939.4/3 = 646.47$

$$SSE = \sum_{j=1}^{k} (n_j - 1)s_j^2 = 9(124.22) + 9(164.68) + 9(105.88) + 9(136.62) = 4{,}782.60$$

$MSE = SSE/(n_T - k) = 4782.6/(40 - 4) = 132.85$

$F = MSTR/MSE = 646.47/132.85 = 4.87$

Using F table (3 degrees of freedom numerator and 36 denominator), p-value is less than .01

Using Excel, the p-value corresponding to $F = 4.87$ is .0061.

Because p-value $\leq \alpha = .05$, we reject the null hypothesis that the mean job satisfaction rating is the same for the four professions.

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47. The Excel Anova Single Factor output is shown below:

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Midcap	10	12.8	1.28	0.0573
Small-cap	10	16.2	1.62	0.144
Hybrid	10	16	1.6	0.5444
Specialty	10	20	2	0.4333

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	2.603	3	0.8677	2.9435	0.0459	2.8663
Within Groups	10.612	36	0.2948			
Total	13.215	39				

Because p-value $\leq \alpha = .05$, we reject the null hypothesis that the mean expense ratios are equal.

48. The Excel Single Factor Tool output is shown below:

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Midwest	16	193.3	12.0813	13.0083
Northeast	16	133.8	8.3625	17.5932
South	25	300.4	12.016	22.2222
West	18	125.8	6.9889	12.4058

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	376.87155	3	125.6238	7.4126	0.0002	2.7336
Within Groups	1203.2533	71	16.9472			
Total	1580.1248	74				

Because the p-value = .0002 is less than $\alpha = .05$, we reject the null hypothesis that the mean rental vacancy rate is the same for each geographic region. The mean vacancy rates were highest (over 12%) in the Midwest and the South.

49.

	Method A	Method B	Method C
Sample Mean	90	84	81
Sample Variance	98.00	168.44	159.78

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$\bar{\bar{x}} = (90 + 84 + 81)/3 = 85$

$$SSTR = \sum_{j=1}^{k} n_j \left(\bar{x}_j - \bar{\bar{x}}\right)^2 = 10(90 - 85)^2 + 10(84 - 85)^2 + 10(81 - 85)^2 = 420$$

$MSTR = SSTR/(k - 1) = 420/2 = 210$

$$SSE = \sum_{j=1}^{k} (n_j - 1)s_j^2 = 9(98.00) + 9(168.44) + 9(159.78) = 3{,}836$$

$MSE = SSE/(n_T - k) = 3{,}836/(30 - 3) = 142.07$

$F = MSTR/MSE = 210/142.07 = 1.48$

Using F table (2 degrees of freedom numerator and 27 denominator), p-value is greater than .10

Using Excel, the p-value corresponding to $F = 1.48$ is .2455.

Because p-value $> \alpha = .05$, we cannot reject the null hypothesis that the means are equal.

50.

	Nonbrowser	Light Browser	Heavy Browser
Sample Mean	4.25	5.25	5.75
Sample Variance	1.07	1.07	1.36

$\bar{\bar{x}} = (4.25 + 5.25 + 5.75)/3 = 5.08$

$$SSTR = \sum_{j=1}^{k} n_j \left(\bar{x}_j - \bar{\bar{x}}\right)^2 = 8(4.25 - 5.08)^2 + 8(5.25 - 5.08)^2 + 8(5.75 - 5.08)^2 = 9.33$$

$MSTR = SSTR/(k - 1) = 9.33/2 = 4.67$

$$SSE = \sum_{j=1}^{k} (n_j - 1)s_j^2 = 7(1.07) + 7(1.07) + 7(1.36) = 24.5$$

$MSE = SSE/(n_T - k) = 24.5/(24 - 3) = 1.17$

$F = MSTR/MSE = 4.67/1.17 = 3.99$

Using F table (2 degrees of freedom numerator and 21 denominator), p-value is between .025 and .05

Using Excel, the p-value corresponding to $F = 3.99$ is .0340.

Because p-value $\leq \alpha = .05$, we reject the null hypothesis that the mean comfort scores are the same for the three groups.

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Chapter 11
Comparisons Involving Proportions and a Test of Independence

Learning Objectives

1. Be able to develop interval estimates and conduct hypothesis tests about the difference between the proportions of two populations.

2. Know the properties of the sampling distribution of the difference between two proportions $(\overline{p}_1 - \overline{p}_2)$.

3. Be able to conduct a goodness of fit test when the population is hypothesized to have a multinomial probability distribution.

4. For a test of independence, be able to set up a contingency table, determine the observed and expected frequencies, and determine if the two variables are independent.

5. Understand the role of the chi-square distribution in conducting tests of goodness of fit and independence.

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Solutions:

1. a. $\bar{p}_1 - \bar{p}_2 = .48 - .36 = .12$

   b. $\bar{p}_1 - \bar{p}_2 \pm z_{.05}\sqrt{\dfrac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \dfrac{\bar{p}_2(1-\bar{p}_2)}{n_2}}$

   $.12 \pm 1.645\sqrt{\dfrac{.48(1-.48)}{400} + \dfrac{.36(1-.36)}{300}}$

   $.12 \pm .0614$ (.0586 to .1814)

   c. $.12 \pm 1.96\sqrt{\dfrac{.48(1-.48)}{400} + \dfrac{.36(1-.36)}{300}}$

   $.12 \pm .0731$ (.0469 to .1931)

2. a. $\bar{p} = \dfrac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2} = \dfrac{200(.22) + 300(.16)}{200 + 300} = .1840$

   $$z = \frac{\bar{p}_1 - \bar{p}_2}{\sqrt{\bar{p}(1-\bar{p})\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} = \frac{.22 - .16}{\sqrt{.1840(1-.1840)\left(\dfrac{1}{200} + \dfrac{1}{300}\right)}} = 1.70$$

   p - value = 1.0000 - .9554 = .0446

   b. p-value $\leq$ .05; reject H_0.

3. a. Professional Golfers: $\bar{p}_1 = 688/1075 = .64$

   Amateur Golfers: $\bar{p}_2 = 696/1200 = .58$

   Professional golfers have the better putting accuracy.

   b. $\bar{p}_1 - \bar{p}_2 = .64 - .58 = .06$

   Professional golfers make 6% more 6-foot putts than the very best amateur golfers.

   c. $\bar{p}_1 - \bar{p}_2 \pm z_{.025}\sqrt{\dfrac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \dfrac{\bar{p}_2(1-\bar{p}_2)}{n_2}}$

   $.64 - .58 \pm 1.96\sqrt{\dfrac{.64(1-.64)}{1075} + \dfrac{.58(1-.58)}{1200}}$

   $.06 \pm .04$ (.02 to .10)

   The confidence interval shows that professional golfers make from 2% to 10% more 6-foot putts than the best amateur golfers.

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4. a. $H_0: p_w \le p_m$

$H_a: p_w > p_m$

b. $\bar{p}_w = 300/811 = .3699$ 37% of women would ask directions

c. $\bar{p}_m = 255/750 = .3400$ 34% of men would ask directions

d. $\bar{p} = \dfrac{n_1\bar{p}_w + n_2\bar{p}_m}{n_1 + n_2} = \dfrac{300+255}{811+750} = .3555$

$$z = \frac{\bar{p}_1 - \bar{p}_2}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{.3699-.3400}{\sqrt{.3555(1-.3555)\left(\frac{1}{811}+\frac{1}{750}\right)}} = 1.23$$

Upper tail p-value is the area to the right of the test statistic

Using normal table with $z = 1.23$: p-value = 1 - .8907 = .1093

p-value $> \alpha$; do not reject H_0

We cannot conclude that women are more likely to ask directions.

5. a. $\bar{p}_1 = 115/250 = .46$ Republicans

$\bar{p}_2 = 98/350 = .28$ Democrats

b. $\bar{p}_1 - \bar{p}_2 = .46 - .28 = .18$

Republicans have a .18, 18%, higher participation rate than Democrats.

c. $$z_{.025}\sqrt{\frac{\bar{p}_1(1-\bar{p}_2)}{n_1}+\frac{\bar{p}_2(1-\bar{p}_2)}{n_2}}$$

$$1.96\sqrt{\frac{.46(1-.46)}{250}+\frac{.28(1-.28)}{350}} = .0777$$

d. Yes, $.18 \pm .0777$ (.1023 to .2577)

Republicans have a 10% to 26% higher participation rate in online surveys than Democrats. Biased survey results of online political surveys are very likely.

6. Let p_1 = the population proportion of delayed departures at Chicago O'Hare

p_2 = the population proportion of delayed departures at Atlanta Hartsfield-Jackson

a. H_0: $p_1 - p_2 = 0$

H_a: $p_1 - p_2 \neq 0$

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b. $\bar{p}_1 = 252/900 = .28$

c. $\bar{p}_2 = 312/1200 = .26$

d. $\bar{p} = \dfrac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2} = \dfrac{252+312}{900+1200} = .2686$

$$z = \frac{\bar{p}_1 - \bar{p}_2}{\sqrt{\bar{p}(1-\bar{p})\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} = \frac{.28 - .26}{\sqrt{.2686(1-.2686)\left(\dfrac{1}{900} + \dfrac{1}{1200}\right)}} = 1.02$$

p-value = 2(1 - .8461) = .3078

Do Not Reject H_0. We cannot conclude that there is a difference between the proportion of delayed departures at the two airports.

7. a. $\bar{p}_1 = 192/300 = .64$

b. $\bar{p}_2 = 117/260 = .45$

c. $\bar{p}_1 - \bar{p}_2 = .64 - .45 = .19$

$$.19 \pm z_{.025}\sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \frac{\bar{p}_2(1-\bar{p}_2)}{n_2}}$$

$$.19 \pm 1.96\sqrt{\frac{.64(1-.64)}{300} + \frac{.45(1-.45)}{260}}$$

$.19 \pm .0813$ (.1087 to .2713)

8. a. $\bar{p}_1$ = proportion of under 30 liking the ad a lot

$\bar{p}_2$ = proportion of 30 to 49 liking the ad a lot

H_0: $p_1 - p_2 = 0$

H_a: $p_1 - p_2 \neq 0$

b. $\bar{p}_1 = 49/100 = .49$

$\bar{p}_2 = 54/150 = .36$

$\bar{p}_1 - \bar{p}_2 = .49 - .36 = .13$

c. $\bar{p} = \dfrac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2} = \dfrac{100(.49)+150(.36)}{100+150} = .412$

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$$z = \frac{\bar{p}_1 - \bar{p}_2}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{.49 - .36}{\sqrt{.412(1-.412)\left(\frac{1}{100} + \frac{1}{150}\right)}} = 2.05$$

p-value = 2(1.0000 - .9798) = .0404

p-value $\leq .05$, reject H_0. There is a difference between the response to the ad by the younger under 30 and the older 30 to 49 age groups.

d. There is a statistically significant difference between the population proportions for the two age groups. The stronger appeal is with the younger, under 30, age group. Miller Lite is most likely pleased and encouraged by the results of the poll. "The Miller Lite Girls" ad ranked among the top three Super Bowl ads in advertising effectiveness. In addition, 49% of the younger, under 30, group liked the ad a lot. While a response of 36% for the older age group was not bad, Miller Lite probably liked, and probably expected, the higher rating among the younger audience. Since a younger audience contains the newer beer drinkers, appealing to the younger audience could bring new customers to the Miller Lite product. The older age group may be less likely to change from their established personal favorite beer because of the commercial.

9. a. H_0: $p_1 - p_2 = 0$

H_a: $p_1 - p_2 \neq 0$

$\bar{p}_1 = 63/150 = .42$

$\bar{p}_2 = 60/200 = .30$

$$\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2} = \frac{63 + 60}{150 + 200} = .3514$$

$$z = \frac{\bar{p}_1 - \bar{p}_2}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{.42 - .30}{\sqrt{.3514(1-.3514)\left(\frac{1}{150} + \frac{1}{200}\right)}} = 2.33$$

p-value = 2(1.0000 - .9901) = .0198

p-value $\leq .05$, reject H_0. There is a difference between the recall rates for the two commercials.

b. $$\bar{p}_1 - \bar{p}_2 \pm z_{.025}\sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \frac{\bar{p}_2(1-\bar{p}_2)}{n_2}}$$

$$.42 - .30 \pm 1.96\sqrt{\frac{.42(1-.42)}{150} + \frac{.30(1-.30)}{200}}$$

$.12 \pm .1014$ (.0186 to .2214)

Commercial A has the better recall rate.

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10. a. H_0: $p_1 - p_2 = 0$

H_a: $p_1 - p_2 \neq 0$

b. $\bar{p}_1 = 141/523 = .2696$ (27%)

$\bar{p}_2 = 81/477 = .1698$ (17%)

c. $\bar{p} = \dfrac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2} = \dfrac{141+81}{523+477} = .2220$

$$z = \frac{\bar{p}_1 - \bar{p}_2}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{.2696-.1698}{\sqrt{.222(1-.222)\left(\frac{1}{523}+\frac{1}{477}\right)}} = 3.79$$

p-value ≈ 0

Reject H_0. There is a significant difference in the population proportions. A higher flying rate in 2003 is observed.

d. It may be that the general population is more acceptable to flying on vacation in 2003. Also, frequent flyer awards and special discount air fares in 2003 may have made 2003 flying more economical.

Note: In 1993, a round trip Newark to San Francisco was $388. In 2003, a special fare for the same trip was $238.

11. a. Expected frequencies: $e_1 = 200\ (.40) = 80$, $e_2 = 200\ (.40) = 80$

$e_3 = 200\ (.20) = 40$

Actual frequencies: $f_1 = 60, f_2 = 120, f_3 = 20$

$$\chi^2 = \frac{(60-80)^2}{80} + \frac{(120-80)^2}{80} + \frac{(20-40)^2}{40}$$
$$= \frac{400}{80} + \frac{1600}{80} + \frac{400}{40}$$
$$= 5 + 20 + 10$$
$$= 35$$

$k - 1 = 2$ degrees of freedom

Using the χ^2 table with $df = 2$, $\chi^2 = 35$ shows the p-value is less than .005.

Using Excel, the p-value corresponding to $\chi^2 = 35$ is approximately 0.

p-value $\leq .01$, reject H_0

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b. $\chi^2_{.01} = 9.210$

Reject H_0 if $\chi^2 \geq 9.210$

$\chi^2 = 35$, reject H_0

12. Expected frequencies: $e_1 = 300\ (.25) = 75$, $e_2 = 300\ (.25) = 75$

$e_3 = 300\ (.25) = 75$, $e_4 = 300\ (.25) = 75$

Actual frequencies: $f_1 = 85, f_2 = 95, f_3 = 50, f_4 = 70$

$$\chi^2 = \frac{(85-75)^2}{75} + \frac{(95-75)^2}{75} + \frac{(50-75)^2}{75} + \frac{(70-75)^2}{75}$$
$$= \frac{100}{75} + \frac{400}{75} + \frac{625}{75} + \frac{25}{75}$$
$$= \frac{1150}{75}$$
$$= 15.33$$

k - 1 = 3 degrees of freedom

Using the χ^2 table with $df = 3$, $\chi^2 = 15.33$ shows the p-value is less than .005.

Using Excel, the p-value corresponding to $\chi^2 = 15.33$ is .0016.

p-value $\leq$ .05, reject H_0

The population proportions are not the same.

13. $H_0 = p_{ABC} = .29, p_{CBS} = .28, p_{NBC} = .25, p_{IND} = .18$

H_a = The proportions are not $p_{ABC} = .29, p_{CBS} = .28, p_{NBC} = .25, p_{IND} = .18$

Expected frequencies: $300\ (.29) = 87$, $300\ (.28) = 84$

$300\ (.25) = 75$, $300\ (.18) = 54$

$e_1 = 87, e_2 = 84, e_3 = 75, e_4 = 54$

Actual frequencies: $f_1 = 95, f_2 = 70, f_3 = 89, f_4 = 46$

$$\chi^2 = \frac{(95-87)^2}{87} + \frac{(70-84)^2}{84} + \frac{(89-75)^2}{75} + \frac{(46-54)^2}{54}$$
$$= 6.87$$

k - 1 = 3 degrees of freedom

Using the χ^2 table with $df = 3$, $\chi^2 = 6.87$ shows the p-value is between .05 and .10.

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Using Excel, the p-value corresponding to $\chi^2 = 6.87$ is .0762.

p-value > .05, do not reject H_0. There has not been a significant change in the viewing audience proportions.

14.

Category	Hypothesized Proportion	Observed Frequency (f_i)	Expected Frequency (e_i)	$(f_i - e_i)^2 / e_i$
Brown	0.30	177	151.8	4.18
Yellow	0.20	135	101.2	11.29
Red	0.20	79	101.2	4.87
Orange	0.10	41	50.6	1.82
Green	0.10	36	50.6	4.21
Blue	0.10	38	50.6	3.14
Total		506		29.51

k - 1 = 5 degrees of freedom

Using the χ^2 table with $df = 5$, $\chi^2 = 29.51$ shows the p-value is less than .005.

Using Excel, the p-value corresponding to $\chi^2 = 29.51$ is approximately 0.

p-value < .05, reject H_0. The percentages reported by the company have changed.

15.

Outlet	Hypothesized Proportion	Observed Frequency (f_i)	Expected Frequency (e_i)	$(f_i - e_i)^2 / e_i$
Wal-Mart	.24	42	33.6	2.10
Dept Stores	.11	20	15.4	1.37
J.C. Penney	.08	8	11.2	0.91
Kohl's	.08	10	11.2	0.13
Mail Order	.12	21	16.8	1.05
Other	.37	39	51.8	3.16
Total		140	140	8.73

Degrees of freedom = 5

Using the χ^2 table with $df = 5$, $\chi^2 = 8.73$ shows the p-value is greater than .10.

Using Excel, the p-value corresponding to $\chi^2 = 8.73$ is .1203.

p-value > .05, cannot reject H_0. We cannot conclude that women shoppers in Atlanta differ from the outlet preferences expressed in the U.S. Shopper Database.

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16. a.

Method	Hypothesized Proportion	Observed Frequency (f_i)	Expected Frequency (e_i)	$(f_i - e_i)^2 / e_i$
Credit Card	.22	46	48.4	.12
Debit Card	.21	67	46.2	9.36
Personal Check	.18	33	39.6	1.10
Cash	.39	74	85.8	1.62
Total		220	220	12.21

Degrees of freedom = 3

Using the χ^2 table with $df = 3$, $\chi^2 = 12.21$ shows the p-value is between .005 and .01.

Using Excel, the p-value corresponding to $\chi^2 = 12.21$ is .0067.

p-value $\leq$ .01, reject H_0. Conclude that the percentages for the methods of in-store payments have changed over the four year period.

b.

		2003	1999	% Change
Credit Card	46/220 =	21%	22%	-1%
Debit Card	67/220 =	30%	21%	+9%
Personal Check	33/220 =	15%	18%	-3%
Cash	74/220 =	34%	39%	-5%

The primary change is that the debit card usage shows the biggest increase in method of payment (up 9%). Cash and personal check have seen the biggest decline in usage, 5% and 3% respectively.

c. 21% + 30% = 51%. Over half of in-store purchases are made using plastic.

17. Expected frequencies: 20% each $n = 60$

$e_1 = 12, e_2 = 12, e_3 = 12, e_4 = 12, e_5 = 12$

Actual frequencies: $f_1 = 5, f_2 = 8, f_3 = 15, f_4 = 20, f_5 = 12$

$$\chi^2 = \frac{(5-12)^2}{12} + \frac{(8-12)^2}{12} + \frac{(15-12)^2}{12} + \frac{(20-12)^2}{12} + \frac{(12-12)^2}{12}$$
$$= 11.50$$

k -1 = 4 degrees of freedom

Using the χ^2 table with $df = 4$, $\chi^2 = 11.50$ shows the p-value is between .01 and .025.

Using Excel, the p-value corresponding to $\chi^2 = 11.50$ is .0215.

p-value $\leq$ .05; reject H_0. Yes, the largest companies differ in performance from the 1000 companies. In general, the largest companies did not do as well as others. 15 of 60 companies (25%) are in the middle group and 20 of 60 companies (33%) are in the next lower group. These both are greater than the 20% expected. Relative few large companies are in the top A and B categories.

Note that this result is for the year 2002. This should not be generalized to other years without additional data.

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18. H_0: $p_1 = .03, p_2 = .28, p_3 = .45, p_4 = .24$

Rating	Observed	Expected	$(f_i - e_i)^2 / e_i$
Excellent	24	.03(400) = 12	12.00
Good	124	.28(400) = 112	1.29
Fair	172	.45(400) = 180	.36
Poor	80	.24(400) = 96	2.67
	400	400	$\chi^2 = 16.31$

Degrees of freedom = k - 1 = 3

Using the χ^2 table with $df = 3$, $\chi^2 = 16.31$ shows the p-value is less than .005.

Using Excel, the p-value corresponding to $\chi^2 = 16.31$ is .0010.

p-value $\leq$ .01, reject H_0. Conclude that the ratings differ. A comparison of observed and expected frequencies show telephone service is slightly better with more excellent and good ratings.

19. H_0 = The column variable is independent of the row variable

H_a = The column variable is not independent of the row variable

Expected Frequencies:

	A	B	C
P	28.5	39.9	45.6
Q	21.5	30.1	34.4

$$\chi^2 = \frac{(20-28.5)^2}{28.5} + \frac{(44-39.9)^2}{39.9} + \frac{(50-45.6)^2}{45.6} + \frac{(30-21.5)^2}{21.5} + \frac{(26-30.1)^2}{30.1} + \frac{(30-34.4)^2}{34.4}$$
$$= 7.86$$

Degrees of freedom = (2-1)(3-1) = 2

Using the χ^2 table with $df = 2$, $\chi^2 = 7.86$ shows the p-value is between .01 and .025.

Using Excel, the p-value corresponding to $\chi^2 = 7.86$ is .0196.

p-value $\leq$ .05, reject H_0. Conclude that the column variable is not independent of the row variable.

20. H_0 = The column variable is independent of the row variable

H_a = The column variable is not independent of the row variable

Expected Frequencies:

	A	B	C
P	17.5000	30.6250	21.8750
Q	28.7500	50.3125	35.9375
R	13.7500	24.0625	17.1875

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$$\chi^2 = \frac{(20-17.5000)^2}{17.5000} + \frac{(30-30.6250)^2}{30.6250} + \cdots + \frac{(30-17.1875)^2}{17.1875}$$
$$= 19.77$$

Degrees of freedom = (3-1)(3-1) = 4

Using the χ^2 table with $df = 4$, $\chi^2 = 19.77$ shows the p-value is less than .005.

Using Excel, the p-value corresponding to $\chi^2 = 19.77$ is .0006.

p-value $\leq$ .05, reject H_0. Conclude that the column variable is not independent of the row variable.

21. H_0 : Type of ticket purchased is independent of the type of flight

H_a: Type of ticket purchased is not independent of the type of flight.

Expected Frequencies:

e_{11} = 35.59 e_{12} = 15.41
e_{21} = 150.73 e_{22} = 65.27
e_{31} = 455.68 e_{32} = 197.32

Ticket	Flight	Observed Frequency (f_i)	Expected Frequency (e_i)	$(f_i - e_i)^2 / e_i$
First	Domestic	29	35.59	1.22
First	International	22	15.41	2.82
Business	Domestic	95	150.73	20.61
Business	International	121	65.27	47.59
Full Fare	Domestic	518	455.68	8.52
Full Fare	International	135	197.32	19.68
Total		920		100.43

Degrees of freedom = (3-1)(2-1) = 2

Using the χ^2 table with $df = 2$, $\chi^2 = 100.43$ shows the p-value is less than .005.

Using Excel, the p-value corresponding to $\chi^2 = 100.43$ is .0000.

p-value $\leq$ .05, reject H_0. Conclude that the type of ticket purchased is not independent of the type of flight.

22. H_0 : Method of payment is independent of age group
H_a : Method of payment is not independent of age group

Observed Frequency (f_{ij})

Payment	18-24	25-34	35-44	45-Over	Total
Plastic	21	27	27	36	111
Cash/Chk	21	36	42	90	189
Total	42	63	69	126	300

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Expected Frequency (e_{ij})

Payment	18-24	25-34	35-44	45-Over	Total
Plastic	15.54	23.31	25.33	46.62	111
Cash/Chk	26.46	39. 69	43.47	79.38	189
Total	42	63	69	126	300

Chi Square $(f_{ij} - e_{ij})^2 / e_{ij}$

Payment	18-24	25-34	35-44	45-Over	Total
Correct	1.92	.58	.08	2.42	5.01
Incorrect	1.13	.34	.05	1.42	2.94
					$\chi^2 = 7.95$

Degrees of freedom = (2-1)(4-1) = 3

Using the χ^2 table with $df = 3$, $\chi^2 = 7.95$ shows the p-value is between.025 and .05.

Using Excel, the p-value corresponding to $\chi^2 = 7.95$ is .0471.

p-value $\leq$ .05, reject H_0. Conclude method of payment is not independent of age group.

b. The estimated probability of using plastic by age group:

Age Group	Probability of Using Plastic
18 to 24	21/42 = .5000
25 to 34	27/63 = .4286
35 to 44	27/69 = .3913
45 and over	36/126 = .2857

The probability of using plastic to make purchases declines by age group. The young consumers, age 18 to 24, have the highest probability of using plastic. This is the only group with a .50 probability of using plastic to make a purchase.

c. Companies such as Visa, MasterCard and Discovery want their cards in the hands of consumers with a high probability of using plastic to make a purchase. Thus, while these companies will want to target all age groups, they should definitely consider specific strategies targeted as getting cards into the hands of the higher use 18 to 24 year old consumers.

23. a. Observed Frequencies

	Health Insurance		
Size of Company	Yes	No	Total
Small	36	14	50
Medium	65	10	75
Large	88	12	100
Total	189	36	225

Expected Frequencies

	Health Insurance		
Size of Company	Yes	No	Total
Small	42	8	50
Medium	63	12	75
Large	84	16	100
Total	189	36	225

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Chi Square

Size of Company	Health Insurance Yes	No	Total
Small	.86	4.50	5.36
Medium	.06	.33	.39
Large	.19	1.00	1.19
			$\chi^2 = 6.94$

Degrees of freedom = (3-1)(2-1) = 2

Using the χ^2 table with $df = 2, \chi^2 = 6.94$ shows the p-value is between .025 and .05.

Using Excel, the p-value corresponding to $\chi^2 = 6.94$ is .0311.

p-value $\leq$ .05, reject H_0. Health insurance coverage is not independent of the size of the company.

b. Percentage of no coverage by company size:

Small 14/50 → 28%
Medium 10/75 → 13%
Large 12/100 → 12%

Small companies have slightly more than twice the percentage of no coverage for medium and large companies.

24. a. Observed Frequency (f_{ij})

Purchase Again	Automobile Chevrolet Impala	Ford Taurus	Honda Accord	Toyota Camry	Total
Yes	49	44	60	46	199
No	37	27	18	19	101
	86	71	78	65	300

Expected Frequency (e_{ij})

Purchase Again	Automobile Chevrolet Impala	Ford Taurus	Honda Accord	Toyota Camry	Total
Yes	57.05	47.10	51.74	43.12	199
No	28.95	23.90	26.26	21.88	101
	86	71	78	65	300

Chi Square $(f_{ij} - e_{ij})^2 / e_{ij}$

Purchase Again	Automobile Chevrolet Impala	Ford Taurus	Honda Accord	Toyota Camry	Total
Yes	1.14	0.20	1.32	0.19	2.85
No	2.24	0.40	2.60	0.38	5.62
					$\chi^2 = 8.47$

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Degrees of freedom = (2 - 1)(4 - 1) = 3

Using the χ^2 table with $df = 3$, $\chi^2 = 8.47$ shows the p-value is between .025 and .05.

Using Excel, the p-value corresponding to $\chi^2 = 8.47$ is .0372.

p-value $\leq$ .05, reject H_0. The owner's intent to purchase the model again is not independent of the automobile.

b. Column Percentages:

Chevrolet Impala	49/86(100) = 57
Ford Taurus	44/71(100) = 62
Honda Accord	60/78(100) = 77
Toyota Camry	46/65(100) = 71

Note: *Consumer Reports* does not report fractional or decimal value overall satisfaction scores.

Ranked in order of overall satisfaction:

Honda Accord, Toyota Camry, Ford Taurus, and Chevrolet Impala

c. The United States models (Impala and Taurus) have satisfaction scores less than the overall satisfaction score for the class, while the Japanese models have satisfaction scores greater than the overall satisfaction score for the class. The satisfaction score is a measure of the likelihood the owner will purchase the model again. There is evidence of greater brand loyalty among the Japanese models. The market share for the United States models may well decline in the future due to the lower owner satisfaction.

25. H_0 : Flying during the snowstorm is independent airline

H_a : Flying during the snowstorm is not independent airline

Observed Frequency (f_{ij})

Flight	American	Continental	Delta	United	Total
Yes	48	69	68	25	210
No	52	41	62	35	190
Total	100	110	130	60	400

Expected Frequency (e_{ij})

Flight	American	Continental	Delta	United	Total
Yes	52.50	57.75	68.25	31.50	210
No	47.50	52.25	61.75	28.50	190
Total	100	110	130	60	400

Chi Square $(f_{ij} - e_{ij})^2 / e_{ij}$

Flight	American	Continental	Delta	United	Total
Yes	.39	2.19	.00	2.34	3.92
No	.43	2.42	.00	1.48	4.33
					χ^2 = 8.25

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Degrees of freedom = (2-1)(4-1) = 3

Using the χ^2 table with $df = 3$, $\chi^2 = 8.25$ shows the p-value is between.025 and .05.

Using Excel, the p-value corresponding to $\chi^2 = 8.25$ is .0411.

p-value $\leq$ .05, reject H_0. The percentage of scheduled flights flown during the snowstorm is not independent of the airline. During this particular storm, the sample data show the following percent of scheduled flights flown: American (48%), Continental (62.7%), Delta (52.3%) and United (41.7%).

Which airline you would choose to fly during similar snowstorm conditions can have different answers for different people. Taking the position that we agree that airlines operate within set safety parameters and fly only if it is safe, we prefer an airline that does the best job of keeping its flights operational during a snowstorm. In this case, Continental and then Delta would be preferred. A very conservative passenger might prefer otherwise, perhaps favoring an airline that flies less and keeps more of its planes on the ground during a snowstorm.

26. a. The sample size is very large: 6448

b. Observed Frequency (f_{ij})

	Country						
Response	G.B.	France	Italy	Spain	Ger.	U.S.	Total
Strongly favor	141	161	298	133	128	204	1065
Favor	348	366	309	222	272	326	1843
Oppose	381	334	219	311	322	316	1883
Strongly Oppose	217	215	219	443	389	174	1657
Total	1087	1076	1045	1109	1111	1020	6448

Expected Frequency (e_{ij})

	Country						
Response	G.B.	France	Italy	Spain	Ger.	U.S.	Total
Strongly favor	180	178	173	183	183	168	1065
Favor	311	307	299	317	318	291	1843
Oppose	317	315	305	324	324	298	1883
Strongly Oppose	279	276	268	285	286	263	1657
Total	1087	1076	1045	1109	1111	1020	6448

$$\chi^2 = \sum_i \sum_j \frac{(f_{ij} - e_{ij})^2}{e_{ij}} = 425.4$$

Degrees of freedom = (4-1)(6-1) = 15

The p-value is approximately 0.

p-value $\leq$ .05, reject H_0. The attitude toward building new nuclear power plants is not independent of the country.

c. We can compute the column percentages from the observed frequencies table to help answer this question.

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Response	Country G.B.	France	Italy	Spain	Ger.	U.S.
Strongly favor	13.0	15.0	28.5	12.0	11.5	20.0
Favor	32.0	34.0	29.5	20.0	24.5	32.0
Oppose	35.0	31.0	21.0	28.0	29.0	31.0
Strongly Oppose	20.0	20.0	21.0	40.0	35.0	17.0
Total	100	100	100	100	100	100

Adding together the percentages of respondents who "Strongly favor" and those who "Favor", we find the following: Great Britain 45%, France 49%, Italy 58%, Spain 32%, Germany 36% and United States 52%.

Italy shows the most support for nuclear power plants with 58% in favor. Spain shows the least support with only 32% in favor. Only Italy and the United States show more than 50% of the respondents in favor of building new nuclear power plants.

27. a. Observed Frequencies

Age	Hours of Sleep Less than 6	6 to 6.9	7 to 7.9	8 or more	Total
49 or younger	38	60	77	65	240
50 or older	36	57	75	92	260
Total	74	117	152	157	500

Expected Frequencies

Age	Hours of Sleep Less than 6	6 to 6.9	7 to 7.9	8 or more	Total
49 or younger	36	56	73	75	240
50 or older	38	61	79	82	260
Total	74	117	152	157	500

Chi Square

Age	Hours of Sleep Less than 6	6 to 6.9	7 to 7.9	8 or more	Total
49 or younger	.17	.26	.22	1.42	2.08
50 or older	.16	.24	.21	1.31	1.92
					$\chi^2 = 4.01$

Degrees of freedom = (2-1)(4-1) = 3

Using the χ^2 table with $df = 3$, $\chi^2 = 4.01$ shows the p-value is greater than .10.

Using Excel, the p-value corresponding to $\chi^2 = 4.01$ is .2604.

p-value > .05, do not reject H_0. Cannot reject the assumption that age and hours of sleep are independent.

b. Since age does not appear to have an effect on sleep on weeknights, use the overall percentages.

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Less than 6 74/500 → 14.8%
6 to 6.9 117/500 → 23.4%
7 to 7.9 152/500 → 30.4%
8 or more 157/500 → 31.4%

28. Observed Frequency (f_{ij})

Work	Anchorage	Atlanta	Minneapolis	Total
Both	57	70	63	190
Only One	33	50	27	110
Total	90	120	90	300

Expected Frequency (e_{ij})

Work	Anchorage	Atlanta	Minneapolis	Total
Both	57	76	57	190
Only One	33	44	33	110
Total	90	120	90	300

Chi Square $(f_{ij} - e_{ij})^2 / e_{ij}$

Work	Anchorage	Atlanta	Minneapolis	Total
Both	.00	.47	.63	1.11
Only One	.00	.82	1.09	1.91
Total	265	160	175	3.01

Degrees of freedom = (3-1)(2-1) = 2

Using the χ^2 table with $df = 2$, $\chi^2 = 3.01$ shows the p-value is greater than .10.

Using Excel, the p-value corresponding to $\chi^2 = 3.01$ is .2220.

p-value > .05, do not reject H_0. Married couples with both the husband and wife working is independent of location. The overall percentage of married couples with both husband and wife working is 190/300 = 63.3%

29.. a. p_1 = population proportion for men

p_2 = population proportion for women

H_0: $p_1 - p_2 = 0$

H_a: $p_1 - p_2 \neq 0$

b. $\bar{p}_1 = 248/800 = .31$

$\bar{p}_2 = 156/600 = .26$

c. $$\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2} = \frac{800(.31) + 600(.26)}{800 + 600} = .2886$$

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$$z = \frac{\bar{p}_1 - \bar{p}_2}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{(.31-.26)}{\sqrt{.2886(1-.2886)\left(\frac{1}{800}+\frac{1}{600}\right)}} = 2.04$$

p-value = 2(1.0000 - .9793) = .0414

p-value $\leq$ .05, reject H_0. Conclude the population proportions are not equal. The proportion is higher for men.

d. $\bar{p}_1 - \bar{p}_2 \pm z_{.025}\sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1}+\frac{\bar{p}_2(1-\bar{p}_2)}{n_2}}$

$(.31-.26) \pm 1.96\sqrt{\frac{.31(1-.31)}{800}+\frac{.26(1-.26)}{600}}$

$.05 \pm .0475$

Margin of Error = .0475

95% Confidence Interval (.0025 to .0975)

30. a. $\bar{p}_1 = 76/400 = .19$

$\bar{p}_2 = 90/900 = .10$

$\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1+n_2} = \frac{76+90}{400+900} = .1277$

$$z = \frac{\bar{p}_1 - \bar{p}_2}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{.19-.10}{\sqrt{.1277(1-.1277)\left(\frac{1}{400}+\frac{1}{900}\right)}} = 4.49$$

p-value ≈ 0

Reject H_0; there is a difference between claim rates.

b. $\bar{p}_1 - \bar{p}_2 \pm z_{.025}\sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1}+\frac{\bar{p}_2(1-\bar{p}_2)}{n_2}}$

$.19-.10 \pm 1.96\sqrt{\frac{.19(1-.19)}{400}+\frac{.10(1-.10)}{900}}$

$.09 \pm .0432$ (.0468 to .1332)

Claim rates are higher for single males.

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31. $\bar{p}_1 = 9/142 = .0634$

$\bar{p}_2 = 5/268 = .0187$

$$\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2} = \frac{9+5}{142+268} = .0341$$

$$z = \frac{\bar{p}_1 - \bar{p}_2}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{.0634 - .0187}{\sqrt{.0341(1-.0341)\left(\frac{1}{142} + \frac{1}{268}\right)}} = 2.37$$

p-value = 2(1.0000 - .9911) = .0178

p-value $\leq$ .02, reject H_0. There is a significant difference in drug resistance between the two states. New Jersey has the higher drug resistance rate.

32. a. March, 2007: $\bar{p}_1 = 70/200 = .35$

March, 2008: $\bar{p}_2 = 70/150 = .47$

b. $\bar{p}_2 - \bar{p}_1 = .47 - .35 = .12$

$$s_{\bar{p}_1 - \bar{p}_2} = \sqrt{\frac{.35(1-.35)}{200} + \frac{.47(1-.47)}{150}} = .0529$$

Confidence interval: $.12 \pm 1.96(.0529)$ or $.12 \pm .1037$ (.0163 to .2237)

c. Since the confidence interval in part (b) does not include 0, I would conclude that occupancy rates are higher in the first week of March, 2008 than in the first week of March, 2007. On the basis of this I would expect occupancy rates to be higher for March, 2008 than for March, 2007.

33. $\bar{p}_1 = .276$ Most recent week
$\bar{p}_2 = .487$ One Week Ago
$\bar{p}_3 = .397$ One Month Ago

a. Point estimate = $\bar{p}_1 - \bar{p}_2 = .276 - .487 = -.211$

Margin of error: $z_{.025}\sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \frac{\bar{p}_2(1-\bar{p}_2)}{n_2}} = 1.96\sqrt{\frac{.276(1-.276)}{240} + \frac{.487(1-.487)}{240}} = .085$

95% confidence interval: $-.211 \pm .085$ (-.296, -.126)

b. H_0: $p_1 - p_3 \geq 0$

H_a: $p_1 - p_3 < 0$

c. $$\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_3}{n_1 + n_3} = \frac{(240)(.276) + (240)(.397)}{240 + 240} = .3365$$

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$$s_{\bar{p}_1-\bar{p}_2} = \sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} = \sqrt{(.3365)(.6635)\left(\frac{2}{240}\right)} = .0431$$

$$z = \frac{.276-.397}{.0431} = -2.81$$

p-value = .0025

With p-value $\leq .01$, we reject H_0 and conclude that bullish sentiment has declined over the past month.

34.

Observed	60	45	59	36
Expected	50	50	50	50

$\chi^2 = 8.04$

Degrees of freedom = 4 - 1 = 3

Using the χ^2 table with $df = 3$, $\chi^2 = 8.04$ shows the p-value is between .025 and .05.

Using Excel, the p-value corresponding to $\chi^2 = 8.04$ is .0452.

p-value $\leq .05$, reject H_0. Conclude that the order potentials are not the same in each sales territory.

35.

Observed	48	323	79	16	63
Expected	37.03	306.82	126.96	21.16	37.03

$$\chi^2 = \frac{(48-37.03)^2}{37.03} + \frac{(323-306.82)^2}{306.82} + \cdots + \frac{(63-37.03)^2}{37.03} = 41.69$$

Degrees of freedom = 5 - 1 = 4

Using the χ^2 table with $df = 2$, $\chi^2 = 41.69$ shows the p-value is less than .005.
Using Excel, the p-value corresponding to $\chi^2 = 41.69$ is .0000.

p-value $\leq .01$, reject H_0. Mutual fund investors' attitudes toward corporate bonds differ from their attitudes toward corporate stock.

36.

Passenger Car	Hypothesized Proportion	Observed Frequency	Expected Frequency	$(f_i - e_i)^2 / e_i$
Toyota Camry	.37	480	444	2.92
Honda Accord	.34	390	408	.79
Ford Taurus	.29	330	348	.93
Total		1200	1200	4.64

Degrees of Freedom: 2

Using the χ^2 table with $df = 2$, $\chi^2 = 4.64$ shows the p-value is between .05 and .10.

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Using Excel, the p-value corresponding to $\chi^2 = 4.64$ is .0983.

p-value > .05, cannot reject H_0. Toyota Camry's market share appears to have increased to 480/1200 = 40%. However, the sample does not justify the conclusion that the market shares have changed from their historical 37%, 34%, 29% levels.

All three manufacturers will want to watch for additional sales reports before drawing a final conclusion.

37.

Observed	13	16	28	17	16
Expected	18	18	18	18	18

$\chi^2 = 7.44$

Degrees of freedom = 5 - 1 = 4

Using the χ^2 table with $df = 4$, $\chi^2 = 7.44$ shows the p-value is greater than .10.

Using Excel, the p-value corresponding to $\chi^2 = 7.44$ is .1144.

p-value > .05, do not reject H_0. The assumption that the number of riders is uniformly distributed cannot be rejected.

38.

Category	Hypothesized Proportion	Observed Frequency (f_i)	Expected Frequency (e_i)	$(f_i - e_i)^2 / e_i$
Very Satisfied	0.28	105	140	8.75
Somewhat Satisfied	0.46	235	230	0.11
Neither	0.12	55	60	0.42
Somewhat Dissatisfied	0.10	90	50	32.00
Very Dissatisfied	0.04	15	20	1.25
Total		500		42.53

Degrees of freedom = 5 - 1 = 4

Using the χ^2 table with $df = 2$, $\chi^2 = 42.53$ shows the p-value is less than .005.

Using Excel, the p-value corresponding to $\chi^2 = 42.53$ is .0000.

p-value $\leq$ .05, reject H_0. Conclude that the job satisfaction for computer programmers is different than the job satisfaction for IS managers.

39. Expected Frequencies:

	Quality	
Shift	Good	Defective
1st	368.44	31.56
2nd	276.33	23.67
3rd	184.22	15.78

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$\chi^2 = 8.10$

Degrees of freedom = (3 - 1)(2 - 1) = 2

Using the χ^2 table with $df = 2$, $\chi^2 = 8.10$ shows the p-value is between .01 and .025.

Using Excel, the p-value corresponding to $\chi^2 = 8.10$ is .0174.

p-value $\leq$ .05, reject H_0. Conclude that shift and quality are not independent.

40. Expected Frequencies:

$e_{11} = 1046.19$ $e_{12} = 632.81$
$e_{21} = 28.66$ $e_{22} = 17.34$
$e_{31} = 258.59$ $e_{32} = 156.41$
$e_{41} = 516.55$ $e_{42} = 312.4$

Employment	Region	Observed Frequency (f_i)	Expected Frequency (e_i)	$(f_i - e_i)^2 / e_i$
Full-Time	Eastern	1105	1046.19	3.31
Full-time	Western	574	632.81	5.46
Part-Time	Eastern	31	28.66	0.19
Part-Time	Western	15	17.34	0.32
Self-Employed	Eastern	229	258.59	3.39
Self-Employed	Western	186	156.41	5.60
Not Employed	Eastern	485	516.55	1.93
Not Employed	Western	344	312.45	3.19
	Total	2969		23.37

Degrees of freedom = (4 - 1)(2 - 1) = 3

Using the χ^2 table with $df = 3$, $\chi^2 = 23.37$ shows the p-value is less than .005.

Using Excel, the p-value corresponding to $\chi^2 = 23.37$ is .0000.

p-value $\leq$ .05, reject H_0. Conclude that employment status is not independent of region.

41. Expected frequencies:

	Loan Approval Decision	
Loan Offices	Approved	Rejected
Miller	24.86	15.14
McMahon	18.64	11.36
Games	31.07	18.93
Runk	12.43	7.57

χ^2 2.21

Degrees of freedom = (4 - 1)(2 - 1) = 3

Using the χ^2 table with $df = 3$, $\chi^2 = 1.21$ shows the p-value is greater than .10.

Using Excel, the p-value corresponding to $\chi^2 = 2.21$ is .5300.

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p-value > .05, do not reject H_0. The loan decision does not appear to be dependent on the officer.

42. a. Column totals: Slower 213, No Preference 21, and Faster 66.

Percentage preferring a slower pace = (213/300)(100) = 71%

Percentage preferring a faster pace = (66/300)(100) = 22%

The combined samples of men and women show a majority would rather live in a place with a slower pace of life.

b. Observed Frequency (f_{ij})

	Preferred Pace of Life			
Respondent	Slower	No Pref	Faster	Total
Men	102	9	39	150
Woman	111	12	27	150
Total	213	21	66	300

Expected Frequency (e_{ij})

	Preferred Pace of Life			
Respondent	Slower	No Pref	Faster	Total
Men	106.5	10.5	33	150
Woman	106.5	10.5	33	150
Total	213	21	66	300

Chi Square $(f_{ij} - e_{ij})^2 / e_{ij}$

	Preferred Pace of Life			
Respondent	Slower	No Pref	Faster	Total
Men	.19	.21	1.09	1.495
Woman	.19	..21	1.09	1.495
				$\chi^2 = 2.99$

Degrees of freedom = (2-1)(3-1) = 2

Using the χ^2 table with $df = 2$, $\chi^2 = 2.99$ shows the p-value is greater than .10.

Using Excel, the p-value corresponding to $\chi^2 = 2.99$ is .2242.

p-value > .05, do not reject H_0. We cannot reject the assumption that the preferred pace of life is independent of the respondent being a man or a woman. That is, there is no statistical evidence to conclude men and women differ with respected to the preferred pace of life.

This is a good example of where it would be desirable to study this further before drawing a conclusion. Including a larger number of men and women in the sample and repeating the analysis should be considered.

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43. Observed Frequencies

Age	Church Attendance Yes	No	Total
20 to 29	31	69	100
30 to 39	63	87	150
40 to 49	94	106	200
50 to 59	72	78	150
Total	260	340	600

Expected Frequencies

Age	Church Attendance Yes	No	Total
20 to 29	43	57	100
30 to 39	65	85	150
40 to 49	87	113	200
50 to 59	65	85	150
Total	260	340	600

Chi Square

Age	Church Attendance Yes	No	
20 to 29	3.51	2.68	6.19
30 to 39	.06	.05	.11
40 to 49	.62	.47	1.10
50 to 59	.75	.58	1.33
			$\chi^2 = 8.73$

Degrees of freedom = 3

Using the χ^2 table with $df = 3$, $\chi^2 = 8.73$ shows the p-value is between .025 and .05.

Using Excel, the p-value corresponding to $\chi^2 = 8.73$ is .0331.

p-value $\leq$ .05, reject H_0. Conclude church attendance is not independent of age.

Attendance by age group:

20 - 29 31/100 → 31%
30 - 39 63/150 → 42%
40 - 49 94/200 → 47%
50 - 59 72/150 → 48%

Church attendance increases as individuals grow older.

44. Expected Frequencies:

	Los Angeles	San Diego	San Francisco	San Jose	Total
Occupied	165.7	124.3	186.4	165.7	642
Vacant	34.3	25.7	38.6	34.3	133
Total	200.0	150.0	225.0	200.0	775

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$$\chi^2 = \frac{(160-165.7)^2}{165.7} + \frac{(116-124.3)^2}{124.3} + \cdots + \frac{(26-34.3)^2}{34.3} = 7.75$$

Degrees of freedom = (2 - 1)(4 - 1) = 3

Using the χ^2 table with $df = 3$, $\chi^2 = 7.75$ shows the p-value between .05 and .10.

Using Excel, the p-value corresponding to $\chi^2 = 7.75$ is .0515.

p-value > .05, do not reject H_0. We cannot conclude that office vacancies are dependent on metropolitan area, but it is close: the p-value is slightly larger than .05.

45. a.

x	Observed Frequencies	Binomial Prob. $n = 4, p = .30$	Expected Frequencies
0	30	.2401	24.01
1	32	.4116	41.16
2	25	.2646	26.46
3	10	.0756	7.56
4	3	.0081	.81
	100		100.00

The expected frequency of $x = 4$ is .81. Combine $x = 3$ and $x = 4$ into one category so that all expected frequencies are 5 or more.

x	Observed Frequencies	Expected Frequencies
0	30	24.01
1	32	41.16
2	25	26.46
3 or 4	13	8.37
	100	100.00

b. $\chi^2 = 6.17$

Degrees of freedom = 4 - 1 = 3

Using the χ^2 table with $df = 3$, $\chi^2 = 6.17$ shows the p-value is greater than .10.

Using Excel, the p-value corresponding to $\chi^2 = 6.17$ is .1036.

p-value > .05, do not reject H_0. Conclude that the assumption of a binomial distribution cannot be rejected.

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Chapter 12
Simple Linear Regression

Learning Objectives

1. Understand how regression analysis can be used to develop an equation that estimates mathematically how two variables are related.

2. Understand the differences between the regression model, the regression equation, and the estimated regression equation.

3. Know how to fit an estimated regression equation to a set of sample data based upon the least-squares method.

4. Be able to determine how good a fit is provided by the estimated regression equation and compute the sample correlation coefficient from the regression analysis output.

5. Understand the assumptions necessary for statistical inference and be able to test for a significant relationship.

6. Know how to develop confidence interval estimates of y given a specific value of x in both the case of a mean value of y and an individual value of y.

7. Learn how to use a residual plot to make a judgement as to the validity of the regression assumptions.

8. Know the definition of the following terms:

   independent and dependent variable
   simple linear regression
   regression model
   regression equation and estimated regression equation
   scatter diagram
   coefficient of determination
   standard error of the estimate
   confidence interval
   prediction interval
   residual plot

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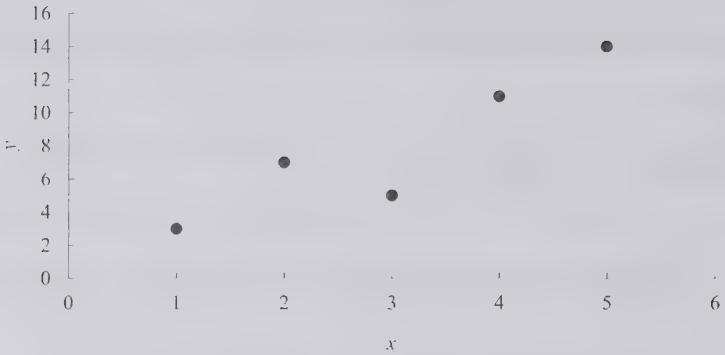
Solutions:

1 a.



b. There appears to be a positive linear relationship between x and y.

c. Many different straight lines can be drawn to provide a linear approximation of the relationship between x *and* y; in part (d) we will determine the equation of a straight line that "best" represents the relationship according to the least squares criterion.

d. $\bar{x} = \dfrac{\Sigma x_i}{n} = \dfrac{15}{5} = 3 \quad \bar{y} = \dfrac{\Sigma y_i}{n} = \dfrac{40}{5} = 8$

$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 26 \quad \Sigma(x_i - \bar{x})^2 = 10$

$b_1 = \dfrac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \dfrac{26}{10} = 2.6$

$b_0 = \bar{y} - b_1\bar{x} = 8 - (2.6)(3) = 0.2$

$\hat{y} = 0.2 + 2.6x$

e. $\hat{y} = 0.2 + 2.6(4) = 10.6$

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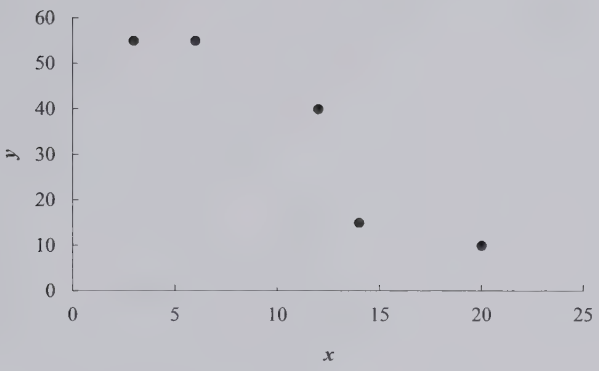
2. a.



b. There appears to be a negative linear relationship between x and y.

c. Many different straight lines can be drawn to provide a linear approximation of the relationship between x *and* y; in part (d) we will determine the equation of a straight line that "best" represents the relationship according to the least squares criterion.

d. $\bar{x} = \dfrac{\Sigma x_i}{n} = \dfrac{55}{5} = 11 \quad \bar{y} = \dfrac{\Sigma y_i}{n} = \dfrac{175}{5} = 35$

$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = -540 \quad \Sigma(x_i - \bar{x})^2 = 180$

$b_1 = \dfrac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \dfrac{-540}{180} = -3$

$b_0 = \bar{y} - b_1\bar{x} = 35 - (-3)(11) = 68$

$\hat{y} = 68 - 3x$

e. $\hat{y} = 68 - 3(10) = 38$

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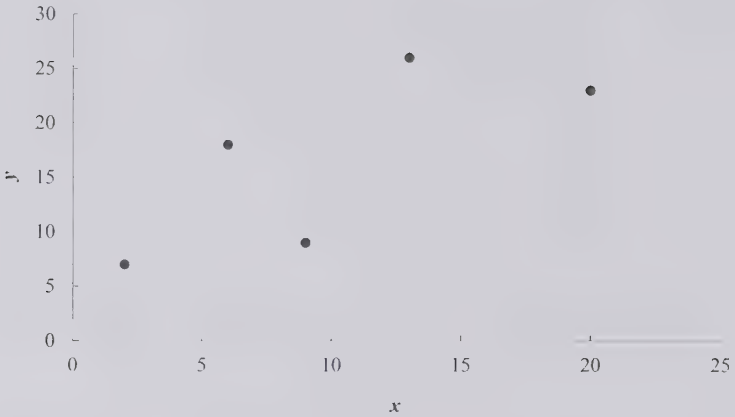
3. a.



b. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{50}{5} = 10 \quad \bar{y} = \frac{\Sigma y_i}{n} = \frac{83}{5} = 16.6$

$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 171 \quad \Sigma(x_i - \bar{x})^2 = 190$

$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{171}{190} = 0.9$

$b_0 = \bar{y} - b_1\bar{x} = 16.6 - (0.9)(10) = 7.6$

$\hat{y} = 7.6 + 0.9x$

c. $\hat{y} = 7.6 + 0.9(6) = 13$

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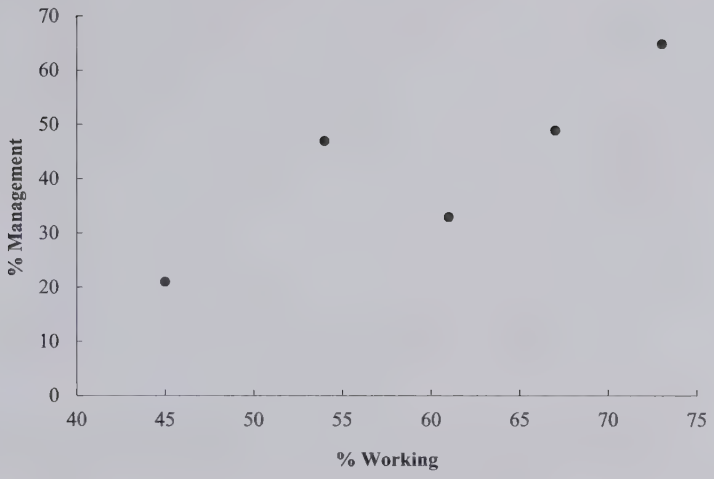
4. a.



b. There appears to be a positive linear relationship between the percentage of women working in the five companies (x) and the percentage of management jobs held by women in that company (y)

c. Many different straight lines can be drawn to provide a linear approximation of the relationship between x and y; in part (d) we will determine the equation of a straight line that "best" represents the relationship according to the least squares criterion.

d. $\bar{x} = \dfrac{\Sigma x_i}{n} = \dfrac{300}{5} = 60 \quad \bar{y} = \dfrac{\Sigma y_i}{n} = \dfrac{215}{5} = 43$

$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 624 \quad \Sigma(x_i - \bar{x})^2 = 480$

$b_1 = \dfrac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \dfrac{624}{480} = 1.3$

$b_0 = \bar{y} - b_1\bar{x} = 43 - 1.3(60) = -35$

$\hat{y} = -35 + 1.3x$

e. $\hat{y} = -35 + 1.3x = -35 + 1.3(60) = 43\%$

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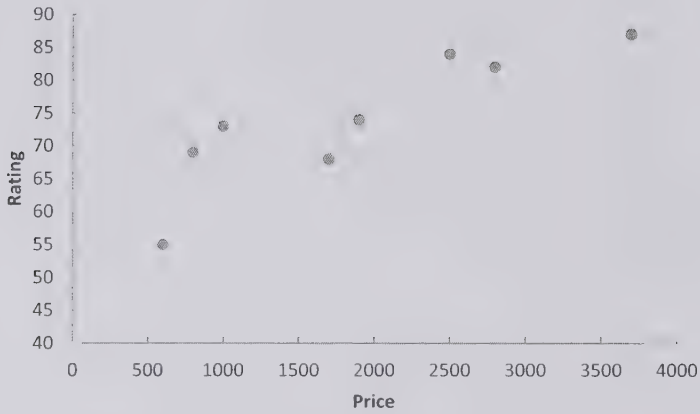
5. a.



b. There appears to be a positive relationship between price and rating. The sign that says "Quality: You Get What You Pay For" does fairly reflect the price-quality relationship for ellipticals.

c. Let x = price ($) and y = rating.

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{1500}{8} = 1875 \quad \bar{y} = \frac{\Sigma y_i}{n} = \frac{592}{8} = 74$$

$$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 68,900 \quad \Sigma(x_i - \bar{x})^2 = 8,155,000$$

$$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{68,900}{8,155,000} = .008449$$

$$b_0 = \bar{y} - b_1\bar{x} = 74 - (.008449)(1875) = 58.158$$

$$\hat{y} = 58.158 + .008449x$$

d. $\hat{y} = 58.158 + .008449x = 58.158 + .008449(1500) = 70.83$ or approximately 71

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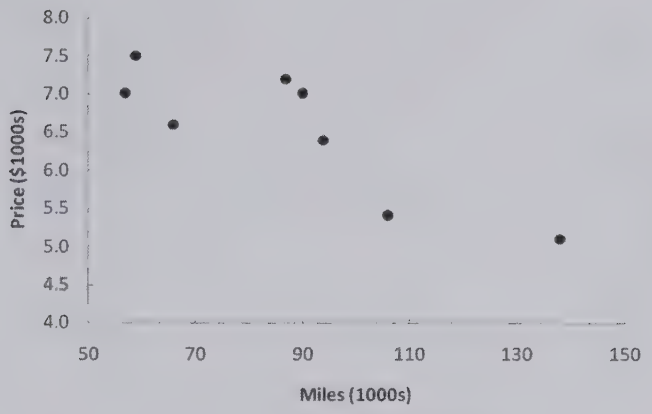
6. a.



b. There appears to be a negative linear relationship between x = miles and y = sales price.

If the car has higher miles, the sales price tends to be lower.

c. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{874}{10} = 87.4 \quad \bar{y} = \frac{\Sigma y_i}{n} = \frac{66.4}{10} = 6.64$

$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = -135.66 \quad \Sigma(x_i - \bar{x})^2 = 5152.4$

$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{-135.66}{5152.40} = -.02633$

$b_0 = \bar{y} - b_1\bar{x} = 6.64 - (-.02633)(87.4) = 8.9412$

$\hat{y} = 8.9412 - .02633x$

d. The slope of the estimated regression equation is -.02633. Thus, a one unit increase in the value of x will result in a decrease in the estimated value of y equal to .02633. Because the data were recorded in thousands, every additional 1000 miles on the car's odometer will result in a $26.33 decrease in the estimated price.

e. $\hat{y} = 8.9412 - .02633(100) = 6.3$ or $6300

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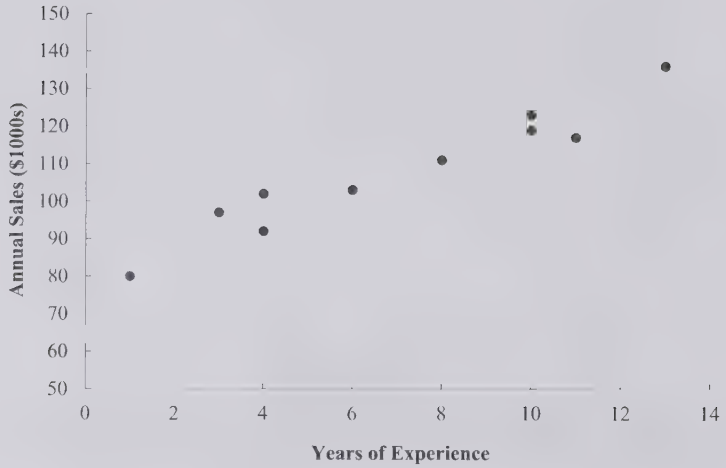
7. a.



b. Let x = years of experience and y = annual sales ($1000s)

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{70}{10} = 7 \quad \bar{y} = \frac{\Sigma y_i}{n} = \frac{1080}{10} = 108$$

$$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 568 \quad \Sigma(x_i - \bar{x})^2 = 142$$

$$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{568}{142} = 4$$

$$b_0 = \bar{y} - b_1\bar{x} = 108 - (4)(7) = 80$$

$$\hat{y} = 80 + 4x$$

c. $\hat{y} = 80 + 4x = 80 + 4(9) = 116$ or $116,000

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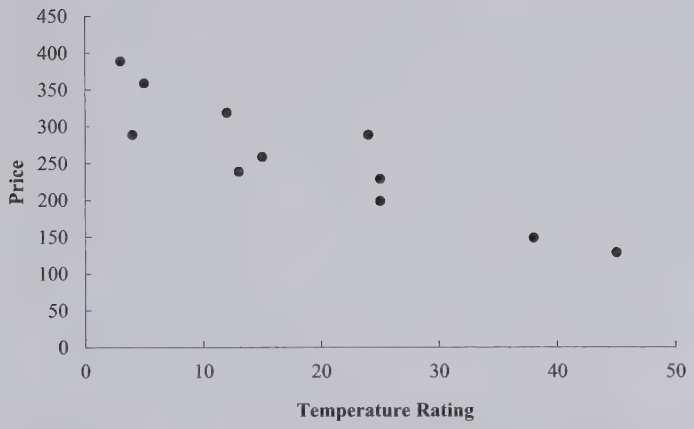
8. a.



b. The scatter diagram and the slope of the estimated regression equation indicate a negative linear relationship between x = temperature rating and y = price. Thus, it appears that sleeping bags with a lower temperature rating cost more than sleeping bags with a higher temperature rating. In other words, it costs more to stay warmer.

c. $\bar{x} = \Sigma x_i / n = 209/11 = 19 \quad \bar{y} = \Sigma y_i / n = 2849/11 = 259$

$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = -10,090 \quad \Sigma(x_i - \bar{x})^2 = 1912$

$$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{-10,090}{1912} = -5.2772$$

$b_0 = \bar{y} - b_1\bar{x} = 259 - (-5.2772)(19) = 359.2668$

$\hat{y} = 359.2668 - 5.2772x$

d. $\hat{y} = 359.2668 - 5.2772x = 359.2668 - 5.2772(20) = 253.72$

Thus, the estimate of the price of sleeping bag with a temperature rating of 20 is approximately $254.

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9. a.

b. There appears to be a positive linear relationship between x = price and y = score.

c. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{2638}{10} = 263.8 \quad \bar{y} = \frac{\Sigma y_i}{n} = \frac{672}{10} = 67.2$

$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 14,601.40 \quad \Sigma(x_i - \bar{x})^2 = 258,695.60$

$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{14,601.40}{258,695.60} = .05644$

$b_0 = \bar{y} - b_1\bar{x} = 67.2 - (.05644)(263.8) = 52.311$

$\hat{y} = 52.311 + .05644x$

d. The slope is .05644. For a \$100 higher price, the score can be expected to increase $100(.05644) = 5.644$, or about 6 points.

e. $\hat{y} = 52.311 + .05644(225) = 65$

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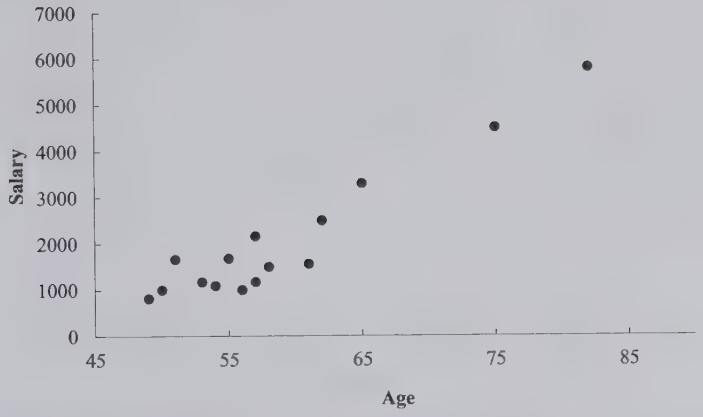
10. a.



b. There appears to be a positive linear relationship between x = age and y = salary.

c. $\bar{x} = \dfrac{\Sigma x_i}{n} = \dfrac{885}{15} = 59 \quad \bar{y} = \dfrac{\Sigma y_i}{n} = \dfrac{30{,}939}{15} = 2062.6$

$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 175{,}265 \quad \Sigma(x_i - \bar{x})^2 = 1174$

$b_1 = \dfrac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \dfrac{175{,}265}{1174} = 149.2888$

$b_0 = \bar{y} - b_1\bar{x} = 2062.6 - (149.2888)(59) = -6745.44$

$\hat{y} = -6745.44 + 149.29x$

d. $\hat{y} = -6745.44 + 149.29x = -6745.44 + 149.29(72) = 4003$ or \$4,003,000

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11. a.

b. There appears to be a positive linear relationship between x = price and y = road-test score.

c. $\bar{x} = \dfrac{\Sigma x_i}{n} = \dfrac{339.6}{12} = 28.3 \quad \bar{y} = \dfrac{\Sigma y_i}{n} = \dfrac{930}{12} = 77.5$

$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 309.90 \quad \Sigma(x_i - \bar{x})^2 = 346.38$

$b_1 = \dfrac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \dfrac{309.90}{346.38} = .8947$

$b_0 = \bar{y} - b_1\bar{x} = 77.5 - (.8947)(28.3) = 52.18$

$\hat{y} = 52.18 + .8947x$

d. The slope is .8947. A sporty car that has a ten thousand dollar higher price can be expected to have a 10(.8947) = 8.947, or approximately a 9 point higher road-test score.

e. $\hat{y} = 52.18 + .8947(36.7) = 85$

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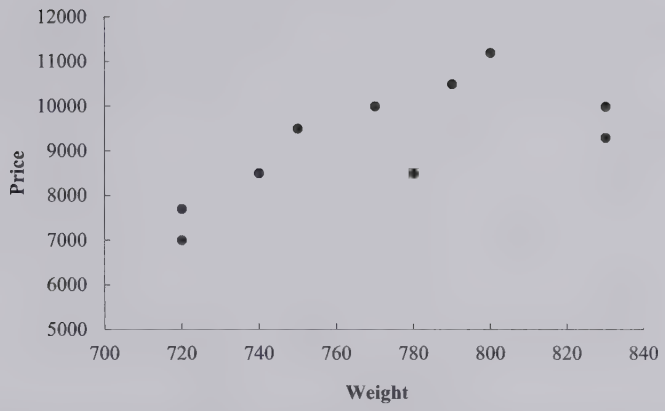
12. a.



b. The scatter diagram indicates a positive linear relationship between x = weight and y = price. Thus, it appears that PWC's that weigh more have a higher price.

c. $\bar{x} = \Sigma x_i / n = 7730/10 = 773 \quad \bar{y} = \Sigma y_i / n = 92,200/10 = 9220$

$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 332,400 \quad \Sigma(x_i - \bar{x})^2 = 14,810$

$$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{332,400}{14,810} = 22.4443$$

$b_0 = \bar{y} - b_1\bar{x} = 9220 - (22.4443)(773) = -8129.4439$

$\hat{y} = -8129.4439 + 22.4443x$

d. $\hat{y} = -8129.4439 + 22.4443x = -8129.4439 + 22.4443(750) = 8703.78$

Thus, the estimate of the price of Jet Ski with a weight of 750 pounds is approximately $8704.

e. No. The relationship between weight and price is not deterministic.

f. The weight of the Kawasaki SX-R 800 is so far below the lowest weight for the data used to develop the estimated regression equation that we would not recommend using the estimated regression equation to predict the price for this model.

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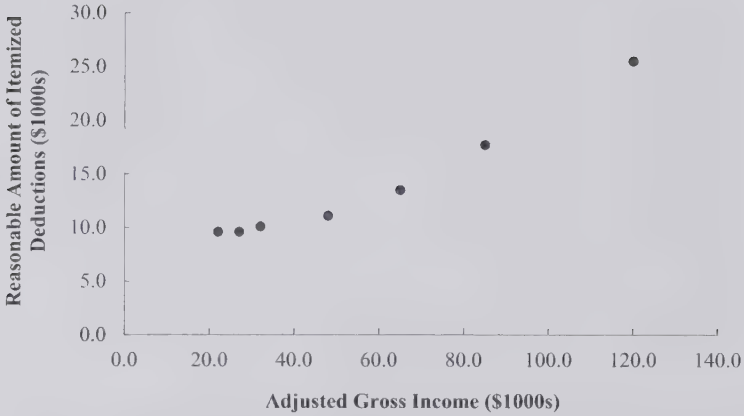
13. a.



b. Let x = adjusted gross income and y = reasonable amount of itemized deductions

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{399}{7} = 57 \quad \bar{y} = \frac{\Sigma y_i}{n} = \frac{97.1}{7} = 13.8714$$

$$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 1233.7 \quad \Sigma(x_i - \bar{x})^2 = 7648$$

$$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{1233.7}{7648} = 0.1613$$

$$b_0 = \bar{y} - b_1\bar{x} = 13.8714 - (0.1613)(57) = 4.6773$$

$$\hat{y} = 4.68 + 0.16x$$

c. $\hat{y} = 4.68 + 0.16x = 4.68 + 0.16(52.5) = 13.08$ or approximately \$13,080.

The agent's request for an audit appears to be justified.

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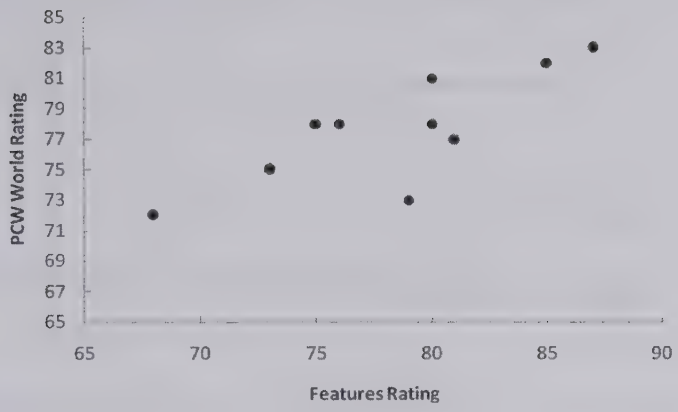
14. a.



b. There appears to be a positive linear relationship between x = features rating and y = PCW World Rating.

c. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{784}{10} = 78.4 \quad \bar{y} = \frac{\Sigma y_i}{n} = \frac{777}{10} = 77.7$

$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 147.20 \quad \Sigma(x_i - \bar{x})^2 = 284.40$

$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{147.20}{284.40} = .51758$

$b_0 = \bar{y} - b_1\bar{x} = 77.7 - (.51758)(78.4) = 37.1217$

$\hat{y} = 37.1217 + .51758x$

d. $\hat{y} = 37.1217 + .51758(70) = 73.35$ or 73

15. a. The estimated regression equation and the mean for the dependent variable are:

$\hat{y}_i = 0.2 + 2.6x_i \qquad \bar{y} = 8$

The sum of squares due to error and the total sum of squares are

$\text{SSE} = \Sigma(y_i - \hat{y}_i)^2 = 12.40 \qquad \text{SST} = \Sigma(y_i - \bar{y})^2 = 80$

Thus, SSR = SST - SSE = 80 - 12.4 = 67.6

b. $r^2 = \text{SSR/SST} = 67.6/80 = .845$

The least squares line provided a very good fit; 84.5% of the variability in y has been explained by the least squares line.

c. $r_{xy} = \sqrt{.845} = +.9192$

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16. a. The estimated regression equation and the mean for the dependent variable are:

$\hat{y}_i = 68 - 3x \qquad \bar{y} = 35$

The sum of squares due to error and the total sum of squares are

$SSE = \sum(y_i - \hat{y}_i)^2 = 230 \qquad SST = \sum(y_i - \bar{y})^2 = 1850$

Thus, SSR = SST - SSE = 1850 - 230 = 1620

b. $r^2 = SSR/SST = 1620/1850 = .876$

The least squares line provided an excellent fit; 87.6% of the variability in y has been explained by the estimated regression equation.

c. $r_{xy} = \sqrt{.876} = -.936$

Note: the sign for r is negative because the slope of the estimated regression equation is negative. ($b_1 = -3$)

17. The estimated regression equation and the mean for the dependent variable are:

$\hat{y}_i = 7.6 + .9x \qquad \bar{y} = 16.6$

The sum of squares due to error and the total sum of squares are

$SSE = \sum(y_i - \hat{y}_i)^2 = 127.3 \qquad SST = \sum(y_i - \bar{y})^2 = 281.2$

Thus, SSR = SST - SSE = 281.2 – 127.3 = 153.9

$r^2 = SSR/SST = 153.9/281.2 = .547$

We see that 54.7% of the variability in y has been explained by the least squares line.

$r_{xy} = \sqrt{.547} = +.740$

18. a. The estimated regression equation and the mean for the dependent variable are:

$\hat{y} = 1790.5 + 581.1x \qquad \bar{y} = 3650$

The sum of squares due to error and the total sum of squares are

$SSE = \sum(y_i - \hat{y}_i)^2 = 85{,}135.14 \qquad SST = \sum(y_i - \bar{y})^2 = 335{,}000$

Thus, SSR = SST - SSE = 335,000 - 85,135.14 = 249,864.86

b. $r^2 = SSR/SST = 249{,}864.86/335{,}000 = .746$

We see that 74.6% of the variability in y has been explained by the least squares line.

c. $r_{xy} = \sqrt{.746} = +.8637$

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19. a. The estimated regression equation and the mean for the dependent variable are:

$\hat{y} = 80 + 4x \qquad \bar{y} = 108$

The sum of squares due to error and the total sum of squares are

$SSE = \Sigma(y_i - \hat{y}_i)^2 = 170 \qquad SST = \Sigma(y_i - \bar{y})^2 = 2442$

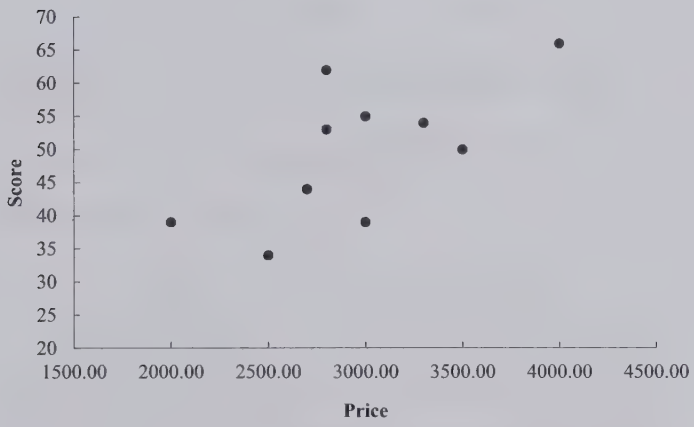
Thus, SSR = SST - SSE = 2442 - 170 = 2272

b. $r^2 = SSR/SST = 2272/2442 = .93$

We see that 93% of the variability in y has been explained by the least squares line.

c. $r_{xy} = \sqrt{.93} = +.96$

20. a.



The scatter diagram indicates a positive linear relationship between x = price and y = score.

$\bar{x} = \Sigma x_i / n = 29,600/10 = 2960 \qquad \bar{y} = \Sigma y_i / n = 496/10 = 49.6$

$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 34,840 \qquad \Sigma(x_i - \bar{x})^2 = 2,744,000$

$$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{34,840}{2,744,000} = .012697$$

$b_0 = \bar{y} - b_1\bar{x} = 49.6 - (.012697)(2960) = 12.0169$

$\hat{y} = 12.0169 + .0127x$

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b. The sum of squares due to error and the total sum of squares are

$SSE = \Sigma(y_i - \hat{y}_i)^2 = 540.0446 \qquad SST = \Sigma(y_i - \bar{y})^2 = 982.4$

Thus, SSR = SST - SSE = 982.4 – 540.0446 = 442.3554

$r^2 = SSR/SST = 442.3554/982.4 = .4503$

The fit provided by the estimated regression equation is not that good; only 45.03% of the variability in y has been explained by the least squares line.

c. $\hat{y} = 12.0169 + .0127x = 12.0169 + .0127(3200) = 52.66$

The estimate of the overall score for a 42-inch plasma television is approximately 53.

21. a. $\bar{x} = \dfrac{\Sigma x_i}{n} = \dfrac{3450}{6} = 575 \qquad \bar{y} = \dfrac{\Sigma y_i}{n} = \dfrac{33,700}{6} = 5616.67$

$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 712,500 \qquad \Sigma(x_i - \bar{x})^2 = 93,750$

$b_1 = \dfrac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \dfrac{712,500}{93,750} = 7.6$

$b_0 = \bar{y} - b_1\bar{x} = 5616.67 - (7.6)(575) = 1246.67$

$\hat{y} = 1246.67 + 7.6x$

b. $7.60

c. The sum of squares due to error and the total sum of squares are:

$SSE = \Sigma(y_i - \hat{y}_i)^2 = 233,333.33 \qquad SST = \Sigma(y_i - \bar{y})^2 = 5,648,333.33$

Thus, SSR = SST - SSE = 5,648,333.33 - 233,333.33 = 5,415,000

$r^2 = SSR/SST = 5,415,000/5,648,333.33 = .9587$

We see that 95.87% of the variability in y has been explained by the estimated regression equation.

d. $\hat{y} = 1246.67 + 7.6x = 1246.67 + 7.6(500) = \$5046.67$

22. a. $\bar{y} = 74 \qquad SSE = 173.88$

The total sum of squares is

$SST = \Sigma(y_i - \bar{y})^2 = 756$

Thus, SSR = SST - SSE = 756 – 173.88 = 582.12

$r^2 = SSR/SST = 582.12/756 = .77$

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b. The estimated regression equation provided a good fit because 77% of the variability in y has been explained by the least squares line.

c. $r_{xy} = \sqrt{.77} = +.88$

This reflects a strong positive linear relationship between price and rating.

23. a. s^2 = MSE = SSE / $(n - 2)$ = 12.4 / 3 = 4.133

b. $s = \sqrt{\text{MSE}} = \sqrt{4.133} = 2.033$

c. $\Sigma(x_i - \bar{x})^2 = 10$

$$s_{b_1} = \frac{s}{\sqrt{\Sigma(x_i - \bar{x})^2}} = \frac{2.033}{\sqrt{10}} = 0.643$$

d. $$t = \frac{b_1}{s_{b_1}} = \frac{2.6}{.643} = 4.044$$

Using t table (3 degrees of freedom), area in tail is between .01 and .025

p-value is between .02 and .05

Using Excel, the p-value corresponding to $t = 4.04$ is .0272.

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$

e. MSR = SSR / 1 = 67.6

F = MSR / MSE = 67.6 / 4.133 = 16.36

Using F table (1 degree of freedom numerator and 3 denominator), p-value is between .025 and .05

Using Excel, the p-value corresponding to $F = 16.36$ is .0272.

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Regression	67.6	1	67.6	16.36	.0272
Error	12.4	3	4.133		
Total	80.0	4			

24. a. s^2 = MSE = SSE/$(n - 2)$ = 230/3 = 76.6667

b. $s = \sqrt{\text{MSE}} = \sqrt{76.6667} = 8.7560$

c. $\Sigma(x_i - \bar{x})^2 = 180$

$$s_{b_1} = \frac{s}{\sqrt{\Sigma(x_i - \bar{x})^2}} = \frac{8.7560}{\sqrt{180}} = 0.6526$$

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d. $t = \dfrac{b_1}{s_{b_1}} = \dfrac{-3}{.653} = -4.59$

Using t table (3 degrees of freedom), area in tail is between .005 and .01; p-value is between .01 and .02

Using Excel, the p-value corresponding to t = -4.59 is .0193.

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$

e. MSR = SSR/1 = 1620

F = MSR/MSE = 1620/76.6667 = 21.13

Using F table (1 degree of freedom numerator and 3 denominator), p-value is between .01 and .025

Using Excel, the p-value corresponding to F = 21.13 is .0193.

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Regression	230	1	230	21.13	.0193
Error	1620	3	76.6667		
Total	1850	4			

25. a. $s^2 = \text{MSE} = \text{SSE}/(n - 2) = 127.3/3 = 42.4333$

$s = \sqrt{\text{MSE}} = \sqrt{42.4333} = 6.5141$

b. $\Sigma(x_i - \bar{x})^2 = 190$

$s_{b_1} = \dfrac{s}{\sqrt{\Sigma(x_i - \bar{x})^2}} = \dfrac{6.5141}{\sqrt{190}} = 0.4726$

$t = \dfrac{b_1}{s_{b_1}} = \dfrac{.9}{.4726} = 1.90$

Using t table (3 degrees of freedom), area in tail is between .05 and .10

p-value is between .10 and .20

Using Excel, the p-value corresponding to t = 1.90 is .1530.

Because p-value $> \alpha$, we cannot reject H_0: $\beta_1 = 0$; x and y do not appear to be related.

c. MSR = SSR/1 = 153.9 /1 = 153.9

F = MSR/MSE = 153.9/42.4333 = 3.63

Using F table (1 degree of freedom numerator and 3 denominator), p-value is greater than .10

Using Excel, the p-value corresponding to F = 3.63 is .1530.

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Because p-value $> \alpha$, we cannot reject H_0: $\beta_1 = 0$; x and y do not appear to be related.

26. a. In solving exercise 18, we found SSE = 85,135.14

s^2 = MSE = SSE/(n - 2) = 85,135.14/4 = 21,283.79

$s = \sqrt{\text{MSE}} = \sqrt{21{,}283.79} = 145.89$

$\Sigma(x_i - \bar{x})^2 = 0.74$

$$s_{b_1} = \frac{s}{\sqrt{\Sigma(x_i - \bar{x})^2}} = \frac{145.89}{\sqrt{0.74}} = 169.59$$

$$t = \frac{b_1}{s_{b_1}} = \frac{581.1}{169.59} = 3.43$$

Using t table (4 degrees of freedom), area in tail is between .01 and .025

p-value is between .02 and .05

Using Excel, the p-value corresponding to t = 3.43 is .0266.

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$; there is a significant relationship between grade point average and monthly salary

b. MSR = SSR/1 = 249,864.86/1 = 249,864.86

F = MSR/MSE = 249,864.86/21,283.79 = 11.74

Using F table (1 degree of freedom numerator and 4 denominator), p-value is between .025 and .05

Using Excel, the p-value corresponding to F = 11.74 is .0266.

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$

c.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Regression	249864.86	1	249864.86	11.74	.0266
Error	85135.14	4	21283.79		
Total	335000	5			

27. a. Let x = number of megapixels and y = price ($)

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{95}{10} = 9.5 \quad \bar{y} = \frac{\Sigma y_i}{n} = \frac{2190}{10} = 219$$

$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 2165$ $\Sigma(x_i - \bar{x})^2 = 56.5$

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$$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{2165}{56.5} = 38.31858$$

$$b_0 = \bar{y} - b_1\bar{x} = 219 - (38.31858)(9.5) = -145.0265$$

$$\hat{y} = -145.0265 + 38.31858x$$

b. SSE = $\Sigma(y_i - \hat{y}_i)^2 = 20,730.27$ SST = $\Sigma(y_i - \bar{y})^2 = 103,690$

Thus, SSR = SST - SSE = 103,690 20,730.27 = 82,959.73

MSR = SSR/1 = 82,959.73

MSE = SSE/(n - 2) = 20,730.27/8 = 2591.28

F = MSR / MSE = 82,959.73/2591.28 = 32.015

Using F table (1 degree of freedom numerator and 8 denominator), p-value is less than .01

Using Excel, the p-value corresponding to F = 32.015 is .000.

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$

Number of megapixels and price are related.

c. r^2 = SSR/SST = 82,959.73/103,690= .80

The estimated regression equation provided a good fit; we should feel comfortable using the estimated regression equation to estimate the price given the number of megapixels.

d. $\hat{y} = -145.0265 + 38.31858(10) = 238.16$ or approximately $238

28. The sum of squares due to error and the total sum of squares are

SSE = $\Sigma(y_i - \hat{y}_i)^2 = 12,953.09$ SST = $\Sigma(y_i - \bar{y})^2 = 66,200$

Thus, SSR = SST - SSE = 66,200 - 12,953.09 = 53,246.91

s^2 = MSE = SSE / (n - 2) = 12,953.09 / 9 = 1439.2322

$s = \sqrt{\text{MSE}} = \sqrt{1439.2322} = 37.9372$

We can use either the t test or F test to determine whether temperature rating and price are related.

We will first illustrate the use of the t test.

Note: from the solution to exercise 10 $\Sigma(x_i - \bar{x})^2 = 1912$

$$s_{b_1} = \frac{s}{\sqrt{\Sigma(x_i - \bar{x})^2}} = \frac{37.9372}{\sqrt{1912}} = .8676$$

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$$t = \frac{b_1}{s_{b_1}} = \frac{-5.2772}{.8676} = -6.0825$$

Using t table (9 degrees of freedom), area in tail is less than .005; p-value is less than .01

Using Excel, the p-value corresponding to t = -6.0825 is .000.

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$

Because we can reject H_0: $\beta_1 = 0$ we conclude that temperature rating and price are related.

Next we illustrate the use of the F test.

MSR = SSR / 1 = 53,246.91

F = MSR / MSE = 53,246.91 / 1439.2322 = 37.00

Using F table (1 degree of freedom numerator and 9 denominator), p-value is less than .01

Using Excel, the p-value corresponding to F = 37.00 is .000.

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$

Because we can reject H_0: $\beta_1 = 0$ we conclude that temperature rating and price are related.

The ANOVA table is shown below.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Regression	53,246.91	1	53,246.91	37.00	.000
Error	12,953.09	9	1439.2322		
Total	66,200	10			

29. SSE = $\Sigma(y_i - \hat{y}_i)^2$ = 233,333.33 SST = $\Sigma(y_i - \bar{y})^2$ = 5,648,333.33

Thus, SSR = SST – SSE= 5,648,333.33 -233,333.33 = 5,415,000

MSE = SSE/(n - 2) = 233,333.33/(6 - 2) = 58,333.33

MSR = SSR/1 = 5,415,000

F = MSR / MSE = 5,415,000 / 58,333.25 = 92.83

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Regression	5,415,000.00	1	5,415,000	92.83	.0006
Error	233,333.33	4	58,333.33		
Total	5,648,333.33	5			

Using F table (1 degree of freedom numerator and 4 denominator), p-value is less than .01

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Using Excel, the p-value corresponding to $F = 92.83$ is .0006.

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$. Production volume and total cost are related.

30. $SSE = \Sigma(y_i - \hat{y}_i)^2 = 173.88$ $SST = \Sigma(y_i - \bar{y})^2 = 756$

Thus, SSR = SST – SSE = 756 – 173.88 = 582.12

$s^2 = MSE = SSE/(n\text{-}2) = 173.88/6 = 28.98$

$s = \sqrt{28.98} = 5.3833$

$\Sigma(x_i - \bar{x})^2 = 8{,}155{,}000$

$$s_{b_1} = \frac{s}{\sqrt{\Sigma(x_i - \bar{x})^2}} = \frac{5.3833}{\sqrt{8{,}155{,}000}} = .001885$$

$$t = \frac{b_1}{s_{b_1}} = \frac{.008449}{.001885} = 4.48$$

Using t table (1 degree of freedom numerator and 8 denominator), area in tail is less than .005

p-value is less than .01

Using Excel, the p-value corresponding to $t = 4.48$ is .0042.

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$

There is a significant relationship between price and rating.

31. SSE = 540.04 and SST = 982.40

Thus, SSR = SST - SSE = 982.40 540.04 = 442.36

$s^2 = MSE = SSE / (n - 2) = 540.04 / 8 = 67.5050$

$s = \sqrt{MSE} = \sqrt{67.5050} = 8.216$

MSR = SSR / 1 = 442.36

F = MSR / MSE = 442.36 / 67.5050 = 6.55

Using F table (1 degree of freedom numerator and 8 denominator), p-value is between .025 and .05

Using Excel, the p-value corresponding to $F = 6.55$ is .034.

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$

Conclusion: price and overall score are related

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32. a. $s = 2.033$

$\bar{x} = 3 \quad \Sigma(x_i - \bar{x})^2 = 10$

$$s_{\hat{y}^*} = s\sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}} = 2.033\sqrt{\frac{1}{5} + \frac{(4-3)^2}{10}} = 1.11$$

b. $\hat{y}^* = .2 + 2.6\,x^* = .2 + 2.6(4) = 10.6$

$\hat{y}^* \pm t_{\alpha/2} s_{\hat{y}^*}$

$10.6 \pm 3.182\,(1.11) = 10.6 \pm 3.53$

or 7.07 to 14.13

c. $$s_{\text{pred}} = s\sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}} = 2.033\sqrt{1 + \frac{1}{5} + \frac{(4-3)^2}{10}} = 2.32$$

d. $\hat{y}^* \pm t_{\alpha/2} s_{\text{pred}}$

$10.6 \pm 3.182\,(2.32) = 10.6 \pm 7.38$

or 3.22 to 17.98

33. a. $s = 8.7560$

b. $\bar{x} = 11 \quad \Sigma(x_i - \bar{x})^2 = 180$

$$s_{\hat{y}^*} = s\sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}} = 8.7560\sqrt{\frac{1}{5} + \frac{(8-11)^2}{180}} = 4.3780$$

$\hat{y}^* = 0.2 + 2.6x^* = 0.2 + 2.6(4) = 10.6$

$\hat{y}^* \pm t_{\alpha/2} s_{\hat{y}^*}$

$44 \pm 3.182\,(4.3780) = 44 \pm 13.93$

or 30.07 to 57.93

c. $$s_{\text{pred}} = s\sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}} = 8.7560\sqrt{1 + \frac{1}{5} + \frac{(8-11)^2}{180}} = 9.7895$$

d. $\hat{y}^* \pm t_{\alpha/2} s_{\text{pred}}$

$44 \pm 3.182(9.7895) = 44 \pm 31.15$

or 12.85 to 75.15

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34. $s = 6.5141$

$\bar{x} = 10 \quad \Sigma(x_i - \bar{x})^2 = 190$

$$s_{\hat{y}^*} = s\sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}} = 6.5141\sqrt{\frac{1}{5} + \frac{(12-10)^2}{190}} = 3.0627$$

$\hat{y}^* = 7.6 + .9x^* = 7.6 + .9(12) = 18.40$

$\hat{y}^* \pm t_{\alpha/2} s_{\hat{y}^*}$

$18.40 \pm 3.182(3.0627) = 18.40 \pm 9.75$

or 8.65 to 28.15

$$s_{\text{pred}} = s\sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}} = 6.5141\sqrt{1 + \frac{1}{5} + \frac{(12-10)^2}{190}} = 7.1982$$

$\hat{y}^* \pm t_{\alpha/2} s_{\text{pred}}$

$18.40 \pm 3.182(7.1982) = 18.40 \pm 22.90$

or -4.50 to 41.30

The two intervals are different because there is more variability associated with predicting an individual value than there is a mean value.

35. a. Note: some of the values shown were computed in exercises 18 and 26.

$s = 145.89$

$\bar{x} = 3.2 \quad \Sigma(x_i - \bar{x})^2 = 0.74$

$$s_{\hat{y}^*} = s\sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}} = 145.89\sqrt{\frac{1}{6} + \frac{(3-3.2)^2}{0.74}} = 68.54$$

$\hat{y}^* = 1790.5 + 581.1x^* = 1790.5 + 581.1(3) = 3533.8$

$\hat{y}^* \pm t_{\alpha/2} s_{\hat{y}^*}$

$3533.8 \pm 2.776\,(68.54) = 3533.8 \pm 190.27$

or \$3343.53 to \$3724.07

b. $$s_{\text{pred}} = s\sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}} = 145.89\sqrt{1 + \frac{1}{6} + \frac{(3-3.2)^2}{0.74}} = 161.19$$

$\hat{y}^* + t_{\alpha/2} s_{\text{pred}}$

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$3533.8 \pm 2.776\ (161.19) = 3533.8 \pm 447.46$

or \$3086.34 to \$3981.26

36. a. $\hat{y}^* = 359.2668 - 5.2772x^* = 359.2668 - 5.2772(30) = 200.95$ or approximately \$201.

b. $s = 37.9372$

$\bar{x} = 19 \quad \Sigma(x_i - \bar{x})^2 = 1912$

$$s_{\hat{y}^*} = s\sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}} = 37.9372\sqrt{\frac{1}{11} + \frac{(30-19)^2}{1912}} = 14.90$$

$\hat{y}^* \pm t_{\alpha/2} s_{\hat{y}^*}$

$200.95 \pm 2.262\ (14.90) = 200.95 \pm 33.70$

or 167.25 to 234.65

c. $$s_{\text{pred}} = s\sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}} = 37.9372\sqrt{1 + \frac{1}{11} + \frac{(30-19)^2}{1912}} = 40.76$$

$\hat{y}^* \pm t_{\alpha/2} s_{\text{pred}}$

$200.95 \pm 2.262\ (40.76) = 200.95 \pm 92.20$

or 108.75 to 293.15

d. As expected, the prediction interval is much wider than the confidence interval. This is due to the fact that it is more difficult to predict the results for one new model with a temperature rating of 30 F° than it is to estimate the mean for all models with a temperature rating of 30 F° .

37. a. $\bar{x} = 57 \quad \Sigma(x_i - \bar{x})^2 = 7648$

$s^2 = 1.88 \quad s = 1.37$

$$s_{\hat{y}^*} = s\sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}} = 1.37\sqrt{\frac{1}{7} + \frac{(52.5-57)^2}{7648}} = 0.52$$

$\hat{y}^* \pm t_{\alpha/2} s_{\hat{y}^*}$

$\hat{y}^* = 4.68 + 0.16\,x^* = 4.68 + 0.16(52.5) = 13.08$

$13.08 \pm 2.571\ (.52) = 13.08 \pm 1.34$

or 11.74 to 14.42 or \$11,740 to \$14,420

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b. $s_{pred} = 1.47$

$13.08 \pm 2.571\ (1.47) = 13.08 \pm 3.78$

or 9.30 to 16.86 or \$9,300 to \$16,860

c. Yes, \$20,400 is much larger than anticipated.

d. Any deductions exceeding the \$16,860 upper limit could suggest an audit.

38. a. $\hat{y}^* = 1246.67 + 7.6(500) = \$5046.67$

b. $\bar{x} = 575 \quad \Sigma(x_i - \bar{x})^2 = 93,750$

$s^2 = \text{MSE} = 58,333.33 \quad s = 241.52$

$$s_{pred} = s\sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}} = 241.52\sqrt{1 + \frac{1}{6} + \frac{(500 - 575)^2}{93,750}} = 267.50$$

$\hat{y}^* \pm t_{\alpha/2} s_{pred}$

$5046.67 \pm 4.604\ (267.50) = 5046.67 \pm 1231.57$

or \$3815.10 to \$6278.24

c. Based on one month, \$6000 is not out of line since \$3815.10 to \$6278.24 is the prediction interval. However, a sequence of five to seven months with consistently high costs should cause concern.

39. a. Let x = miles of track and y = weekday ridership in thousands.

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{203}{7} = 29 \quad \bar{y} = \frac{\Sigma y_i}{n} = \frac{309}{7} = 44.1429$$

$$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 1471 \quad \Sigma(x_i - \bar{x})^2 = 838$$

$$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{1471}{838} = 1.7554$$

$$b_0 = \bar{y} - b_1\bar{x} = 44.1429 - (1.7554)(29) = -6.76$$

$$\hat{y} = -6.76 + 1.755x$$

b. SST =3620.9 SSE = 1038.7 SSR = 2582.1

$r^2 = \text{SSR/SST} = 2582.1/3620.9 = .713$

The estimated regression equation explained 71.3% of the variability in y; a good fit.

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c. $s^2 = \text{MSE} = 1038.7/5 = 207.7$

$s = \sqrt{207.7} = 14.41$

$$s_{\hat{y}^*} = s\sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}} = 14.41\sqrt{\frac{1}{7} + \frac{(30-29)^2}{838}} = 5.47$$

$\hat{y}^* = -6.76 + 1.755x^* = -6.76 + 1.755(30) = 45.9$

$45.9 \pm 2.571(5.47) = 45.9 \pm 14.1$

or 31.8 to 60

d. $$s_{\text{pred}} = s\sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\Sigma(x_i - \bar{x})^2}} = 14.41\sqrt{1 + \frac{1}{7} + \frac{(30-29)^2}{838}} = 15.41$$

$\hat{y}^* \pm t_{\alpha/2} s_{\text{pred}}$

$45.9 \pm 2.571(15.41) = 45.9 \pm 39.6$

or 6.3 to 85.5

The prediction interval is so wide that it would not be of much value in the planning process. A larger data set would be beneficial.

40. a. 9

b. $\hat{y} = 20.0 + 7.21x$

c. $t = 5.29$

From the t table (7 degrees of freedom), area in tail is less than .005.

p-value is less than .01

Actual p-value = .000

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$

d. SSE = SST - SSR = 51,984.1 - 41,587.3 = 10,396.8

MSE = 10,396.8 / 7 = 1,485.3

F = MSR / MSE = 41,587.3 / 1,485.3 = 28.00

From the F table (1 degree of freedom numerator and 7 denominator), p-value is less than .01

Actual p-value = .001

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$.

e. $\hat{y} = 20.0 + 7.21x = 20.0 + 7.21(50) = 380.5$ or $380,500

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41. a. $\hat{y} = 6.1092 + 0.8951x$

b. $t = \dfrac{b_1}{s_{b_1}} = \dfrac{0.8951}{0.149} = 6.01$

From the t table (8 degrees of freedom), area in tail is less than .005.

p-value is less than .01

Actual p-value = .000

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$; monthly maintenance expense is related to usage.

c. r^2 = SSR/SST = 1575.76/1924.90 = 0.82. A good fit.

42. a. $\hat{y} = 80.0 + 50.0x$

b. F = MSR / MSE = 6828.6 / 82.1 = 83.17

From the F table (1 degree of freedom numerator and 28 denominator), p-value is less than .01

Actual p-value = .000

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$.

Branch office sales are related to the salespersons.

c. $t = \dfrac{50}{5.482} = 9.12$

From the t table (28 degrees of freedom), area in tail is less than .005.

p-value is less than .01

Actual p-value = .000

Because p-value $\leq \alpha$, we reject H_0: $\beta_1 = 0$

d. $680,000

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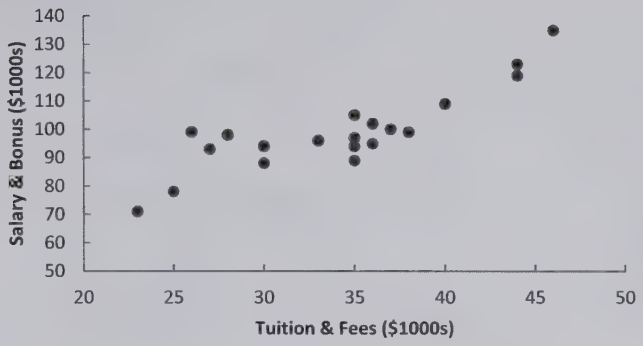
43. a.



b. There appears to be a positive relationship between the two variables. Students that graduate from the schools with higher tuition and fees tend to receive a higher starting salary and bonus.

The Excel output is shown below:

Regression Statistics	
Multiple R	0.8591
R Square	0.7381
Adjusted R Square	0.7236
Standard Error	7.6087
Observations	20

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	2937.1252	2937.1252	50.7336	1.22948E-06
Residual	18	1042.0748	57.8930		
Total	19	3979.2			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	33.7880	9.3398	3.6176	0.0020	14.1658	53.4102
Tuition & Fees ($1000s)	1.9154	0.2689	7.1228	1.22948E-06	1.3505	2.4804

d. The p-value = .000 < α = .05 (t or F); significant relationship

e. r^2 = .7381. The least squares line provided a good fit; approximately 74% of the variability in salary and bonus can be explained by the linear relationship with tuition and fees.

f. $\hat{y} = 33.788 + 1.9154(43) = 116.15$ or approximately $116,000.

Note to Instructor: The average starting salary and bonus reported by U.S. News & World Report for the University of Virginia was $121,000.

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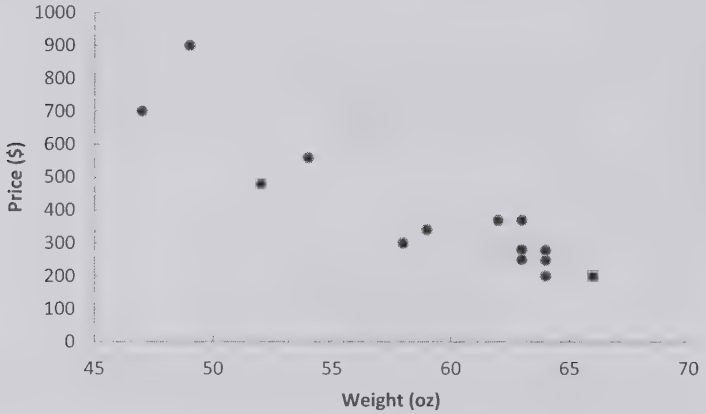
44. a. Scatter diagram:



b. There appears to be a negative linear relationship between the two variables. The heavier helmets tend to be less expensive.

c. The Excel output is shown below:

Regression Statistics	
Multiple R	0.8800
R Square	0.7743
Adjusted R Square	0.7602
Standard Error	91.8098
Observations	18

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	462761.145	462761.15	54.9008	1.47771E-06
Residual	16	134864.6328	8429.0395		
Total	17	597625.7778			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	2044.3809	226.3543	9.0318	1.111E-07	1564.5313	2524.2306
Weight	-28.3499	3.8261	-7.4095	1.478E-06	-36.4609	-20.2388

$\hat{y}$ = 2044.4 − 28.35 Weight

d. Significant relationship: p-value = .000 < α = .05

e. $r^2 = 0.774$; A good fit

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45. a. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{70}{5} = 14 \quad \bar{y} = \frac{\Sigma y_i}{n} = \frac{76}{5} = 15.2$

$\Sigma(x_i - \bar{x})(y_i - \bar{y}) = 200 \quad \Sigma(x_i - \bar{x})^2 = 126$

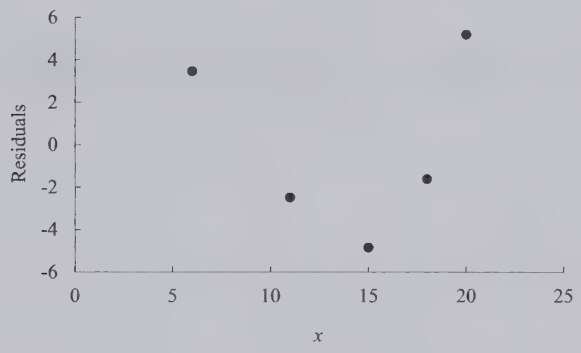
$b_1 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{200}{126} = 1.5873$

$b_0 = \bar{y} - b_1\bar{x} = 15.2 - (1.5873)(14) = -7.0222$

$\hat{y} = -7.02 + 1.59x$

b. The residuals are 3.48, -2.47, -4.83, -1.6, and 5.22

c.



With only 5 observations it is difficult to determine if the assumptions are satisfied. However, the plot does suggest curvature in the residuals that would indicate that the error term assumptions are not satisfied. The scatter diagram for these data also indicates that the underlying relationship between x and y may be curvilinear.

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46. a. $\hat{y} = 2.32 + .64x$

b.

The assumption that the variance is the same for all values of x is questionable. The variance appears to increase for larger values of x.

47. a. Let x = advertising expenditures and y = revenue

$\hat{y} = 29.4 + 1.55x$

b. SST = 1002 SSE = 310.28 SSR = 691.72

MSR = SSR / 1 = 691.72

MSE = SSE / $(n - 2)$ = 310.28/ 5 = 62.0554

F = MSR / MSE = 691.72/ 62.0554= 11.15

Using F table (1 degree of freedom numerator and 5 denominator), p-value is between .01 and .025

Using Excel, the p-value corresponding to F = 11.15 is .0206.

Because p-value $\leq \alpha$ = .05, we conclude that the two variables are related.

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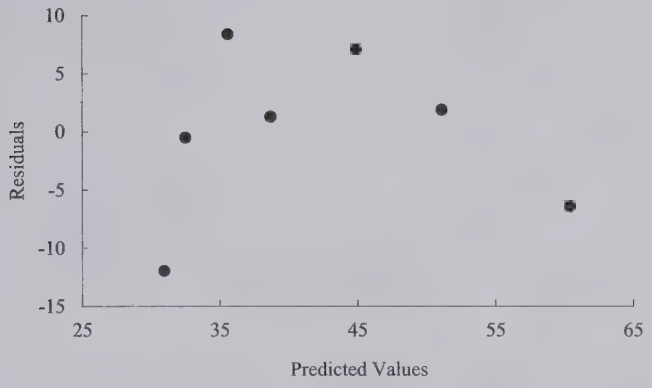
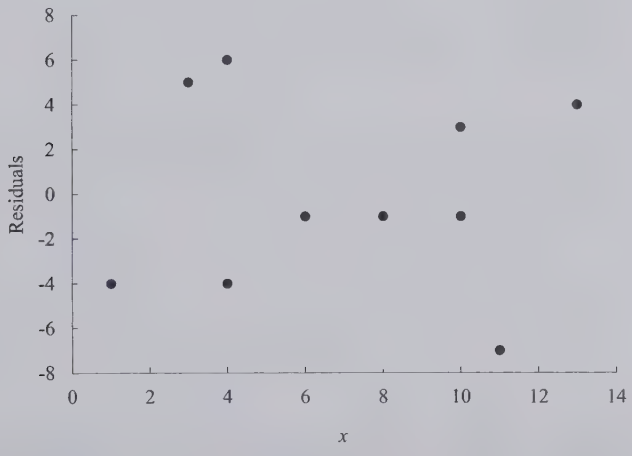
c.



d. The residual plot leads us to question the assumption of a linear relationship between x and y. Even though the relationship is significant at the .05 level of significance, it would be extremely dangerous to extrapolate beyond the range of the data.

48. a. $\hat{y} = 80 + 4x$

b.



c. The assumptions concerning the error term appear reasonable.

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49. a. The Excel output follows:

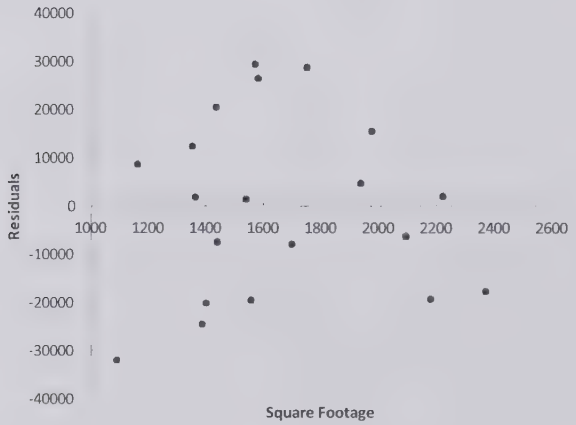
Regression Statistics	
Multiple R	0.7547
R Square	0.5695
Adjusted R Square	0.5456
Standard Error	19166
Observations	20

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	8748562231	8748562231	23.8163	0.0001
Residual	18	6612039769367335542.7			
Total	19	15360602000			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	22635.949	20460.0712	1.1063	0.2831	-20349.0654	65620.9636
Square Footage	58.9595	12.0814	4.8802	0.0001	33.5775	84.3416

b.



c. The residual plot leads us to question the assumption of a linear relationship between square footage and price. Therefore, even though the relationship is very significant (p-value = .0001), using the estimated regression equation make predictions of the price for a house with square footage beyond the range of the data is not recommended.

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50. a. The Excel output is shown below:

Regression Statistics	
Multiple R	0.8624
R Square	0.7438
Adjusted R Square	0.7118
Standard Error	1.4193
Observations	10

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	46.7838	46.7838	23.2233	0.0013
Residual	8	16.1162	2.0145		
Total	9	62.9			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	9.2649	1.0991	8.4293	2.99E-05
Shares	0.7105	0.1474	4.8191	0.0013

b. Since the p-value corresponding to $F = 23.223 = .0013 < \alpha = .05$, the relationship is significant.

c. $r^2 = .744$; a good fit. The least squares line explained 74.4% of the variability in Price.

d. $\hat{y} = 9.26 + .711(6) = 13.53$

51. a. The Excel output is shown below:

Regression Statistics	
Multiple R	0.8772
R Square	0.7694
Adjusted R Square	0.7606
Standard Error	12.0064
Observations	28

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	12507.0298	12507.0298	86.7612	9.10403E-10
Residual	26	3748.0199	144.154611		
Total	27	16255.0497			

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	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-2.9874	5.7914	-0.5158	0.6103	-14.8919	8.9170
Fair Value ($)	0.9113	0.0978	9.3146	9.10403E-10	0.7102	1.1124

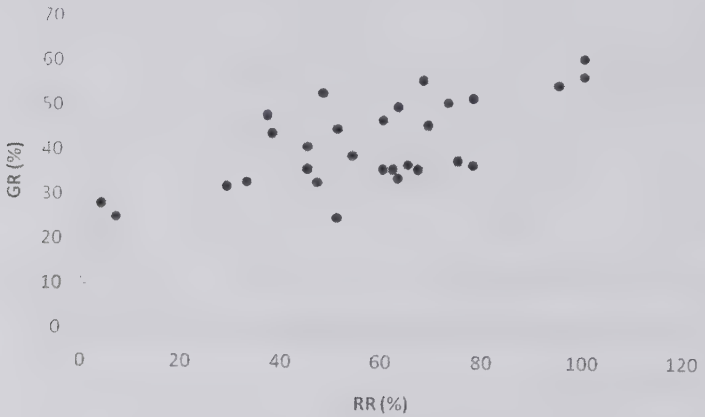
$\hat{y}$ = -2.9874 + .9113 Fair Value ($)

b. Significant relationship: p-value = .000 < α = .05

c. $\hat{y}$ = -2.9874 + .9113 Fair Value ($) = -2.9874 + .9113(50) = 42.5776 or approximately $42.58

d. The estimated regression equation should provide a good estimate because r^2 = 0.7694

52. a.



The scatter diagram indicates a positive linear relationship between the two variables. Online universities with higher retention rates tend to have higher graduation rates.

b. The Excel output follows:

Regression Statistics	
Multiple R	0.6702
R Square	0.4492
Adjusted R Square	0.4288
Standard Error	7.4561
Observations	29

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	1224.2860	1224.2860	22.0221	6.95491E-05
Residual	27	1501.0244	55.5935		
Total	28	2725.3103			

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	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	25.4229	3.7463	6.7862	2.7441E-07	17.7362	33.1096
RR(%)	0.2845	0.0606	4.6928	6.95491E-05	0.1601	0.4089

c. Because the p-value = .000 < α =.05, the relationship is significant.

d. The estimated regression equation is able to explain 44.92% of the variability in the graduation rate based upon the linear relationship with the retention rate. It is not a great fit, but given the type of data, the fit is reasonably good.

e. With a retention rate of 51% it does appear that the graduation rate of 25% is low as compared to the results for other online universities. The president of South University should be concerned after looking at the data. Using the estimated regression equation, we estimate that the graduation rate at South University should be 25.4229 + .2845(51) = 40%.

f. With a retention rate of only 4%, the president of the University of Phoenix should be concerned after looking at the data.

53. The Excel output is shown below:

Regression Statistics	
Multiple R	0.9253
R Square	0.8562
Adjusted R Square	0.8382
Standard Error	4.2496
Observations	10

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	860.0509486	860.0509	47.6238	0.0001
Residual	8	144.4740514	18.0593		
Total	9	1004.525			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	10.5280	3.7449	2.8113	0.0228
Weekly Usage	0.9534	0.1382	6.9010	0.0001

a. $\hat{y} = 10.528 + .9534x$

b. Since the p-value corresponding to $F = 47.62 = .0001 < \alpha = .05$, we reject H_0: $\beta_1 = 0$.

c. The 95% prediction interval is 28.74 to 49.52 or \$2874 to \$4952

d. Yes, since the expected expense is \$3913.

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54. a. The Excel output is shown below:

Regression Statistics	
Multiple R	0.8597
R Square	0.7391
Adjusted R Square	0.6739
Standard Error	1.4891
Observations	6

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	25.1304	25.1304	11.3333	0.0281
Residual	4	8.8696	2.2174		
Total	5	34			

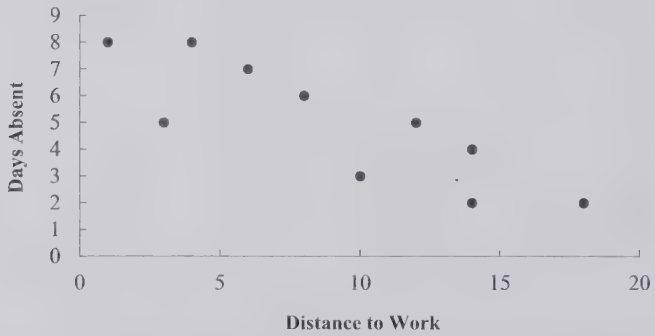
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	22.1739	1.6527	13.4164	0.0002
Line Speed	-0.1478	0.0439	-3.3665	0.0281

b. Since the p-value corresponding to $F = 11.33 = .0281 < \alpha = .05$, the relationship is significant.

c. $r^2 = .739$; a good fit. The least squares line explained 73.9% of the variability in the number of defects.

d. The 95% confidence interval is 12.294 to 17.271.

55. a. The scatter diagram follows:



A negative linear relationship appears to be reasonable.

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b. The Excel output is shown below:

Regression Statistics	
Multiple R	0.8431
R Square	0.7109
Adjusted R Square	0.6747
Standard Error	1.2894
Observations	10

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	32.6993	32.6993	19.6677	0.0022
Residual	8	13.3007	1.6626		
Total	9	46			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	8.0978	0.8088	10.0119	8.41E-06
Distance to Work	-0.3442	0.0776	-4.4348	0.0022

c. Since the *p*-value corresponding to $F = 419.67$ is $.0022 < \alpha = .05$. We reject $H_0 : \beta_1 = 0$.

d. $r^2 = .711$. The estimated regression equation explained 71.1% of the variability in y; this is a reasonably good fit.

e. The 95% confidence interval is 5.195 to 7.559 or approximately 5.2 to 7.6 days.

56. a. The Excel output is shown below:

Regression Statistics	
Multiple R	0.9341
R Square	0.8725
Adjusted R Square	0.8566
Standard Error	75.4983
Observations	10

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	312050	312050	54.7456	7.62662E-05
Residual	8	45600	5700		
Total	9	357650			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	220	58.4808	3.7619	0.0055
Age	131.6667	17.7951	7.3990	7.63E-05

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b. Since the p-value corresponding to $F = 54.75$ is $.000 < \alpha = .05$, we reject H_0: $\beta_1 = 0$.

c. $r^2 = .873$. The least squares line provided a very good fit.

d. The 95% prediction interval is 559.5 to 933.9 or \$559.50 to \$933.90

57. a. The Excel output is shown below:

Regression Statistics	
Multiple R	0.9369
R Square	0.8777
Adjusted R Square	0.8624
Standard Error	7.5231
Observations	10

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	3249.720752	3249.721	57.4182	6.43959E-05
Residual	8	452.7792483	56.5974		
Total	9	3702.5			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	5.8470	7.9717	0.7335	0.4842
Hours Studying	0.8295	0.1095	7.5775	6.44E-05

b. Since the p-value corresponding to $F = 57.42$ is $.000 < \alpha = .05$, we reject H_0: $\beta_1 = 0$.

c. 84.65 points

d. The 95% prediction interval is 65.35 to 103.96

58. A portion of the Excel Regression tool output for this problem follows.

Regression Statistics	
Multiple R	0.6852
R Square	0.4695
Adjusted R Square	0.4032
Standard Error	2.6641
Observations	10

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ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	50.2554	50.2554	7.0807	0.0288
Residual	8	56.7806	7.0976		
Total	9	107.036			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.2747	0.9004	0.3051	0.7681
S&P 500	0.9498	0.3569	2.6609	0.0288

a. $\hat{y} = 0.2747 + 0.9498$ S&P 500 Market beta = .95

b. Since the p-value = 0.029 is less than $\alpha = .05$, the relationship is significant.

c. $r^2 = .470$. The least squares line does not provide a very good fit.

d. Xerox has higher risk.

59. a. The Excel output is shown below:

Regression Statistics	
Multiple R	0.4659
R Square	0.2171
Adjusted R Square	0.1736
Standard Error	0.2088
Observations	20

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.2175	0.2175	4.9901	0.0384
Residual	18	0.7845	0.0436		
Total	19	1.002			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-0.4710	0.5842	-0.8061	0.4307
Adjusted Gross Income	3.86778E-05	1.73143E-05	2.2339	0.0384

b. Since the p-value = 0.0384 is less than $\alpha = .05$, the relationship is significant.

c. $r^2 = .217$. The least squares line does not provide a very good fit.

d. The 95% confidence interval is .7729 to .9927.

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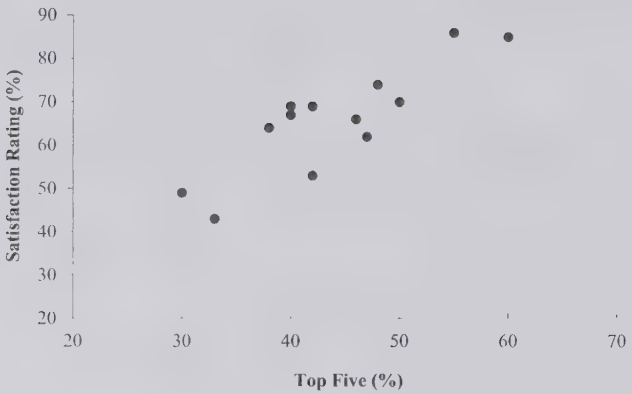
60. a.



b. There appears to be a positive linear relationship between the two variables.

A portion of the Excel Regression tool output for this problem follows.

Regression Statistics	
Multiple R	0.8610
R Square	0.7413
Adjusted R Square	0.7178
Standard Error	6.6265
Observations	13

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	1383.9115	1383.911	31.5169	0.0002
Residual	11	483.0116	43.9101		
Total	12	1866.9231			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	9.3742	10.2391	0.9155	0.3795
Top Five (%)	1.2875	0.2293	5.6140	0.0002

c. $\hat{y} = 9.3742 + 1.2875$ Top Five (%)

d. Since the p-value = .0002 < α = .05, the relationship is significant.

e. $r^2 = .7413$; a good fit. The least squares line explained 74.13% of the variability in the satisfaction rating.

f. $r = \sqrt{r^2} = \sqrt{.7413} = .86$

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Chapter 13
Multiple Regression

Learning Objectives

1. Understand how multiple regression analysis can be used to develop relationships involving one dependent variable and several independent variables.

2. Be able to interpret the coefficients in a multiple regression analysis.

3. Know the assumptions necessary to conduct statistical tests involving the hypothesized regression model.

4. Understand the role of Excel in performing multiple regression analysis.

5. Be able to interpret and use Excel's Regression tool output to develop the estimated regression equation.

6. Be able to determine how good a fit is provided by the estimated regression equation.

7. Be able to test for the significance of the regression equation.

8. Understand how multicollinearity affects multiple regression analysis.

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Solutions:

1. a. $b_1 = .5906$ is an estimate of the change in y corresponding to a 1 unit change in x_1 when x_2 is held constant.

   $b_2 = .4980$ is an estimate of the change in y corresponding to a 1 unit change in x_2 when x_1 is held constant.

   b. $\hat{y} = 29.1270 + 5906(180) + .4980(310) = 289.815$

2. a. The Excel output is shown below:

Regression Statistics	
Multiple R	0.8124
R Square	0.6600
Adjusted R Square	0.6175
Standard Error	25.4009
Observations	10

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	10021.24739	10021.25	15.5318	0.0043
Residual	8	5161.652607	645.2066		
Total	9	15182.9			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	45.0594	25.4181	1.7727	0.1142
X1	1.9436	0.4932	3.9410	0.0043

An estimate of y when $x_1 = 45$ is

$\hat{y} = 45.0594 + 1.9436(45) = 132.52$

b. The Excel output is shown below:

Regression Statistics	
Multiple R	0.4707
R Square	0.2215
Adjusted R Square	0.1242
Standard Error	38.4374
Observations	10

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ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	3363.4142	3363.414	2.2765	0.1698
Residual	8	11819.4858	1477.436		
Total	9	15182.9			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	85.2171	38.3520	2.2220	0.0570
X2	4.3215	2.8642	1.5088	0.1698

An estimate of y when $x_2 = 15$ is

$\hat{y} = 85.2171 + 4.3215(15) = 150.04$

c. The Excel output is shown below:

Regression Statistics	
Multiple R	0.9620
R Square	0.9255
Adjusted R Square	0.9042
Standard Error	12.7096
Observations	10

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	14052.15497	7026.077	43.4957	0.0001
Residual	7	1130.745026	161.535		
Total	9	15182.9			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-18.3683	17.97150328	-1.0221	0.3408
X1	2.0102	0.2471	8.1345	8.19E-05
X2	4.7378	0.9484	4.9954	0.0016

An estimate of y when $x_1 = 45$ and $x_2 = 15$ is

$\hat{y} = -18.3683 + 2.0102(45) + 4.7378(15) = 143.16$

3. a. $b_1 = 3.8$ is an estimate of the change in y corresponding to a 1 unit change in x_1 when x_2, x_3, and x_4 are held constant.

$b_2 = -2.3$ is an estimate of the change in y corresponding to a 1 unit change in x_2 when x_1, x_3, and x_4 are held constant.

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$b_3 = 7.6$ is an estimate of the change in y corresponding to a 1 unit change in x_3 when x_1, x_2, and x_4 are held constant.

$b_4 = 2.7$ is an estimate of the change in y corresponding to a 1 unit change in x_4 when x_1, x_2, and x_3 are held constant.

b. $\hat{y} = 17.6 + 3.8(10) - 2.3(5) + 7.6(1) + 2.7(2) = 57.1$

4. a. $\hat{y} = 25 + 10(15) + 8(10) = 255$; sales estimate: \$255,000

b. Sales can be expected to increase by \$10 for every dollar increase in inventory investment when advertising expenditure is held constant. Sales can be expected to increase by \$8 for every dollar increase in advertising expenditure when inventory investment is held constant.

5. a. The Excel output is shown below:

Regression Statistics	
Multiple R	0.8078
R Square	0.6526
Adjusted R Square	0.5946
Standard Error	1.2152
Observations	8

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	16.6401	16.6401	11.2688	0.0153
Residual	6	8.8599	1.4767		
Total	7	25.5			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	88.6377	1.5824	56.0159	2.174E-09
Television Advertising (\$1000s)	1.6039	0.4778	3.3569	0.0153

$\hat{y} = 88.6377 + 1.6039x_1$

where x_1 = television advertising (\$1000s)

b. The Excel output is shown below:

Regression Statistics	
Multiple R	0.9587
R Square	0.9190
Adjusted R Square	0.8866
Standard Error	0.6426
Observations	8

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ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	23.4354	11.7177	28.3778	0.0019
Residual	5	2.0646	0.4129		
Total	7	25.5			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	83.2301	1.5739	52.8825	4.57E-08
Television Advertising ($1000s)	2.2902	0.3041	7.5319	0.0007
Newspaper Advertising ($1000s)	1.3010	0.3207	4.0567	0.0098

$\hat{y} = 83.2301 + 2.2902x_1 + 1.3010x_2$

where

x_1 = television advertising ($1000s)

x_2 = newspaper advertising ($1000s)

c. It is 1.6039 in part (a) and 2.2902 in part (b); in part (a) the coefficient is an estimate of the change in revenue due to a one-unit change in television advertising expenditures; in part (b) it represents an estimate of the change in revenue due to a one-unit change in television advertising expenditures with the amount of newspaper advertising is held constant

d. Revenue = 83.2301 + 2.2902(3.5) + 1.3010(1.8) = $93.59 or $93,590

6. a. A portion of the Excel output is shown below:

Regression Statistics	
Multiple R	0.7831
R Square	0.6133
Adjusted R Square	0.5918
Standard Error	9.2459
Observations	20

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	2440.4426	2440.443	28.5477	4.44604E-05
Residual	18	1538.7574	85.4865		
Total	19	3979.2			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-20.0228	22.4094	-0.8935	0.3834	-67.1032	27.0576
Recruiter Score	33.7264	6.3123	5.3430	4.45E-05	20.4648	46.9880

The estimated regression equation is $\hat{y}$ = - 20.0228 + 33.7264 Recruiter Score

Note that a higher average recruiter score results in a higher starting salary and bonus.

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b. A portion of the Excel output is shown below:

Regression Statistics	
Multiple R	0.8904
R Square	0.7928
Adjusted R Square	0.7684
Standard Error	6.9645
Observations	20

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	3154.6231	1577.3115	32.5189	1.54775E-06
Residual	17	824.5769	48.5045		
Total	19	3979.2			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.8790	17.7371	0.0496	0.9611	-36.5431	38.3011
Recruiter Score	14.5907	6.8903	2.1176	0.0493	0.0534	29.1281
Tuition & Fees ($)	1.3687	0.3567	3.8372	0.0013	0.6162	2.1213

The estimated regression equation is
$\hat{y}$ = 0.8790 + 14.5907 Recruiter Score + 1.3687 Tuition & Fees ($1000s)

c. The estimated regression coefficient for the recruiter assessment score is not the same in parts (a) and (b). In part (a) the estimated regression coefficient is 33.7264 and in part (b) it is 14.5907. In part (a) the estimated coefficient is the change in salary & bonus ($1000s) due to a one point increase in the recruiter assessment score with nothing known about the out-of-state tuition and fees. In part (b), the coefficient is the marginal change in salary & bonus ($1000s) due to a one point increase in the recruiter assessment score with out-of-state tuition and fees held constant.

d. $\hat{y}$ =.8790 +14.5907(4.1) +1.3687(43) = 119.5550 or approximately $120,000

Note to Instructor: The average starting salary and bonus reported by U.S. News & World Report for the University of Virginia business school graduates was $121,000.

7. a. A portion of the Excel output is shown below:

Regression Statistics	
Multiple R	0.7432
R Square	0.5524
Adjusted R Square	0.4965
Standard Error	2.5922
Observations	10

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ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	66.3434	66.3434	9.8731	0.0138
Residual	8	53.7566	6.7196		
Total	9	120.1			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	66.0623	3.7934	17.4153	1.2052E-07	57.3148	74.8098
Performance	0.1699	0.0541	3.1422	0.0138	0.0452	0.2946

The estimated regression equation is
$\hat{y}$ = 66.0623 + 0.1699 Performance

b. A portion of the Excel output is shown below:

Regression Statistics	
Multiple R	0.9148
R Square	0.8369
Adjusted R Square	0.7903
Standard Error	1.6729
Observations	10

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	100.5109	50.255	17.9584	0.0018
Residual	7	19.5891	2.7984		
Total	9	120.1			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	39.9820	7.8551	5.09	0.0014	21.4077	58.5562
Performance	0.1134	0.0385	2.9482	0.0215	0.0224	0.2043
Features	0.3820	0.1093	3.4942	0.0101	0.1235	0.6406

The estimated regression equation is
$\hat{y}$ = 39.9820 + 0.1134 Performance + 0.3820 Features

c. $\hat{y}$ = 39.9820 + .1134(80) + .3820(70) = 75.7940 or 76

8. a. A portion of the Excel output is shown below:

Regression Statistics	
Multiple R	0.3597
R Square	0.1294
Adjusted R Square	0.1013
Standard Error	3717.8542
Observations	33

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ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	63665063.15	63665063	4.6059	0.0398
Residual	31	428495630.7	13822440		
Total	32	492160693.9			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	31054.26	2216.6088	14.0098	5.917E-15	26533.4562	35575.0631
Reliability	-1328.722	619.1214	-2.1461	0.0398	-2591.4283	-66.0154

The estimated regression equation is $\hat{y} = 31054.26 - 1328.722$ Reliability

Because the p-value = .0398 < α = .05, there is a significant relationship between price and the reliability rating.

b. A portion of the Excel output is shown below:

Regression Statistics	
Multiple R	0.4921
R Square	0.2421
Adjusted R Square	0.1916
Standard Error	3526.0357
Observations	33

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	119172859.8	59586430	4.7926	0.0156
Residual	30	372987834	12432928		
Total	32	492160693.9			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	21312.9147	5066.9762	4.2062	0.0002	10964.7689	31661.0606
Road-Test Score	136.6908	64.6918	2.1130	0.0430	4.5726	268.8090
Reliability	-1446.3449	589.8114	-2.4522	0.0202	-2650.9005	-241.7892

The estimated regression equation is
$\hat{y} = 21{,}312.9147 + 136.6908$ Road-Test Score $- 1446.3449$ Reliability

c. $\hat{y} = 21{,}312.9147 + 136.6908(80) - 1446.3449(4) = 26{,}462.7991$ or approximately \$26,463

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9. a. A portion of the Excel Regression tool output is shown below:

Regression Statistics	
Multiple R	0.7728
R Square	0.5972
Adjusted R Square	0.5498
Standard Error	1.5954
Observations	20

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	64.1567	32.0783	12.6033	0.0004
Residual	17	43.2688	2.5452		
Total	19	107.4255			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	64.9659	9.0087	7.2115	1.45E-06
Beam	-0.3896	0.0958	-4.0671	0.0008
HP	0.0511	0.0131	3.8927	0.0012

b. $\hat{y} = 64.9659 - .3896 \text{ Beam} + .0511 \text{ HP} = 64.9659 - .3896(85) + .0511(330) = 48.7$

Thus, an estimate of the top speed for the Svfara SV609 is 48.7 mph.

10. a. A portion of the Excel output is shown below:

Regression Statistics	
Multiple R	0.4482
R Square	0.2008
Adjusted R Square	0.1712
Standard Error	0.1266
Observations	29

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.1088	0.1088	6.7857	0.0148
Residual	27	0.4330	0.0160		
Total	28	0.5418			

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	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-1.2207	0.6617	-1.8448	0.0761
FG%	3.9576	1.5193	2.6049	0.0148

$\hat{y}$ = -1.2207 + 3.9576 FG%

b. An increase of 1% in the percentage of field goals made will increase the percentage of games won by 3.9576(.01) = .039576 or approximately .04.

c. A portion of the Excel output is shown below:

Regression Statistics	
Multiple R	0.7508
R Square	0.5638
Adjusted R Square	0.5114
Standard Error	0.0972
Observations	29

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.3055	0.1018	10.7697	9.94428E-05
Residual	25	0.2364	0.0095		
Total	28	0.5418			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-1.2346	0.6003	-2.0568	0.0503
FG%	4.8166	1.1830	4.0713	0.0004
Opp 3 Pt%	-2.5895	0.7041	-3.6777	0.0011
Opp TO	0.0344	0.0125	2.7469	0.0110

$\hat{y}$ = -1.2346 + 4.8166 FG% - 2.5895 Opp 3 PT% + .0344 Opp TO

d. To increase the percentage of games won a team needs to increase the percentage of field goals made, decrease the percentage of three-point shots made by the team's opponent, and increase the number of turnovers committed by the team's opponent.

e. $\hat{y}$ = -1.2346 + 4.8166(.45) -2.5895(.34) + .0344(17) = .6372

11. a. SSE = SST - SSR = 6,724.125 - 6,216.375 = 507.75

b. $R^2 = \frac{\text{SSR}}{\text{SST}} = \frac{6,216.375}{6,724.125} = .924$

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c. $R_a^2 = 1-(1-R^2)\dfrac{n-1}{n-p-1} = 1-(1-.924)\dfrac{10-1}{10-2-1} = .902$

d. The estimated regression equation provided an excellent fit.

12. a. $R^2 = \dfrac{\text{SSR}}{\text{SST}} = \dfrac{14,052.2}{15,182.9} = .926$

b. $R_a^2 = 1-(1-R^2)\dfrac{n-1}{n-p-1} = 1-(1-.926)\dfrac{10-1}{10-2-1} = .905$

c. Yes; after adjusting for the number of independent variables in the model, we see that 90.5% of the variability in y has been accounted for.

13. a. $R^2 = \dfrac{\text{SSR}}{\text{SST}} = \dfrac{1760}{1805} = .975$

b. $R_a^2 = 1-(1-R^2)\dfrac{n-1}{n-p-1} = 1-(1-.975)\dfrac{30-1}{30-4-1} = .971$

c. The estimated regression equation provided an excellent fit.

14. a. $R^2 = \dfrac{\text{SSR}}{\text{SST}} = \dfrac{12,000}{16,000} = .75$

b. $R_a^2 = 1-(1-R^2)\dfrac{n-1}{n-p-1} = 1-.25\dfrac{9}{7} = .68$

c. The adjusted coefficient of determination shows that 68% of the variability has been explained by the two independent variables; thus, we conclude that the model does not explain a large amount of variability.

15. a. $R^2 = \dfrac{\text{SSR}}{\text{SST}} = \dfrac{23.435}{25.5} = .919$

$R_a^2 = 1-(1-R^2)\dfrac{n-1}{n-p-1} = 1-(1-.919)\dfrac{8-1}{8-2-1} = .887$

b. Multiple regression analysis is preferred since both R^2 and R_a^2 show an increased percentage of the variability of y explained when both independent variables are used.

16. Note: the Excel output is shown in the solution to Exercise 6

a. The coefficient of determination is $r^2 = .6133$; this is not too bad a fit using only one independent variable.

b. Using both independent variables provides a better fit. $R^2 = .793$ and $R_a^2 = .768$

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The multiple linear regression equation with recruiter assessment score and the out-of-state tuition and fees as the independent variables will provide a better estimate of the starting salary and bonus of the graduates.

17. a. $R^2 = \frac{\text{SSR}}{\text{SST}} = \frac{64.1567}{107.4255} = .5972$

$$R_a^2 = 1-(1-R^2)\frac{n-1}{n-p-1} = 1-(1-.5972)\frac{20-1}{20-2-1} = .5498$$

b. The fit is not very good

18. a. $r^2 = .5638$ and $R_a^2 = .5114$

b. Although the fit is not very good, the estimated regression equation does explain over 50% of the variability in the dependent variable.

19. a. MSR = SSR/p = 6,216.375/2 = 3,108.188

$$\text{MSE} = \frac{\text{SSE}}{n-p-1} = \frac{507.75}{10-2-1} = 72.536$$

b. F = MSR/MSE = 3,108.188/72.536 = 42.85

p-value (2 degrees of freedom numerator and 7 denominator) = .0001

Because the p-value $\leq \alpha = .05$, the overall model is significant.

c. $t = .5906/.0813 = 7.26$

p-value (7 degrees of freedom) = .0002

Because the p-value $\leq \alpha = .05$, β_1 is significant.

d. $t = .4980/.0567 = 8.78$

p-value (7 degrees of freedom) = .0001

Because the p-value $\leq \alpha = .05$, β_2 is significant.

20. A portion of the Excel output is shown below.

Regression Statistics	
Multiple R	0.9620
R Square	0.9255
Adjusted R Square	0.9042
Standard Error	12.7096
Observations	10

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ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	14052.15497	7026.077	43.4957	0.0001
Residual	7	1130.745026	161.535		
Total	9	15182.9			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-18.36826758	17.97150328	-1.0221	0.3408
X1	2.0102	0.2471	8.1345	8.19E-05
X2	4.7378	0.9484	4.9954	0.0016

a. Since the *p*-value corresponding to $F = 43.4957$ is $.0001 < \alpha = .05$, we reject H_0: $\beta_1 = \beta_2 = 0$; there is a significant relationship.

b. Since the *p*-value corresponding to $t = 8.1345$ is $.0000 < \alpha = .05$, we reject H_0: $\beta_1 = 0$; β_1 is significant.

c. Since the *p*-value corresponding to $t = 4.9954$ is $.0016 < \alpha = .05$, we reject H_0: $\beta_2 = 0$; β_2 is significant.

21. a. In the two independent variable case the coefficient of x_1 represents the expected change in y corresponding to a one unit increase in x_1 when x_2 is held constant. In the single independent variable case the coefficient of x_1 represents the expected change in y corresponding to a one unit increase in x_1.

b. Yes. If x_1 and x_2 are correlated, one would expect a change in the coefficient of x_1 when x_2 is dropped from the model.

22. a. SSE = SST - SSR = 16000 - 12000 = 4000

$$s^2 = \frac{\text{SSE}}{n-p-1} = \frac{4000}{7} = 571.43$$

$$\text{MSR} = \frac{\text{SSR}}{p} = \frac{12000}{2} = 6000$$

b. F = MSR/MSE = 6000/571.43 = 10.50

p-value (2 degrees of freedom numerator and 7 denominator) = .0078

Because the *p*-value $\leq \alpha = .05$, we reject H_0. There is a significant relationship among the variables.

23. a. $F = 28.3778$

p-value (2 degrees of freedom, numerator and 5 denominator) = .0019

Because the *p*-value $\leq \alpha = .01$, we reject H_0.

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b. $t = 7.5319$

p-value = .0007

Because the p-value $\leq \alpha = .05$, β_1 is significant and x_1 should not be dropped from the model.

c. $t = 4.0567$

p-value = .0098

Because the p-value $\leq \alpha = .05$, β_2 is significant and x_2 should not be dropped from the model.

24. a. A portion of the Regression tool output is shown below:

Regression Statistics	
Multiple R	0.5646
R Square	0.3187
Adjusted R Square	0.2809
Standard Error	0.3286
Observations	39

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	1.8188	0.9094	8.4210	0.001
Residual	36	3.8877	0.1080		
Total	38	5.7065			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-0.6820	0.5044	-1.3523	0.1847
Revenue	0.0498	0.0135	3.7042	0.0007
%Wins	0.0147	0.0063	2.3338	0.0253

b. Because the p-value = .001< $\alpha = .05$, there is a significant relationship.

c. For Revenue: Because the p-value = .0007 < $\alpha = .05$, Revenue is significant.

For %Wins: Because the p-value = .0253 < $\alpha = .05$, %Wins is significant.

25. a. A portion of the Excel output is shown below:

Regression Statistics	
Multiple R	0.8498
R Square	0.7222
Adjusted R Square	0.6702
Standard Error	744.8942
Observations	20

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ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	23083887.82	7694629.3	13.8675	0.0001
Residual	16	8877877.129	554867.32		
Total	19	31961764.95			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	18861.259	1758.2643	10.7272	1.025E-08	15133.9048	22588.6124
MPG	-281.6487	74.8026	-3.7652	0.0017	-440.2232	-123.0742
Reliability	207.5047	172.0310	1.2062	0.2453	-157.1848	572.1942
Safety	1700.4603	285.8616	5.9485	2.042E-05	1094.4608	2306.4599

The estimated regression equation is

$\hat{y}$ = 18861.259 – 281.6487 MPG + 207.5047 Reliability + 1700.4603 Safety

b. Because the *p*-value = .0001 < α = .05, there is a significant relationship.

c. MPG: Because the *p*-value = .0017 < α = .05, MPG is significant.

Reliability: Because the *p*-value = .2453 > α = .05, Reliability is not significant.

Safety: Because the *p*-value = .000 < α = .05, Safety is significant.

Recommend removing Reliability from the estimated regression equation.

d. A portion of the Excel output after removing Reliability is shown below:

Regression Statistics	
Multiple R	0.8349
R Square	0.6970
Adjusted R Square	0.6613
Standard Error	754.7953
Observations	20

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	22276594.46	11138297	19.5506	3.91337E-05
Residual	17	9685170.493	569715.91		
Total	19	31961764.95			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	18769.427	1779.9641	10.5448	7.075E-09	15014.0313	22524.8235
MPG	-256.2143	72.7230	-3.5232	0.0026	-409.6464	-102.7821
Safety	1771.1027	283.5170	6.2469	8.851E-06	1172.9341	2369.2714

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The estimated regression equation is

$\hat{y}$ = 18769.457 – 256.2143 MPG + 1771.1027 Safety

The coefficient of determination for the estimated regression equation developed in part (a) is .7222. After removing Reliability, the coefficient of determination is .6970. There is very little difference in the fit provided by the two estimated regression equations. But, because Reliability is not significant, this result is expected. The estimated regression equation shown in part (d) is recommended.

26. a. Since the *p*-value corresponding to $F = 10.7697$ is $.0000 < \alpha = .05$, there is a significant relationship between the percentage of games won and the independent variables.

    b. All of the independent variables are significant because the *p*-values corresponding to the *t* test are all less than $\alpha = .05$ (see computer output in 10c).

27. a. $\hat{y} = 29.1270 + .5906(180) + .4980(310) = 289.8150$

    b. The point estimate for an individual value is $\hat{y} = 289.8150$, the same as the point estimate of the mean value.

28. a. $\hat{y} = -18.4 + 2.01(45) + 4.74(15) = 143.15$

    b. Using StatTools, the 95% prediction interval is 111.16 to 175.16.

29. a. $\hat{y} = 83.2 + 2.29(3.5) + 1.30(1.8) = 93.555$ or $93,555

    b. Using StatTools, the prediction interval estimate: 91.774 to 95.401 or $91,774 to $95,401

30. a. $\hat{y} = 64.9659 - .3896(85) + .0511(330) = 48.7$ or approximately 49

    b. The 95% prediction interval for the Svfara SV609 is 44.815 to 52.589.

31. a. A portion of the Excel output is shown below:

Regression Statistics	
Multiple R	0.9299
R Square	0.8647
Adjusted R Square	0.8241
Standard Error	0.1895
Observations	14

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	2.2958	0.7653	21.3067	0.0001
Residual	10	0.3592	0.0359		
Total	13	2.6550			

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	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-0.0554	1.1284	-0.0491	0.9618	-2.5696	2.4589
Handling	0.2760	0.1213	2.2756	0.0461	0.0058	0.5463
Dependability	0.4469	0.1111	4.0234	0.0024	0.1994	0.6943
Fit and Finish	0.2703	0.0953	2.8370	0.0176	0.0580	0.4825

$\hat{y}$ = -.0554 + .2760 Handling + .4469 Dependability + .2703 Fit and Finish

b. $\hat{y}$ = -.0554 + .2760 Handling + .4469 Dependability + .2703 Fit and Finish

$\hat{y}$ = -.0554 + .2760(8.28) + .4469(9.06) + .2703(8.07) = 8.46

d. 95% prediction interval is 7.926 to 8.993

e. The point estimate 8.46 is very close to the rating of 8.65. Note that the 95% prediction interval includes the overall rating reported by Car and Driver.

32. a. $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$ where

$x_2 = 0$ if level 1 and 1 if level 2

b. $E(y) = \beta_0 + \beta_1 x_1 + \beta_2(0) = \beta_0 + \beta_1 x_1$

c. $E(y) = \beta_0 + \beta_1 x_1 + \beta_2(1) = \beta_0 + \beta_1 x_1 + \beta_2$

d. $\beta_2 = E(y \mid \text{level 2}) - E(y \mid \text{level 1})$

β_1 is the change in $E(y)$ for a 1 unit change in x_1 holding x_2 constant.

33. a. two

b. $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$ where

x_2	x_3	Level
0	0	1
1	0	2
0	1	3

c. $E(y \mid \text{level 1}) = \beta_0 + \beta_1 x_1 + \beta_2(0) + \beta_3(0) = \beta_0 + \beta_1 x_1$

$E(y \mid \text{level 2}) = \beta_0 + \beta_1 x_1 + \beta_2(1) + \beta_3(0) = \beta_0 + \beta_1 x_1 + \beta_2$

$E(y \mid \text{level 3}) = \beta_0 + \beta_1 x_1 + \beta_2(0) + \beta_3(0) = \beta_0 + \beta_1 x_1 + \beta_3$

$\beta_2 = E(y \mid \text{level 2}) - E(y \mid \text{level 1})$

$\beta_3 = E(y \mid \text{level 3}) - E(y \mid \text{level 1})$

β_1 is the change in $E(y)$ for a 1 unit change in x_1 holding x_2 and x_3 constant.

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34. a. $15,300

b. Estimate of sales = 10.1 - 4.2(2) + 6.8(8) + 15.3(0) = 56.1 or $56,100

c. Estimate of sales = 10.1 - 4.2(1) + 6.8(3) + 15.3(1) = 41.6 or $41,600

35. a. Let Type = 0 if a mechanical repair
Type = 1 if an electrical repair

The Excel output is shown below:

Regression Statistics	
Multiple R	0.2952
R Square	0.0871
Adjusted R Square	-0.0270
Standard Error	1.0934
Observations	10

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.9127	0.9127	0.7635	0.4077
Residual	8	9.5633	1.1954		
Total	9	10.476			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	3.45	0.5467	6.3109	0.0002
Type	0.6167	0.7058	0.8738	0.4077

$\hat{y}$ = 3.45 + .6167 Type

b. The estimated regression equation did not provide a good fit. In fact, the *p*-value of .4077 shows that the relationship is not significant for any reasonable value of α.

c. Person = 0 if Bob Jones performed the service and Person = 1 if Dave Newton performed the service. The Excel output is shown below:

Regression Statistics	
Multiple R	0.7816
R Square	0.6109
Adjusted R Square	0.5623
Standard Error	0.7138
Observations	10

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	6.4	6.4	12.5613	0.0076
Residual	8	4.076	0.5095		
Total	9	10.476			

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	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	4.62	0.3192	14.4729	5.08E-07
Person	-1.6	0.4514	-3.5442	0.0076

$\hat{y} = 4.62 - 1.6$ Person

d. We see that 61.1% of the variability in repair time has been explained by the repair person that performed the service; an acceptable, but not good, fit.

36. a. The Excel output is shown below:

Regression Statistics	
Multiple R	0.9488
R Square	0.900199692
Adjusted R Square	0.850299539
Standard Error	0.4174
Observations	10

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	9.4305	3.1435	18.0400	0.0021
Residual	6	1.0455	0.1743		
Total	9	10.476			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	1.8602	0.7286	2.5529	0.0433
Months Since Last Service	0.2914	0.0836	3.4862	0.0130
Type	1.1024	0.3033	3.6342	0.0109
Person	-0.6091	0.3879	-1.5701	0.1674

$\hat{y} = 1.8602 + .2914$ Months $+ 1.1024$ Type $- .6091$ Person

b. Since the *p*-value corresponding to $F = 18.04$ is $.0021 < \alpha = .05$, the overall model is statistically significant.

c. The *p*-value corresponding to $t = -1.5701$ is $.1674 > \alpha = .05$; thus, the addition of Person is not statistically significant. Person is highly correlated with Months (the sample correlation coefficient is -.691); thus, once the effect of Months has been accounted for, Person will not add much to the model.

37. a. A portion of the Excel output follows:

Regression Statistics	
Multiple R	0.5867
R Square	0.3442
Adjusted R Square	0.3097
Standard Error	3.0257
Observations	21

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ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	91.2902	91.2902	9.9715	0.0052
Residual	19	173.9479	9.1552		
Total	20	265.2381			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	69.2760	3.4005	20.3725	2.27644E-14	62.1588	76.3933
Price ($)	0.5586	0.1769	3.1578	0.0052	0.1883	0.9288

The estimated regression equation is $\hat{y} = 69.2760 + 0.5586$ Price

b. Because the p-value = .0052 < α = .05, there is a significant relationship.

c. Let Type_Italian = 1 if the restaurant is an Italian restaurant; 0 otherwise

d. A portion of the Excel output follows:

Regression Statistics	
Multiple R	0.7254
R Square	0.5262
Adjusted R Square	0.4736
Standard Error	2.6422
Observations	21

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	139.5771	69.789	9.9967	0.0012
Residual	18	125.6610	6.9812		
Total	20	265.2380952			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	67.4049	3.0535	22.075	1.74112E-14	60.9898	73.8199
Price ($)	0.5734	0.1546	3.7097	0.0016	0.2487	0.8982
Type_Italian	3.0382	1.1552	2.63	0.0170	0.6112	5.4652

The estimated regression equation is
$\hat{y} = 67.4049 + 0.5734$ Price $+ 3.0382$ Type_Italian

e. For the Type_Italian dummy variable, the p-value = .0170 < α = .05; thus, type of restaurant is a significant factor in overall customer satisfaction.

f. The estimated regression equation computed in part (d) is
$\hat{y} = 67.4049 + 0.5734$ Price $+ 3.0382$ Type_Italian

For a seafood/steakhouse Type_Italian = 0 and the estimated score is
$\hat{y} = 67.4049 + .5734(20) + 3.0382(0) = 78.8729$

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For an Italian restaurant Type_Italian = 1 and the estimated score is
$\hat{y} = 67.4049 + .5734(20) + 3.0382(1) = 81.9111$

Thus, the satisfaction score increases by 3.0382 points.

38. a. The Excel output is shown below:

Regression Statistics	
Multiple R	0.9346
R Square	0.8735
Adjusted R Square	0.8498
Standard Error	5.7566
Observations	20

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	3660.7396	1220.247	36.8230	2.06404E-07
Residual	16	530.2104	33.1382		
Total	19	4190.95			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-91.7595	15.2228	-6.0278	1.76E-05
Age	1.0767	0.1660	6.4878	7.49E-06
Pressure	0.2518	0.0452	5.5680	4.24E-05
Smoker	8.7399	3.0008	2.9125	0.0102

$\hat{y} = -91.7595 + 1.0767$ Age $+ .2518$ Pressure $+ 8.7399$ Smoker

b. Since the *p*-value corresponding to $t = 2.9125$ is $.0102 < \alpha = .05$, smoking is a significant factor.

c. The point estimate is 34.27; the 95% prediction interval is 21.35 to 47.18. Thus, the probability of a stroke (.2135 to .4718 at the 95% confidence level) appears to be quite high. The physician would probably recommend that Art quit smoking and begin some type of treatment designed to reduce his blood pressure.

39. a. The Excel output is shown below:

Regression Statistics	
Multiple R	0.7946
R Square	0.6314
Adjusted R Square	0.5393
Standard Error	7.2693
Observations	6

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	362.1304802	362.1305	6.8530	0.0589
Residual	4	211.3695198	52.8424		
Total	5	573.5			

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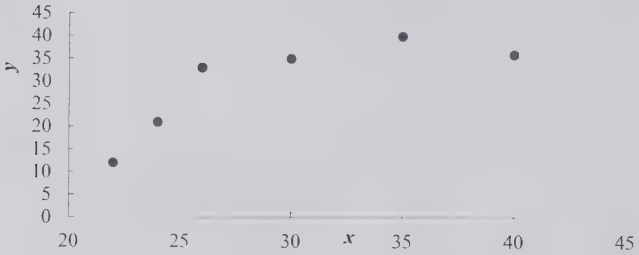
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-6.7745	14.1709	-0.4781	0.6576
x	1.2296	0.4697	2.6178	0.0589

b. Since the *p*-value corresponding to $F = 6.8530$ is $0.0589 > \alpha = .05$, the relationship is not significant.

c.



The scatter diagram suggests that a curvilinear relationship may be appropriate.

d. The Excel output is shown below:

Regression Statistics	
Multiple R	0.9720
R Square	0.9448
Adjusted R Square	0.9080
Standard Error	3.2482
Observations	6

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	541.8473	270.9236	25.6778	0.0130
Residual	3	31.6527	10.5509		
Total	5	573.5			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-168.8848	39.7862	-4.2448	0.0239
x	12.1870	2.6632	4.5760	0.0196
xsq	-0.1770	0.0429	-4.1271	0.0258

e. Since the *p*-value corresponding to $F = 25.6778$ is $.013 < \alpha = .05$, the relationship is significant.

f. $\hat{y} = -168.8848 + 12.1870(25) - 0.1770(25)^2 = 25.165$

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40. a. The Excel output is shown below:

Regression Statistics	
Multiple R	0.7833
R Square	0.6136
Adjusted R Square	0.4848
Standard Error	3.5311
Observations	5

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	59.3939	59.3939	4.7634	0.1171
Residual	3	37.4061	12.4687		
Total	4	96.8			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	9.3152	4.1961	2.2200	0.1130
x	0.4242	0.1944	2.1825	0.1171

The high p-value (.117) indicates a weak relationship; note that 61.4% of the variability in y has been explained by x.

b. The Excel output is shown below:

Regression Statistics	
Multiple R	0.9830
R Square	0.9662
Adjusted R Square	0.9324
Standard Error	1.2788
Observations	5

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	93.5293	46.7647	28.5963	0.0338
Residual	2	3.2707	1.6353		
Total	4	96.8			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-8.1014	4.1038	-1.9741	0.1871
x	2.4127	0.4409	5.4724	0.0318
xsq	-0.0480	0.0105	-4.5688	0.0447

At the .05 level of significance, the relationship is significant; the fit is excellent.

c. $\hat{y} = -8.1014 + 2.4127(20) - 0.0480(20)^2 = 20.953$

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41. a. The Excel output is shown below:

Regression Statistics	
Multiple R	0.9426
R Square	0.8884
Adjusted R Square	0.8606
Standard Error	32.2940
Observations	6

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	33223.2143	33223.2143	31.8564	0.0049
Residual	4	4171.6190	1042.9048		
Total	5	37394.8333			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	943.0476	59.3802	15.8815	9.18737E-05
x	8.7143	1.5440	5.6441	0.0049

The p-value of .0049 < α = .01; reject H_0

b. The Excel output is shown below:

Regression Statistics	
Multiple R	0.9899
R Square	0.9799
Adjusted R Square	0.9665
Standard Error	15.8264
Observations	6

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	36643.4048	18321.7024	73.1475	0.0028
Residual	3	751.4286	250.4762		
Total	5	37394.8333			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	432.5714	141.1763	3.0641	0.0548
x	37.4286	7.8074	4.7940	0.0173
xsq	-0.3829	0.1036	-3.6952	0.0344

Because the p = value corresponding to F = 73.1475 is .0028 < α = .05, the relationship is significant.

c. $\hat{y} = 432.5714 + 37.4286x - .3829x^2 = 432.5714 + 37.4286(38) - .3829(38)^2 = 1302$

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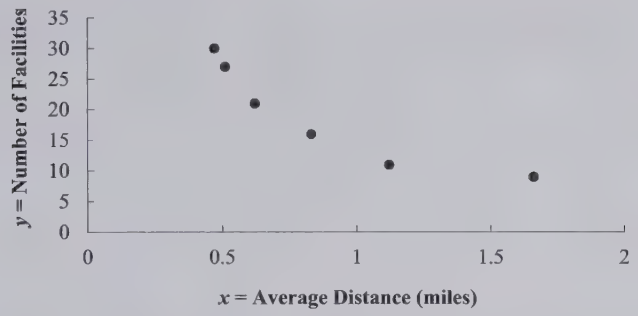
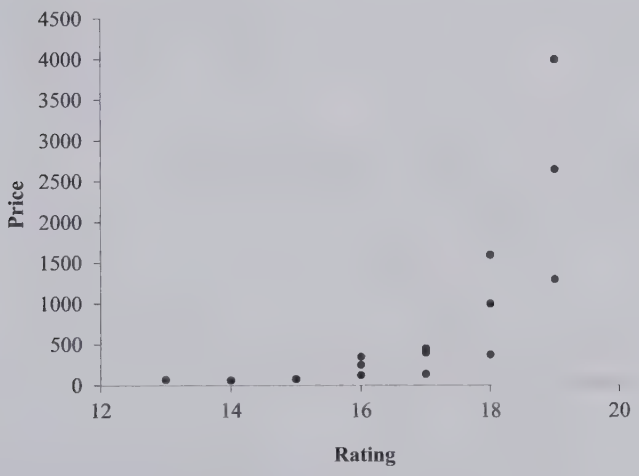
42. a. The scatter diagram is shown below:



b. No; the relationship appears to be curvilinear.

c. $\hat{y} = 2.90 - 0.185x + .00351x^2 \quad R_a^2 = .91$

43. a. The scatter diagram is shown below:



A simple linear regression model does not appear to be appropriate. There appears to be a curvilinear relationship between the two variables.

b. A portion of the Regression tool output is shown below:

Regression Statistics	
Multiple R	0.8380
R Square	0.7023
Adjusted R Square	0.6527
Standard Error	668.3117
Observations	15

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ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	12643603.96	6321802	14.1541	0.0007
Residual	12	5359686.271	446640.52		
Total	14	18003290.23			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	33828.505	13656.8051	2.4770	0.0291
Rating	-4570.609	1688.1238	-2.7075	0.0190
RatingSq	153.555	51.7173	2.9691	0.0117

44. a. The expected increase in final college grade point average corresponding to a one point increase in high school grade point average is .0235 when SAT mathematics score does not change. Similarly, the expected increase in final college grade point average corresponding to a one point increase in the SAT mathematics score is .00486 when the high school grade point average does not change.

b. $\hat{y} = -1.41 + .0235(84) + .00486(540) = 3.19$

45. a. Job satisfaction can be expected to decrease by 8.69 units with a one unit increase in length of service if the wage rate does not change. A dollar increase in the wage rate is associated with a 13.5 point increase in the job satisfaction score when the length of service does not change.

b. $\hat{y} = 14.4 - 8.69(4) + 13.5(6.5) = 67.39$

46. a. The computer output with the missing values filled in is as follows:

Regression Statistics	
Multiple R	0.9607
R Square	0.923
Adjusted R Square	0.9102
Standard Error	3.35
Observations	15

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	1612	806	71.82	2.10068E-07
Residual	12	134.67	11.2225		
Total	14	1746.67			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	8.103	2.667	3.0382	0.0103
X1	7.602	2.105	3.6114	0.0036
X2	3.111	0.613	5.0750	0.0003

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$\hat{y} = 8.103 + 7.602\ X1 + 3.111\ X2$

b. The p-value (2 degrees of freedom numerator and 12 denominator) corresponding to $F = 71.82$ is .0000

Because the p-value $\leq \alpha = .05$, there is a significant relationship.

c. For β_1: The p-value (12 degrees of freedom) corresponding to $t = 3.6114$ is .0036

Because the p-value $\leq \alpha = .05$, reject H_0: $\beta_1 = 0$

For β_2: The p-value (12 degrees of freedom) corresponding to $t = 5.0750$ is .0003

Because the p-value $\leq \alpha = .05$, reject H_0: $\beta_2 = 0$

47. a. The regression equation is

Regression Statistics	
Multiple R	0.9681
R Square	0.9373
Adjusted R Square	0.9194
Standard Error	0.1298
Observations	10

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	1.7621	0.8810	52.3053	6.17838E-05
Residual	7	0.1179	0.0168		
Total	9	1.88			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-1.4053	0.4848	-2.8987	0.0230
X1	0.0235	0.0087	2.7078	0.0303
X2	0.0049	0.0011	4.5125	0.0028

$\hat{y} = -1.4053 + .0235\ X1 + .0049\ X2$

b. $F = 52.3053$

p-value (2 degrees of freedom numerator and 7 degrees of freedom denominator) = .0000

Because the p-value $\leq \alpha = .05$, there is a significant relationship.

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c. $R^2 = \frac{SSR}{SST} = .937$

$R_a^2 = 1-(1-.937)\frac{9}{7} = .919$

good fit

d. For β_1: the p-value corresponding to $t = 2.7078$ is .0303

Because the p-value is $\leq \alpha = .05$, reject H_0: $\beta_1 = 0$

For β_2: the p-value corresponding to $t = 4.5125$ is .0028

Because the p-value is $\leq \alpha = .05$, reject H_0: $\beta_2 = 0$

48. a. The regression equation is

Regression Statistics	
Multiple R	0.9493
R Square	0.9012
Adjusted R Square	0.8616
Standard Error	3.773
Observations	8

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	648.83	324.415	22.7916	0.0031
Residual	5	71.17	14.234		
Total	7	720			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	14.4	8.191	1.7580	0.1391
X1	-8.69	1.555	-5.5884	0.0025
X2	13.517	2.085	6.4830	0.0013

$\hat{y} = 14.4 - 8.69\,X1 + 13.517\,X2$

b. The p-value (5 degrees of freedom) corresponding to $F = 22.7916$ is .0031

Because the p-value is $\leq \alpha = .05$, there is a significant relationship.

c. $R^2 = \frac{SSR}{SST} = .901$

$R_a^2 = 1-(1-.901)\frac{7}{5} = .861$

good fit

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d. For β_1: the p-value (5 degrees of freedom) corresponding to $t = -5.5884$ is .0025.

Because the p-value is $\leq \alpha = .05$, reject H_0: $\beta_1 = 0$

For β_2: the p-value corresponding to $t = 6.4830$ is .0013

Because the p-value is $\leq \alpha = .05$, reject H_0: $\beta_2 = 0$

49. a. A portion of the Excel output follows:

Regression Statistics	
Multiple R	0.9182
R Square	0.8430
Adjusted R Square	0.8332
Standard Error	0.8411
Observations	18

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	60.7866	60.7866	85.9295	7.80448E-08
Residual	16	11.3184	0.7074		
Total	17	72.105			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-7.5218	1.4668	-5.1282	0.0001	-10.6312	-4.4124
Steering	1.8151	0.1958	9.2698	7.804E-08	1.4000	2.2302

The estimated regression equation is $\hat{y} = -7.5218 + 1.8151$ Steering

Because the p-value = .000 < $\alpha = .05$, there is a significant relationship.

b. The estimated regression equation provided a good fit; 84.3 % of the variability in the Buy Again rating was explained by the linear effect of the Steering rating.

c. A portion of the Excel output follows:

Regression Statistics	
Multiple R	0.9653
R Square	0.9318
Adjusted R Square	0.9227
Standard Error	0.5727
Observations	18

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	67.1848	33.5924	102.4121	1.79936E-09
Residual	15	4.9202	0.3280		
Total	17	72.105			

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	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-5.3877	1.1095	-4.8558	0.0002	-7.7526	-3.0228
Steering	0.6899	0.2875	2.3992	0.0299	0.0770	1.3028
Tread Wear	0.9113	0.2063	4.4166	0.0005	0.4715	1.3511

The estimated regression equation is $\hat{y} = -5.3877 + 0.6899$ Steering $+ 0.9113$ Treadwear

d. For the Treadwear independent variable, the p-value = .0005 < α = .05; thus, the addition of Treadwear is significant.

50. a. A portion of the Excel output is shown below:

Regression Statistics	
Multiple R	0.6667
R Square	0.4446
Adjusted R Square	0.4193
Standard Error	4.5109
Observations	24

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	358.2899	358.2899	17.6076	0.0004
Residual	22	447.6684	20.3486		
Total	23	805.9583			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	67.6762	2.3047	29.3646	3.85E-19
Price	0.0046	0.0011	4.1961	0.0004

b. Because the p-value = .0004 < α = .05, there is a significant relationship.

c. A portion of the Regression tool output is shown below:

Regression Statistics	
Multiple R	0.7852
R Square	0.6166
Adjusted R Square	0.5591
Standard Error	3.9307
Observations	24

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ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	496.9487067	165.6496	10.7213	0.0002
Residual	20	309.0096266	15.4505		
Total	23	805.9583333			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	65.6597	2.5069	26.1921	5.94E-17
Price	0.0023	0.0012	1.8845	0.0741
Quality-E	10.2097	3.4379	2.9697	0.0076
Quality-VG	5.9246	2.7586	2.1477	0.0442

d. Because the p-value = .0002 < α = .05, there is a significant overall relationship.

e. Price, Quality-E, and Quality-VG are all significant because for each independent variable the corresponding p-value is less than α = .10.

f. In part (c) we developed the following estimated regression equation:

Score = 65.6597 + 0.0023 Price + 10.2097 Quality-E + 5.9246 Quality-VG

For a good quality rating Quality-E = 0 and Quality-VG = 0

Score = 65.6597 + 0.0023 (2000) + 10.2097(0) + 5.9246(0) = 70.2597 or approximately 70

For a very good quality rating Quality-E = 0 and Quality-VG = 1

Score = 65.6597 + 0.0023 (2000) + 10.2097(0) + 5.9246(1) = 76.1843 or approximately 76

The change in going from a good to a very good quality rating is 5.9246 or approximately 6

51. a. Type of Fund is a categorical variable with three levels. Let FundDE = 1 for a domestic equity fund and FundIE = 1 for an international fund.

A portion of the Excel output is shown below:

Regression Statistics	
Multiple R	0.7838
R Square	0.6144
Adjusted R Square	0.5960
Standard Error	5.5978
Observations	45

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ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	2096.8489	1048.4245	33.4584	2.03818E-09
Residual	42	1316.0771	31.3352		
Total	44	3412.9260			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	4.9090	1.7702	2.7732	0.0082	1.3366	8.4814
FundDE	10.4658	2.0722	5.0505	9.033E-06	6.2839	14.6477
FundIE	21.6823	2.6553	8.1658	3.288E-10	16.3237	27.0408

The estimated regression equation is

$\hat{y} = 4.9090 + 10.4658$ FundDE $+ 21.6823$ FundIE

Since the p-value corresponding to $F = 33.4584$ is $.0000 < \alpha = .05$, there is a significant relationship.

b. R Square = .6144. A reasonably good fit using only Type of Fund.

c. A portion of the Excel output follows:

Regression Statistics	
Multiple R	0.8135
R Square	0.6617
Adjusted R Square	0.6279
Standard Error	5.3726
Observations	45

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	4	2258.3432	564.5858	19.5598	5.48647E-09
Residual	40	1154.5827	28.8646		
Total	44	3412.9260			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	1.1899	2.3781	0.5004	0.6196	-3.6164	5.9961
FundDE	6.8969	2.7651	2.4942	0.0169	1.3083	12.4854
FundIE	17.6800	3.3161	5.3315	4.096E-06	10.9778	24.3821
Net Asset Value ($)	0.0265	0.0670	0.3950	0.6950	-0.1089	0.1619
Expense Ratio (%)	6.4564	2.7593	2.3399	0.0244	0.8798	12.0331

The estimated regression equation is

$\hat{y} = 1.1899 + 6.8969$ FundDE $+ 17.6800$ FundIE $+ 0.0265$ Net Asset Value ($)
$+ 6.4564$ Expense Ratio (%)

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Since the p-value corresponding to $F = 19.5558$ is $.0000 < \alpha = .05$, there is a significant relationship.

For Net Asset Value ($), the p-value corresponding to $t = .3950$ is $.6950 > \square = .05$, Net Asset Value ($) is not significant and can be deleted from the model.

d. Morningstar Rank is a categorical variable. The data set only contains funds with four ranks (2-Star through –5Star), so three dummy variables are needed. Let 3StarRank = 1 for a 3-StarRank, 4StarRank = 1 for a 4-StarRank, and 5StarRank = 1 for a 5-StarRank.

A portion of the Excel output follows:

Regression Statistics	
Multiple R	0.8501
R Square	0.7227
Adjusted R Square	0.6789
Standard Error	4.9904
Observations	45

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	6	2466.5721	411.0954	16.5072	2.96759E-09
Residual	38	946.3539	24.9040		
Total	44	3412.9260			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-4.6074	3.2909	-1.4000	0.1696	-11.2694	2.0547
FundDE	8.1713	2.2754	3.5912	0.0009	3.5650	12.7776
FundIE	19.5194	2.7795	7.0227	2.292E-08	13.8926	25.1461
Expense Ratio (%)	5.5197	2.5862	2.1343	0.0393	0.2843	10.7552
3StarRank	5.9237	2.8250	2.0969	0.0427	0.2048	11.6426
4StarRank	8.2367	2.8474	2.8927	0.0063	2.4725	14.0009
5StarRank	6.6241	3.1425	2.1079	0.0417	0.2624	12.9858

The estimated regression equation is

$\hat{y}$ = -4.6074 + 8.1713 FundDE + 19.5194 FundIE +5.5197 Expense Ratio (%) + 5.9237 3StarRank + 8.2367 4StarRank + 6.6241 5StarRank

At the .05 level of significance, all the independent variables are significant.

e. $\hat{y}$ = -4.6074 + 8.1713(1) + 19.5194(0) +5.5197(1.05) + 5.9237(1) + 8.2367(0) +6.62415(0) = 15.28%

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52. a. A portion of the Excel output follows:

Regression Statistics	
Multiple R	0.8333
R Square	0.6945
Adjusted R Square	0.6935
Standard Error	2.2212
Observations	311

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	3464.8213	3464.8213	702.3017	1.51247E-81
Residual	309	1524.4584	4.9335		
Total	310	4989.2797			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	35.3950	0.3818	92.6977	1.47E-227	34.6437	36.1464
Displacement	-2.8821	0.1088	-26.5010	1.512E-81	-3.0961	-2.6681

The estimated regression equation is
$\hat{y} = 35.3950 - 2.8821$ Displacement

Since the p-value corresponding to $F = 702.3017$ is $.0000 < \alpha = .05$, there is a significant relationship.

b. The Excel output follows:

Regression Statistics	
Multiple R	0.8827
R Square	0.7791
Adjusted R Square	0.7769
Standard Error	1.8947
Observations	311

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	3887.1290	1295.7097	360.9151	2.6788E-100
Residual	307	1102.1508	3.5901		
Total	310	4989.2797			

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	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	30.2289	0.5948	50.8225	1.63E-151	29.0585	31.3993
ClassMidsize	3.4563	0.3233	10.6900	6.956E-23	2.8201	4.0925
ClassLarge	1.7078	0.4048	4.2190	3.235E-05	0.9113	2.5043
Displacement	-1.9347	0.1293	-14.9591	2.216E-38	-2.1891	-1.6802

The estimated regression equation is
$\hat{y}$ = 35.2289 + 3.4563 ClassMidsize +1.7078 ClassLarge – 1.9347 Displacement

c. For ClassMidsize, the *p*-value corresponding to $t = 10.6900$ is $.000 < \alpha = .05$; significant. For ClassLarge, the *p*-value corresponding to $t = 4.2190$ is $.000 < \alpha = .05$; significant. The addition of the dummy variables is significant.

d. The Excel output follows:

Regression Statistics	
Multiple R	0.8976
R Square	0.8057
Adjusted R Square	0.8031
Standard Error	1.7801
Observations	311

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	4	4019.6456	1004.9114	317.1329	1.7602E-107
Residual	306	969.6341	3.1687		
Total	310	4989.2797			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	30.9548	0.5700	54.3099	3.66E-159	29.8333	32.0764
ClassMidsize	2.9832	0.3124	9.5479	4.436E-19	2.3684	3.5980
ClassLarge	1.5088	0.3815	3.9545	9.534E-05	0.7580	2.2595
Displacement	-1.8533	0.1222	-15.1719	3.72E-39	-2.0937	-1.6129
FuelPremium	-1.4306	0.2212	-6.4668	3.954E-10	-1.8659	-0.9953

The estimated regression equation is
$\hat{y}$ = 30.9548 + 2.9832 ClassMidsize +1.5088 ClassLarge – 1.8533 Displacement
– 1.4306 FuelPremium

e. Since the *p*-value corresponding to $F = 317.1329$ is $.0000 < \alpha = .05$, there is a significant overall relationship. Because the *p*-values for each independent variable are also $< \alpha = .05$, each of the independent variables is significant.

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Chapter 14
Statistical Methods for Quality Control

Learning Objectives

1. Learn about the importance of total quality, quality control, and how statistical methods can assist in the quality control process.

2. Be able to construct quality control charts and understand how they are used for statistical process control.

3. Learn about acceptance sampling procedures.

4. Know the difference between consumer's risk and producer's risk.

5. Know what is meant by multiple sampling plans.

6. Know the definitions of the following terms:

total quality	*p* chart
Six Sigma	*np* chart
assignable causes	lot
common causes	acceptance sampling
control charts	producer's risk
upper control limit (UCL)	consumer's risk
lower control limit (LCL)	acceptance criterion
$\bar{x}$ chart	operating characteristic (OC) curve
R chart	multiple sampling plan

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Solutions:

1. a. For $n = 4$

$$\text{UCL} = \mu + 3(\sigma/\sqrt{n}) = 12.5 + 3(.8/\sqrt{4}) = 13.7$$
$$\text{LCL} = \mu - 3(\sigma/\sqrt{n}) = 12.5 - 3(.8/\sqrt{4}) = 11.3$$

b. For $n = 8$

$$\text{UCL} = \mu + 3(.8/\sqrt{8}) = 13.35$$
$$\text{LCL} = \mu - 3(.8/\sqrt{8}) \quad 11.65$$

For $n = 16$

$$\text{UCL} = \mu + 3(.8/\sqrt{16}) = 13.10$$
$$\text{LCL} = \mu - 3(.8/\sqrt{16}) = 11.90$$

c. UCL and LCL become closer together as n increases. If the process is in control, the larger samples should have less variance and should fall closer to 12.5.

2. a. $\mu = \dfrac{677.5}{25(5)} = 5.42$

b. $\text{UCL} = \mu + 3(\sigma/\sqrt{n}) = 5.42 + 3(.5/\sqrt{5}) = 6.09$
$\text{LCL} = \mu - 3(\sigma/\sqrt{n}) = 5.42 - 3(.5/\sqrt{5}) = 4.75$

3. a. $p = \dfrac{135}{25(100)} = 0.0540$

b. $\sigma_{\bar{p}} = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{0.0540(0.9460)}{100}} = 0.0226$

c. $\text{UCL} = p + 3\sigma_{\bar{p}} = 0.0540 + 3(0.0226) = 0.1218$
$\text{LCL} = p - 3\sigma_{\bar{p}} = 0.0540 - 3(0.0226) = -0.0138$

Use LCL = 0

4. R Chart:

$$\text{UCL} = \bar{R}D_4 = 1.6(1.864) = 2.98$$
$$\text{LCL} = \bar{R}D_3 = 1.6(0.136) = 0.22$$

$\bar{\bar{x}}$ Chart:

$$\text{UCL} = \bar{\bar{x}} + A_2\bar{R} = 28.5 + 0.373(1.6) = 29.10$$
$$\text{LCL} = \bar{\bar{x}} - A_2\bar{R} = 28.5 - 0.373(1.6) = 27.90$$

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5. a. $UCL = \mu + 3(\sigma/\sqrt{n}) = 128.5 + 3(.4/\sqrt{6}) = 128.99$
$LCL = \mu - 3(\sigma/\sqrt{n}) = 128.5 - 3(.4/\sqrt{6}) = 128.01$

b. $\bar{x} = \Sigma x_i / n = \frac{772.4}{6} = 128.73$ in control

c. $\bar{x} = \Sigma x_i / n = \frac{774.3}{6} = 129.05$ out of control

6. Process Mean $= \frac{20.12 + 19.90}{2} = 20.01$

$UCL = \mu + 3(\sigma/\sqrt{n}) = 20.01 + 3(\sigma/\sqrt{5}) = 20.12$

Solve for σ:

$$\sigma = \frac{(20.12 - 20.01)\sqrt{5}}{3} = 0.082$$

7.

Sample Number	Observations			$\bar{x}_i$	R_i
1	31	42	28	33.67	14
2	26	18	35	26.33	17
3	25	30	34	29.67	9
4	17	25	21	21.00	8
5	38	29	35	34.00	9
6	41	42	36	39.67	6
7	21	17	29	22.33	12
8	32	26	28	28.67	6
9	41	34	33	36.00	8
10	29	17	30	25.33	13
11	26	31	40	32.33	14
12	23	19	25	22.33	6
13	17	24	32	24.33	15
14	43	35	17	31.67	26
15	18	25	29	24.00	11
16	30	42	31	34.33	12
17	28	36	32	32.00	8
18	40	29	31	33.33	11
19	18	29	28	25.00	11
20	22	34	26	27.33	12

$\bar{R} = 11.4$ and $\bar{\bar{x}} = 29.17$

R Chart:

$UCL = \bar{R}D_4 = 11.4(2.574) = 29.34$
$LCL = \bar{R}D_3 = 11.4(0) = 0$

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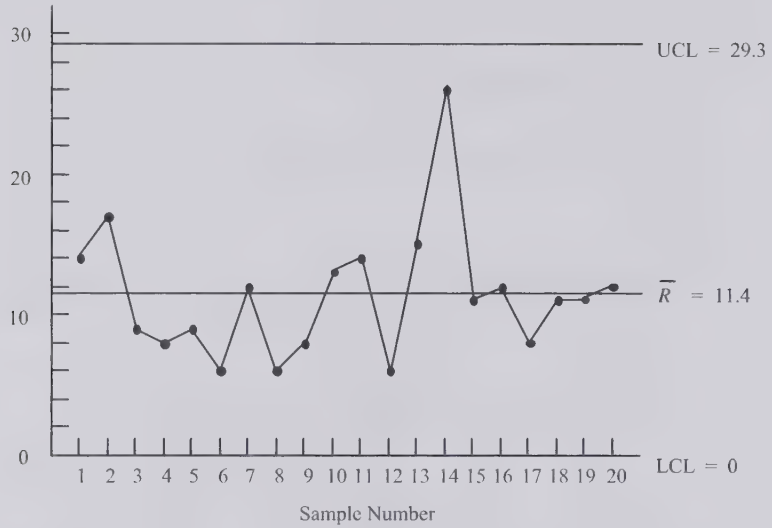
$\bar{x}$ Chart:

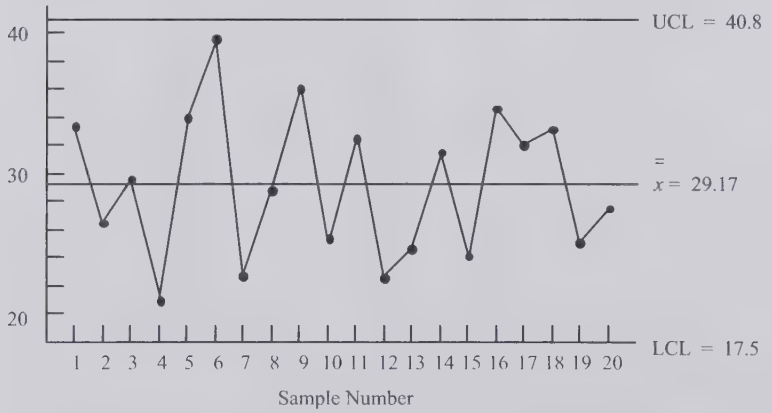
$\text{UCL} = \bar{\bar{x}} + A_2\bar{R} = 29.17 + 1.023(11.4) = 40.8$

$\text{LCL} = \bar{\bar{x}} - A_2\bar{R} = 29.17 - 1.023(11.4) = 17.5$

R Chart:



$\bar{x}$ Chart:



8. a. $p = \dfrac{141}{20(150)} = 0.0470$

b. $\sigma_{\bar{p}} = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{0.0470(0.9530)}{150}} = 0.0173$

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$UCL = p + 3\sigma_{\bar{p}} = 0.0470 + 3(0.0173) = 0.0989$
$LCL = p - 3\sigma_{\bar{p}} = 0.0470 - 3(0.0173) = -0.0049$

Use LCL = 0

c. $\bar{p} = \dfrac{12}{150} = 0.08$

Process should be considered in control.

d. $p = .047, n = 150$

$UCL = np + 3\sqrt{np(1-p)} = 150(0.047) + 3\sqrt{150(0.047)(0.953)} = 14.826$
$LCL = np - 3\sqrt{np(1-p)} = 150(0.047) - 3\sqrt{150(0.047)(0.953)} = -0.726$

Thus, the process is out of control if more than 14 defective packages are found in a sample of 150.

e. Process should be considered to be in control since 12 defective packages were found.

f. The *np* chart may be preferred because a decision can be made by simply counting the number of defective packages.

9. a. Total defectives: 165

$$p = \frac{165}{20(200)} = 0.0413$$

b. $\sigma_{\bar{p}} = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{0.0413(0.9587)}{200}} = 0.0141$

$UCL = p + 3\sigma_{\bar{p}} = 0.0413 + 3(0.0141) = 0.0836$
$LCL = p - 3\sigma_{\bar{p}} = 0.0413 + 3(0.0141) = -0.0010$

Use LCL = 0

c. $\bar{p} = \dfrac{20}{200} = 0.10$ Out of control

d. $p = .0413, n = 200$

$UCL = np + 3\sqrt{np(1-p)} = 200(0.0413) + 3\sqrt{200(0.0413)(0.9587)} = 16.702$
$LCL = np - 3\sqrt{np(1-p)} = 200(0.0413) - 3\sqrt{200(0.0413)(0.9587)} = 0.1821$

e. The process is out of control since 20 defective pistons were found.

10. $$f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

When $p = .02$, the probability of accepting the lot is

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$$f(0) = \frac{25!}{0!(25-0)!}(0.02)^0(1-0.02)^{25} = 0.6035$$

When p = .06, the probability of accepting the lot is

$$f(0) = \frac{25!}{0!(25-0)!}(0.06)^0(1-0.06)^{25} = 0.2129$$

11. a. Using binomial probabilities with n = 20 and p_0 = .02.

P (Accept lot) = $f(0)$ = .6676

Producer's risk: α = 1 - .6676 = .3324

b. P (Accept lot) = $f(0)$ = .2901

Producer's risk: α = 1 - .2901 = .7099

12. At p_0 = .02, the n = 20 and c = 1 plan provides

P (Accept lot) = $f(0) + f(1)$ = .6676 + .2725 = .9401

Producer's risk: α = 1 - .9401 = .0599

At p_0 = .06, the n = 20 and c = 1 plan provides

P (Accept lot) = $f(0) + f(1)$ = .2901 + .3703 = .6604

Producer's risk: α = 1 - .6604 = .3396

For a given sample size, the producer's risk decreases as the acceptance number c is increased.

13. a. Using binomial probabilities with n = 20 and p_0 = .03.

P(Accept lot) = $f(0) + f(1)$
= .5438 + .3364 = .8802

Producer's risk: α = 1 - .8802 = .1198

b. With n = 20 and p_1 = .15.

P(Accept lot) = $f(0) + f(1)$
= .0388 + .1368 = .1756

Consumer's risk: β = .1756

c. The consumer's risk is acceptable; however, the producer's risk associated with the n = 20, c = 1 plan is a little larger than desired.

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14.

	c	P (Accept) $p_0 = .05$	Producer's Risk α	P (accept) $p_1 = .30$	Consumer's Risk β
($n = 10$)	0	.5987	.4013	.0282	.0282
	1	.9138	.0862	.1493	.1493
	2	.9884	.0116	.3828	.3828
($n = 15$)	0	.4633	.5367	.0047	.0047
	1	.8291	.1709	.0352	.0352
	2	.9639	.0361	.1268	.1268
	3	.9946	.0054	.2968	.2968
($n = 20$)	0	.3585	.6415	.0008	.0008
	1	.7359	.2641	.0076	.0076
	2	.9246	.0754	.0354	.0354
	3	.9842	.0158	.1070	.1070

The plan with $n = 15$, $c = 2$ is close with $\alpha = .0361$ and $\beta = .1268$. However, the plan with $n = 20$, $c = 3$ is necessary to meet both requirements.

15. a. P (Accept) shown for p values below:

c	$p = .01$	$p = .05$	$p = .08$	$p = .10$	$p = .15$
0	.8179	.3585	.1887	.1216	.0388
1	.9831	.7359	.5169	.3918	.1756
2	.9990	.9246	.7880	.6770	.4049

The operating characteristic curves would show the P (Accept) versus p for each value of c.

b. P (Accept)

c	At $p_0 = .01$	Producer's Risk	At $p_1 = .08$	Consumer's Risk
0	.8179	.1821	.1887	.1887
1	.9831	.0169	.5169	.5169
2	.9990	.0010	.7880	.7880

16. a. $\mu = \dfrac{\Sigma \bar{x}}{20} = \dfrac{1908}{20} = 95.4$

b. $\text{UCL} = \mu + 3(\sigma / \sqrt{n}) = 95.4 + 3(.50 / \sqrt{5}) = 96.07$
$\text{LCL} = \mu - 3(\sigma / \sqrt{n}) = 95.4 - 3(.50 / \sqrt{5}) = 94.73$

c. No; all were in control

17. a. For $n = 10$

$\text{UCL} = \mu + 3(\sigma / \sqrt{n}) = 350 + 3(15 / \sqrt{10}) = 364.23$
$\text{LCL} = \mu - 3(\sigma / \sqrt{n}) = 350 - 3(15 / \sqrt{10}) = 335.77$

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For $n = 20$

UCL = 350 + 3(15 / $\sqrt{20}$) = 360.06
LCL = 350 - 3(15 / $\sqrt{20}$) = 339.94

For $n = 30$

UCL = 350 + 3(15 / $\sqrt{30}$) = 358.22
LCL = 350 - 3(15 / $\sqrt{30}$) = 343.78

b. Both control limits come closer to the process mean as the sample size is increased.

c. The process will be declared out of control and adjusted when the process is in control.

d. The process will be judged in control and allowed to continue when the process is out of control.

e. The controls limits for each sample size were computed using $z = 3$. Because $P(z \leq -3) = .0013$, P(Type I error) = 2(.0013) = .0026

f. Increasing the sample size provides a more accurate estimate of the process mean and reduces the probability of making a Type II error.

18. *R* Chart:

UCL = $\bar{R}D_4 = 2(2.115) = 4.23$
LCL = $\bar{R}D_3 = 2(0) = 0$

$\bar{x}$ Chart:

UCL = $\bar{\bar{x}} + A_2\bar{R} = 5.42 + 0.577(2) = 6.57$
LCL = $\bar{\bar{x}} - A_2\bar{R} = 5.42 - 0.577(2) = 4.27$

Estimate of Standard Deviation:

$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{2}{2.326} = 0.86$$

19. $\bar{R} = 0.665$ $\bar{\bar{x}} = 95.398$

$\bar{x}$ Chart:

UCL = $\bar{\bar{x}} + A_2\bar{R} = 95.398 + 0.577(0.665) = 95.782$
LCL = $\bar{\bar{x}} - A_2\bar{R} = 95.398 - 0.577(0.665) = 95.014$

R Chart:

UCL = $\bar{R}D_4 = 0.665(2.114) = 1.406$
LCL = $\bar{R}D_3 = 0.665(0) = 0$

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The *R* chart indicated the process variability is in control. All sample ranges are within the control limits. However, the process mean is out of control. Sample 11 ($\bar{x} = 95.80$) and Sample 17 ($\bar{x}$ =94.82) fall outside the control limits.

20. $\bar{R} = .053 \quad \bar{\bar{x}} = 3.082$

$\bar{x}$ Chart:

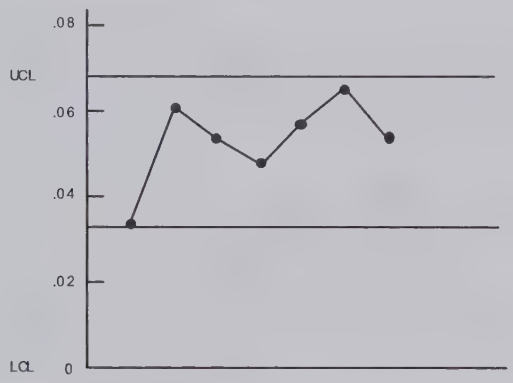
$\text{UCL} = \bar{\bar{x}} + A_2\bar{R} = 3.082 + 0.577(0.053) = 3.112$
$\text{LCL} = \bar{\bar{x}} - A_2\bar{R} = 3.082 - 0.577(0.053) = 3.051$

R Chart:

$\text{UCL} = \bar{R}D_4 = 0.053(2.115) = 0.1121$
$\text{LCL} = \bar{R}D_3 = 0.053(0) = 0$

All sample averages and sample ranges are within the control limits for both charts.

21. a.



Warning: Process should be checked. All points are within control limits; however, all points are also greater than the process proportion defective.

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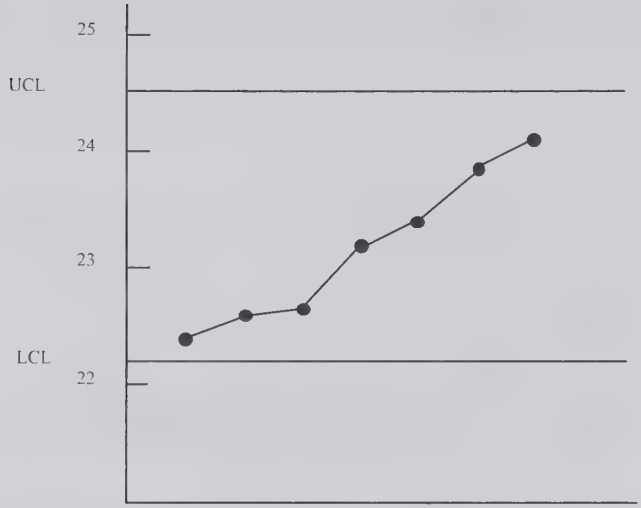
b.



Warning: Process should be checked. All points are within control limits yet the trend in points show a movement or shift toward UCL out-of-control point.

22. a. $p = .04$

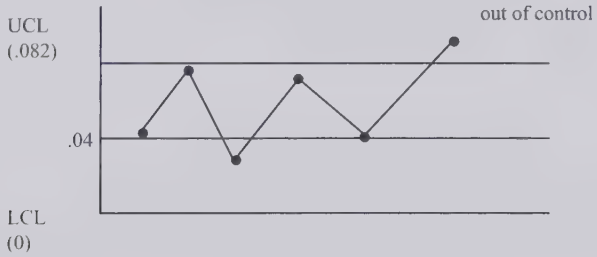
$$\sigma_p = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.04(0.96)}{200}} = 0.0139$$

$UCL = p + 3\sigma_{\bar{p}} = 0.04 + 3(0.0139) = 0.0817$

$LCL = p - 3\sigma_{\bar{p}} = 0.04 - 3(0.0139) = -0.0017$

Use LCL = 0

b.



For month 1 $\bar{p} = 10/200 = 0.05$. Other monthly values are .075, .03, .065, .04, and .085. Only the last month with $\bar{p} = 0.085$ is an out-of-control situation. The seesaw (i.e. zigzag) pattern of the points is also not considered normal for an in-control process.

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23. a. Use binomial probabilities with $n = 10$.

At $p_0 = .05$,

$P(\text{Accept lot}) = f(0) + f(1) + f(2)$
$= .5987 + .3151 + .0746 = .9884$

Producer's Risk: $\alpha = 1 - .9884 = .0116$

At $p_1 = .20$,

$P(\text{Accept lot}) = f(0) + f(1) + f(2)$
$= .1074 + .2684 + .3020 = .6778$

Consumer's risk: $\beta = .6778$

b. The consumer's risk is unacceptably high. Too many bad lots would be accepted.

c. Reducing c would help, but increasing the sample size appears to be the best solution.

24. a. P (Accept) are shown below: (Using $n = 15$)

	$p = .01$	$p = .02$	$p = .03$	$p = .04$	$p = .05$
$f(0)$	.8601	.7386	.6333	.5421	.4633
$f(1)$	.1303	.2261	.2938	.3388	.3658
	.9904	.9647	.9271	.8809	.8291
$\alpha = 1 - P$ (Accept)	.0096	.0353	.0729	.1191	.1709

Using $p_0 = .03$ since α is close to .075. Thus, .03 is the fraction defective where the producer will tolerate a .075 probability of rejecting a good lot (only .03 defective).

b.

	$p = .25$
$f(0)$	.0134
$f(1)$	.0668
$\beta =$	.0802

25. a. P (Accept) when $n = 25$ and $c = 0$. Use the binomial probability function with

$$f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

or

$$f(0) = \frac{25!}{0!25!} p^0 (1-p)^{25} = (1-p)^{25}$$

If	$f(0)$
$p = .01$	.7778
$p = .03$	.4670
$p = .10$	.0718
$p = .20$	.0038

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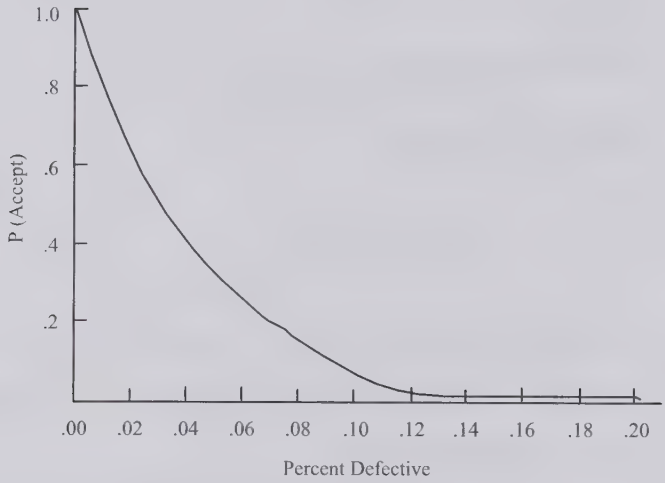
b.



c. $1 - f(0) = 1 - .778 = .222$

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Chapter 15
Time Series Analysis and Forecasting

Learning Objectives

1. Be able to construct a time series plot and identify the underlying pattern in the data.
2. Understand how to measure forecast accuracy.
3. Be able to use smoothing techniques such as moving averages and exponential smoothing to forecast a time series with a horizontal pattern.
4. Know how simple linear regression can be used to forecast a time series with a linear trend.
5. Be able to develop a quadratic trend equation and an exponential trend equation to forecast a time series with a curvilinear or nonlinear trend.
6. Know how to develop forecasts for a time series that has a seasonal pattern.
7. Know how time series decomposition can be used to separate or decompose a time series into season, trend, and irregular components.
8. Be able to deseasonalize a time series.
9. Know the definition of the following terms:

time series	mean squared error
time series plot	mean absolute percentage error
horizontal pattern	moving average
stationary time series	weighted moving average
trend pattern	smoothing constant
seasonal pattern	time series decomposition
cyclical pattern	additive model
mean absolute error	multiplicative model

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Solutions:

1. The following table shows the calculations for parts (a), (b), and (c).

Week	Time Series Value	Forecast	Forecast Error	Absolute Value of Forecast Error	Squared Forecast Error	Percentage Error	Absolute Value of Percentage Error
1	18						
2	13	18	-5	5	25	-38.46	38.46
3	16	13	3	3	9	18.75	18.75
4	11	16	-5	5	25	-45.45	45.45
5	17	11	6	6	36	35.29	35.29
6	14	17	-3	3	9	-21.43	21.43
			Totals	22	104	-51.30	159.38

   a. MAE = 22/5 = 4.4

   b. MSE = 104/5 = 20.8

   c. MAPE = 159.38/5 = 31.88

   d. Forecast for week 7 is 14

2. The following table shows the calculations for parts (a), (b), and (c).

Week	Time Series Value	Forecast	Forecast Error	Absolute Value of Forecast Error	Squared Forecast Error	Percentage Error	Absolute Value of Percentage Error
1	18						
2	13	18.00	-5.00	5.00	25.00	-38.46	38.46
3	16	15.50	0.50	0.50	0.25	3.13	3.13
4	11	15.67	-4.67	4.67	21.81	-42.45	42.45
5	17	14.50	2.50	2.50	6.25	14.71	14.71
6	14	15.00	-1.00	1.00	1.00	-7.14	7.14
			Totals	13.67	54.31	-70.21	105.86

   a. MAE = 13.67/5 = 2.73

   b. MSE = 54.31/5 = 10.86

   c. MAPE = 105.89/5 = 21.18

   d. Forecast for week 7 is (18 + 13 + 16 + 11 + 17 + 14) / 6 = 14.83

3. The following table shows the measures of forecast error for both methods.

	Exercise 1	Exercise 2
MAE	4.40	2.73
MSE	20.80	10.86
MAPE	31.88	21.18

   For each measure of forecast accuracy the average of all the historical data provided more accurate forecasts than simply using the most recent value.

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4. a.

Month	Time Series Value	Forecast	Forecast Error	Squared Forecast Error
1	24			
2	13	24	-11	121
3	20	13	7	49
4	12	20	-8	64
5	19	12	7	49
6	23	19	4	16
7	15	23	-8	64
			Total	363

MSE = 363/6 = 60.5

Forecast for month 8 = 15

b.

Week	Time Series Value	Forecast	Forecast Error	Squared Forecast Error
1	24			
2	13	24.00	-11.00	121.00
3	20	18.50	1.50	2.25
4	12	19.00	-7.00	49.00
5	19	17.25	1.75	3.06
6	23	17.60	5.40	29.16
7	15	18.50	-3.50	12.25
			Total	216.72

MSE = 216.72/6 = 36.12

Forecast for month 8 = (24 + 13 + 20 + 12 + 19 + 23 + 15) / 7 = 18

c. The average of all the previous values is better because MSE is smaller.

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5. a.

The data appear to follow a horizontal pattern.

b. Three-week moving average.

Week	Time Series Value	Forecast	Forecast Error	Squared Forecast Error
1	18			
2	13			
3	16			
4	11	15.67	-4.67	21.78
5	17	13.33	3.67	13.44
6	14	14.67	-0.67	0.44
			Total	35.67

MSE = 35.67/3 = 11.89.

The forecast for week 7 = (11 + 17 + 14) / 3 = 14

c. Smoothing constant = .2

Week	Time Series Value	Forecast	Forecast Error	Squared Forecast Error
1	18			
2	13	18.00	-5.00	25.00
3	16	17.00	-1.00	1.00
4	11	16.80	-5.80	33.64
5	17	15.64	1.36	1.85
6	14	15.91	-1.91	3.66
			Total	65.15

MSE = 65.15/5 = 13.03

The forecast for week 7 is .2(14) + (1 - .2)15.91 = 15.53

d. The three-week moving average provides a better forecast since it has a smaller MSE.

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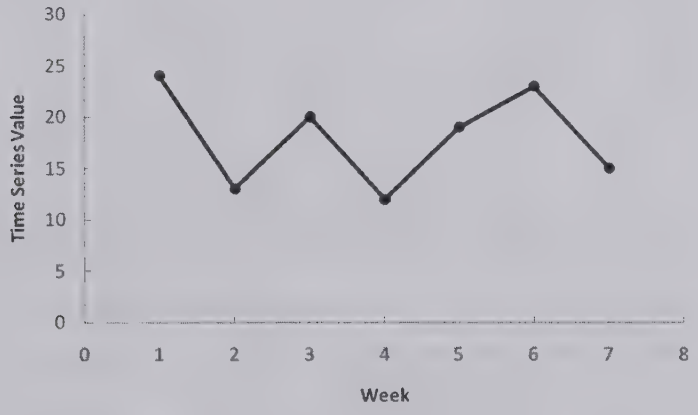
e. Smoothing constant = .4

Week	Time Series Value	Forecast	Forecast Error	Squared Forecast Error
1	18			
2	13	18.00	-5.00	25.00
3	16	16.00	0.00	0.00
4	11	16.00	-5.00	25.00
5	17	14.00	3.00	9.00
6	14	15.20	-1.20	1.44
			Total	60.44

MSE = 60.44/5 = 12.09

The exponential smoothing forecast using $\alpha = .4$ provides a better forecast than the exponential smoothing forecast using $\alpha = .2$ since it has a smaller MSE.

6. a.



The data appear to follow a horizontal pattern.

Three-week moving average.

Week	Time Series Value	Forecast	Forecast Error	Squared Forecast Error
1	24			
2	13			
3	20			
4	12	19.00	-7.00	49.00
5	19	15.00	4.00	16.00
6	23	17.00	6.00	36.00
7	15	18.00	-3.00	9.00
			Total	110.00

MSE = 110/4 = 27.5.

The forecast for week 8 = (19 + 23 + 15) / 3 = 19

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b. Smoothing constant = .2

Week	Time Series Value	Forecast	Forecast Error	Squared Forecast Error
1	24			
2	13	24.00	-11.00	121.00
3	20	21.80	-1.80	3.24
4	12	21.44	-9.44	89.11
5	19	19.55	-0.55	0.30
6	23	19.44	3.56	12.66
7	15	20.15	-5.15	26.56
			Total	252.87

MSE = 252.87/6 = 42.15

The forecast for week 8 is .2(15) + (1 - .2)20.15 = 19.12

c. The three-week moving average provides a better forecast since it has a smaller MSE.

d. Smoothing constant = .4

Week	Time Series Value	Forecast	Forecast Error	Squared Value of Forecast Error
1	24			
2	13	24.00	-11.00	121.00
3	20	19.60	0.40	0.16
4	12	19.76	-7.76	60.22
5	19	16.66	2.34	5.49
6	23	17.59	5.41	29.23
7	15	19.76	-4.76	22.62
			Total	238.72

MSE = 238.72/6 = 39.79

The exponential smoothing forecast using $\alpha = .4$ provides a better forecast than the exponential smoothing forecast using $\alpha = .2$ since it has a smaller MSE.

7. a.

Week	Time-Series Value	4-Week Moving Average Forecast	$(\text{Error})^2$	5-Week Moving Average Forecast	$(\text{Error})^2$
1	17				
2	21				
3	19				
4	23				
5	18	20.00	4.00		
6	16	20.25	18.06	19.60	12.96
7	20	19.00	1.00	19.40	0.36
8	18	19.25	1.56	19.20	1.44
9	22	18.00	16.00	19.00	9.00
10	20	19.00	1.00	18.80	1.44
11	15	20.00	25.00	19.20	17.64
12	22	18.75	10.56	19.00	9.00
		Totals	77.18		51.84

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b. MSE(4-Week) = 77.18 / 8 = 9.65

MSE(5-Week) = 51.84 / 7 = 7.41

c. For the limited data provided, the 5-week moving average provides the smallest MSE.

8. a.

Week	Time-Series Value	Weighted Moving Average Forecast	Forecast Error	$(\text{Error})^2$
1	17			
2	21			
3	19			
4	23	19.33	3.67	13.47
5	18	21.33	-3.33	11.09
6	16	19.83	-3.83	14.67
7	20	17.83	2.17	4.71
8	18	18.33	-0.33	0.11
9	22	18.33	3.67	13.47
10	20	20.33	-0.33	0.11
11	15	20.33	-5.33	28.41
12	22	17.83	4.17	17.39
			Total	103.43

b. MSE = 103.43 / 9 = 11.49

Prefer the unweighted moving average here; it has a smaller MSE.

c. You could always find a weighted moving average at least as good as the unweighted one. Actually the unweighted moving average is a special case of the weighted ones where the weights are equal.

9. The following tables show the calculations for $\alpha = .1$

Week	Time Series Value	Forecast	Forecast Error	Absolute Value of Forecast Error	Squared Forecast Error	Percentage Error	Absolute Value of Percentage Error
1	17						
2	21	17.00	4.00	4.00	16.00	19.05	19.05
3	19	17.40	1.60	1.60	2.56	8.42	8.42
4	23	17.56	5.44	5.44	29.59	23.65	23.65
5	18	18.10	-0.10	0.10	0.01	-0.56	0.56
6	16	18.09	-2.09	2.09	4.37	-13.06	13.06
7	20	17.88	2.12	2.12	4.49	10.60	10.60
8	18	18.10	-0.10	0.10	0.01	-0.56	0.56
9	22	18.09	3.91	3.91	15.29	17.77	17.77
10	20	18.48	1.52	1.52	2.31	7.60	7.60
11	15	18.63	-3.63	3.63	13.18	-24.20	24.20
12	22	18.27	3.73	3.73	13.91	16.95	16.95
			Totals	28.24	101.72	65.67	142.42

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The following tables show the calculations for $\alpha = .2$

Week	Time Series Value	Forecast	Forecast Error	Absolute Value of Forecast Error	Squared Forecast Error	Percentage Error	Absolute Value of Percentage Error
1	17						
2	21	17.00	4.00	4.00	16.00	19.05	19.05
3	19	17.80	1.20	1.20	1.44	6.32	6.32
4	23	18.04	4.96	4.96	24.60	21.57	21.57
5	18	19.03	-1.03	1.03	1.06	-5.72	5.72
6	16	18.83	-2.83	2.83	8.01	-17.69	17.69
7	20	18.26	1.74	1.74	3.03	8.70	8.70
8	18	18.61	-0.61	0.61	0.37	-3.39	3.39
9	22	18.49	3.51	3.51	12.32	15.95	15.95
10	20	19.19	0.81	0.81	0.66	4.05	4.05
11	15	19.35	-4.35	4.35	18.92	-29.00	29.00
12	22	18.48	3.52	3.52	12.39	16.00	16.00
			Totals	28.56	98.80	35.84	147.44

a. MSE for $\alpha = .1 = 101.72/11 = 9.25$

MSE for $\alpha = .2 = 98.80/11 = 8.98$

$\alpha = .2$ provides more accurate forecasts based upon MSE

b. MAE for $\alpha = .1 = 28.24/11 = 2.57$

MAE for $\alpha = .2 = 28.56/11 = 2.60$

$\alpha = .1$ provides more accurate forecasts based upon MAE; but, they are very close.

c. MAPE for $\alpha = .1 = 142.42/11 = 12.95\%$

MAPE for $\alpha = .2 = 147.44/11 = 13.40\%$

$\alpha = .1$ provides more accurate forecasts based upon MAPE

10. a. $F_{13} = .2Y_{12} + .16Y_{11} + .64(.2Y_{10} + .8F_{10}) = .2Y_{12} + .16Y_{11} + .128Y_{10} + .512F_{10}$

$F_{13} = .2Y_{12} + .16Y_{11} + .128Y_{10} + .512(.2Y_9 + .8F_9) = .2Y_{12} + .16Y_{11} + .128Y_{10} + .1024Y_9 + .4096F_9$

$F_{13} = .2Y_{12} + .16Y_{11} + .128Y_{10} + .1024Y_9 + .4096(.2Y_8 + .8F_8) = .2Y_{12} + .16Y_{11} + .128Y_{10} + .1024Y_9 + .08192Y_8 + .32768F_8$

b. The more recent data receives the greater weight or importance in determining the forecast. The moving averages method weights the last n data values equally in determining the forecast.

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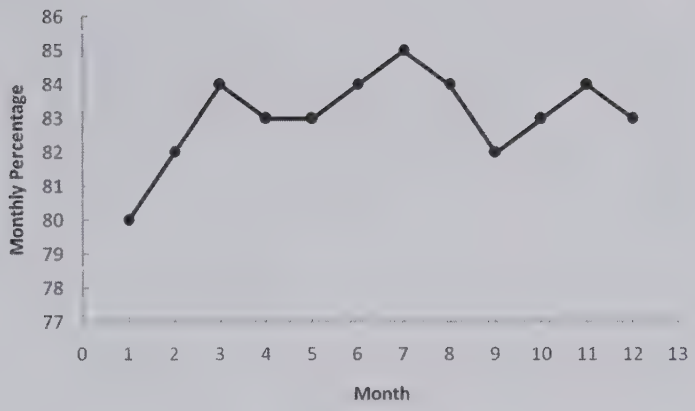
11. a.



The first two time series values may be an indication that the time series has shifted to a new higher level, as shown by the remainig 10 values. But, overall, the time series plot exhibits a horizontal pattern.

b.

Month	Y_t	3-Month Moving Averages Forecast	$(\text{Error})^2$	$\alpha = 2$ Forecast	$(\text{Error})^2$
1	80				
2	82			80.00	4.00
3	84			80.40	12.96
4	83	82.00	1.00	81.12	3.53
5	83	83.00	0.00	81.50	2.25
6	84	83.33	0.45	81.80	4.84
7	85	83.33	2.79	82.24	7.62
8	84	84.00	0.00	82.79	1.46
9	82	84.33	5.43	83.03	1.06
10	83	83.67	0.45	82.83	0.03
11	84	83.00	1.00	82.86	1.30
12	83	83.00	0.00	83.09	0.01
		Totals	11.12		39.06

MSE(3-Month) = 11.12 / 9 = 1.24

MSE(α = .2) = 39.06 / 11 = 3.55

A 3-month moving average provides the most accurate forecast using MSE

c. 3-month moving average forecast = (83 + 84 + 83) / 3 = 83.3

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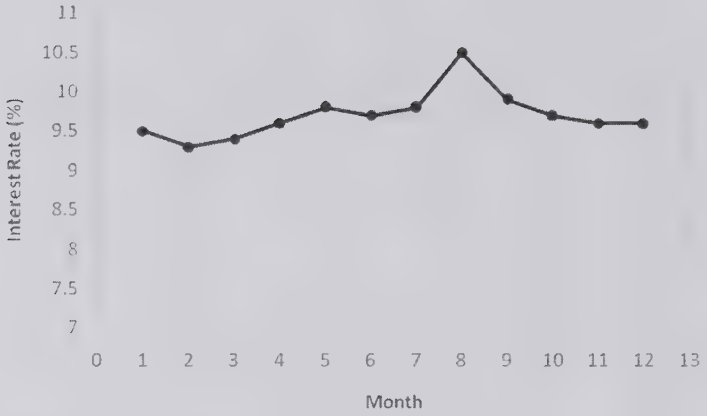
12. a.



The data appear to follow a horizontal pattern.

b.

Month	Time-Series Value	3-Month Moving Average Forecast	$(\text{Error})^2$	4-Month Moving Average Forecast	$(\text{Error})^2$
1	9.5				
2	9.3				
3	9.4				
4	9.6	9.40	0.04		
5	9.8	9.43	0.14	9.45	0.12
6	9.7	9.60	0.01	9.53	0.03
7	9.8	9.70	0.01	9.63	0.03
8	10.5	9.77	0.53	9.73	0.59
9	9.9	10.00	0.01	9.95	0.00
10	9.7	10.07	0.14	9.98	0.08
11	9.6	10.03	0.18	9.97	0.14
12	9.6	9.73	0.02	9.92	0.10
		Totals	1.08		1.09

MSE(3-Month) = 1.08 / 9 = .12

MSE(4-Month) = 1.09 / 8 = .14

Use 3-Month moving averages.

c. Forecast = (9.7 + 9.6 + 9.6) / 3 = 9.63

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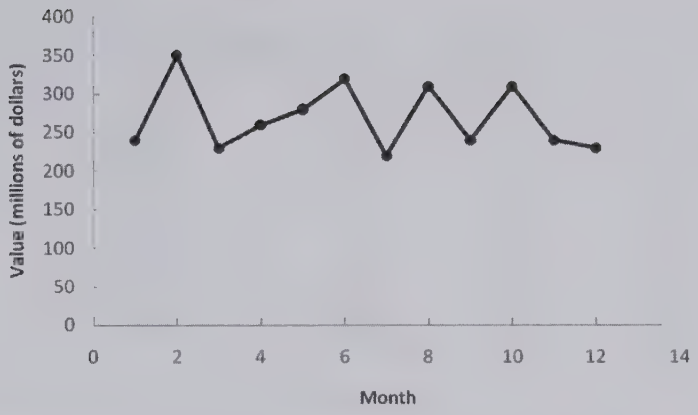
13. a.



The data appear to follow a horizontal pattern.

b.

Month	Time-Series Value	3-Month Moving Average Forecast	$(\text{Error})^2$	$\alpha = .2$ Forecast	$(\text{Error})^2$
1	240				
2	350			240.00	12100.00
3	230			262.00	1024.00
4	260	273.33	177.69	255.60	19.36
5	280	280.00	0.00	256.48	553.19
6	320	256.67	4010.69	261.18	3459.79
7	220	286.67	4444.89	272.95	2803.70
8	310	273.33	1344.69	262.36	2269.57
9	240	283.33	1877.49	271.89	1016.97
10	310	256.67	2844.09	265.51	1979.36
11	240	286.67	2178.09	274.41	1184.05
12	230	263.33	1110.89	267.53	1408.50
		Totals	17,988.52		27,818.49

MSE(3-Month) = 17,988.52 / 9 = 1998.72

MSE(α = .2) = 27,818.49 / 11 = 2528.95

Based on the above MSE values, the 3-month moving averages appear better. However, exponential smoothing was penalized by including month 2 which was difficult for any method to forecast. Using only the errors for months 4 to 12, the MSE for exponential smoothing is:

MSE(α = .2) = 14,694.49 / 9 = 1632.72

Thus, exponential smoothing was better considering months 4 to 12.

c. Using exponential smoothing,

$F_{13} = \alpha\, Y_{12} + (1 - \alpha)F_{12} = .20(230) + .80(267.53) = 260$

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14. a.

The data appear to follow a horizontal pattern.

b. Smoothing constant = .3.

Month t	Time-Series Value Y_t	Forecast F_t	Forecast Error $Y_t - F_t$	Squared Error $(Y_t - F_t)^2$
1	105			
2	135	105.00	30.00	900.00
3	120	114.00	6.00	36.00
4	105	115.80	-10.80	116.64
5	90	112.56	-22.56	508.95
6	120	105.79	14.21	201.92
7	145	110.05	34.95	1221.50
8	140	120.54	19.46	378.69
9	100	126.38	-26.38	695.90
10	80	118.46	-38.46	1479.17
11	100	106.92	-6.92	47.89
12	110	104.85	5.15	26.52
			Total	5613.18

MSE = 5613.18 / 11 = 510.29

Forecast for month 13: $F_{13} = .3(110) + .7(104.85) = 106.4$

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c. Smoothing constant = .5

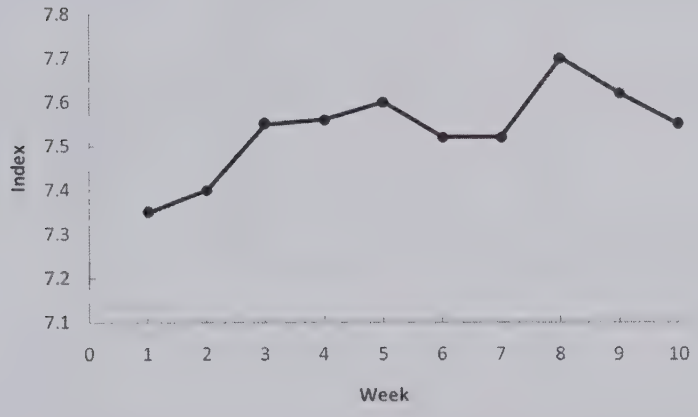
Month t	Time-Series Value Y_t	Forecast F_t	Forecast Error $Y_t - F_t$	Squared Error $(Y_t - F_t)^2$
1	105			
2	135	105	30.00	900.00
3	120	120	0.00	0.00
4	105	120	-15.00	225.00
5	90	112.50	-22.50	506.25
6	120	101.25	18.75	351.56
7	145	110.63	34.37	1181.30
8	140	127.81	12.19	148.60
9	100	133.91	-33.91	1149.89
10	80	116.95	-36.95	1365.30
11	100	98.48	1.52	2.31
12	110	99.24	10.76	115.78
				5945.99

MSE = 5945.99 / 11 = 540.55

Forecast for month 13: $F_{13} = .5(110) + .5(99.24) = 104.62$

Conclusion: a smoothing constant of .3 is better than a smoothing constant of .5 since the MSE is less for 0.3.

15. a.



You might think the time series plot shown above exhibits some trend. But, this is simply due to the fact that the smallest value on the vertical axis is 7.1, as shown by the following version of the plot.

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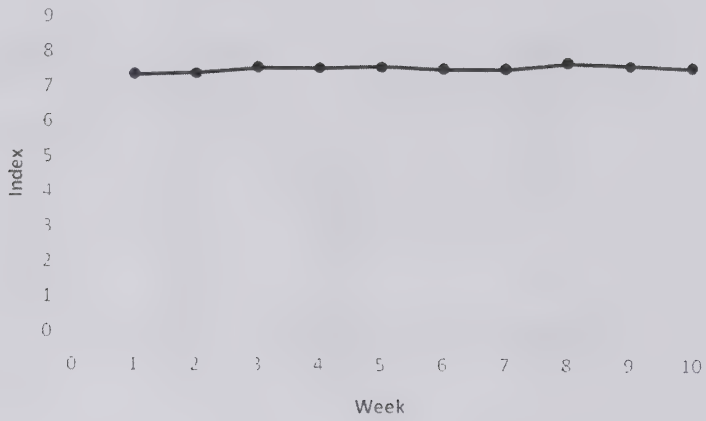

In other words, the time series plot shows an underlying horizontal pattern.

b/c.

Week	Time-Series Value	$\alpha = .2$ Forecast	$(\text{Error})^2$	$\alpha = .3$ Forecast	$(\text{Error})^2$
1	7.35				
2	7.40	7.35	.0025	7.35	.0025
3	7.55	7.36	.0361	7.36	.0361
4	7.56	7.40	.0256	7.42	.0196
5	7.60	7.43	.0289	7.46	.0196
6	7.52	7.46	.0036	7.50	.0004
7	7.52	7.48	.0016	7.51	.0001
8	7.70	7.48	.0484	7.51	.0361
9	7.62	7.53	.0081	7.57	.0025
10	7.55	7.55	.0000	7.58	.0009
			.1548		.1178

d. MSE($\alpha = .2$) = .1548 / 9 = .0172

MSE($\alpha = .3$) = .1178 / 9 = .0131

Use $\alpha = .3$.

$F_{11} = .3Y_{10} + .7F_{10} = .3(7.55) + .7(7.58) = 7.57$

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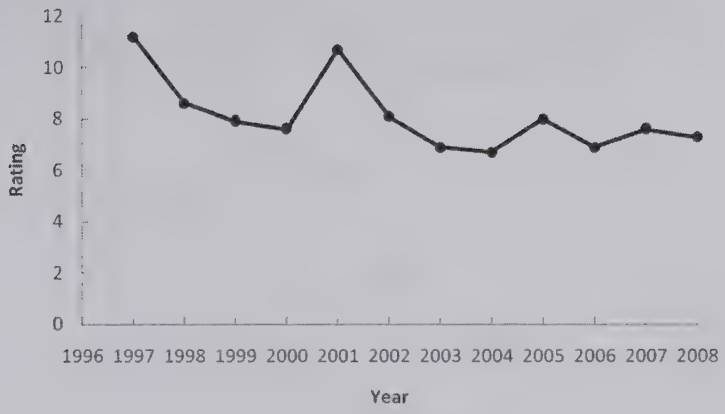
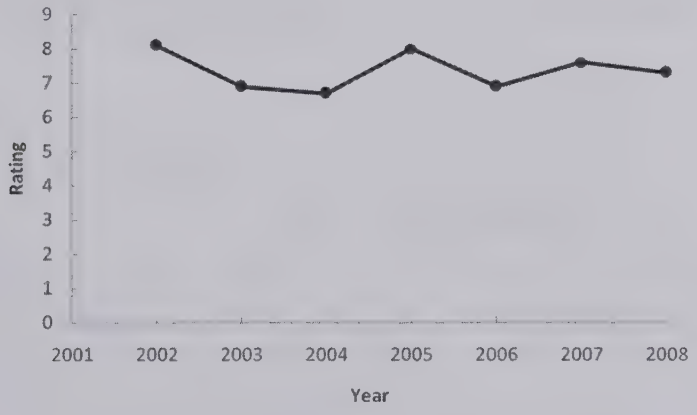
16. a.



The time series plot indicates a possible linear trend in the data. This could be due to decreasing viewer interest in watching the Masters. But, closer inspection of the data indicates that the two highest ratings correspond to years 1997 and 2001, years in which Tiger Woods won the tournament. In fact, four of the five highest ratings occurred when Tiger Woods has won the tournament. So, instead of an underlying linear trend in the time series, the pattern observed may be simply due to the effect Tiger Woods has on ratings and not necessarily on any long-term decrease in viewer interest.

b. The methods disucssed in this section are only applicable for a time series that has a horizontal pattern. So, if there is really a long-term linear trend in the data, the methods disucssed in this section are not appropriate.

c. The following time series plot shows the ratings for years 2002 – 2008.



The time series plot for the data for years 2002 – 2008 exhibits a horizontal pattern. It seems reasonable to conclude that the extreme values observed in 1997 and 2001 are more attributable to viewer interest in the performance of Tiger Woods. Basing the forecast on years 2002 – 2008 does seem reasonable. But, because of the injury that Tiger Woods experienced in the 2008 season, if he

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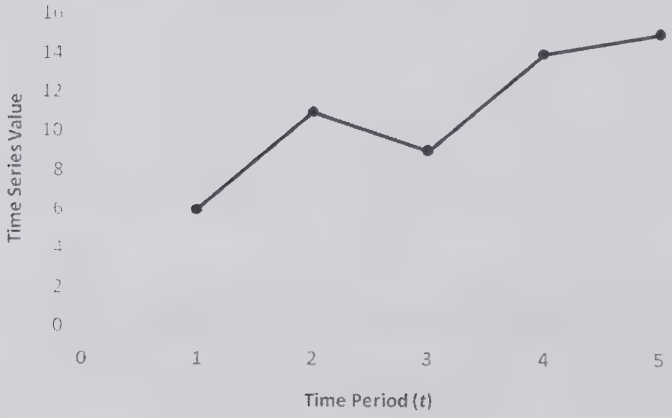
is able to play in the 2009 Masters then the rating for 2009 may be significantly higher than suggested by the data for years 2002 – 2008. These types of issues are what make forecasting in practice so difficult. For the methods to work, we have to be able to assume that the pattern in the past is appropriate for the future. But, because of the great influence Tiger Woods has on viewer interest, making this assumption for this time series may not be appropriate.

17. a.



The time series plot shows a linear trend.

b. $\bar{t} = \dfrac{\sum_{t=1}^{n} t}{n} = \dfrac{15}{5} = 3 \qquad \bar{Y} = \dfrac{\sum_{t=1}^{n} Y_t}{n} = \dfrac{55}{5} = 11$

$\Sigma(t-\bar{t})(Y_t-\bar{Y}) = 21 \qquad \Sigma(t-\bar{t})^2 = 10$

$$b_1 = \frac{\sum_{t=1}^{n}(t-\bar{t})(Y_t-\bar{Y})}{\sum_{t=1}^{n}(t-\bar{t})^2} = \frac{21}{10} = 2.1$$

$b_0 = \bar{Y} - b_1\bar{t} = 11 - (2.1)(3) = 4.7$

$T_t = 4.7 + 2.1t$

c. $T_6 = 4.7 + 2.1(6) = 17.3$

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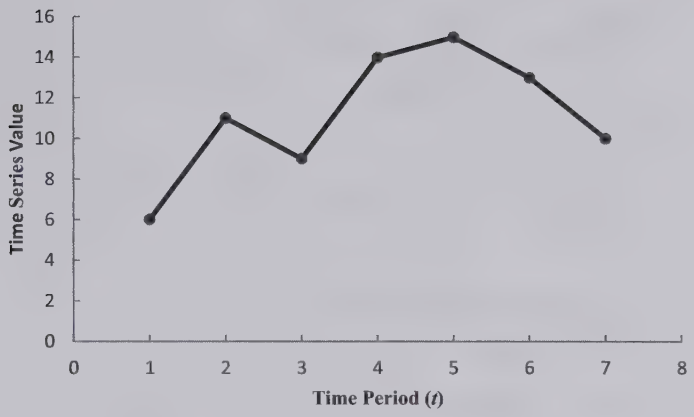
18. a.



The time series plot exhibits a curvilinear trend.

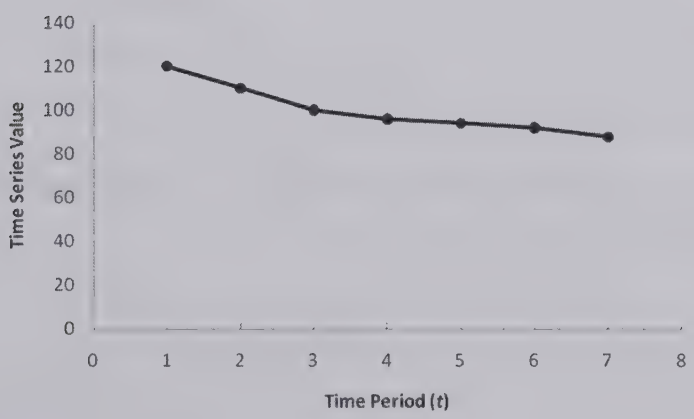
b. Using Excel's chart tools the quadratic trend equation is $T_t = 1.1429 + 5.3571t - .5714t^2$

c. Forecast for $t = 8$ is $T_8 = 1.1429 + 5.3571(8) - .5714(8)^2 = 7.4$

d. Using the linear trend equation $T_8 = 4.7 + 2.1(8) = 21.5$

The forecast obtained using the linear trend equation provides a much different forecast because it has not picked up the change that has occurred in the time series in periods 7 and 8. Each time a new time series value becomes available we need to develop a new time series plot and make a new decision as to what the underlying pattern of the time series is.

19. a.



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The time series plot shows a linear trend.

b. $\bar{t} = \dfrac{\sum_{t=1}^{n} t}{n} = \dfrac{28}{7} = 4 \qquad \bar{Y} = \dfrac{\sum_{t=1}^{n} Y_t}{n} = \dfrac{700}{7} = 100$

$\Sigma(t-\bar{t})(Y_t-\bar{Y}) = -138 \qquad \Sigma(t-\bar{t})^2 = 28$

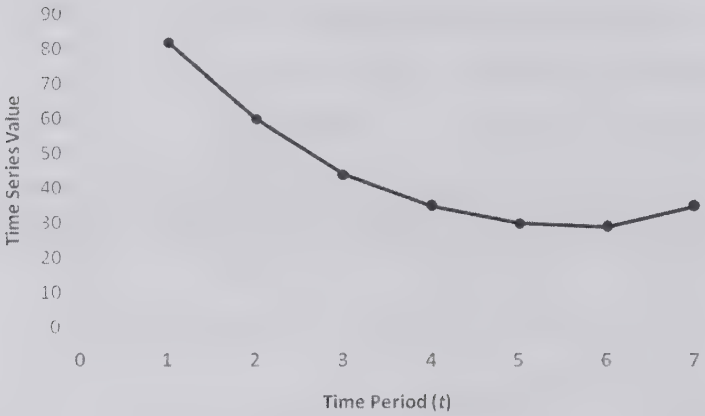
$$b_1 = \frac{\sum_{t=1}^{n}(t-\bar{t})(Y_t-\bar{Y})}{\sum_{t=1}^{n}(t-\bar{t})^2} = \frac{-138}{28} = -4.9286$$

$b_0 = \bar{Y} - b_1\bar{t} = 100 - (-4.9286)(4) = 119.714$

$T_t = 119.714 - 4.9286t$

c. $T_8 = 119.714 - 4.9286(8) = 80.29$

20. a.



The time series plot exhibits a curvilinear trend.

b. Using Excel's Regression tool, the quadratic trend equation is $T_t = 107.857 - 28.9881t + 2.65476t^2$

c. $T_t = 107.857 - 28.9881(8) + 2.65476(8)^2 = 45.86$

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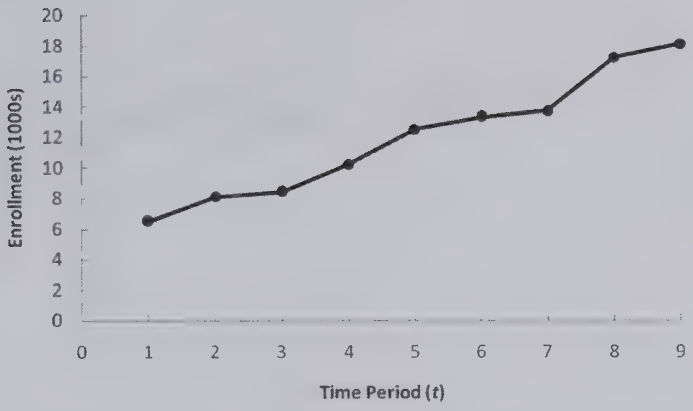
21. a.



The time series plot shows a linear trend

b. $\bar{t} = \dfrac{\sum_{t=1}^{n} t}{n} = \dfrac{45}{9} = 5 \qquad \bar{Y} = \dfrac{\sum_{t=1}^{n} Y_t}{n} = \dfrac{108}{9} = 12$

$\Sigma(t-\bar{t})(Y_t-\bar{Y}) = 87.4 \qquad \Sigma(t-\bar{t})^2 = 60$

$$b_1 = \frac{\sum_{t=1}^{n}(t-\bar{t})(Y_t-\bar{Y})}{\sum_{t=1}^{n}(t-\bar{t})^2} = \frac{87.4}{60} = 1.4567$$

$b_0 = \bar{Y} - b_1\bar{t} = 12 - (1.4567)(5) = 4.7165$

$T_t = 4.7165 + 1.4567t$

c. $T_{10} = 4.7165 + 1.4567(10) = 19.28$

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22. a.

The time series plot shows a downward linear trend

b. $\bar{t} = \dfrac{\sum_{t=1}^{n} t}{n} = \dfrac{28}{7} = 4 \qquad \bar{Y} = \dfrac{\sum_{t=1}^{n} Y_t}{n} = \dfrac{77}{7} = 11$

$\Sigma(t-\bar{t})(Y_t-\bar{Y}) = -19.6 \qquad \Sigma(t-\bar{t})^2 = 28$

$$b_1 = \frac{\sum_{t=1}^{n}(t-\bar{t})(Y_t-\bar{Y})}{\sum_{t=1}^{n}(t-\bar{t})^2} = \frac{-19.6}{28} = -.7$$

$b_0 = \bar{Y} - b_1\bar{t} = 11 - (-.7)(4) = 13.8$

$T_t = 13.8 - .7t$

c. 2010 corresponds to time period $t = 8$. $T_8 = 13.8 - .7(8) = 8.2$

d. If SCF can continue to decrease the percentage of funds spent on administrative and fund-raising by .7% per year, the forecast of expenses for 2015 is 4.70%.

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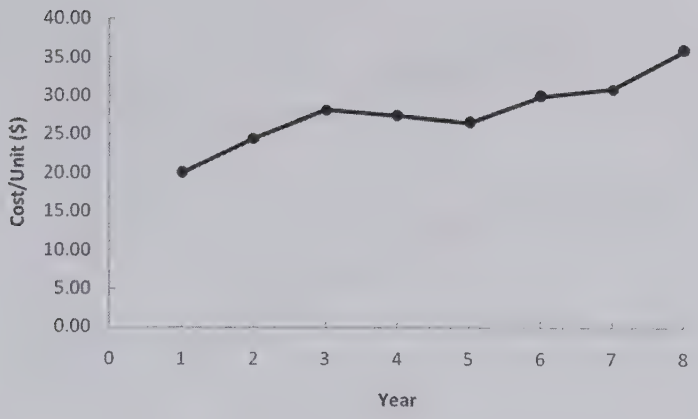
23. a.



The time series plot shows an upward linear trend

b. $\bar{t} = \dfrac{\sum_{t=1}^{n} t}{n} = \dfrac{36}{8} = 4.5 \quad \bar{Y} = \dfrac{\sum_{t=1}^{n} Y_t}{n} = \dfrac{223.8}{8} = 27.98$

$\Sigma(t-\bar{t})(Y_t-\bar{Y}) = 74.5 \quad \Sigma(t-\bar{t})^2 = 42$

$$b_1 = \frac{\sum_{t=1}^{n}(t-\bar{t})(Y_t-\bar{Y})}{\sum_{t=1}^{n}(t-\bar{t})^2} = \frac{74.5}{42} = 1.7738$$

$b_0 = \bar{Y} - b_1\bar{t} = 27.98 - (1.7738)(4.5) = 19.9979$

$T_t = 19.9979 + 1.7738t$

c. The average cost/unit has been increasing by approximately $1.77 per year.

d. $T_9 = 19.9979 + 1.7738(9) = 35.96$

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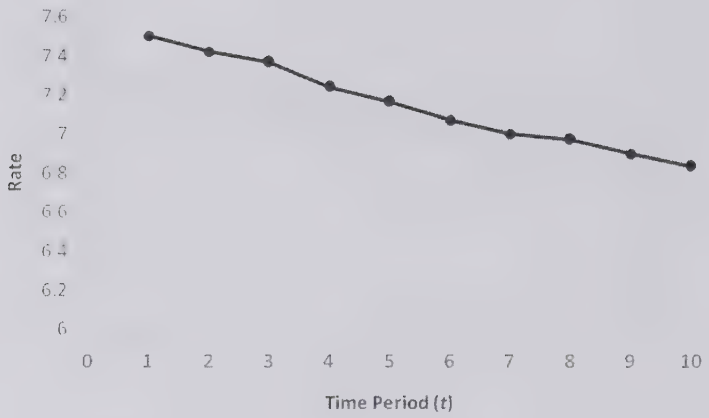
24. a.



The time series plot shows a linear trend.

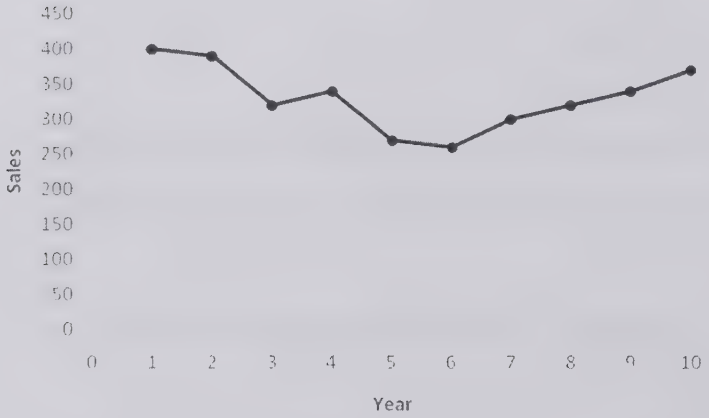
b. Using Excel's Regression tool, the linear trend equation is $T_t = 7.5623 - .0754\,1t$

c. A forecast for August corresponds to $t = 11$.

$T_{11} = 7.5623 - .0754\,1(11) = 6.7328$

d. Given the uncertainty in global market conditions, making a prediction for December using only time is not recommended.

25. a.



A linear trend is not appropriate.

b. Using Excel's Regression tool, the quadratic trend equation is $T_t = 472.7 - 62.9t + 5.303t^2$

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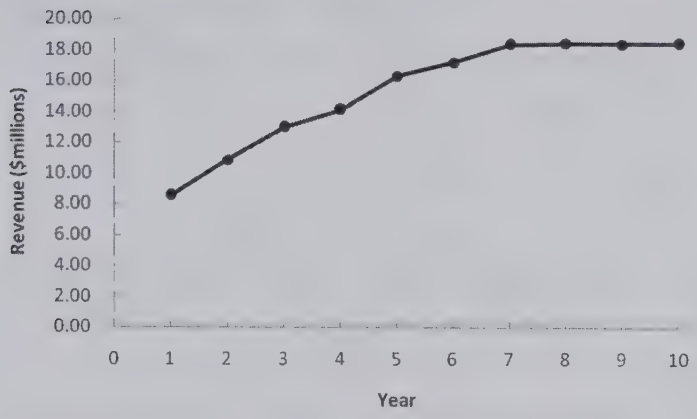
c. $T_{11} = 472.7 - 62.9(11) + 5.303(11)^2 = 422$

d. Sales appear to have bottomed-out in year 6 and appear to be increasing in a linear fashion for years 6 – 10. So, another alternative would be to use the data for years 6 – 10 (or 5 -10) to develop a linear trend equation to forecast sales in year 11.

26. a.

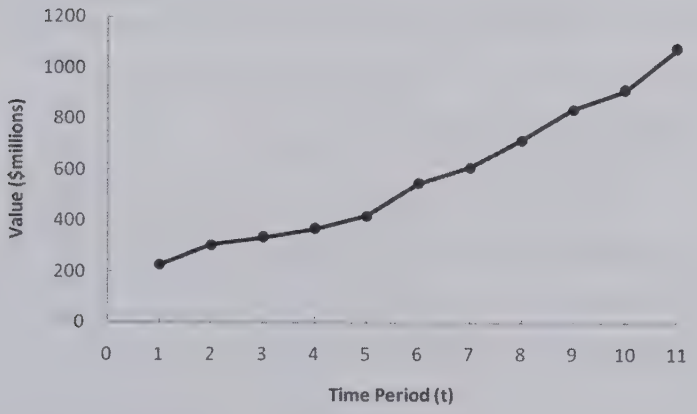


A linear trend is not appropriate.

b. Using Excel's Regression tool, the quadratic trend equation is $T_t = 5.702 + 2.889t - .1618t^2$

c. $T_{11} = 5.702 + 2.889(11) - .1618(11)^2 = 17.90$

27. a.



The time series plot indicates a slight curvature in the data.

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b. Using Excel's Regression tool, the linear trend equation is $T_t = 82.09 + 82.45t$

Using the regression output, MSE = 2981.7

c. Using Excel's Regression tool, the quadratic trend equation is $T_t = 217.2 + 20.10t + 5.196t^2$

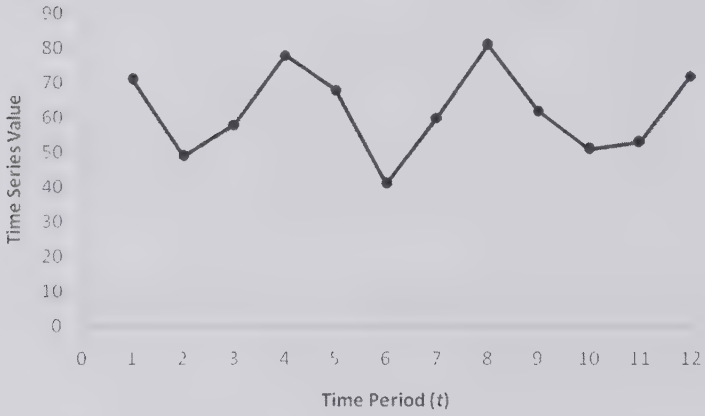
Using the regression output, MSE = 459.0

d. We recommend using the quadratic trend equation because it provides the best fit (smallest MSE).

e. Using the quadratic trend equation the estimate of value for 2009 (t = 12) is

$T_{12} = 217.2 + 20.10(12) + 5.196(12)^2 = 1207$

28. a.



The time series plot shows a horizontal pattern. But, there is a seasonal pattern in the data. For instance, in each year the lowest value occurs in quarter 2 and the highest value occurs in quarter 4.

b. Using Excel's Regression tool, the estimated multiple regression equation is:

Value = 77.0 - 10.0 Qtr1 - 30.0 Qtr2 - 20.0 Qtr3

c. The quarterly forecasts for next year are as follows:

Quarter 1 forecast = 77.0 - 10.0(1) - 30.0(0) - 20.0(0) = 67

Quarter 2 forecast = 77.0 - 10.0(0) - 30.0(1) - 20.0(0) = 47

Quarter 3 forecast = 77.0 - 10.0(0) - 30.0(0) - 20.0(1) = 57

Quarter 4 forecast = 77.0 - 10.0(0) - 30.0(0) - 20.0(0) = 77

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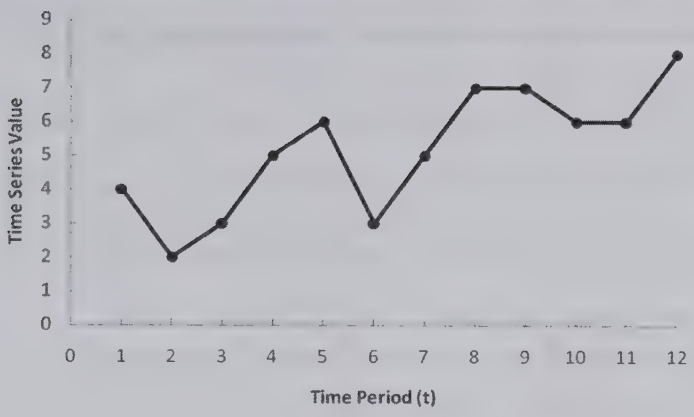
29. a.



The time series plot shows a linear trend and a seasonal pattern in the data.

b. Using Excel's Regression tool, the estimated multiple regression equation is:

Value = 3.42 + 0.219 Qtr1 - 2.19 Qtr2 - 1.59 Qtr3 + 0.406*t*

c. The quarterly forecasts for next year (*t* = 13, 14, 15, and 16) are as follows:

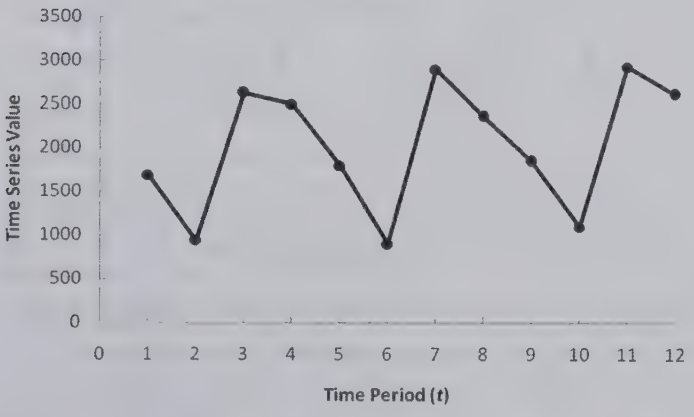
Quarter 1 forecast = 3.42 + 0.219(1) - 2.19(0) - 1.59(0) + 0.406(13) = 8.92

Quarter 2 forecast = 3.42 + 0.219(0) - 2.19(1) - 1.59(0) + 0.406(14) = 6.91

Quarter 3 forecast = 3.42 + 0.219(0) - 2.19(0) - 1.59(1) + 0.406(15) = 7.92

Quarter 4 forecast = 3.42 + 0.219(0) - 2.19(0) - 1.59(0) + 0.406(16) = 9.92

30. a.



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There appears to be a seasonal pattern in the data and perhaps a moderate upward linear trend.

b. Using Excel's Regression tool, the estimated multiple regression equation is:

Value = 2492 - 712 Qtr1 - 1512 Qtr2 + 327 Qtr3

c. The quarterly forecasts for next year are as follows:

Quarter 1 forecast = 2492 – 712(1) – 1512(0) + 327(0) = 1780

Quarter 2 forecast = 2492 – 712(0) – 1512(1) + 327(0) = 980

Quarter 3 forecast = 2492 – 712(0) – 1512(0) + 327(1) = 2819

Quarter 4 forecast = 2492 – 712(0) – 1512(0) + 327(0) = 2492

d. Using Excel's Regression tool, the estimated multiple regression equation is:

Value = 2307 - 642 Qtr1 - 1465 Qtr2 + 350 Qtr3 + 23.1t

The quarterly forecasts for next year are as follows:

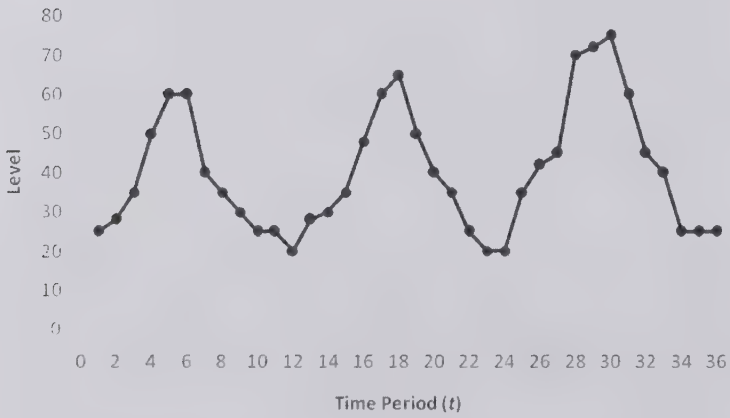
Quarter 1 forecast = 2307 – 642(1) – 1465(0) + 350(0) + 23.1(17) = 2058

Quarter 2 forecast = 2307 – 642(0) – 1465(1) + 350(0) + 23.1(18) = 1258

Quarter 3 forecast = 2307 – 642(0) – 1465(0) + 350(1) + 23.1(19) = 3096

Quarter 4 forecast = 2307 – 642(0) – 1465(0) + 350(0) + 23.1(20) = 2769

31. a.



The time series plot indicates a seasonal pattern in the data and perhaps a slight upward linear trend.

b. Using Excel's Regression tool, the estimated multiple regression equation is:

Level = 21.7 + 7.67 Hour1 + 11.7 Hour2 + 16.7 Hour3 + 34.3 Hour4 + 42.3 Hour5 + 45.0 Hour6 + 28.3 Hour7 + 18.3 Hour8 + 13.3 Hour9 + 3.33 Hour10 + 1.67 Hour11

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c. The hourly forecasts for the next day can be obtained very easily using the estimated regression equation. For instance, setting Hour1 = 1 and the rest of the dummy variables equal to 0 provides the forecast for the first hour; setting Hour2 = 1 and the rest of the dummy variables equal to 0 provides the forecast for the second hour; and so on.

Forecast for hour 1 = 21.667 + 7.667(1) + 11.667(0) + 16.667(0) + 34.333 (0) + 42.333(0) + 45.000(0) + 28.333(0) + 18.333(0) + 13.333(0) + 3.333(0) + 1.667(0) = 29.33

Forecast for hour 2 = 21.667 + 7.667(0) + 11.667(1) + 16.667(0) + 34.333 (0) + 42.333(0) + 45.000(0) + 28.333(0) + 18.333(0) + 13.333(0) + 3.333(0) + 1.667(0) = 33.33

The forecasts for the remaining hours can be obtained similarly. But, since there is no trend the data the hourly forecasts can also be computed by simply taking the average of the three time series values for each hour.

Hour	July 15	July 16	July 17	Average
1	25	28	35	29.33
2	28	30	42	33.33
3	35	35	45	38.33
4	50	48	70	56.00
5	60	60	72	64.00
6	60	65	75	66.67
7	40	50	60	50.00
8	35	40	45	40.00
9	30	35	40	35.00
10	25	25	25	25.00
11	25	20	25	23.33
12	20	20	25	21.67

In other words, the forecast for hour 1 is the average of the three observations for hour 1 on July 15, 16, and 17, or 29.33; the forecast for hour 2 is the average of the three observations for hour 1 on July 15, 16, and 17, or 33.33; and so on. Note that the forecast for the last hour is 21.67, the value of b_0 in the estimated regression equation.

d. Using Excel's Regression tool, the estimated multiple regression equation is:

Level = 11.2 + 12.5 Hour1 + 16.0 Hour2 + 20.6 Hour3 + 37.8 Hour4 + 45.4 Hour5 + 47.6 Hour6 + 30.5 Hour7 + 20.1 Hour8 + 14.6 Hour9 + 4.21 Hour10 + 2.10 Hour11 + 0.437t

Hour 1 on July 18 corresponds to Hour1 = 1 and t = 37.

Forecast for hour 1 on July 18 = 11.167 + 12.479(1) + .4375(37) = 39.834

Hour 2 on July 18 corresponds to Hour2 = 1 and t = 38.

Forecast for hour 2 on July 18 = 11.167 + 16.042(1) + .4375(38) = 43.834

The forecasts for the other hours are computed in a similar manner. The following table shows the forecasts for the 12 hours on July 18.

Hour 1	39.834
Hour 2	43.834
Hour 3	48.834
Hour 4	66.500
Hour 5	74.501
Hour 6	77.167

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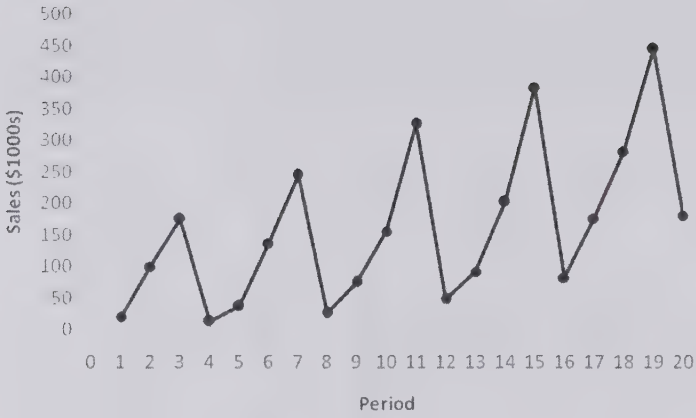
Hour 7	60.501
Hour 8	50.500
Hour 9	45.501
Hour 10	35.500
Hour 11	33.834
Hour 12	32.167

32. a.



The time series plot shows both a linear trend and seasonal effects.

b. Using Excel's Regression tool, the estimated regression equation is:

Revenue = 70.0 + 10.0 Qtr1 + 105 Qtr2 + 245 Qtr3

Quarter 1 forecast = 70.0 + 10.0(1) + 105(0) + 245(0) = 80

Quarter 2 forecast = 70.0 + 10.0(0) + 105(1) + 245(0) = 175

Quarter 3 forecast = 70.0 + 10.0(0) + 105(0) + 245(1) = 315

Quarter 4 forecast = 70.0 + 10.0(0) + 105(0) + 245(0) = 70

c. Using Excel's Regression tool, the estimated multiple regression equation is:

Revenue = - 70.1 + 45.0 Qtr1 + 128 Qtr2 + 257 Qtr3 + 11.7 Period

Quarter 1 forecast = -70.1 + 45.0(1) + 128(0) + 257(0) + 11.7(21) = 221

Quarter 2 forecast = -70.1 + 45.0(0) + 128(1) + 257(0) + 11.7(22) = 315

Quarter 3 forecast = -70.1 + 45.0(0) + 128(0) + 257(1) + 11.7(23) = 456

Quarter 4 forecast = -70.1 + 45.0(0) + 128(0) + 257(0) + 11.7(24) = 211

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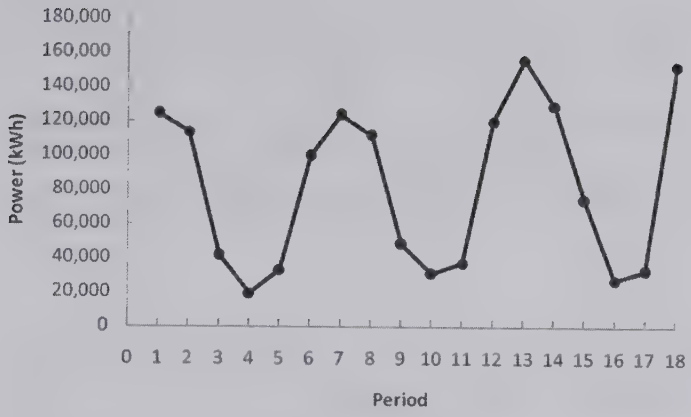
33. a.



The time series plot indicates a seasonal effect. Power consumption is lowest in the time period 12-4 A.M., steadily increases to the highest value in the 12-4 P.M. time period, and then decreases again. There may also be some linear trend in the data.

b. Using Excel's Regression tool, the estimated multiple regression equation is:

Power = 54445 - 28505 Time1 - 20137 Time2 + 69538 Time3 + 80221 Time4 + 63605 Time5

c. The estimate of Timko's power usage from noon to 8:00 P.M. on Thursday is

12-4 P.M. Forecast = 54445 – 28505(0) - 20137(0)+ 69538(0)+ 80221(1)+ 63605(0) = 134,666

4-8 P.M. Forecast = 54445 – 28505(0) - 20137(0)+ 69538(0)+ 80221(0)+ 63605(1) = 118,050

d. Using Excel's Regression tool, the estimated multiple regression equation is:

Power = 36918 - 30452 Time1 - 24032 Time2 + 63696 Time3 + 84116 Time4 + 65553 Time5 + 1947 Period

e. The estimate of Timko's power usage from noon to 8:00 P.M. on Thursday Periods 19 and 20 is

12-4 P.M. Forecast = 36918 - 30452(0)- 24032(0)+ 63696(0)+ 84116(1) + 65553(0)+ 1947(19) = 158,027

4-8 P.M. Forecast = 36918 - 30452(0)- 24032(0)+ 63696(0)+ 84116(0) + 65553(1)+ 1947(20) = 141,411

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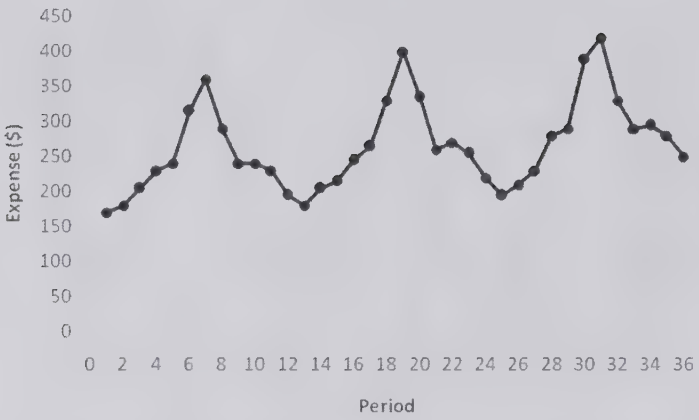
34. a.



The time series plot shows seasonal and linear trend effects.

b. Note: Jan = 1 if January, 0 otherwise; Feb = 1 if February, 0 otherwise; and so on.

Using Excel's Regression tool, the estimated multiple regression equation is:

Expense = 175 - 18.4 Jan - 3.72 Feb + 12.7 Mar + 45.7 Apr + 57.1 May + 135 Jun + 181 Jul + 105 Aug + 47.6 Sep + 50.6 Oct + 35.3 Nov + 1.96 Period

c. Note: The next time period in the time series is Period = 37 (January of Year 4).

January forecast = 175 - 18.4(1) - 3.72(0) + 12.7(0) + 45.7(0) + 57.1(0) + 135(0) + 181(0) + 105(0) + 47.6(0) + 50.6(0) + 35.3(0) + 1.96(37) = 229

February forecast = 175 - 18.4(0) - 3.72(1) + 12.7(0) + 45.7(0) + 57.1(0) + 135(0) + 181(0) + 105(0) + 47.6(0) + 50.6(0) + 35.3(0) + 1.96(38) = 246

March forecast = 175 - 18.4(0) - 3.72(0) + 12.7(1) + 45.7(0) + 57.1(0) + 135(0) + 181(0) + 105(0) + 47.6(0) + 50.6(0) + 35.3(0) + 1.96(39) = 264

April forecast = 175 - 18.4(0) - 3.72(0) + 12.7(0) + 45.7(1) + 57.1(0) + 135(0) + 181(0) + 105(0) + 47.6(0) + 50.6(0) + 35.3(0) + 1.96(40) = 299

May forecast = 175 - 18.4(0) - 3.72(0) + 12.7(0) + 45.7(0) + 57.1(1) + 135(0) + 181(0) + 105(0) + 47.6(0) + 50.6(0) + 35.3(0) + 1.96(41) = 312

June forecast = 175 - 18.4(0) - 3.72(0) + 12.7(0) + 45.7(0) + 57.1(0) + 135(1) + 181(0) + 105(0) + 47.6(0) + 50.6(0) + 35.3(0) + 1.96(42) = 392

July forecast = 175 - 18.4(0) - 3.72(0) + 12.7(0) + 45.7(0) + 57.1(0) + 135(0) + 181(1) + 105(0) + 47.6(0) + 50.6(0) + 35.3(0) + 1.96(43) = 440

August forecast = 175 - 18.4(0) - 3.72(0) + 12.7(0) + 45.7(0) + 57.1(0) + 135(0) + 181(0) + 105(1) + 47.6(0) + 50.6(0) + 35.3(0) + 1.96(44) = 366

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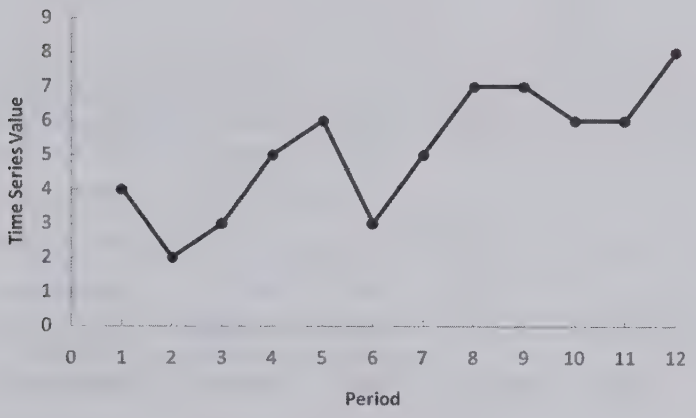
September forecast = 175 - 18.4(0) - 3.72(0) + 12.7(0) + 45.7(0) + 57.1(0) + 135(0) + 181(0) + 105(0) + 47.6(1) + 50.6(0) + 35.3(0) + 1.96(45) = 311

October forecast = 175 - 18.4(0) - 3.72(0) + 12.7(0) + 45.7(0) + 57.1(0) + 135(0) + 181(0) + 105(0) + 47.6(0) + 50.6(1) + 35.3(0) + 1.96(46) = 316

November forecast = 175 - 18.4(0) - 3.72(0) + 12.7(0) + 45.7(0) + 57.1(0) + 135(0) + 181(0) + 105(0) + 47.6(0) + 50.6(0) + 35.3(1) + 1.96(47) = 302

December forecast = 175 - 18.4(0) - 3.72(0) + 12.7(0) + 45.7(0) + 57.1(0) + 135(0) + 181(0) + 105(0) + 47.6(0) + 50.6(0) + 35.3(0) + 1.96(48) = 269

35. a.



The time series plot indicates a linear trend and a seasonal pattern.

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b.

Year	Quarter	Time Series Value	Four-Quarter Moving Average	Centered Moving Average
1	1	4		
	2	2		
			3.50	
	3	3		3.750
			4.00	
	4	5		4.125
			4.25	
2	1	6		4.500
			4.75	
	2	3		5.000
			5.25	
	3	5		5.375
			5.50	
	4	7		5.875
			6.25	
3	1	7		6.375
			6.50	
	2	6		6.625
			6.75	
	3	6		
	4	8		

c.

Year	Quarter	Time Series Value	Centered Moving Average	Seasonal-Irregular Value
1	1	4		
	2	2		
	3	3	3.750	0.800
	4	5	4.125	1.212
2	1	6	4.500	1.333
	2	3	5.000	0.600
	3	5	5.375	0.930
	4	7	5.875	1.191
3	1	7	6.375	1.098
	2	6	6.625	0.906
	3	6		
	4	8		

Quarter	Seasonal-Irregular Values		Seasonal Index	Adjusted Seasonal Index
1	1.333	1.098	1.216	1.205
2	0.600	0.906	0.753	0.746
3	0.800	0.930	0.865	0.857
4	1.212	1.191	1.202	1.191
		Total	4.036	

Note: Adjustment for seasonal index = 4.000 / 4.036 = 0.991

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36. a.

Year	Quarter	Time Series Value	Adjusted Seasonal Index	Deseasonalized Value
1	1	4	1.205	3.320
	2	2	0.746	2.681
	3	3	0.858	3.501
	4	5	1.191	4.198
2	1	6	1.205	4.979
	2	3	0.746	4.021
	3	5	0.858	5.834
	4	7	1.191	5.877
3	1	7	1.205	5.809
	2	6	0.746	8.043
	3	6	0.858	7.001
	4	8	1.191	6.717

b. Let Period = 1 denote the time series value in Year 1 – Quarter 1; Period = 2 denote the time series value in Year 1 – Quarter 2; and so on. Using Excel's Regression tool, the estimated regression equation obtained treating Period as the independent variable and the Deseasonlized Values as the values of the dependent variable follows.

Deseasonalized Value = 2.42 + 0.422 Period

c. The quarterly deseasonalized trend forecasts for Year 4 (Periods 13, 14, 15, and 16) are as follows:

Forecast for quarter 1 = 2.42 + 0.422(13) = 7.906

Forecast for quarter 2 = 2.42 + 0.422(14) = 8.328

Forecast for quarter 3 = 2.42 + 0.422(15) = 8.750

Forecast for quarter 4 = 2.42 + 0.422(16) = 9.172

d. Adjusting the quarterly deseasonalized trend forecasts provides the following quarterly estimates:

Forecast for quarter 1 = 7.906(1.205) = 9.527

Forecast for quarter 2 = 8.328(.746) = 6.213

Forecast for quarter 3 = 8.750(.857) = 7.499

Forecast for quarter 4 = 9.172(1.191) = 10.924

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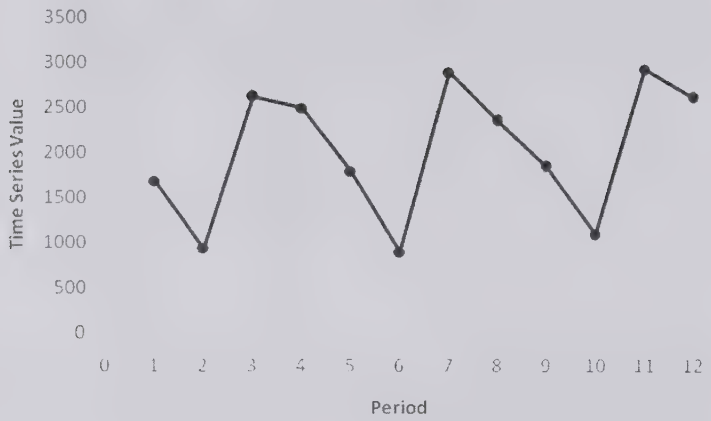
37. a.



The time series plot indicates linear trend and seasonal pattern.

b.

Year	Quarter	Time Series Value	Four-Quarter Moving Average	Centered Moving Average
1	1	1690		
	2	940		
			1938.75	
	3	2625		1952.500
			1966.25	
	4	2500		1961.250
			1956.25	
2	1	1800		1990.625
			2025.00	
	2	900		2007.500
			1990.00	
	3	2900		1996.250
			2002.50	
	4	2360		2027.500
			2052.50	
3	1	1850		2056.250
			2060.00	
	2	1100		2091.875
			2123.75	
	3	2930		
	4	2615		

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c.

Year	Quarter	Time Series Value	Centered Moving Average	Seasonal-Irregular Value
1	1	1690		
	2	940		
	3	2625	1952.500	1.344
	4	2500	1961.250	1.275
2	1	1800	1990.625	0.904
	2	900	2007.500	0.448
	3	2900	1996.250	1.453
	4	2360	2027.500	1.164
3	1	1850	2056.250	0.900
	2	1100	2091.875	0.526
	3	2930		
	4	2615		

Quarter	Seasonal-Irregular Values		Seasonal Index	Adjusted Seasonal Index
1	0.904	0.900	0.902	0.900
2	0.448	0.526	0.487	0.486
3	1.344	1.453	1.399	1.396
4	1.275	1.164	1.219	1.217
		Total	4.007	

Note: Adjustment for seasonal index = 4.000 / 4.007 = 0.998

d. The largest school effect is in the third quarter which corresponds to back-to-school demand during July, August, and September of each year.

e.

Year	Quarter	Time Series Value	Adjusted Seasonal Index	Deseasonalized Value
1	1	1690	0.900	1877.778
	2	940	0.486	1934.156
	3	2625	1.396	1880.372
	4	2500	1.217	2054.232
2	1	1800	0.900	2000.000
	2	900	0.486	1851.852
	3	2900	1.396	2077.364
	4	2360	1.217	1939.195
3	1	1850	0.900	2055.556
	2	1100	0.486	2263.374
	3	2930	1.396	2098.854
	4	2615	1.217	2148.726

f. Let Period = 1 denote the time series value in Year 1 – Quarter 1; Period = 2 denote the time series value in Year 1 – Quarter 2; and so on. Using Excel's Regression tool, the estimated regression equation obtained treating Period as the independent variable and the Deseasonlized Values as the values of the dependent variable follows.

Deseasonalized Value = 1852 + 25.2 Period

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The quarterly deseasonalized trend forecasts for Year 4 (Periods 13, 14, 15, and 16) are as follows:

Forecast for quarter 1 = 1852 + 25.2(13) = 2179.6

Forecast for quarter 2 = 1852 + 25.2 (14) = 2204.8

Forecast for quarter 3 = 1852 + 25.2 (15) = 2230.0

Forecast for quarter 4 = 1852 + 25.2 (16) = 2255.2

g. Adjusting the quarterly deseasonalized trend forecasts provides the following quarterly estimates:

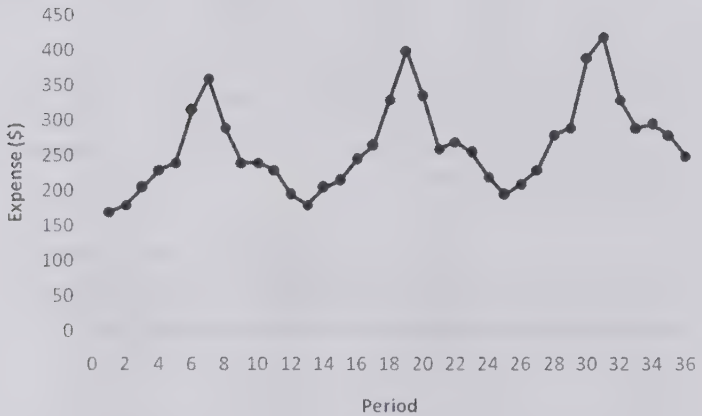
Forecast for quarter 1 = 2179.6(.900) = 1962

Forecast for quarter 2 = 2204.8(.486) = 1072

Forecast for quarter 3 = 2230.0(1.396) = 3113

Forecast for quarter 4 = 2255.2(1.217) = 2745

38. a.



The time series plot shows a linear trend and seasonal effects.

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b.

Month	Expense	Centered Moving Average	Seasonal-Irregular Value
1	170		
2	180		
3	205		
4	230		
5	240		
6	315		
7	360	241.67	1.49
8	290	243.13	1.19
9	240	244.58	0.98
10	240	245.63	0.98
11	230	247.29	0.93
12	195	248.96	0.78
13	180	251.25	0.72
14	205	254.79	0.80
15	215	257.50	0.83
16	245	259.58	0.94
17	265	261.88	1.01
18	330	263.96	1.25
19	400	265.63	1.51
20	335	266.46	1.26
21	260	267.29	0.97
22	270	269.38	1.00
23	255	271.88	0.94
24	220	275.42	0.80
25	195	278.75	0.70
26	210	279.38	0.75
27	230	280.42	0.82
28	280	282.71	0.99
29	290	284.79	1.02
30	390	287.08	1.36
31	420		
32	330		
33	290		
34	295		
35	280		
36	250		

Month	Seasonal-Irregular Values		Seasonal Index
1	0.72	0.70	0.71
2	0.80	0.75	0.78
3	0.83	0.82	0.83
4	0.94	0.99	0.97
5	1.01	1.02	1.02
6	1.25	1.36	1.30
7	1.49	1.51	1.50
8	1.19	1.26	1.23
9	0.98	0.97	0.98
10	0.98	1.00	0.99
11	0.93	0.94	0.93
12	0.78	0.80	0.79
		Total	12.03

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Notes: 1. Adjustment for seasonal index = 12 / 12.03 = 0.998

2. Because the seasonal-irregular values and the seasonal index values were rounded to two decimal places to simplify the presentation, the adjustment is really not necessary in this problem since it implies more accuracy than is warranted.

c.

Month	Expense	Seasonal Index	Deseasonalized Expense
1	170	0.71	239.44
2	180	0.78	230.77
3	205	0.83	246.99
4	230	0.97	237.11
5	240	1.02	235.29
6	315	1.3	242.31
7	360	1.5	240.00
8	290	1.23	235.77
9	240	0.98	244.90
10	240	0.99	242.42
11	230	0.93	247.31
12	195	0.79	246.84
13	180	0.71	253.52
14	205	0.78	262.82
15	215	0.83	259.04
16	245	0.97	252.58
17	265	1.02	259.80
18	330	1.3	253.85
19	400	1.5	266.67
20	335	1.23	272.36
21	260	0.98	265.31
22	270	0.99	272.73
23	255	0.93	274.19
24	220	0.79	278.48
25	195	0.71	274.65
26	210	0.78	269.23
27	230	0.83	277.11
28	280	0.97	288.66
29	290	1.02	284.31
30	390	1.3	300.00
31	420	1.5	280.00
32	330	1.23	268.29
33	290	0.98	295.92
34	295	0.99	297.98
35	280	0.93	301.08
36	250	0.79	316.46

d. Let Period = 1 denote the time series value in January – Year 1; Period = 2 denote the time series value in February – Year 2; and so on. Using Excel's Regression tool, the estimated regression equation obtained treating Period as the independent variable and the Deseasonlized Values as the values of the dependent variable follows.

Deseasonalized Expense = 228 + 1.96 Period

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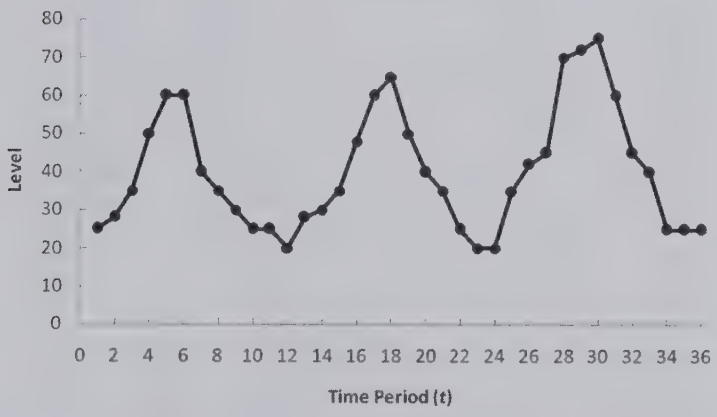
e. The linear trend estimates for the deseasonalized time series and the adjustment based upon the seasonal effects are shown below.

Month	Deseasonalized Trend Forecast	Seasonal Index	Monthly Forecast
January	300.52	0.71	213.37
February	302.48	0.78	235.93
March	304.44	0.83	252.69
April	306.4	0.97	297.21
May	308.36	1.02	314.53
June	310.32	1.30	403.42
July	312.28	1.50	468.42
August	314.24	1.23	386.52
September	316.2	0.98	309.88
October	318.16	0.99	314.98
November	320.12	0.93	297.71
December	322.08	0.79	254.44

For instance, using the estimated regression equation the deseasonalized trend forecast for January in Year 4 (Period = 37) is 228 + 1.96(37) = 300.52. Since the January seasonal index is .71, the forecast for January is .71(300.52) = $213.37, or approximately $213. The other monthly forecasts were computed in a similar fashion.

39. a.



The time series plot indicates a seasonal pattern in the data and perhaps a slight upward linear trend.

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b.

Day	Hour	Reading	Centered Moving Average	Seasonal-Irregular Value
July 15	1	25		
	2	28		
	3	35		
	4	50		
	5	60		
	6	60		
	7	40	36.208	1.105
	8	35	36.417	0.961
	9	30	36.500	0.822
	10	25	36.417	0.686
	11	25	36.333	0.688
	12	20	36.542	0.547
July 16	1	28	37.167	0.753
	2	30	37.792	0.794
	3	35	38.208	0.916
	4	48	38.417	1.249
	5	60	38.208	1.570
	6	65	38.000	1.711
	7	50	38.292	1.306
	8	40	39.083	1.023
	9	35	40.000	0.875
	10	25	41.333	0.605
	11	20	42.750	0.468
	12	20	43.667	0.458
July 17	1	35	44.500	0.787
	2	42	45.125	0.931
	3	45	45.542	0.988
	4	70	45.750	1.530
	5	72	45.958	1.567
	6	75	46.375	1.617
	7	60		
	8	45		
	9	40		
	10	25		
	11	25		
	12	25		

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Hour	Seasonal-Irregular Values		Seasonal Index	Adjusted Seasonal Index
1	0.787	0.753	0.770	0.771
2	0.931	0.794	0.862	0.864
3	0.988	0.916	0.952	0.954
4	1.530	1.249	1.390	1.392
5	1.567	1.570	1.568	1.571
6	1.617	1.711	1.664	1.667
7	1.105	1.306	1.205	1.207
8	0.961	1.023	0.992	0.994
9	0.822	0.875	0.848	0.850
10	0.686	0.605	0.646	0.647
11	0.688	0.468	0.578	0.579
12	0.547	0.458	0.503	0.504

c. The adjusted seasonal indexes can now be used to deseasonalize the data as shown below.

Day	Hour	Reading	Centered Moving Average	Seasonal-Irregular Value	Seasonal Index	Deseasonalized Reading
July 15	1	25			0.771	32.412
	2	28			0.864	32.414
	3	35			0.954	36.697
	4	50			1.392	35.914
	5	60			1.571	38.186
	6	60			1.667	35.996
	7	40	36.208	1.105	1.207	33.129
	8	35	36.417	0.961	0.994	35.210
	9	30	36.500	0.822	0.850	35.295
	10	25	36.417	0.686	0.647	38.651
	11	25	36.333	0.688	0.579	43.179
	12	20	36.542	0.547	0.504	39.717
July 16	1	28	37.167	0.753	0.771	36.302
	2	30	37.792	0.794	0.864	34.729
	3	35	38.208	0.916	0.954	36.697
	4	48	38.417	1.249	1.392	34.477
	5	60	38.208	1.570	1.571	38.186
	6	65	38.000	1.711	1.667	38.996
	7	50	38.292	1.306	1.207	41.412
	8	40	39.083	1.023	0.994	40.240
	9	35	40.000	0.875	0.850	41.178
	10	25	41.333	0.605	0.647	38.651
	11	20	42.750	0.468	0.579	34.543
	12	20	43.667	0.458	0.504	39.717

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July 17	1	35	44.500	0.787	0.771	45.377
	2	42	45.125	0.931	0.864	48.621
	3	45	45.542	0.988	0.954	47.182
	4	70	45.750	1.530	1.392	50.279
	5	72	45.958	1.567	1.571	45.823
	6	75	46.375	1.617	1.667	44.995
	7	60			1.207	49.694
	8	45			0.994	45.270
	9	40			0.850	47.061
	10	25			0.647	38.651
	11	25			0.579	43.179
	12	25			0.504	49.646

d. Using Excel's Regression tool, the trend line fitted to the deseasonalized data is:

Deseasonalized Reading = 33.0 + 0.392*t*

e. Using the estimated regression equation Deseasonalized Reading = 33.0 + 0.392*t*, the linear trend estimates for the deseasonalized time series and the adjustment based upon the hourly effects are shown below.

t	Hour	Deseasonalized Trend Forecast	Seasonal Index	Hourly Forecast
37	1	47.504	0.770	37
38	2	47.896	0.862	41
39	3	48.288	0.952	46
40	4	48.68	1.390	68
41	5	49.072	1.568	77
42	6	49.464	1.664	82
43	7	49.856	1.205	60
44	8	50.248	0.992	50
45	9	50.64	0.848	43
46	10	51.032	0.646	33
47	11	51.424	0.578	30
48	12	51.816	0.503	26

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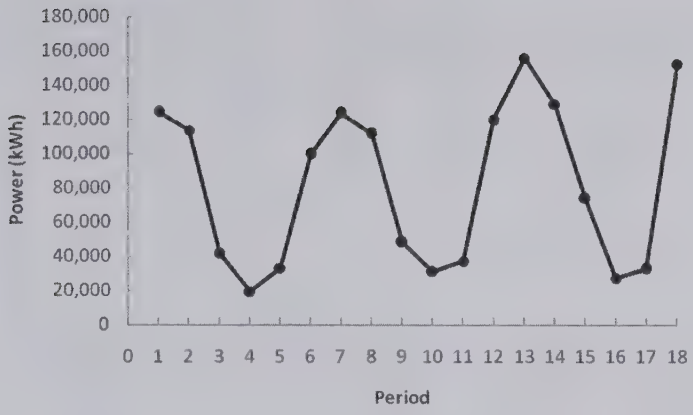
40. a.



The time series plot indicates a seasonal effect. Power consumption is lowest in the time period 12-4 A.M., steadily increases to the highest value in the 12-4 P.M. time period, and then decreases again. There may also be some linear trend in the data.

b.

Day	Time Period	Power	Centered Moving Average	Seasonal-Irregular Values
Monday	12-4 p.m.	124299		
Monday	4-8 p.m.	113545		
Monday	8-12 midnight	41300		
Tuesday	12-4 a.m.	19281	71803.3	0.2685
Tuesday	4-8 a.m.	33195	71598.2	0.4636
Tuesday	8-12 noon	99516	72013.5	1.3819
Tuesday	12-4 p.m.	123666	73575.2	1.6808
Tuesday	4-8 p.m.	111717	74887.4	1.4918
Tuesday	8-12 midnight	48112	76910.0	0.6256
Wednesday	12-4 a.m.	31209	81311.6	0.3838
Wednesday	4-8 a.m.	37014	85439.8	0.4332
Wednesday	8-12 noon	119968	89021.8	1.3476
Wednesday	12-4 p.m.	156033	90849.4	1.7175
Wednesday	4-8 p.m.	128889	90167.9	1.4294
Wednesday	8-12 midnight	73923	92517.8	0.7990
Thursday	12-4 a.m.	27330		
Thursday	4-8 a.m.	32715		
Thursday	8-12 noon	152465		

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Time Period	Seasonal-Irregular Values		Seasonal Index	Adjusted Seasonal Index
12-4 a.m.	0.3838	0.2685	0.3262	0.3256
4-8 a.m.	0.4332	0.4636	0.4484	0.4476
8-12 noon	1.3476	1.3819	1.3648	1.3622
12-4 p.m.	1.6808	1.7175	1.6992	1.6959
4-8 p.m.	1.4918	1.4294	1.4606	1.4578
8-12 midnight	0.6256	0.7990	0.7123	0.7109
		Total	6.0114	

c.

Day	Time Period	Power	Adjusted Seasonal Index	Deseasonalized Power
Monday	12-4 p.m.	124299	1.6959	73292.80
Monday	4-8 p.m.	113545	1.4578	77885.67
Monday	8-12 midnight	41300	0.7109	58092.48
Tuesday	12-4 a.m.	19281	0.3256	59225.36
Tuesday	4-8 a.m.	33195	0.4476	74166.93
Tuesday	8-12 noon	99516	1.3622	73056.72
Tuesday	12-4 p.m.	123666	1.6959	72919.55
Tuesday	4-8 p.m.	111717	1.4578	76631.76
Tuesday	8-12 midnight	48112	0.7109	67674.22
Wednesday	12-4 a.m.	31209	0.3256	95864.54
Wednesday	4-8 a.m.	37014	0.4476	82699.65
Wednesday	8-12 noon	119968	1.3622	88070.95
Wednesday	12-4 p.m.	156033	1.6959	92004.73
Wednesday	4-8 p.m.	128889	1.4578	88410.82
Wednesday	8-12 midnight	73923	0.7109	103979.91
Thursday	12-4 a.m.	27330	0.3256	83949.43
Thursday	4-8 a.m.	32715	0.4476	73094.48
Thursday	8-12 noon	152465	1.3622	111927.66

Using Excel's Regression tool to fit a linear trend equation to the deseasonalized time series provides the following estimated regression equation:

Deseasonalized Power = 63108 + 1854t

Using the deseasonalized linear trend equation and the seasonal indexes computed in part (b) we can forecast power consumption for the 12-4 P.M. time period and the 4-8 P.M. time period.

12-4 P.M. (corresponds to $t = 19$)

Deseasonalized Power = 63108 + 1854(19) = 98,334

Seasonal Index for this period = 1.6959
Forecast for 12-4 P.M. = 1.6959(98,334) = 166,764.63 or approximately 166,765 kWh

4-8 P.M. (corresponds to $t = 20$)

Deseasonalized Power = 63108 + 1854(20) = 100,188

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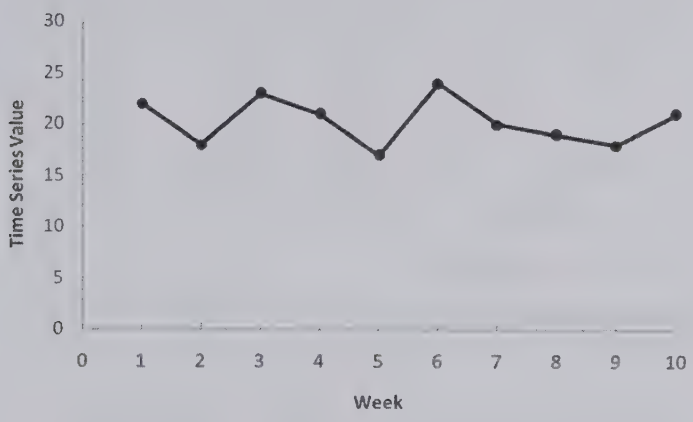
Seasonal Index for this period = 1.4578

Forecast for 4-8 P.M. = 1.4578(100,188) = 146,054.07 or approximately 146,054 kWh

Thus, the forecast of power consumption from noon to 8 P.M. is 166,765 + 146,054 = 312,819 kWh

41. a.



The time series plot indicates a horizontal pattern.

b. Three-week moving average.

Week	Time Series Value	Forecast	Forecast Error	Squared Forecast Error
1	22			
2	18			
3	23			
4	21	21.00	0.00	0.00
5	17	20.67	-3.67	13.44
6	24	20.33	3.67	13.44
7	20	20.67	-0.67	0.44
8	19	20.33	-1.33	1.78
9	18	21.00	-3.00	9.00
10	21	19.00	2.00	4.00
			Total	42.11

MSE = 42.11 / 7 = 6.02

F_{11} = (19 + 18 + 21) / 3 = 19.33

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c. Exponential smoothing using $\alpha = .2$

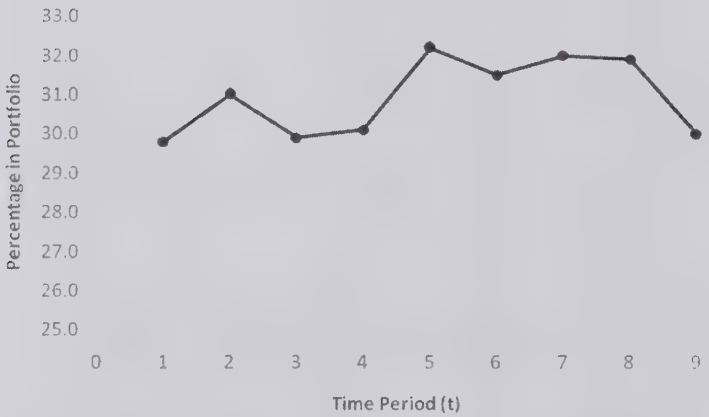
Week	Time Series Value	Forecast	Forecast Error	Squared Value of Forecast Error
1	22			
2	18	22.00	-4.00	16.00
3	23	21.20	1.80	3.24
4	21	21.56	-0.56	0.31
5	17	21.45	-4.45	19.78
6	24	20.56	3.44	11.84
7	20	21.25	-1.25	1.55
8	19	21.00	-2.00	3.99
9	18	20.60	-2.60	6.75
10	21	20.08	0.92	0.85
			Total	64.33

MSE = 64.33/ / 9 = 7.15

$F_{11} = 0.2(21) + 0.8(20.08) = 20.26$

d. The 3-month moving average is preferable. It has a smaller MSE.

42. a.



The time series plot indicates a horizontal pattern.

b.

Period	Time Series Value	$\alpha = .2$ Forecasts	$\alpha = .3$ Forecasts	$\alpha = .4$ Forecasts
1	29.8			
2	31.0	29.80	29.80	29.80
3	29.9	30.04	30.16	30.28
4	30.1	30.01	30.08	30.13
5	32.2	30.03	30.09	30.12
6	31.5	30.46	30.72	30.95
7	32.0	30.67	30.95	31.17
8	31.9	30.94	31.27	31.50
9	30.0	31.13	31.46	31.66

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MSE($\alpha = .2$) = 11.22/8 = 1.40

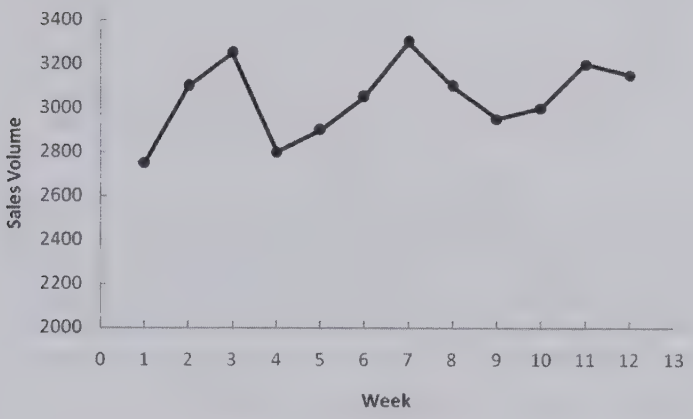
MSE($\alpha = .3$) = 10.19/8 = 1.27

MSE($\alpha = .4$) = 9.83/8 = 1.23

A smoothing constant of $\alpha = .4$ provides the best forecast because it has a smaller MSE

c. Using $\alpha = .4$, $F_{10} = .4(30) + .6(31.66) = 31.00$

43. a.



The time series plot indicates a horizontal pattern.

b.

Week	Sales Volume	Forecast	Forecast Error	Squared Value of Forecast Error
1	2750			
2	3100	2750.00	350.000	122,500.00
3	3250	2890.00	360.000	129,600.00
4	2800	3034.00	-234.000	54,756.00
5	2900	2940.40	-40.400	1,632.16
6	3050	2924.24	125.760	15,815.58
7	3300	2974.54	325.456	105,921.61
8	3100	3104.73	-4.726	22.34
9	2950	3102.84	-152.836	23,358.79
10	3000	3041.70	-41.702	1,739.02
11	3200	3025.02	174.979	30,617.68
12	3150	3095.01	54.987	3,023.62
			Total	488,986.80

Note: MSE = 488,986.80/11 = 44,453

Forecast for week 13 = .4(3150) + .6(3095.01) = 3117.01 or 3117 half-gallons of milk.

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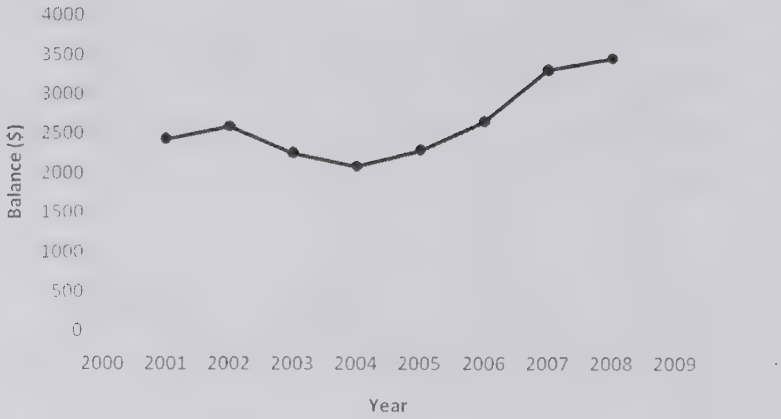
44. a.



There appears to be an increasing trend in the data.

b. Using Excel's Regression tool, the estimated regression equation is:

Balance($) = 1984 + 146$t$

Note: $t = 1$ corresponds to 2001, $t = 2$ corresponds to 2002, and so on.

The forecast for 2009 ($t = 9$) is Balance ($) = 1984 + 146(9) = $3298

c. Using Excel's Regression tool, the estimated multiple regression equation is:

Balance($) = 2924 - 419$t$ + 62.7t^2

Note: $t = 1$ corresponds to 2001, $t = 2$ corresponds to 2002, and so on.

The forecast for 2009 ($t = 9$) is Balance ($) = 2924 – 419(9) + 62.7$(9)^2$ = $4232

d. Linear trend equation: MSE = 143,487 Quadratic trend equation: MSE = 40,183

The quadratic trend equation provides the best forecast accuracy for the historical data.

e. The following graph displays shows the linear trend equation and quadratic trend equation on the time series plot. Note that the difference between the quadratic trend forecast for 2009 and the linear trend forecast for 2009 is $4232 - $3298 = $934.

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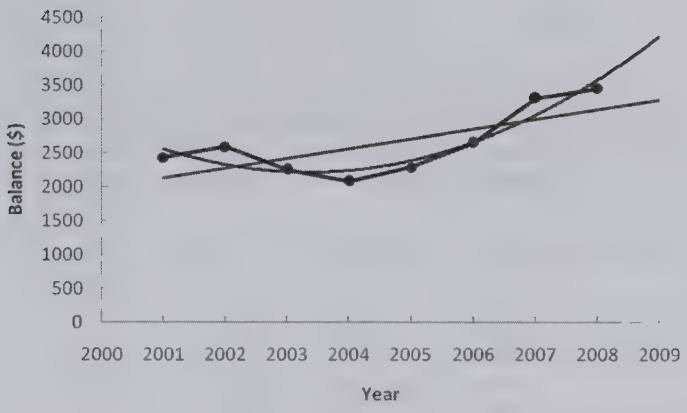
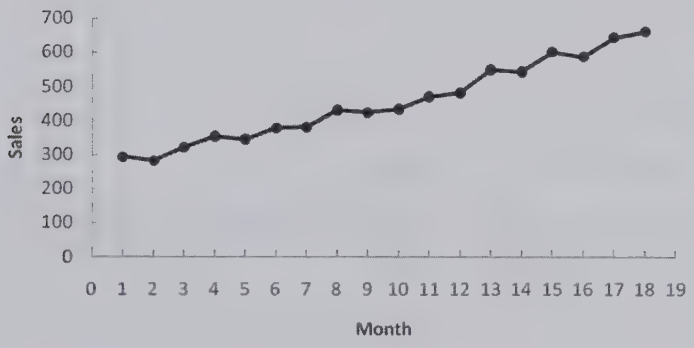


The key question is whether we believe that the average balance required will continue to rise as rapidly as the quadratic trend equation suggests. Looking more closely at the time series data, note that if we looked only at the time series values for 2001 – 2006, we would observe an underlying horizontal pattern. In fact, it looks like the average balance bottomed out in 2004 and has started to increase in a linear fashion over the time period 2004 – 2008. As a result, it may be worth considering fitting just a linear trend equation to the time series data for 2004 2008. In any event, we would not recommend use of a quadratic trend equation even though it provided the best fit for the historical data.

45. a.



The time series plot shows a linear trend.

b.

Smoothing Constant	MSE
α = .3	4,492.37
α = .4	2,964.67
α = .5	2,160.31

The α = .5 smoothing constant is better because it has the smallest MSE.

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c. $T_t = 244.778 + 22.088t$

MSE = 357.81

d. Trend projection provides much better forecasts because it has the smallest MSE. The reason MSE is smaller for trend projection is that sales are increasing over time; as a result, exponential smoothing continuously underestimates the value of sales. If you look at the forecast errors for exponential smoothing you will see that the forecast errors are positive for periods 2 through 18.

46. a. The following table shows the calculations using a smoothing constant of .4.

Month	Sales ($1000s)	Forecast
January	185.72	
February	167.84	185.72
March	205.11	178.57
April	210.36	189.18
May	255.57	197.65
June	261.19	220.82

The forecast for July is .4(261.19) + .6(220.82) = 236.97

Forecast for August, using forecast for July as the actual sales in July, is 236.97.

Exponential smoothing provides the same forecast for every period in the future. This is why it is not usually recommended for long-term forecasting.

b. Using Excel's Regression tool, the linear trend equation is:

$T_t = 149.72 + 18.451t$

Forecast for July ($t = 7$) is 149.72 + 18.451(7) = 278.88

Forecast for August ($t = 8$) is 149.72 + 18.451(8) = is 297.33

c. The proposed settlement is not fair since it does not account for the upward trend in sales. Based upon trend projection, the settlement should be based on forecasted lost sales of $278,880 in July and $297,330 in August.

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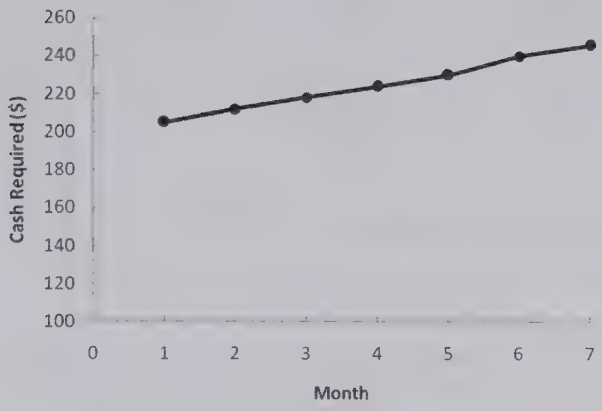
47. a.



The time series plot indicates a linear trend.

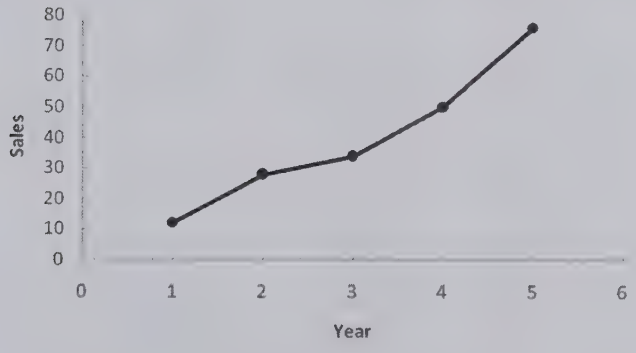
b. Using Excel's Regression tool, the estimated regression equation is:

Cash Required ($1000s) = 198 + 6.82 Month

Forecast for month 8 = 197.714 + 6.8214(8) = 252

Forecast for month 9 = 197.714 + 6.8214(9) = 259

48. a.



The time series plot shows a linear trend.

b. $\bar{t} = \frac{\sum_{t=1}^{n} t}{n} = \frac{15}{5} = 3 \quad \bar{Y} = \frac{\sum_{t=1}^{n} Y_t}{n} = \frac{200}{5} = 40$

$\Sigma(t-\bar{t})(Y_t-\bar{Y}) = 150 \quad \Sigma(t-\bar{t})^2 = 10$

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$$b_1 = \frac{\sum_{t=1}^{n}(t-\bar{t})(Y_t-\bar{Y})}{\sum_{t=1}^{n}(t-\bar{t})^2} = \frac{150}{10} = 15$$

$b_0 = \bar{Y} - b_1\bar{t} = 40 - (15)(3) = -5$

$T_t = -5 + 15t$

The slope of 15 indicates that the average increase in sales is 15 pianos per year.

c. Forecast for year 6 is $T_6 = -5 + 15(6) = 85$

Forecast for year 7 is $T_6 = -5 + 15(7) = 100$

49. a. Using Excel's Regression tool, the estimated multiple regression equation is:

Sales = 7.15 - 5.59 Qtr1 - 10.9 Qtr2 - 11.5 Qtr3 + 0.937t

b. Quarterly forecasts for next year correspond to t = 21, 22, 23, and 24.

Forecast for Quarter 1 (t = 21) = 7.150 -5.587(1) -10.925(0) -11.463(0) + .9375(21) = 21.25

Forecast for Quarter 2 (t = 22) = 7.150 -5.587(0) -10.925(1) -11.463(0) + .9375(22) = 16.85

Forecast for Quarter 3 (t = 23) = 7.150 -5.587(0) -10.925(0) -11.463(1) + .9375(23) = 17.25

Forecast for Quarter 4 (t = 24) = 7.150 -5.587(0) -10.925(0) -11.463(0) + .9375(24) = 29.65

50. a.

t	Sales	Centered Moving Average	Seasonal-Irregular Value
1	4		
2	2		
3	1	3.250	0.308
4	5	3.750	1.333
5	6	4.375	1.371
6	4	5.875	0.681
7	4	7.500	0.533
8	14	7.875	1.778
9	10	7.875	1.270
10	3	8.250	0.364
11	5	8.750	0.571
12	16	9.750	1.641
13	12	10.750	1.116
14	9	11.750	0.766
15	7	13.250	0.528
16	22	14.125	1.558
17	18	15.000	1.200
18	10	17.375	0.576
19	13		
20	35		

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Quarter	Seasonal-Irregular Values	Seasonal Index
1	1.371, 1.270, 1.116, 1.200	1.2394
2	0.681, 0.364, 0.776, 0.576	0.5965
3	0.308, 0.533, 0.571, 0.528	0.4852
4	1.333, 1.778, 1.641, 1.558	1.5774
	Total	3.8985

Quarter	Adjusted Seasonal Index
1	1.2717
2	0.6120
3	0.4978
4	1.6185

Note: Adjustment for seasonal index = 4 / 3.8985 = 1.0260

b. The largest effect is in quarter 4; this seems reasonable since retail sales are generally higher during October, November, and December.

51. a. Note: the adjusted seasonal indexes were computed in Exercise 50

Year	Quarter	Sales	Adjusted Seasonal Index	Deseasonalized Sales
1	1	4	1.2717	3.1454
	2	2	0.6120	3.2680
	3	1	0.4978	2.0088
	4	5	1.6185	3.0893
2	1	6	1.2717	4.7181
	2	4	0.6120	6.5359
	3	4	0.4978	8.0354
	4	14	1.6185	8.6500
3	1	10	1.2717	7.8635
	2	3	0.6120	4.9020
	3	5	0.4978	10.0442
	4	16	1.6185	9.8857
4	1	12	1.2717	9.4362
	2	9	0.6120	14.7059
	3	7	0.4978	14.0619
	4	22	1.6185	13.5928
5	1	18	1.2717	14.1543
	2	10	0.6120	16.3399
	3	13	0.4978	26.1149
	4	35	1.6185	21.6250

Using Excel's Regression tool, the estimated regression equation is:

Deseasonalized Sales = - 0.36 + 0.997t

b. The quarterly trend forecasts for next year correspond to t = 21, 22, 23, and 24.

Forecast for Quarter 1 (t = 21) = -.356 + .9966(21) = 20.57

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Forecast for Quarter 2 ($t = 22$) = -.356 + .9966(22) = 21.57

Forecast for Quarter 3 ($t = 23$) = -.356 + .9966(23) = 22.57

Forecast for Quarter 4 ($t = 24$) = -.356 + .9966(24) = 23.56

c. Multiplying the quarterly trend forecasts by the adjusted seasonal indexes provides the forecasts for next year.

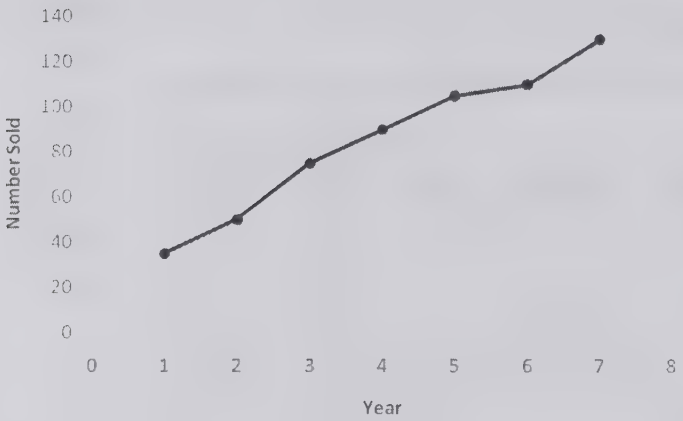
Forecast for Quarter 1 ($t = 21$) = 20.57(1.2717) = 26.2

Forecast for Quarter 2 ($t = 22$) = 21.57(.6120) = 13. 2

Forecast for Quarter 3 ($t = 23$) = 22.57(.4978) = 11.2

Forecast for Quarter 4 ($t = 24$) = 23.56(1.6185) = 38.1

52. a.



A linear trend pattern appears to be present in the time series plot.

b. Using Excel's Regression tool, the estimated regression equation is:

Number Sold = 22.9 + 15.5 Year

c. Forecast in year 8 = 22.857 +15.536(8) = 147.15 or approximately 147 units.

53. a. Using Excel's Regression tool, the estimated multiple regression equation is:

Sales = 0.036 + 4.91 Qtr1 + 14.5 Qtr2 + 9.11 Qtr3 + 0.971t

b. The quarterly forecast for next year correspond to $t = 29$, 30, 31, and 32.

Forecast for Quarter 1 ($t = 29$) = 0.0357 + 4.9129(1) + 14.5134(0) + 9.1138(0) + 0.971(29) = 33.1

Forecast for Quarter 2 ($t = 30$) = 0.0357 + 4.9129(0) + 14.5134(1) + 9.1138(0) + 0.971(30) = 43.7

Forecast for Quarter 3 ($t = 31$) = 0.0357 + 4.9129(0) + 14.5134(0) + 9.1138(1) + 0.971(31) = 39.3

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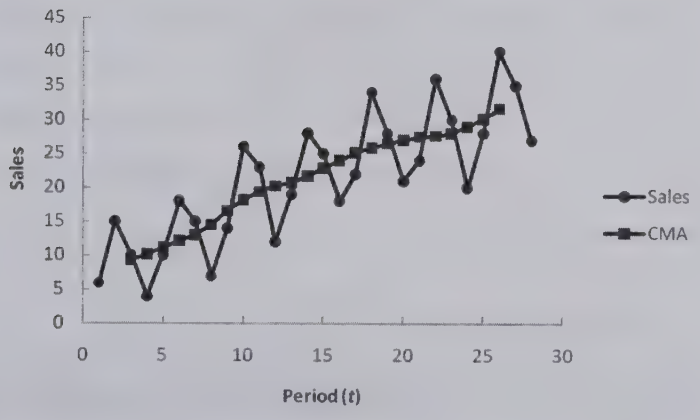
Forecast for Quarter 4 ($t = 32$) = 0.0357 + 4.9129(0) + 14.5134(0) + 9.1138(0) + 0.971(32) = 31.1

54. a.

t	Sales	Centered Moving Average
1	6	
2	15	
3	10	9.250
4	4	10.125
5	10	11.125
6	18	12.125
7	15	13.000
8	7	14.500
9	14	16.500
10	26	18.125
11	23	19.375
12	12	20.250
13	19	20.750
14	28	21.750
15	25	22.875
16	18	24.000
17	22	25.125
18	34	25.875
19	28	26.500
20	21	27.000
21	24	27.500
22	36	27.625
23	30	28.000
24	20	29.000
25	28	30.125
26	40	31.625
27	35	
28	27	

b.



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The centered moving average values smooth out the time series by removing seasonal effects and some of the random variability. The centered moving average time series shows the trend in the data.

c.

t	Sales	Centered Moving Average	Seasonal-Irregular Value
1	6		
2	15		
3	10	9.250	1.081
4	4	10.125	0.395
5	10	11.125	0.899
6	18	12.125	1.485
7	15	13.000	1.154
8	7	14.500	0.483
9	14	16.500	0.848
10	26	18.125	1.434
11	23	19.375	1.187
12	12	20.250	0.593
13	19	20.750	0.916
14	28	21.750	1.287
15	25	22.875	1.093
16	18	24.000	0.750
17	22	25.125	0.876
18	34	25.875	1.314
19	28	26.500	1.057
20	21	27.000	0.778
21	24	27.500	0.873
22	36	27.625	1.303
23	30	28.000	1.071
24	20	29.000	0.690
25	28	30.125	0.929
26	40	31.625	1.265
27	35		
28	27		

Quarter	Seasonal-Irregular Component Values		Seasonal Index
1	0.899, 0.848, 0.916, 0.876, 0.873, 0.929		0.890
2	1.485, 1.434, 1.287, 1.314, 1.303, 1.265		1.348
3	1.081, 1.154, 1.187, 1.093, 1.057, 1.071		1.107
4	0.395, 0.483, 0.593, 0.750, 0.778, 0.690		0.615
		Total	3.960

Quarter	Adjusted Seasonal Index
1	0.899
2	1.362
3	1.118
4	0.621

Note: Adjustment for seasonal index = 4.00 / 3.96 = 1.0101

d. Hudson Marine experiences the largest seasonal increase in quarter 2. Since this quarter occurs prior to the peak summer boating season, this result seems reasonable. But the largest seasonal effect is the seasonal decrease in quarter 4. This is also reasonable because of decreased boating in the fall and winter.

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55. a.

Year	Quarter	Sales	Adjusted Seasonal Index	Deseasonalized Sales
1	1	6	0.899	6.673
	2	15	1.362	11.016
	3	10	1.118	8.942
	4	4	0.621	6.443
2	1	10	0.899	11.122
	2	18	1.362	13.219
	3	15	1.118	13.413
	4	7	0.621	11.275
3	1	14	0.899	15.571
	2	26	1.362	19.094
	3	23	1.118	20.566
	4	12	0.621	19.328
4	1	19	0.899	21.132
	2	28	1.362	20.563
	3	25	1.118	22.355
	4	18	0.621	28.993
5	1	22	0.899	24.468
	2	34	1.362	24.969
	3	28	1.118	25.037
	4	21	0.621	33.825
6	1	24	0.899	26.692
	2	36	1.362	26.438
	3	30	1.118	26.825
	4	20	0.621	32.214
7	1	28	0.899	31.141
	2	40	1.362	29.376
	3	35	1.118	31.296
	4	27	0.621	43.489

Using Excel's Regression tool, the estimated multiple regression equation is:

Deseasonalized Sales = $6.33 + 1.05t$

b. The quarterly forecast for next year correspond to $t = 29$, 30, 31, and 32.

Forecast for Quarter 1 ($t = 29$) = $6.332 + 1.055(29) = 36.93$

Forecast for Quarter 2 ($t = 30$) = $6.332 + 1.055(30) = 37.98$

Forecast for Quarter 3 ($t = 31$) = $6.332 + 1.055(31) = 39.04$
Forecast for Quarter 4 ($t = 32$) = $6.332 + 1.055(32) = 40.09$

c. Multiplying the quarterly trend forecasts by the adjusted seasonal indexes provides the forecasts for next year.

Forecast for Quarter 1 ($t = 29$) = $36.93(.899) = 33.2$

Forecast for Quarter 2 ($t = 30$) = $37.98(1.362) = 51.7$

Forecast for Quarter 3 ($t = 31$) = $39.04(1.118) = 43.65$

Forecast for Quarter 4 ($t = 32$) = $40.09(.621) = 24.9$

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Essentials of Modern Business Statistics, 5e, WEBfiles

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